Numerical Experiments for the Naive AMIAS Algorithm

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This file replicates the numerical experiments in Appendix B.1 of the following paper:

ℓ_0 Trend Filtering by Canhong Wen, Xueqin Wang, Aijun Zhang.

1 Load Packages and Functions Needed

```
library(AMIAS)
library("gridExtra")
```

```
## Warning: package 'gridExtra' was built under R version 3.6.3
```

```
library(ggpubr)
```

```
## Warning: package 'ggpubr' was built under R version 3.6.3
```

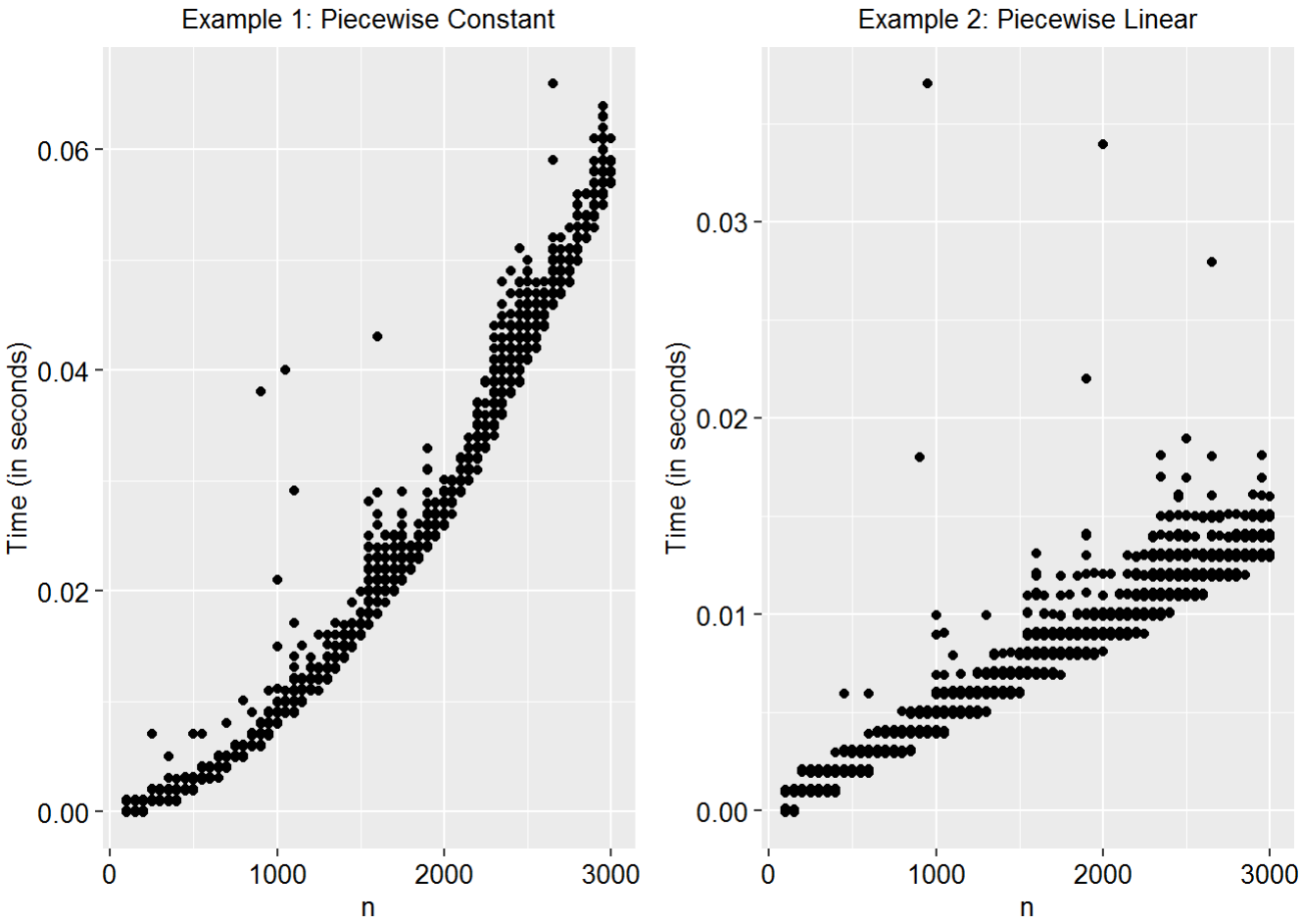
```
# ----------------------------------
# Toy Example: Piecewise Constant/Linear Simulation
# ----------------------------------
ToyEx <- function(n, sigma=0.1, q=0, seed=NA) {
  tau = 0.5
  if (!is.na(seed)) set.seed(seed)
  x = seq(1/n, 1,length.out = n)
  if (q==0){
    y0 = 0*x; y0[x>tau] = 1
  }
  if (q==1) {
    y0 = 2*(tau-x); y0[x>tau] = 2*(x[x>tau]-tau)
  }
  y = y0 + sigma*rnorm(n)
  return(list(y = y, x = x, y0 = y0, tau = tau))
}

# ----------------------------------
# Simu Example: Piecewise Constant/Linear Simulation
#     Equal-spaced knots or Random Knots
# ----------------------------------
SimuEx <- function(n, sigma=0.1, q=0, nknot=4, seed=NA, RandKnot=FALSE, AdaKnot=FALSE) {
  if (!is.na(seed)) set.seed(seed)
  x = seq(1/n, 1,length.out = n)
  A=round(seq(0, n, length.out=nknot+2))[seq(2,nknot+1)]
  if(RandKnot) A = sort(sample(seq(6, n-5, 5), nknot))
  if(AdaKnot) A = round(seq(1, sqrt(n), length.out=nknot+2)^2)[seq(2,nknot+1)]
  tau = x[A]
  tau1 = c(0, tau, 1)
  if (q==0){
    aa = 1-seq(1,nknot+1)%%2
    y0 = 0*x
    for (j in 1:(nknot+1)) y0[x>tau1[j] & x<=tau1[j+1]] = aa[j]
  }
  if (q==1) {
    aa = 2*(-1)^seq(1,nknot+1)
    phi = rep(1, n)
    for (j in 1:(nknot+1)) phi = cbind(phi,pmin(pmax(x-tau1[j], 0), tau1[j+1]-tau1[j]))
    y0 = phi%*%c(0.5+1/(nknot+1),aa)
  }
  y = y0 + sigma*rnorm(n)
  return(list(y = y, x = x, y0 = y0, tau = tau, SetA = A))
}
```

