Online Supplement
***KPG* Complexity Proof**

We perform a reduction from the *DeNegre Bilevel Knapsack Problem* (BKP) below, which is Σ_2^p-complete (Caprara et al. 2014).

Definition 1 (BKP). *Given two m-dimensional non-negative integer vectors a and b and two non-negative integers A and B, the BKP asks whether there exists a binary vector x – with $\sum_{j=1}^{m} a_j x_j \leq A$ – satisfying $\sum_{j=1}^{m} b_j y_j (1 - x_j) \leq B - 1$ for any binary vector y such that $\sum_{j=1}^{m} b_j y_j \leq B$.*

Without loss of generality, we assume $a_j \leq A$ for any j. If this is not the case, we can always modify the original BKP instance as follows: (i.) we replace A with $2A+1$, any $a_j \leq A$ with $2a_j$, and any $a_j > A$ with $(2A+1)$, and (ii.) we add a new element $m+1$ (i.e., a new item), with $a_{m+1} = 1$ and $b_{m+1} = B$. In any solution of this modified instance, we must have $x_{m+1} = 1$, otherwise $\sum_{j=1}^{m+1} b_j y_j (1 - x_j) \leq B - 1$ would never hold since $\sum_{j=1}^{m+1} b_j y_j (1 - x_j) = B$ when $x_{m+1} = 0$ and $y_{m+1} = 1$. Setting $x_{m+1} = 1$ gives a residual capacity $2A$ for the packing constraint of x. Indeed, every subset of x variables with original $a_j \leq A$ that was satisfying $\sum_{j=1}^{m} a_j x_j \leq A$ now satisfies $\sum_{j=1}^{m} 2a_j x_j \leq 2A$. On the contrary, we cannot select any x_j variable with original $a_j > A$. Thus, a solution (if any) to the original instance corresponds to a solution to the modified instance, and vice versa.

Proof of Theorem 2. First, deciding if KPG admits a PNE is in Σ_2^p, as we ask whether there exists a strategy profile where every player cannot improve its payoff with any of its strategies, and we can compute the payoff of such strategies in polynomial time. Given a BKP instance, we construct a KPG instance with 2 players as follows. We consider $m+1$ items and associate the elements of vectors x and y with the first m elements of vectors x^1 and x^2, respectively. Then, player 1 solves the problem in (1), whereas player 2 solves the problem in (2).

$$\max_{x^1} \{ \sum_{j=1}^{m} b_j x_j^1 x_j^2 + x_{m+1}^1 x_{m+1}^2 : \sum_{j=1}^{m} a_j x_j^1 \leq A,\ x^1 \in \{0,1\}^{m+1} \}. \tag{1}$$

$$\max_{x^2} \{ (B-1) x_{m+1}^2 + \sum_{j=1}^{m} b_j x_j^2 - \sum_{j=1}^{m} b_j x_j^2 x_j^1 : \sum_{j=1}^{m} b_j x_j^2 + B x_{m+1}^2 \leq B,\ x^2 \in \{0,1\}^{m+1} \}. \tag{2}$$

In order to prove the theorem, we show that the *KPG* instance has a *PNE* if and only if the corresponding BKP instance admits a solution.

BKP admits a solution. We assume the BKP instance has a solution $\overline{x}$. We prove that $\hat{x}^1 = (\overline{x}, 1)$, $\hat{x}^2 = (\overline{0}, 1)$ (with $\overline{0}$ being an m-dimensional vector of zeros) is a *PNE*. First, both the strategies $\hat{x}^1$ and $\hat{x}^2$ are feasible by construction. Given $\hat{x}^2$, player 1 attains the maximum payoff of 1 by playing strategy $\hat{x}^1$. The strategy $\hat{x}^2$ yields a payoff of $B-1$ for player 2 when player 1 plays $\hat{x}^1$. Player 2 cannot profitably deviate by setting $x^2_{m+1} = 0$. This follows from the fact that the BKP instance has a solution $\overline{x}$ and, given that $\hat{x}^1_j = \overline{x}_j$ for $j = 1, \dots, m$, the following inequality must hold.

$$\sum_{j=1}^m b_j x^2_j - \sum_{j=1}^m b_j x^2_j \hat{x}^1_j \leq B - 1.$$

Thus, the pair of strategies $(\hat{x}^1, \hat{x}^2)$ is also a *PNE* for the *KPG* instance.

