Appendix A: Considered benchmark problems

A.1. CFLP

In this problem, there is a set of l locations where a facility can be opened, and a set of r customers that each have to be assigned to a location. Two decisions have to be made: which locations to open and which customers to assign to which facilities. Both opening a facility and assigning a customer to an open facility induces a cost. Moreover, the demand being handled in a facility j cannot exceed a threshold t_j. Finally, the company may chose to ignore some of the customers to reduce their costs.

The first objective is to minimize the cost of assigning customers to facilities. The second objective consists in minimizing the opening cost of the facilities. The third objective aims to maximize the overall demand of the customers that is satisfied.

Let $y_j = 1$ if a facility is opened at location j, and $y_j = 0$ otherwise, $\forall j \in \{1, ..., l\}$. Furthermore, let $x_{ij} = 1$ if customer i is assigned to location j, and $x_{ij} = 0$ otherwise, $\forall i \in \{1, ..., r\}, \forall j \in \{1, ..., l\}$. Finally, $z_i = 1$ if customer i is served, 0 otherwise, $\forall i \in \{1, ..., r\}$.

$$
\begin{aligned}
\min \ & \sum_{i=1}^{r} \sum_{j=1}^{l} c_{ij} x_{ij} & \\
\min \ & \sum_{j=1}^{l} f_j y_j & \\
\max \ & \sum_{i=1}^{r} d_i z_i & \\
\text{s.t.} \ & \sum_{j=1}^{l} x_{ij} = z_i & \forall i \in \{1, \dots, r\} \\
& x_{ij} \leq y_j & \forall i \in \{1, \dots, r\}, \ j \in \{1, \dots, l\} \\
& \sum_{i=1}^{r} d_i x_{ij} \leq t_j y_j & \forall j \in \{1, \dots, l\} \\
& x_{ij} \in \{0, 1\} & \forall i \in \{1, \dots, r\}, \ j \in \{1, \dots, l\} \\
& y_j \in \{0, 1\} & \forall j \in \{1, \dots, l\} \\
& z_i \in \{0, 1\} & \forall i \in \{1, \dots, r\}
\end{aligned}
$$

Instances are taken from An et al. (2022).

A.2. KP

In the Knapsack problem, a subset of items has to be selected from a set of n items. Each item i has a weight w_i, and there is a limit b on the total weight of the items being selected. Moreover, each item i has a utility c_i^k in objective k, and the goal is to maximize the utility of the subset of items selected over all objective functions.

Let $x_i = 1$ if item i is selected, 0 otherwise. The multi-objective Knapsack Problem (KP) with p objectives can be formulated as follows:

$$
\begin{aligned}
\min \ & \sum_{i=1}^{n} c_i^k x_i && \forall k \in \{1,...,p\} \\
\text{s.t.} \ & \sum_{i=1}^{n} w_i x_i \le b \\
& x_i \in \{0,1\} && \forall i \in \{1,\dots,n\}
\end{aligned}
$$

Instances are taken from Kirlik and Sayın (2014).

A.3. UFLP

In this problem, there is a set of l locations where a facility can be opened, and a set of r customers that each have to be assigned to a location. Two decisions have to be made: which locations to open and which customers to assign to which facilities. Both opening a facility and assigning a customer to an open facility induces a cost, and the overall cost has to be minimized. Let $y_j = 1$ if a facility is opened at location j, and $y_j = 0$ otherwise, $\forall j \in \{1,...,l\}$. Furthermore, let $x_{ij} = 1$ if customer i is assigned to location j, and $x_{ij} = 0$ otherwise, $\forall i \in \{1,...,r\}, \forall j \in \{1,...,l\}$.

The multi-objective Uncapacitated Facility Location Problem (UFLP) with p objectives can be formulated as the following MOCO problem

$$
\begin{aligned}
\min \ & \sum_{i=1}^{r}\sum_{j=1}^{l} c_{ij}^k x_{ij} + \sum_{j=1}^{l} f_j^k y_j && \forall k \in \{1,...,p\} \\
\text{s.t.} \ & \sum_{j=1}^{l} x_{ij} = 1 && \forall i \in \{1,\dots,r\} \\
& x_{ij} \le y_j && \forall i \in \{1,\dots,r\},\ j \in \{1,\dots,l\} \\
& x_{ij} \in \{0,1\} && \forall i \in \{1,\dots,r\},\ j \in \{1,\dots,l\} \\
& y_j \in \{0,1\} && \forall j \in \{1,\dots,l\}
\end{aligned}
$$

Instances are taken from Forget et al. (2022b).