2 Figure B.1: Scatterplots of runtime versus sample size *n*

```
nlist <- seq(100, 3000, 50)
runtime <- array(0, c(length( nlist),100, 2))

for(i in seq_along(nlist)){
  n <- nlist[i]
  for(seed in 1:100){

    # Piecewise constant case
    # Toy Piecewise constant/linear: one knot only
    q=0; sigma=0.1; nknot=1
    data = ToyEx(n=n, q=q, sigma=sigma, seed=seed)
    start_time <- Sys.time()
    amias(data$y, D_type="tf0", k=nknot)
    end_time <- Sys.time()
    runtime[i, seed, 1] <- end_time - start_time

    q=1; sigma=0.1;  nknot=1
    data = ToyEx(n=n, q=q, sigma=sigma, seed=seed)
    start_time <- Sys.time()
    amias(data$y, D_type="tfq", q=q, k=nknot)
    end_time <- Sys.time()
    runtime[i, seed, 2] <- end_time - start_time

  }
}
save.image("runtime_amias.RData")
```

```
########################
## Runtime in amias
load("runtime_amias.RData")

t1 <- as.numeric(t(runtime[,,1]))
data <- cbind(rep(nlist, each=100), t1)
data <- as.data.frame(data)
colnames(data) <- c("n", "time")
library(ggplot2)
p1 <- ggplot(data = data, aes( x= n, y= time)) + geom_jitter(height = 0.0001, width=0) +
  # geom_smooth(method = "lm", formula = y ~ x + I(x^2), se= FALSE, size=2, color = "deepskyblue") +
  ggtitle("Example 1: Piecewise Constant") +
  labs(x="n", y = "Time (in seconds)") +
  theme(axis.text.x = element_text(size = 10,color="black"),
        axis.title.x = element_text(size = 10,color="black"),
        axis.text.y = element_text(size = 10,color="black"),
        axis.title.y = element_text(size = 10,color="black"),
        plot.title = element_text(size = 10, hjust = 0.5))



t1 <- as.numeric(t(runtime[,,2]))
t1[which.max(t1)] <- NA
data <- cbind(rep(nlist, each=100), t1)
data <- as.data.frame(data)
colnames(data) <- c("n", "time")
library(ggplot2)
p2 <- ggplot(data = data, aes( x= n, y= time)) + geom_jitter(height = 0.0001, width=0) +
  # geom_smooth(method=lm, se= FALSE, size=2, color = "deepskyblue") +
  ggtitle("Example 2: Piecewise Linear") +
  labs(x="n", y = "Time (in seconds)") +
  theme(axis.text.x = element_text(size = 10,color="black"),
        axis.title.x = element_text(size = 10,color="black"),
        axis.text.y = element_text(size = 10,color="black"),
        axis.title.y = element_text(size = 10,color="black"),
        plot.title = element_text(size = 10, hjust = 0.5))

# png("figs/amias_time_n.png", pointsize=6, width=850, height=400, res=120)
grid.arrange(p1, p2, ncol=2)
```

```
## Warning: Removed 1 rows containing missing values (geom_point).
```