BKP has no solution. If the BKP instance has no solution, player 2 never plays $x^2_{m+1} = 1$ in a best-response, as it can always obtain a payoff of B with variables $x^2_1, \dots, x^2_m$ for any player 1's feasible strategy. Consider any player 2's best-response $\hat{x}^2$, with $\hat{x}^2_{m+1} = 0$, and assume the *KPG* instance has a *PNE* $(\hat{x}^1, \hat{x}^2)$. Then, in the player 1's best-response $\hat{x}^1$, there exists at least one $\hat{x}^1_j = 1$ when $\hat{x}^2_j = 1$ and $b_j > 0$ (since $a_j \leq A$ for any j). However, in this case, player 2 would deviate from $\hat{x}^2$, since $\hat{x}^2$ gives a payoff $< B$ under $\hat{x}^1$. Thus, no *PNE* exists in the *KPG* instance. □

Potential *KPG*

Proof of Theorem 3. To prove our result, we first consider the case where $c^i > 0$ for each player i. Let the function $\phi(x^1, \dots, x^n) : \prod_{i=1}^n \mathcal{X}^i \to \mathbb{R}$ be such that

$$\phi(x^1, \dots, x^n) = \sum_{i=1}^n \frac{1}{c^i} \Big(\sum_{j=1}^m p^i_j x^i_j \Big) + \sum_{l=1}^{n-1} \sum_{k=l+1}^n \Gamma(x^l, x^k), \tag{3}$$

where $\Gamma(x^l, x^k) = \sum_{j=1}^m x^l_j x^k_j$ is the number of items selected by both player l and player k in their strategies x^l and x^k. We prove that ϕ is a potential function by showing that, given two strategies $\bar{x}^i$ and $\hat{x}^i$ of any player i and any given strategies x^{-i} of the other players, we have $\phi(\bar{x}^i, x^{-i}) - \phi(\hat{x}^i, x^{-i}) > 0$ if and only if $u^i(\bar{x}^i, x^{-i}) - u^i(\hat{x}^i, x^{-i}) > 0$. Given the definition of ϕ, we have that

$$\phi(\bar{x}^i, x^{-i}) - \phi(\hat{x}^i, x^{-i}) = \frac{1}{c^i} \sum_{j=1}^m p^i_j \bar{x}^i_j + \sum_{l=1}^{i-1} \Gamma(x^l, \bar{x}^i) + \sum_{k=i+1}^n \Gamma(\bar{x}^i, x^k)$$

$$
\begin{aligned}
&- \frac{1}{c^i}\sum_{j=1}^{m} p^i_j \hat{x}^i_j - \sum_{l=1}^{i-1}\Gamma(x^l,\hat{x}^i) - \sum_{k=i+1}^{n}\Gamma(\hat{x}^i,x^k) \\
&= \frac{1}{c^i}\Big(\sum_{j=1}^{m} p^i_j \bar{x}^i_j + c^i\sum_{l=1}^{i-1}\Gamma(x^l,\bar{x}^i) + c^i\sum_{k=i+1}^{n}\Gamma(\bar{x}^i,x^k)\Big) \\
&- \frac{1}{c^i}\Big(\sum_{j=1}^{m} p^i_j \hat{x}^i_j + c^i\sum_{l=1}^{i-1}\Gamma(x^l,\hat{x}^i) + c^i\sum_{k=i+1}^{n}\Gamma(\hat{x}^i,x^k)\Big) \\
&= \frac{1}{c^i}\big(u^i(\bar{x}^i,x^{-i}) - u^i(\hat{x}^i,x^{-i})\big).
\end{aligned}
$$

It follows that ϕ is a potential function for the *KPG*. On the other hand, if $c^i < 0$ for each player i, by similar arguments, we can show that $-\phi$ is a potential function. □

Extended Computational Results

In the following sections, we report the full results of our computational tests. The columns are similar to the ones reported in the previous tables, possibly with the following additions (i.) *#It* indicating the number of iterations of our algorithm, and (ii.) PNE * reporting the social welfare of the most efficient *PNE*, (iii.) PNE ° reporting the social welfare of the less efficient *PNE* (if computed), (iv.) *OSW* reporting the optimal social welfare in the game, and (v.) *Bound* reporting the last proven bound on $\mathcal{Q}$ before the latter becomes infeasible (or the algorithm hits the time limit), irrespective on whether the algorithm enumerated *PNEa* or not.

Full *KPG* Results

We report the two tables with the full *KPG* results. In the first column of Tables 1 and 2 we add the field I to specify the instance type. Specifically, the knapsack capacity of player i is given by $\sum_{j=1}(w^i_j)I/10$.

