Electronic Companion

Appendix A: Example: optimal timed route

Assume that the node sequence S is $(0,u,v,n+1)$. S_1 and S_2 are the timed routes of S with the minimal carbon emissions, i.e., $c_{S_1}=c_{S_2}$. The departure time of node u in S_1 is earlier than that in S_2 (i.e., $t_u^1<t_u^2$), and the departure time of node v in S_1 is later than that in S_2 (i.e., $t_v^1>t_v^2$). For simplicity, let S_1 be $((0,t_0),(u,t_u^1=1),(v,t_v^1=5),(n+1))$ and S_2 be $((0,t_0),(u,t_u^2=2),(v,t_v^2=4),(n+1))$. Figure EC.1 shows the details of the two timed routes S_1 and S_2.

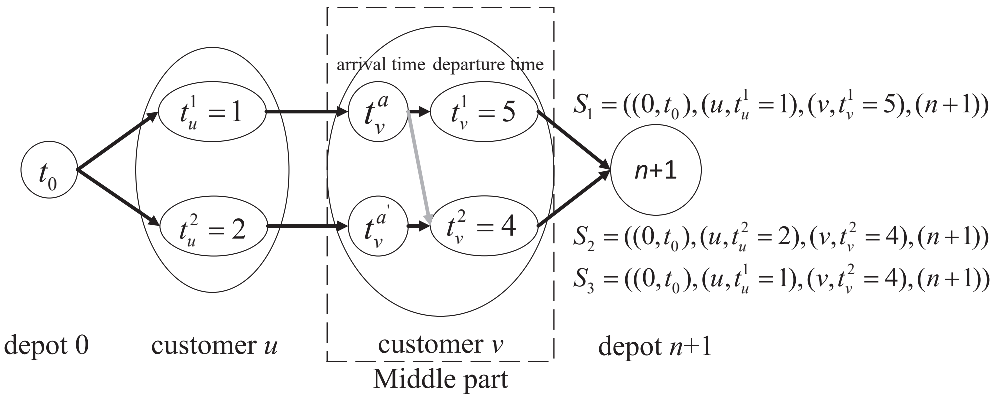

Figure EC.1 Timed routes S_1 and S_2 associated with node sequence $(0,u,v,n+1)$

In Figure EC.1, there are two TD arcs of arc (u,v): TD arc 1 $((u,t_u^1=1),(v,t_v^a))$ and TD arc 2 $((u,t_u^2=2),(v,t_v^{a'}))$.

In the TDGVRPTW, the first-in-first-out (FIFO) property is used. Thus, leaving a node earlier guarantees that the vehicle will arrive earlier at destination (Ichoua et al. 2003, Desaulniers et al. 2014). And the arrival time t_v^a of v in TD arc 1 with departure time $t_u^1=1$ must be not later than the arrival time $t_v^{a'}$ with $t_u^2=2$. That is to say $t_v^a\le t_v^{a'}$.

For S_2, the departure time of v in S_2 is $t_v^2=4$. For S_1, since $t_v^a\le t_v^{a'}$, the departure time of v in S_1 can also be $t_v^2=4$, as shown in the gray line in the middle part of Figure EC.1. Therefore, another timed route $S_3=((0,t_0),(u,t_u^1),(v,t_v^2),(n+1))$ must exist. S_3 has the same carbon emissions as that of S_1 and S_2. For S_3, each node has the earliest departure time of the node in S_1, S_2 and S_3. Thus, S_3 is the optimal timed route because S_3 has the earliest time to return to the depot of all timed routes with the minimal carbon emissions.

The optimal timed route S_3 can be obtained using lexicographical order as follows.

Step 1: Obtain the timed routes with the earliest departure time of u, i.e., S_1 and S_3. S_2 can be eliminated.

Step 2: Obtain the timed routes with the earliest departure time of v, i.e., S_3. S_1 can be eliminated.

In all the timed routes with the same carbon emissions, the timed route with the earliest departure time of each node has the earliest time to return to the depot.

Appendix B: Proof of Theorem 1

Proof. Let $S_{r'} = ((0,t_0),(i_1,t_{i_1}),\ldots,(i_j,t'_{i_j}),(i_k,t_{i_k}),\ldots,(i_z,t_{i_z}),(n+1))$ be the optimal timed route of the node sequence $S=(0,i_1,\ldots,i_j,i_k,\ldots,i_z,n+1)$. Assume that at least one TD arc in $S_{r'}$ is the dominated TD arc.