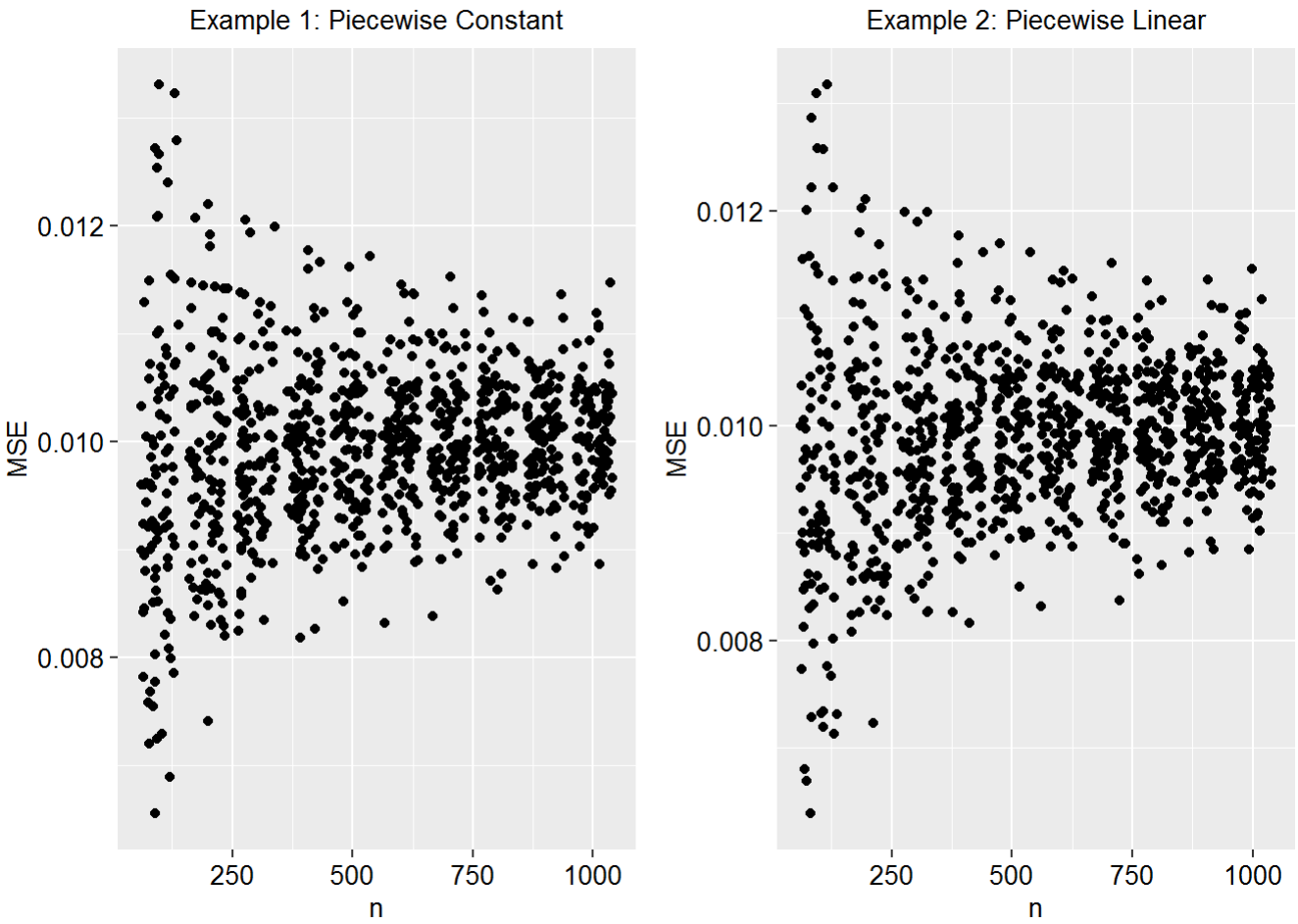



```
# dev.off()
```

3 Figure B.2: Jitter plots of MSE versus sample size *N* in Example 1 and Example 2

```
nlist <- seq(100, 1000, 100)
mse <- array(0, c(length(nlist),100, 4))

for(i in seq_along(nlist)){
  n <- nlist[i]
  print(n)
  for(seed in 1:100){
    # Piecewise constant case
    sigma=0.1; q=0; nknot = 1
    data = ToyEx(n=n, q=q, sigma=sigma, seed=seed)

    op <- par(mfrow=c(1,4))
    resL0 <- amias(data$y, D_type="tf0", k=nknot)
    mse[i,seed,1] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = sample(n-q-1, nknot))
    mse[i,seed,2] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)
    par(op)

    sigma=0.1; q=1; nknot = 1
    data = ToyEx(n=n, q=q, sigma=sigma, seed=seed)

    resL0 <- amias(data$y, D_type="tfq", q=q, k=nknot)
    mse[i,seed,3] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0 <- amias(data$y, D_type="tfq", q=q,  k=nknot, A = sample(n-q-1, nknot))
    mse[i,seed,4] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)



  }
}
save.image("amias_mse_n_single.RData")
```

```
load("amias_mse_n_single.RData")

title <- c("Example 1: Piecewise Constant","Example 1: Piecewise Constant",  "Example 2: Piecewise Linear", "Example 2: Piecewise Linear")
p <- list()
for(j in 1:4){
  t1 <- as.numeric(t(mse[,,j]))
  data <- cbind(rep(nlist, each=100), t1)
  data <- as.data.frame(data)
  colnames(data) <- c("n", "time")
  library(ggplot2)
  p[[j]] <- ggplot(data = data, aes( x= n, y= time)) + geom_jitter()+
    ggtitle(title[j]) +
    labs(x="n", y = "MSE") +
    theme(axis.text.x = element_text(size = 10,color="black"),
          axis.title.x = element_text(size = 10,color="black"),
          axis.text.y = element_text(size = 10,color="black"),
          axis.title.y = element_text(size = 10,color="black"),
          plot.title = element_text(size = 10, hjust = 0.5))
}

# png("figs/amias_mse_n_1.png", pointsize=6, width=850, height=400, res=120)
grid.arrange(p[[1]], p[[3]], ncol=2)
```