(*n*, *m*, *d*, *I*)	***PoS***	**#EI**	**#EI_P**	**#EI_D**	**#It**	**Time**	**Time-1st**	***PNE* ***	***OSW***	**Bound**
(2, 25, A, 2)	1.106	12	0	8	3	0.036	0.035	1884	2084	1884
(2, 25, A, 5)	1.025	20	0	0	3	0.095	0.093	3086	3163	3086
(2, 25, A, 8)	1.000	12	0	1	2	0.035	0.032	4883	4883	4883
(2, 25, B, 2)	1.021	14	0	0	3	0.067	0.065	1609	1643	1609
(2, 25, B, 5)	1.025	28	0	4	4	0.250	0.182	3456	3542	3459
(2, 25, B, 8)	-	10	0	7	2	0.038	0.035	4624	4624	4624
(2, 25, C, 2)	-	16	7	5	4	0.153	-	-	1480	1329
(2, 25, C, 5)	-	62	12	17	11	0.967	-	-	2083	1863
(2, 25, C, 8)	1.064	10	10	1	4	0.037	0.036	2739	2914	2739
(2, 50, A, 2)	1.024	24	0	3	4	0.213	0.208	3824	3914	3824
(2, 50, A, 5)	1.035	16	0	2	3	0.214	0.212	6404	6626	6404
(2, 50, A, 8)	1.016	20	0	3	4	0.205	0.204	6703	6809	6703
(2, 50, B, 2)	1.004	10	0	0	2	0.043	0.040	3930	3946	3930
(2, 50, B, 5)	1.004	42	0	34	5	0.853	0.620	6931	6962	6936
(2, 50, B, 8)	1.008	28	0	25	6	0.620	0.501	9294	9372	9294
(2, 50, C, 2)	1.018	8	25	0	3	0.087	0.086	3173	3230	3173
(2, 50, C, 5)	-	112	25	22	17	17.190	-	-	5654	4923
(2, 50, C, 8)	1.134	98	31	12	18	1.749	1.747	5358	6074	5358
(2, 75, A, 2)	1.008	36	0	19	4	0.407	0.401	5784	5831	5784
(2, 75, A, 5)	1.004	40	0	49	4	1.025	0.572	12701	12757	12702
(2, 75, A, 8)	1.001	38	0	25	3	0.359	0.342	16319	16337	16319
(2, 75, B, 2)	1.033	122	0	41	12	12.483	9.045	5690	5880	5694
(2, 75, B, 5)	1.015	72	0	35	8	5.865	1.420	10293	10449	10297
(2, 75, B, 8)	1.010	108	0	26	12	6.691	6.664	13769	13910	13769
(2, 75, C, 2)	1.061	94	43	57	9	3.072	3.064	4356	4623	4356
(2, 75, C, 5)	-	136	35	87	18	134.899	-	-	7908	6934
(2, 75, C, 8)	1.089	108	37	57	18	5.289	4.865	8455	9207	8467
(2, 100, A, 2)	1.007	38	0	29	5	1.409	1.153	8302	8357	8313
(2, 100, A, 5)	1.002	20	0	4	2	0.355	0.188	18271	18301	18274
(2, 100, A, 8)	1.011	18	0	11	3	0.521	0.398	18516	18723	18519
(2, 100, B, 2)	1.018	78	0	11	8	4.294	4.281	8156	8303	8156
(2, 100, B, 5)	1.010	500	0	203	42	655.957	425.088	14246	14390	14248
(2, 100, B, 8)	1.002	38	0	25	5	0.997	0.988	19054	19084	19054
(2, 100, C, 2)	1.048	116	49	33	13	15.873	11.332	5808	6084	5817
(2, 100, C, 5)	-	464	53	260	30	tl	-	-	9611	8958
(2, 100, C, 8)	-	1512	64	66	110	tl	-	-	9791	9007