Appendix B: Additional computational results

			CB		FOB		NOB	
p	pb	n	NVF	VF	NVF	VF	NVF	VF
3	CFLP	65	7906.6 (0)	2382.6 (0)	11786.2 (0)	1389.4 (0)	15090.4 (0)	15072.4 (0)
		230	198405.7 (6)	65681.8 (4)	405597.4 (7)	49365.9 (1)	78613.1 (10)	32593.7 (10)
		495	6380.6 (10)	4357.9 (10)	33545.9 (10)	4497.9 (10)	4245.2 (10)	2934.4 (10)
	KP	40	104268.4 (0)	15918.4 (0)	136980.2 (0)	13023.6 (0)	137544.2 (0)	71502.4 (0)
		50	283955 (0)	34498.6 (0)	385177 (0)	27261.8 (0)	385625.2 (0)	174757.2 (0)
		60	804121.5 (1)	121213.2 (0)	1128878.7 (2)	95023.4 (0)	623624.4 (9)	428664.6 (7)
		70	1310720.2 (7)	289369.3 (3)	1839920.2 (7)	226918 (2)	617265.9 (10)	456791.8 (10)
		80	1368612.3 (10)	377936.3 (5)	1812818.1 (10)	303356 (5)	499527.9 (10)	367138.9 (10)
	UFLP	56	49212.2 (0)	20885.2 (0)	216010.6 (0)	14512.6 (0)	63239 (0)	62944.2 (0)
		72	111566.6 (0)	47018.2 (0)	531514 (0)	33598.6 (0)	151994.2 (0)	151525.4 (0)
		90	229470.8 (0)	97638.6 (0)	1103596 (5)	72544.4 (0)	275612.6 (6)	235776.7 (9)
		110	195819.1 (10)	157547.9 (5)	681660.1 (10)	73505.6 (10)	71013.9 (10)	72963.3 (10)
4	KP	20	7651.6 (0)	2103.6 (0)	8442.4 (0)	1899.4 (0)	8998 (0)	5043.2 (0)
		30	43529.8 (0)	8469.2 (0)	49641 (0)	7589.4 (0)	53294.2 (0)	26693.4 (0)
		40	330758.7 (2)	82242.8 (2)	430534.1 (2)	72322.6 (1)	340657.3 (7)	221146.5 (3)
	UFLP	42	62617.8 (0)	29925.4 (0)	199163 (0)	22728.8 (0)	68111.2 (0)	68070.2 (0)
		56	71905 (10)	49605.7 (10)	172122.8 (10)	32013.5 (10)	183316.4 (10)	100648.1 (10)

Table 7 The average number of nodes explored over 10 instances for each problem class, number of objectives, number of variables, and configuration. The number in brackets is the number of instances unsolved. Note that when the number of unsolved instances is high, the number of nodes explored may be low due to the fact that the algorithm could not explore a large number of nodes within the time limit of one hour.