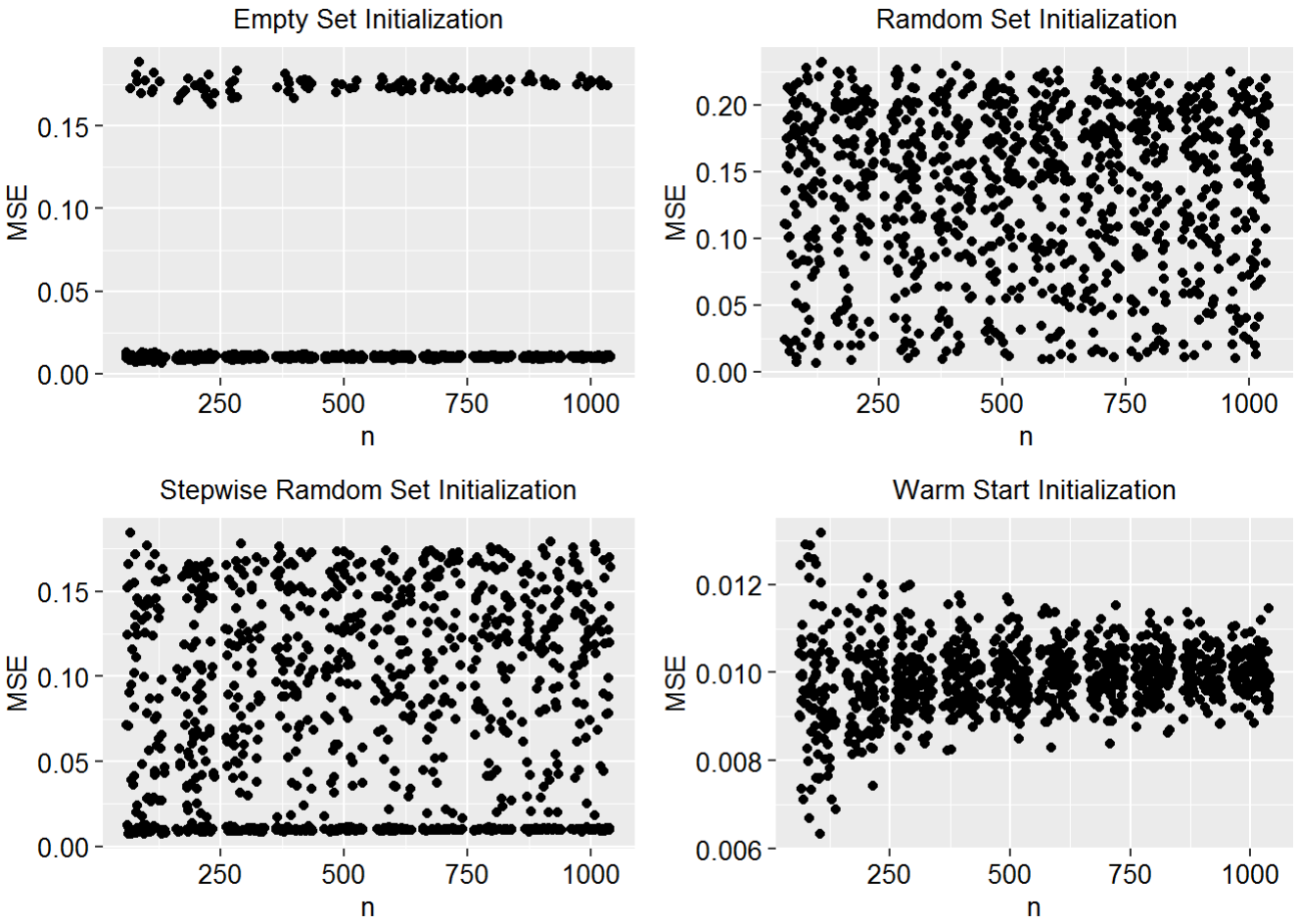


```
# dev.off()
```

4 Figure B.3: Jitter plotS of MSE versus sample size *n* for different initialization strategies in Example 3 with two knots

```
nlist <- seq(100, 1000, 100)
mse <- array(0, c(length(nlist),100, 4))

for(i in seq_along(nlist)){
  n <- nlist[i]
  print(n)
  for(seed in 1:100){
    # Piecewise constant case
    sigma=0.1; q=0; nknot=2;
    data = SimuEx(n=n, sigma=sigma, q=q, nknot=nknot, seed=seed)

    op <- par(mfrow=c(1,4))
    resL0 <- amias(data$y, D_type="tf0", k=nknot)
    mse[i,seed,1] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = sample(n-q-1, nknot))
    mse[i,seed,2] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0_1 <- amias(data$y, D_type="tf0", k=1)
    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = c(resL0_1$A, sample(setdiff(1:(n-q-1), resL0_1$A), 1)))
    mse[i,seed,3] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0_1 <- amias(data$y, D_type="tf0", k=1)
    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = c(resL0_1$A, which.max(abs(resL0_1$u))))
    mse[i,seed,4] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)
    par(op)


  }
}
save.image("amias_mse_n.RData")
```

```
load("amias_mse_n.RData")
title <- c("Empty Set Initialization", "Ramdom Set Initialization", "Stepwise Ramdom Set Initialization", "Warm Start In
itialization")
p <- list()
for(j in 1:4){
  t1 <- as.numeric(t(mse[,,j]))
  data <- cbind(rep(nlist, each=100), t1)
  data <- as.data.frame(data)
  colnames(data) <- c("n", "time")
  library(ggplot2)
  p[[j]] <- ggplot(data = data, aes( x= n, y= time)) + geom_jitter()+
    ggtitle(title[j]) +
    labs(x="n", y = "MSE") +
    theme(axis.text.x = element_text(size = 10,color="black"),
          axis.title.x = element_text(size = 10,color="black"),
          axis.text.y = element_text(size = 10,color="black"),
          axis.title.y = element_text(size = 10,color="black"),
          plot.title = element_text(size = 10, hjust = 0.5))
}

# png("figs/amias_mse_n.png", pointsize=6, width=850, height=800, res=120)
grid.arrange(p[[1]], p[[2]],p[[3]], p[[4]], ncol=2)
```