Table 1 **Full results for the KPG with $n = 2$.**

(n, m, d, I)	*PoS*	#EI	#EI_P	#EI_D	#It	Time	Time-1st	*PNE* *	*OSW*	Bound
(3, 25, A, 2)	1.010	21	0	0	3	0.166	0.164	3738	3777	3738
(3, 25, A, 5)	1.004	30	0	0	2	0.151	0.144	5480	5500	5480
(3, 25, A, 8)	1.011	42	0	28	3	0.323	0.316	9592	9693	9592
(3, 25, B, 2)	1.034	27	0	3	3	0.223	0.219	4535	4691	4535
(3, 25, B, 5)	1.005	45	0	18	3	0.394	0.388	7293	7329	7293
(3, 25, B, 8)	1.008	60	0	23	4	0.387	0.378	10346	10433	10346
(3, 25, C, 2)	1.259	78	6	5	8	6.765	5.643	2152	2710	2165
(3, 25, C, 5)	-	159	24	64	13	82.115	-	-	4980	3935
(3, 25, C, 8)	-	36	59	32	4	0.449	-	-	5735	4414
(3, 50, A, 2)	1.033	99	0	17	5	3.739	3.727	6769	6995	6769
(3, 50, A, 5)	1.037	69	0	6	5	2.413	2.043	11345	11764	11346
(3, 50, A, 8)	1.007	117	0	50	8	49.004	29.269	17283	17406	17283
(3, 50, B, 2)	1.011	36	0	1	4	1.976	1.971	7549	7634	7549
(3, 50, B, 5)	1.015	468	0	99	29	tl	483.842	13571	13781	13573
(3, 50, B, 8)	1.011	114	0	33	9	77.373	15.220	19680	19896	19697
(3, 50, C, 2)	-	231	37	108	15	934.599	-	-	5215	3764
(3, 50, C, 5)	-	159	64	397	10	211.139	-	-	9148	7564
(3, 50, C, 8)	-	54	88	169	5	0.977	-	-	11002	9194
(3, 75, A, 2)	1.003	60	0	70	4	9.057	1.342	14664	14711	14672
(3, 75, A, 5)	1.041	45	0	1	3	0.842	0.827	13869	14434	13869
(3, 75, A, 8)	1.002	87	0	286	5	4.056	4.032	26468	26519	26468
(3, 75, B, 2)	-	444	0	130	18	tl	-	-	11508	11229
(3, 75, B, 5)	1.002	81	0	97	4	5.206	5.180	23139	23194	23139
(3, 75, B, 8)	1.011	309	0	51	18	1143.710	540.207	30118	30438	30118
(3, 75, C, 2)	-	357	36	177	15	tl	-	-	7242	6568
(3, 75, C, 5)	-	141	152	654	7	175.807	-	-	13553	11175
(3, 75, C, 8)	-	21	74	128	3	0.517	-	-	16736	14235
(3, 100, A, 2)	-	333	0	15	15	tl	-	-	15164	14825
(3, 100, A, 5)	1.003	408	0	28	21	tl	1330.340	32673	32766	32677
(3, 100, A, 8)	1.006	42	0	391	3	1.959	1.915	37607	37826	37607
(3, 100, B, 2)	-	516	0	297	21	tl	-	-	15946	15679
(3, 100, B, 5)	-	291	0	81	12	tl	-	-	29393	29119
(3, 100, B, 8)	-	630	0	127	29	tl	-	-	40282	40082
(3, 100, C, 2)	-	288	45	285	12	tl	-	-	11222	10045
(3, 100, C, 5)	-	234	226	2059	10	tl	-	-	18272	15941
(3, 100, C, 8)	-	30	242	715	3	0.932	-	-	20653	16855

Table 2 **Full results for the KPG with $n = 3$.**

Full *NFG* Results

In Table 3, we report the full results for the *NFG*. In the second and third columns, we report the weights of player 1 and 2 as w^1 and w^2, respectively. We remark that $w^3 = 1 - w^1 - w^2$.