			% LB set						% Probing						% Other					
			CB		FOB		NOB		CB		FOB		NOB		CB		FOB		NOB	
p	pb	n	NVF	VF	NVF	VF	NVF	VF	NVF	VF	NVF	VF	NVF	VF	NVF	VF	NVF	VF	NVF	VF
3	CFLP	65	90.0	55.1	86.8	43.2	89.3	43.6	0.0	41.0	0.0	53.4	0.0	51.4	10.0	3.9	13.2	3.4	10.7	5.0
		230	88.5	48.4	80.8	32.4	95.7	47.1	0.0	48.3	0.0	64.8	0.0	51.1	11.5	3.3	19.2	2.8	4.3	1.8
		495	98.3	73.3	93.3	53.2	98.7	70.0	0.0	25.5	0.0	45.8	0.0	29.0	1.7	1.2	6.7	1.1	1.3	0.9
	KP	40	90.6	69.9	88.3	62.5	95.0	68.5	0.0	25.6	0.0	32.8	0.0	28.0	9.4	4.5	11.7	4.7	5.0	3.5
		50	89.1	68.3	86.5	59.8	95.0	68.5	0.0	26.8	0.0	35.2	0.0	28.0	10.9	4.9	13.5	5.1	5.0	3.5
		60	87.5	66.3	84.0	56.5	94.8	66.2	0.0	27.8	0.0	37.7	0.0	29.8	12.5	5.9	16.0	5.9	5.2	3.9
		70	85.8	63.2	81.4	50.6	94.3	59.2	0.0	29.8	0.0	42.6	0.0	36.4	14.2	7.0	18.6	6.8	5.7	4.4
		80	83.7	62.0	79.5	49.2	94.3	62.0	0.0	30.4	0.0	42.6	0.0	33.7	16.3	7.6	20.5	8.2	5.7	4.2
	UFLP	56	78.1	62.4	69.7	35.0	83.5	59.8	0.0	21.6	0.0	54.1	0.0	26.6	21.9	16.1	30.3	11.0	16.5	13.6
		72	72.7	57.4	67.2	29.1	87.2	61.7	0.0	22.6	0.0	57.9	0.0	24.2	27.3	19.9	32.8	13.1	12.8	14.1
		90	75.6	57.7	58.2	24.9	84.7	60.8	0.0	21.8	0.0	61.1	0.0	23.1	24.4	20.5	41.8	14.0	15.3	16.1
		110	63.8	51.1	47.1	18.5	84.7	66.9	0.0	21.9	0.0	66.6	0.0	17.9	36.2	27.0	52.9	14.9	15.3	15.2
4	KP	20	93.8	85.9	93.0	84.3	95.8	82.1	0.0	9.2	0.0	10.8	0.0	14.7	6.2	4.9	7.0	4.9	4.2	3.2
		30	91.6	82.3	90.3	79.8	95.7	81.8	0.0	10.8	0.0	13.0	0.0	14.7	8.4	6.9	9.7	7.2	4.3	3.5
		40	85.7	78.2	83.3	73.7	94.5	82.8	0.0	8.9	0.0	11.7	0.0	11.9	14.3	13.0	16.7	14.6	5.5	5.3
	UFLP	42	53.6	41.1	51.5	35.4	70.5	43.0	0.0	15.0	0.0	26.0	0.0	32.4	46.4	43.9	48.5	38.6	29.5	24.6
		56	36.2	29.3	30.1	20.1	50.6	33.2	0.0	12.6	0.0	30.2	0.0	35.2	63.8	58.1	69.9	49.7	49.4	31.5

Table 8 Comparison of different objective branching settings (cone branching (CB), full objective branching (FOB), no objective branching (NOB)) in combination with (VF) and without (NVF) probing concerning the average percentage of CPU time spent in different parts of the algorithm over 10 instances for each problem class, number of objectives, and number of variables. *% LB set* **represents the share of CPU time spent in the computation of lower bound sets.** *% Probing* **represents the proportion of CPU time dedicated to probing.** *% Other* **is the percentage of CPU time spent in other parts of the algorithms such as dominance test, creation of sub-problems, node selection, etc.**

p	pb	n	CPU		# Nodes		% CPU
			BBGAP	BBWSN	BBGAP	BBWSN	BBGAP
3	CFLP	65	19.6	10.0	2064.6 (0)	1278.8 (0)	1.8
		230	3718.7	1988.0	34494.4 (10)	40765.4 (1)	3.9
		495	3602.3	3601.7	1918.1 (10)	4496.5 (10)	5.1
	KP	40	124.8	46.7	5826.8 (0)	7405.8 (0)	9.8
		50	286.5	116.5	12459.3 (0)	14972 (0)	12.9
		60	1034.9	457.8	33760 (0)	43623 (0)	22.4
		70	2348.9	1414.1	57832.4 (4)	105508.2 (0)	22.2
		80	3260.9	2491.5	66489.8 (5)	154884.5 (3)	21.1
	UFLP	56	235.5	177.3	7524.1 (0)	12282.8 (0)	22.6
		72	791.1	564.4	15755.7 (0)	26574.4 (0)	25.8
		90	2539.7	1590.5	30022.4 (0)	52141.6 (0)	26.8
		110	3601.1	3381.7	17823.2 (10)	80985.7 (5)	34.2
4	KP	20	35.9	7.0	923.5 (0)	1325.6 (0)	8.8
		30	150.5	42.0	3640.7 (0)	5105.6 (0)	25.8
		40	2176.1	854.8	14339.3 (4)	41642 (0)	51.0
	UFLP	42	3530.1	558.9	7984.4 (7)	22393.8 (0)	48.3
		56	3600.0	3600.0	350.25 (10)	39079.1 (10)	50.8

Table 9 Comparison of node selection rules BBWSN and BBGAP. Columns *CPU* **give the average CPU time expressed in seconds over 10 instances for each problem class, number of objectives, and number of variables. Columns** *# Nodes* **provide the average number of nodes explored as well as the number of unsolved instances (indicated in brackets). Finally, Column** *% CPU* **represents the percentage of the total CPU time spent in updating gaps in configuration BBGAP**