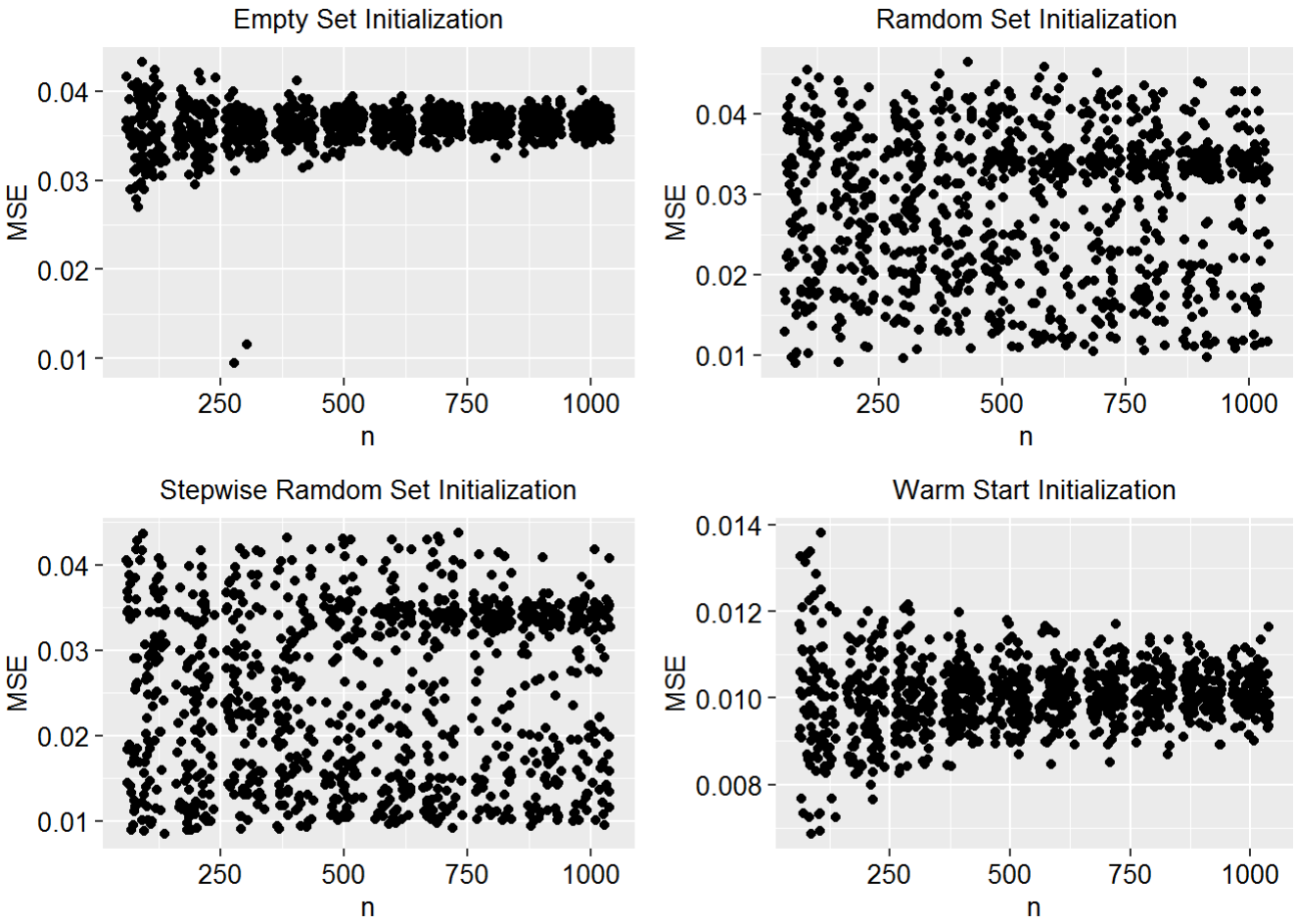



```
# dev.off()
```

5 Figure B.4: Jitter plots of MSE versus sample size *n* for different initialization strategies in Example 4 with two knots

```
nlist <- seq(100, 1000, 100)
mse <- array(0, c(length(nlist),100, 4))

for(i in seq_along(nlist)){
  n <- nlist[i]
  print(n)
  for(seed in 1:100){
    # Piecewise constant case
    sigma=0.1; q=1; nknot=2;
    data = SimuEx(n=n, sigma=sigma, q=q, nknot=nknot, seed=seed)

    op <- par(mfrow=c(1,4))
    resL0 <- amias(data$y, D_type="tfq", q=1, k=nknot)
    mse[i,seed,1] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0 <- amias(data$y, D_type="tfq", q=1, k=nknot, A = sample(n-q-1, nknot))
    mse[i,seed,2] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0_1 <- amias(data$y, D_type="tfq", q=1, k=1)
    resL0 <- amias(data$y, D_type="tfq", q=1, k=nknot, A = c(resL0_1$A, sample(setdiff(1:(n-q-1), resL0_1$A), 1)))
    mse[i,seed,3] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0_1 <- amias(data$y, D_type="tfq", q=1, k=1)
    resL0 <- amias(data$y, D_type="tfq", q=1,  k=nknot, A = c(resL0_1$A, which.max(abs(resL0_1$u))))
    mse[i,seed,4] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)
    par(op)
  }
}

save.image("amias_mse_n_q=1.RData")
```

```
load("amias_mse_n_q=1.RData")
title <- c("Empty Set Initialization", "Ramdom Set Initialization", "Stepwise Ramdom Set Initialization", "Warm Start In
itialization")
p <- list()
for(j in 1:4){
  t1 <- as.numeric(t(mse[,,j]))
  data <- cbind(rep(nlist, each=100), t1)
  data <- as.data.frame(data)
  colnames(data) <- c("n", "time")
  library(ggplot2)
  p[[j]] <- ggplot(data = data, aes( x= n, y= time)) + geom_jitter()+
    ggtitle(title[j]) +
    labs(x="n", y = "MSE") +
    theme(axis.text.x = element_text(size = 10,color="black"),
          axis.title.x = element_text(size = 10,color="black"),
          axis.text.y = element_text(size = 10,color="black"),
          axis.title.y = element_text(size = 10,color="black"),
          plot.title = element_text(size = 10, hjust = 0.5))
}

# png("figs/amias_mse_n_q=1.png", pointsize=6, width=850, height=800, res=120)
grid.arrange(p[[1]], p[[2]],p[[3]], p[[4]], ncol=2)
```




```
# dev.off()
```