$(\|V\|, \|E\|)$	w^1	w^2	*PoS*	#EI	#It	Time	Time-1st	*PNE* *	*OSW*	Bound
(50, 99)	0.33	0.33	1.061	5	3	0.037	0.036	980	924	980
(50, 99)	0.6	0.2	1.245	8	3	0.040	0.039	1150	924	1150
(50, 99)	0.45	0.45	1.061	5	3	0.034	0.034	980	924	980
(100, 206)	0.33	0.33	1.000	3	2	0.047	0.041	1320	1320	1320
(100, 206)	0.6	0.2	1.000	2	2	0.046	0.040	1320	1320	1320
(100, 206)	0.45	0.45	1.000	2	2	0.047	0.040	1320	1320	1320
(150, 308)	0.33	0.33	1.015	8	4	0.996	0.222	2049	2019	2042
(150, 308)	0.6	0.2	1.015	5	4	0.354	0.353	2049	2019	2049
(150, 308)	0.45	0.45	1.015	5	3	0.565	0.190	2049	2019	2041
(200, 416)	0.33	0.33	1.000	1	2	0.109	0.096	2336	2336	2336
(200, 416)	0.6	0.2	1.007	12	5	2.828	1.696	2352	2336	2346
(200, 416)	0.45	0.45	1.187	22	10	6.908	1.529	2352	2336	2349
(250, 517)	0.33	0.33	1.027	137	37	144.392	33.653	2730	2672	2729
(250, 517)	0.6	0.2	1.027	47	17	43.991	13.430	2730	2672	2729
(250, 517)	0.45	0.45	1.012	10	5	2.111	1.122	2703	2672	2693
(300, 626)	0.33	0.33	1.060	36	10	14.877	2.068	3587	3567	3587
(300, 626)	0.6	0.2	1.053	26	11	21.300	5.701	3587	3567	3585
(300, 626)	0.45	0.45	1.000	1	2	0.161	0.140	3567	3567	3567
(350, 730)	0.33	0.33	1.003	15	5	9.664	3.100	3678	3669	3677
(350, 730)	0.6	0.2	1.014	41	11	31.889	18.997	3687	3669	3687
(350, 730)	0.45	0.45	1.000	1	2	0.197	0.173	3669	3669	3669
(400, 822)	0.33	0.33	1.207	100	29	163.047	0.228	4348	4319	4347
(400, 822)	0.6	0.2	1.016	543	116	tl	584.854	4387	4319	4373
(400, 822)	0.45	0.45	1.007	103	26	121.910	100.997	4348	4319	4346
(450, 934)	0.33	0.33	1.159	0	2	0.304	0.250	4827	4827	4827
(450, 934)	0.6	0.2	1.021	575	119	tl	7.284	4925	4827	4866
(450, 934)	0.45	0.45	1.159	609	115	tl	0.281	4934	4827	4864
(500, 1060)	0.33	0.33	1.004	66	29	198.440	5.191	5535	5512	5534
(500, 1060)	0.6	0.2	1.004	20	8	20.951	11.231	5535	5512	5535
(500, 1060)	0.45	0.45	1.005	21	12	41.808	5.321	5535	5512	5534

Table 3 Full results for the NFG.

Full *CFLD* Results

We report the results for a set of instances from Crönert and Minner (2022, table 2) (i.e., $\beta = 0.5$ and $d_{max} = 20$). We report the full set of our results in Table 4, where, in the second and third columns, we report the budget of player 1 and 2 as B^1 and B^2. When $n = 3$, $B^3 = 10$.

n	B^1	B^2	*PoS*	#EI	#It	Time	Time-1st	*PNE* *	*OSW*	Bound
2	**10**	**10**	1.000	2	2	0.038	0.036	69	69	69
2	**10**	**20**	1.000	2	2	0.259	0.256	109	109	109
2	**10**	**30**	1.000	2	2	0.871	0.869	153	153	153
2	**10**	**40**	1.000	2	2	1.026	1.025	186	186	186
2	**10**	**50**	1.000	2	2	0.545	0.544	212	212	212
2	**10**	**60**	1.000	2	2	0.627	0.626	232	232	232
2	**10**	**70**	1.013	4	3	2.081	2.079	236	239	236
2	**10**	**80**	1.047	8	4	4.908	4.905	236	247	236
2	**10**	**90**	1.029	8	4	3.532	3.529	245	252	245
2	**10**	**100**	1.028	8	4	3.706	3.701	247	254	247
2	**20**	**20**	1.000	2	2	0.426	0.424	136	136	136
2	**20**	**30**	1.000	2	2	1.153	1.151	180	180	180
2	**20**	**40**	1.000	2	2	0.867	0.865	210	210	210
2	**20**	**50**	1.000	4	3	1.760	1.758	232	232	232
2	**20**	**60**	1.013	10	5	11.852	6.770	236	239	238
2	**20**	**70**	1.038	6	3	7.494	7.194	234	243	234
2	**20**	**80**	-	6	4	9.530	-	-	252	243
2	**20**	**90**	-	8	4	14.304	-	-	254	247
2	**20**	**100**	-	6	3	19.163	-	-	254	252
2	**30**	**30**	1.000	2	2	2.583	2.580	202	202	202
2	**30**	**40**	1.000	2	2	1.852	1.849	232	232	232
2	**30**	**50**	1.030	14	5	13.268	6.067	236	243	238
2	**30**	**60**	1.065	14	7	37.077	37.067	232	247	232
2	**30**	**70**	1.050	8	4	38.741	38.384	240	252	240
2	**30**	**80**	1.058	16	5	515.179	270.395	240	254	241
2	**30**	**90**	-	10	6	1327.610	-	-	254	240
2	**30**	**100**	-	8	5	778.459	-	-	254	247
2	**40**	**40**	1.138	24	9	491.695	154.949	210	239	216
2	**40**	**50**	1.038	16	6	128.764	23.469	238	247	240
2	**40**	**60**	1.050	18	9	344.539	98.475	240	252	240
2	**40**	**70**	1.058	14	6	808.094	418.576	240	254	245
2	**40**	**80**	1.058	10	5	1636.910	779.146	240	254	243
3	**10**	**10**	1.072	6	3	0.360	0.358	69	74	69
3	**10**	**20**	1.000	3	2	0.180	0.178	136	136	136
3	**10**	**30**	1.000	3	2	0.522	0.518	180	180	180
3	**10**	**40**	1.000	3	2	0.494	0.492	210	210	210
3	**10**	**50**	1.000	3	2	0.631	0.628	232	232	232
3	**10**	**60**	1.022	9	3	2.037	1.978	232	237	232
3	**10**	**70**	1.030	9	4	4.772	4.769	236	243	236
3	**10**	**80**	1.029	9	4	3.437	3.433	245	252	245
3	**10**	**90**	1.037	9	4	6.679	6.676	245	254	245
3	**10**	**100**	-	9	4	16.522	-	-	254	249
3	**20**	**20**	1.000	3	2	1.520	1.517	158	158	158
3	**20**	**30**	1.037	18	4	6.161	5.795	187	194	187
3	**20**	**40**	1.032	9	3	3.088	3.018	217	224	217
3	**20**	**50**	1.000	3	2	1.931	1.929	239	239	239
3	**20**	**60**	1.030	15	5	19.666	19.662	236	243	236
3	**20**	**70**	1.068	6	3	5.114	5.111	236	252	236

3	**20**	**80**	1.058	9	4	14.024	14.021	240	254	240
3	**20**	**90**	-	9	3	35.657	-	-	254	252
3	**20**	**100**	-	9	4	162.306	-	-	254	252
3	**30**	**30**	1.000	18	4	11.841	11.838	216	216	216
3	**30**	**40**	1.102	27	8	54.111	35.612	216	238	218
3	**30**	**50**	1.038	24	8	51.141	51.135	238	247	238
3	**30**	**60**	1.050	18	6	109.115	109.110	240	252	240
3	**30**	**70**	1.058	24	8	226.185	226.180	240	254	240
3	**30**	**80**	1.058	15	5	833.079	434.912	240	254	247
3	**30**	**90**	-	9	3	1153.500	-	-	254	254
3	**30**	**100**	1.058	12	5	tl	3021.700	240	254	243
3	**40**	**40**	1.038	36	8	222.003	221.639	238	247	238
3	**40**	**50**	1.050	27	8	207.155	206.729	240	252	240
3	**40**	**60**	1.076	24	9	2848.410	1696.300	236	254	244
3	**40**	**70**	1.058	24	8	2441.450	833.853	240	254	240
3	**40**	**80**	-	15	6	tl	-	-	254	252

Table 4: Full results for the *CFLD* from the instances of Crönert and Minner (2022, Table 2).

Full *qIPG*s Results

We report the full results for the instances of Schwarze and Stein (2022) in Table 5, and the ones generated following the scheme of Sagratella (2016) in Table 6. In the latter table, we refer to Sagratella (2016) for an overview on the instances acronyms.