6 Figure B.5 Jitter plots of MSE versus sample size *n* for two stepwise initialization strategies in Example 3 with two knots

```
nlist <- seq(100, 1000, 100)
mse <- array(0, c(length(nlist),100, 2))

for(i in seq_along(nlist)){
  n <- nlist[i]
  print(n)
  for(seed in 1:100){
    # Piecewise constant case
    sigma=0.1; q=0; nknot=2;
    data = SimuEx(n=n, sigma=sigma, q=q, nknot=nknot, seed=seed)

    op <- par(mfrow=c(1,2))

    resL0_1 <- amias(data$y, D_type="tf0", k=1)
    y <- data$y
    L <- sum((y-resL0_1$alpha)^2)

    alpha <- loss <- rep(0, length=n)
    for(j in 2:(n-1)){
      if(j<resL0_1$A){
        alpha[1:j] <- mean(y[1:j])
        alpha[(j+1):resL0_1$A] <- mean(y[(j+1):resL0_1$A])
        alpha[(resL0_1$A+1):n] <- mean(y[(resL0_1$A+1):n])
        n1 <- j
        n2 <- resL0_1$A - j
      }else if(j>resL0_1$A){
        alpha[1:resL0_1$A] <- mean(y[1:resL0_1$A])
        alpha[(resL0_1$A+1):j] <- mean(y[(resL0_1$A+1):j])
        alpha[(j+1):n] <- mean(y[(j+1):n])
        n1 <- j - resL0_1$A
        n2 <- n-j
      }
      loss[j] <- L - sum((y-alpha)^2)
    }



    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = c(resL0_1$A, which.max(loss)))
    mse[i,seed,1] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)

    resL0_1 <- amias(data$y, D_type="tf0", k=1)
    resL0 <- amias(data$y, D_type="tf0", k=nknot, A = c(resL0_1$A, which.max(abs(resL0_1$u))))
    mse[i,seed,2] <- mean((as.numeric(data$y)-resL0$alpha)^2)
    plot(resL0)
    par(op)
  }
```

```
}
save.image("amias_mse_n_2.RData")
```

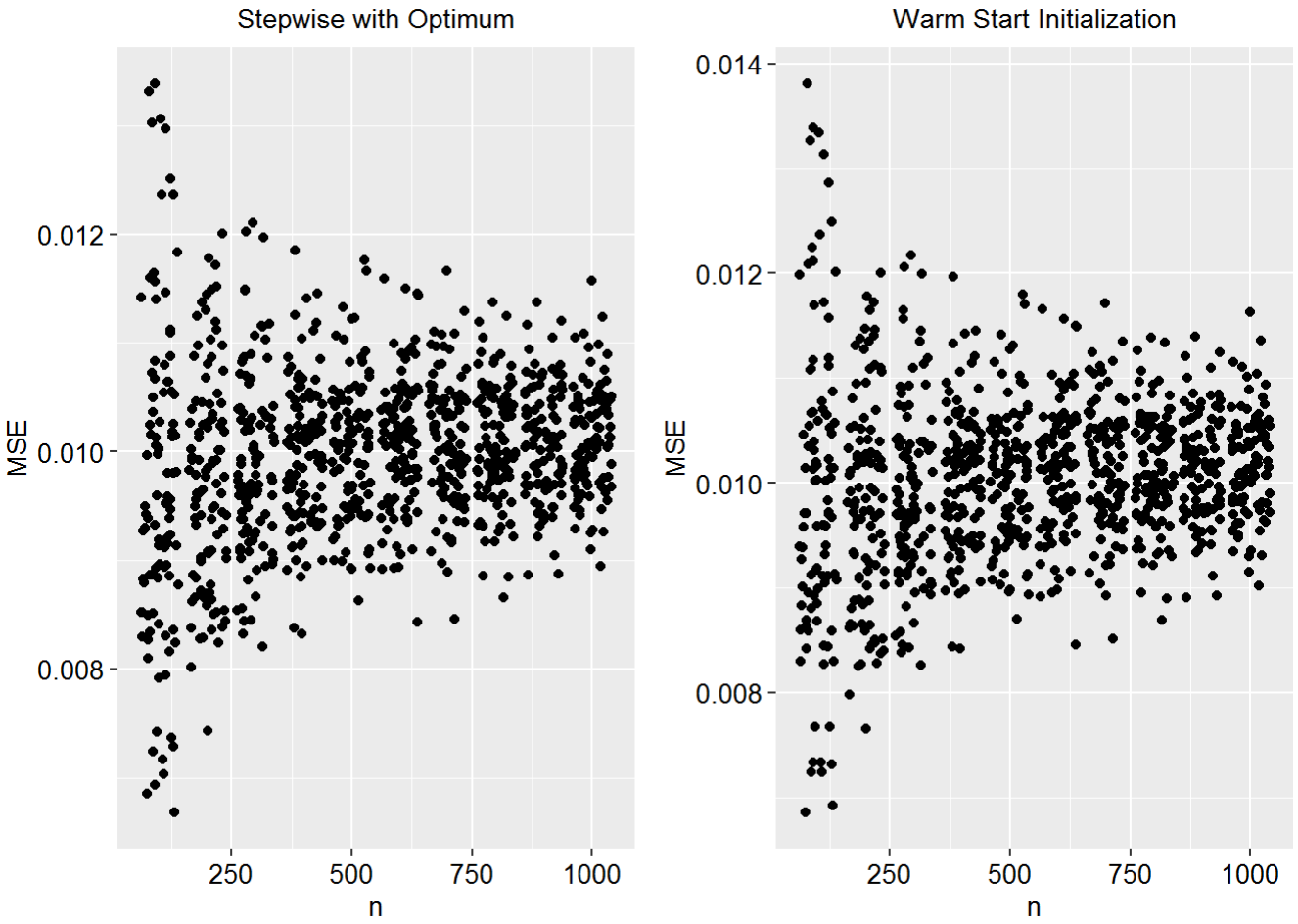
```
load("amias_mse_n_2.RData")
title <- c("Stepwise with Optimum", "Warm Start Initialization")
p <- list()
for(j in 1:2){
  t1 <- as.numeric(t(mse[,,j]))
  data <- cbind(rep(nlist, each=100), t1)
  data <- as.data.frame(data)
  colnames(data) <- c("n", "time")
  library(ggplot2)
  p[[j]] <- ggplot(data = data, aes( x= n, y= time)) +
    geom_point(position = position_jitter(seed = 1))+
    ggtitle(title[j]) +
    labs(x="n", y = "MSE") +
    theme(axis.text.x = element_text(size = 10,color="black"),
          axis.title.x = element_text(size = 10,color="black"),
          axis.text.y = element_text(size = 10,color="black"),
          axis.title.y = element_text(size = 10,color="black"),
          plot.title = element_text(size = 10, hjust = 0.5))
}

# png("figs/amias_mse_n_2.png", pointsize=6, width=850, height=400, res=120)
grid.arrange(p[[1]], p[[2]], ncol=2)
```

```
# dev.off()
```

7 Figure B.6 Jitter plots of MSE versus sample size *n* for two stepwise initialization strategies in Example 4 with two knots

```
library(segmented)
nlist <- seq(100, 1000, 100)
mse <- array(0, c(length(nlist),100, 2))

for(i in seq_along(nlist)){
  n <- nlist[i]
  print(n)
  for(seed in 1:100){
    # Piecewise constant case
    sigma=0.1; q=1; nknot=2;
    data = SimuEx(n=n, sigma=sigma, q=q, nknot=nknot, seed=seed)

    #op <- par(mfrow=c(1,2))

    resL0_1 <- amias(data$y, D_type="tfq", q=1, k=1)
    y <- data$y
    L <- sum((y-resL0_1$alpha)^2)

    alpha <- loss <- rep(0, length=n)
    for(j in 6:(n-5)){
      if(abs(j-resL0_1$A)>1){
        x <- 1:n
        o<-lm(y~x)
        os<-segmented(o, psi=c(j, resL0_1$A), control = seg.control(it.max=0))
        alpha <- predict(os)
        loss[j] <- L - sum((y-alpha)^2)
      }
    }

    resL0 <- amias(data$y, D_type="tfq", q=1, k=nknot, A = c(resL0_1$A, which.max(loss)))
    mse[i,seed,1] <- mean((as.numeric(data$y)-resL0$alpha)^2)

    resL0_1 <- amias(data$y, D_type="tfq", q=1, k=1)
    resL0 <- amias(data$y, D_type="tfq", q=1,  k=nknot, A = c(resL0_1$A, which.max(abs(resL0_1$u))))
    mse[i,seed,2] <- mean((as.numeric(data$y)-resL0$alpha)^2)
  }
}

save.image("amias_mse_n_3.RData")
```

```
load("amias_mse_n_3.RData")
title <- c("Stepwise with Optimum", "Warm Start Initialization")
p <- list()
for(j in 1:2){
  t1 <- as.numeric(t(mse[,,j]))
  data <- cbind(rep(nlist, each=100), t1)
  data <- as.data.frame(data)
  colnames(data) <- c("n", "time")
  library(ggplot2)
  p[[j]] <- ggplot(data = data, aes( x= n, y= time)) +
    geom_point(position = position_jitter(seed = 1))+
    ggtitle(title[j]) +
    labs(x="n", y = "MSE") +
    theme(axis.text.x = element_text(size = 10,color="black"),
          axis.title.x = element_text(size = 10,color="black"),
          axis.text.y = element_text(size = 10,color="black"),
          axis.title.y = element_text(size = 10,color="black"),
          plot.title = element_text(size = 10, hjust = 0.5))
}

# png("figs/amias_mse_n_3.png", pointsize=6, width=850, height=400, res=120)
grid.arrange(p[[1]], p[[2]], ncol=2)
```