Instance	#EQs	*PoS*	*PoA*	#EI	#It	Time	Time-1st	*PNE* *	*PNE* °	*OSW*	Bound
C22_3	2	1.0815	1.1238	14	4	0.3692	0.2835	-22.7030	-21.8475	-24.5524	-22.7030
C22_2	1	1.0000	1.0000	2	2	0.0949	0.0811	-0.3146	-0.3146	-0.3146	-0.3146
C22_1	2	1.4233	2.1559	24	6	0.6366	0.1852	-13.5053	-8.9158	-19.2216	0.0196
C22_4	0	-	-	16	3	0.3097	-	-	-	-15.1462	-6.8359
C23_1	2	1.0353	1.6333	28	5	0.7040	0.2652	-10.7928	-6.8413	-11.1737	0.0000
C23_3	2	1.4506	3.0534	24	7	1.0306	0.5635	-22.3566	-10.6215	-32.4315	-10.6215
C23_2	0	-	-	26	6	1.0065	-	-	-	-22.3275	-0.3407
C23_6	1	23.2815	23.2815	16	4	0.7469	0.6326	-0.3396	-0.3396	-7.9063	-0.3572
C23_7	1	1.0101	1.0101	6	3	0.4038	0.2817	-4.5242	-4.5242	-4.5698	-4.5242
C23_5	0	-	-	20	6	1.0198	-	-	-	-8.2644	-0.2266
C23_4	0	-	-	26	6	0.9098	-	-	-	-44.4346	-4.6428
C23_8	2	1.0153	1.5113	68	13	3.2209	0.3646	-74.4543	-50.0193	-75.5936	-3.5526
C24_4	0	-	-	32	5	4.6024	-	-	-	-46.2197	-1.3690
C24_3	0	-	-	48	7	4.2759	-	-	-	-49.3061	-2.2944
C24_2	0	-	-	40	5	2.8302	-	-	-	-50.0571	-1.5728
C24_1	1	1.3206	1.3206	20	3	1.8845	1.1885	-6.4656	-6.4656	-8.5384	-6.4825
C25_4	1	1.4166	1.4166	34	7	34.4549	4.6820	-50.3544	-50.3544	-71.3315	-5.5704
C25_1	3	1.2068	11.3913	64	10	32.8601	3.6125	-22.4829	-2.3818	-27.1321	-2.3818
C25_3	1	2.0289	2.0289	60	10	26.9686	4.6411	-45.6431	-45.6431	-92.6057	-8.0414
C25_2	1	4.1130	4.1130	66	12	49.2170	17.9271	-10.2162	-10.2162	-42.0192	-2.1366
C32_1	2	3.1976	6.0787	30	4	0.7915	0.6078	-21.6314	-11.3788	-69.1684	-21.6314
C32_2	1	1.1101	1.1101	15	5	1.1502	0.4799	-28.0541	-28.0541	-31.1421	-9.9981
C32_3	3	1.1778	-	63	9	2.2256	0.8736	-45.5016	0.0000	-53.5937	-14.9792
C32_4	0	-	-	60	4	0.6875	-	-	-	-30.6557	-8.8992
C33_2	1	8.3676	8.3676	117	12	23.1855	15.4598	-9.2113	-9.2113	-77.0768	-3.7804
C33_3	1	1.7099	1.7099	102	10	25.2901	1.6631	-57.6349	-57.6349	-98.5491	-1.3043
C33_1	1	1.2914	1.2914	129	17	55.9786	4.1433	-138.1190	-138.1190	-178.3720	-6.0334
C33_4	0	-	-	87	9	38.0066	-	-	-	-118.7970	-6.0558
NC22_1	2	2.2947	2.2947	18	4	0.4126	0.1822	-8.7456	-8.7456	-20.0687	-8.7456
NC22_2	1	1.9081	1.9081	14	4	0.4483	0.3664	-12.2614	-12.2614	-23.3957	-12.2614
NC22_3	1	2.3939	2.3939	16	5	0.4243	0.3463	-22.1224	-22.1224	-52.9584	-22.1224