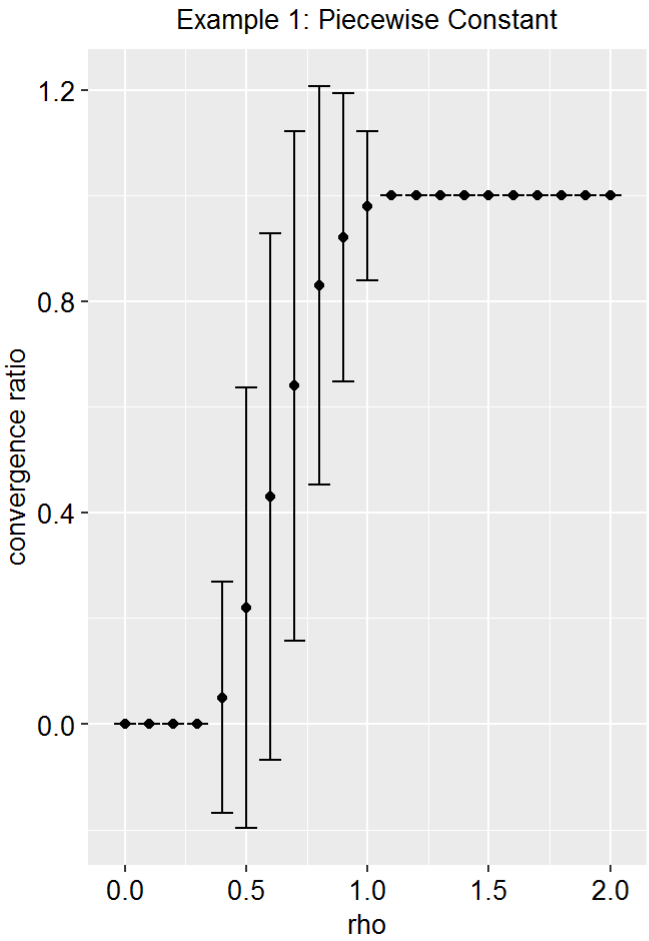


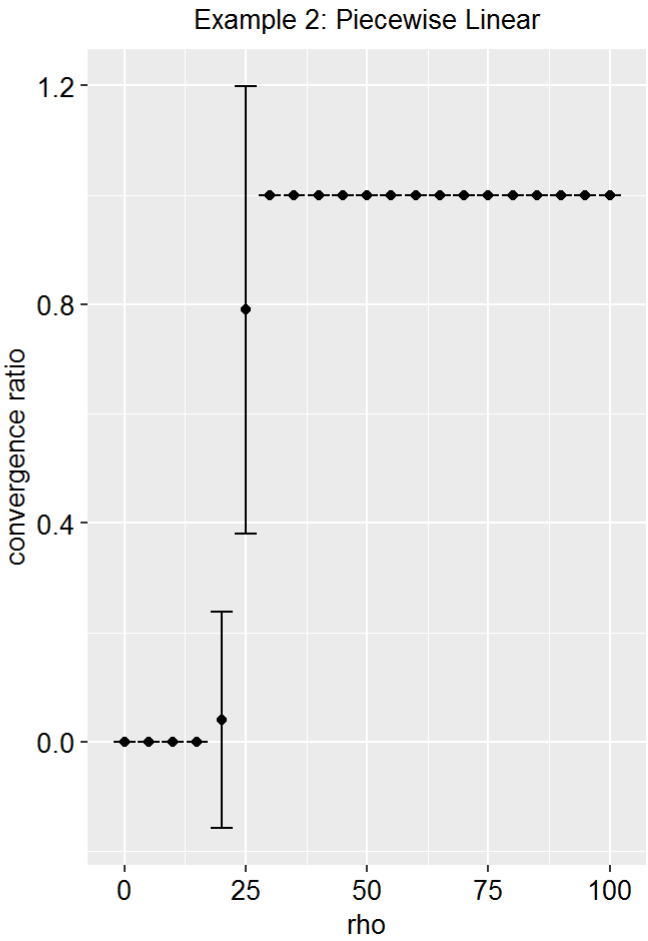

```
# dev.off()
```

8 Figure B.7: Error bars of the proportion of convergence against the mixing parameter ρ

```
source("utils.R")
source("amiasutils.R")
# amias rho vs convergence
c1 <- c2 <- c()
for(i in 1:100){
  if (i%%20==0) print(i)
  data1 = SimuEx(n=100, sigma=0.1, q=0, nknot=1, seed=i)
  data2 = SimuEx(n=100, sigma=0.1, q=1, nknot=1, seed=i)
  rho1 <- seq(0,2,0.1)
  rho2 <- seq(0,data2$n,5)

  r1 <- r2 <- c()
  for(rho in rho1){
    res1 = amias_R(y = data1$y, D = DiffMat(data1$n, data1$nknot), A = c(), k = data1$nknot, rho = rho, q = data1$q)
    if(res1$iter==20){
      r1 <- c(r1, 0)
    }else{
      r1 <- c(r1, 1)
    }
  }
  for(rho in rho2){
    res2 = amias_R(y = data2$y, D = DiffMat(data2$n, data2$nknot), A = c(), k = data2$nknot, rho = rho, q = data2$q)
    if(res2$iter==20){
      r2 <- c(r2, 0)
    }else{
      r2 <- c(r2, 1)
    }
  }
  c1 <- rbind(c1, r1)
  c2 <- rbind(c2, r2)
}
save.image("amias_rho_conv.RData")
```

```
load("amias_rho_conv.RData")
p1 = convsteps(t(c1), rho1, "Example 1: Piecewise Constant", 'rho', 'convergence ratio')
p2 = convsteps(t(c2), rho2, "Example 2: Piecewise Linear", 'rho', 'convergence ratio')

# png("figs/amias_rho_conv.png", pointsize=8, width=850, height=400, res=120)
ggarrange(p1, p2, ncol = 2)
```


Example 1: Piecewise Constant
convergence ratio
1.2
0.8
0.4
0.0
0.0
0.5
1.0
1.5
2.0
rho


Example 2: Piecewise Linear
convergence ratio
1.2
0.8
0.4
0.0
0
25
50
75
100
rho

```
# dev.off()
```