NC22_4	0	-	-	16	4	0.4330	-	-	-	-34.0944	-22.9080
NC23_8	0	-	-	18	5	0.8855	-	-	-	-57.4117	-31.8276
NC23_2	1	1.4346	1.4346	10	4	1.0951	0.2406	-29.1437	-29.1437	-41.8083	-15.2740
NC23_3	0	-	-	20	4	0.7570	-	-	-	-79.2272	-28.3022
NC23_1	1	1.6194	1.6194	36	8	1.5463	0.6476	-61.1489	-61.1489	-99.0215	-2.1164
NC23_4	3	1.1508	1.9143	32	9	1.4491	0.3539	-74.7629	-44.9448	-86.0367	-35.2390
NC23_5	2	1.0962	1.7415	10	4	0.9394	0.3543	-86.4907	-54.4442	-94.8133	-54.4442
NC23_7	0	-	-	24	4	1.0328	-	-	-	-46.1839	-14.7437
NC23_6	0	-	-	12	3	0.8541	-	-	-	-39.4816	-29.1236
NC24_4	0	-	-	34	5	2.0809	-	-	-	-71.6710	-62.7970
NC24_1	1	1.0190	1.0190	16	4	1.4601	0.8236	-128.9180	-128.9180	-131.3660	-98.6356
NC24_2	0	-	-	10	3	1.0317	-	-	-	-59.2505	-50.3392
NC24_3	0	-	-	18	4	1.4827	-	-	-	-81.1047	-62.3756
NC25_4	1	1.4370	1.4370	14	6	3.9086	2.9218	-116.9060	-116.9060	-167.9990	-116.9060
NC25_2	2	1.0487	1.1744	30	8	7.1719	0.9324	-183.7380	-164.0840	-192.6940	-72.1818
NC25_3	2	1.4358	1.9921	32	8	25.6082	3.2763	-121.5220	-87.5895	-174.4870	-87.5895
NC25_1	0	-	-	38	6	19.4117	-	-	-	-163.5600	-62.2675
NC32_3	2	1.0000	1.4850	24	5	1.2589	0.4827	-101.4570	-68.3218	-101.4570	2.6985
NC32_2	2	1.0652	1.4657	15	4	0.5213	0.2782	-43.2125	-31.4043	-46.0281	-31.4043
NC32_1	0	-	-	36	4	0.7526	-	-	-	-66.5208	-21.4082
NC32_4	4	1.0145	1.8493	45	9	1.5802	0.3484	-77.9484	-42.7617	-79.0771	-20.4243
NC33_1	2	1.0042	1.0451	33	6	6.7043	1.3057	-184.7260	-177.4950	-185.4930	-99.5192
NC33_3	0	-	-	42	7	4.3455	-	-	-	-104.1130	-21.0555
NC33_2	1	1.3586	1.3586	54	6	6.8289	3.3613	-90.6533	-90.6533	-123.1600	-71.4323
NC33_4	1	1.9431	1.9431	57	8	6.5654	2.6089	-120.5760	-120.5760	-234.2880	-41.7204

Table 5: Full results for the *qIPG* from the instances of Schwarze and Stein (2022).

Instance	#EQs	#EI	#It	Time	Time-1st	*PNE* *	*PNE* °	*OSW*
2-1-A-H	3	110	24	0.889	0.057	-74.0	0.0	-2128000.0
2-1-B-H	2	102	16	0.581	0.158	-8.5	0.0	-8500000.0
2-1-A-L	5	188	30	2.171	0.175	-74.0	0.0	-4376000.0
2-1-B-L	2	136	23	0.743	0.100	-27.5	-0.5	-6477570.0
2-2-A-H	7	50	12	1.769	0.148	-425.5	0.0	-1438.0
2-2-B-H	8	166	23	11.707	4.969	-924.5	-0.5	-2712.0
2-2-A-L	1	16	3	0.327	0.247	0.0	0.0	-124.0
2-2-B-L	7	112	19	6.500	0.205	-7289.5	0.0	-8560.0
2-3-A-H	4	20	6	0.458	0.140	-283.0	0.0	-1118.0
2-3-B-H	3	54	8	0.989	0.475	-25.5	0.0	-3138.0
2-3-A-L	1	8	3	0.179	0.137	0.0	0.0	-270.0
2-3-B-L	1	14	3	0.250	0.188	0.0	0.0	-750.5
3-1-A-H	6	228	25	5.137	0.964	-4776.0	0.0	-76091.5
3-1-B-H	8	246	31	4.465	0.647	-957.0	0.0	-234695.0
3-1-A-L	3	159	18	3.220	1.273	-618.5	0.0	-93872.0
3-1-B-L	1	105	12	1.204	0.392	0.0	0.0	-71595.0
3-2-A-H	1	33	5	0.760	0.395	0.0	0.0	-1962.5
3-2-B-H	1	15	3	0.078	0.069	0.0	0.0	-1080.0
3-2-A-L	8	84	11	3.390	0.629	-1558.0	0.0	-3032.5
3-2-B-L	4	51	8	1.269	0.447	-125.0	0.0	-2044.0
4-1-A-H	4	76	7	2.140	1.077	-249.0	0.0	-552.5
4-1-B-H	13	152	16	4.654	1.689	-3603.0	0.0	-6115.5
4-1-A-L	13	116	10	5.927	0.869	-1462.0	0.0	-1804.0
4-1-B-L	11	132	12	2.863	0.238	-1677.5	0.0	-5817.5
6-1-A-H	3	66	5	0.595	0.425	-36.5	0.0	-1437.5
6-1-B-H	2	54	6	0.457	0.302	-17.0	0.0	-1715.0
6-1-A-L	3	60	5	0.711	0.192	-440.0	0.0	-2795.0
6-1-B-L	6	138	8	2.059	0.602	-363.5	0.0	-10510.0

Table 6 Full results for the qIPG from the instances of Sagratella (2016).

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