According to this assumption, $((i_j,t'_{i_j}),(i_k,t^{a'}_{i_k}))\in A_{i_j i_k}$ is the dominated TD arc of $S_{r'}$. Let$((i_j,t_{i_j}),(i_k,t^a_{i_k}))\in A_{i_j i_k}$ be the non-dominated TD arc; therefore, $t_{i_j}\le t'_{i_j}$ and $c_{i_j i_k}(t_{i_j})\le c_{i_j i_k}(t'_{i_j})$, where at least one inequality is strict.

Let $S_r = ((0,t_0),(i_1,t_{i_1}),\ldots,(i_j,t_{i_j}),(i_k,t_{i_k}),\ldots,(i_z,t_{i_z}),(n+1))$ be the timed route with node sequence S. The carbon emissions c_{S_r} of S_r are equal to $c_{0i_1}(t_0)+\ldots+c_{i_j i_k}(t_{i_j})+\ldots+c_{i_z n+1}(t_{i_z})$. In addition, the carbon emissions $c_{S_{r'}}$ of $S_{r'}$ are equal to $c_{0i_1}(t_0)+\ldots+c_{i_j i_k}(t'_{i_j})+\ldots+c_{i_z n+1}(t_{i_z})$. There are two discussion cases:

Case 1: $c_{i_j i_k}(t_{i_j})<c_{i_j i_k}(t'_{i_j})$. $c_{i_j i_k}(t_{i_j})<c_{i_j i_k}(t'_{i_j})$, $c_{S_r}<c_{S_{r'}}$. Thus, $S_{r'}$ is a non-optimal timed route. However, this contradicts the assumption that $S_{r'}$ is the optimal timed route.

Case 2: $c_{i_j i_k}(t_{i_j})=c_{i_j i_k}(t'_{i_j})$. $c_{i_j i_k}(t_{i_j})=c_{i_j i_k}(t'_{i_j})$, so $c_{S_r}=c_{S_{r'}}$. However, $t_{i_j}<t'_{i_j}$, $((i_j,t_{i_j}),(i_k,t^a_{i_k}))$ contains all possible extension. Thus, $((i_j,t_{i_j}),(i_k,t^a_{i_k}))$ is retained and $((i_j,t'_{i_j}),(i_k,t^{a'}_{i_k}))$ is eliminated.

These two cases are exhaustive. The assumption that the optimal timed route is composed of the dominated TD arcs in Case 1 leads to a contradiction, and the dominated TD arc can be replaced by the non-dominated TD arc in Case 2. Thus, the optimal timed route is composed of non-dominated TD arcs.

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Appendix C: Slope of the function

Proof. Let $t_i\in[W'_k,W_k]$ be the departure time of node $i\in V$ where W'_k and W_k are adjacent breakpoints, $k=1,2,\ldots,m$. Evidently, for the situation in which the vehicle speed on arc $(i,j)\in A$ changes from v_k to v_{k+1} (e.g., $t_i\in[W'_1,W_1]$ in Figure 2), the vehicle must travel at speed v_k until it reaches the breakpoint where vehicle speed changes from v_k to v_{k+1} and the traveling distance is d_k. From there, the vehicle must travel at speed v_{k+1} for the remainder of its trip to reach node j, and traveling distance is d_{k+1}. Thus, $d_k+d_{k+1}=d_{ij}$ where d_k is the traveling distance at which the vehicle speed is v_k.

Let $t'_i\in[W'_k,W_k]$ and $\triangle t=t'_i-t_i$. Therefore, the slope of the function of $[W'_k,W_k]$ is equal to $\frac{c_{ij}(t'_i)-c_{ij}(t_i)}{\triangle t}=\frac{\gamma f\beta\{[v_k^2(d_k-\triangle t v_k)+v_{k+1}^2(d_{k+1}+\triangle t v_k]-(v_k^2 d_k+v_{k+1}^2 d_{k+1})\}}{\triangle t}=\gamma f\beta(v_k v_{k+1}^2-v_k^3)$.

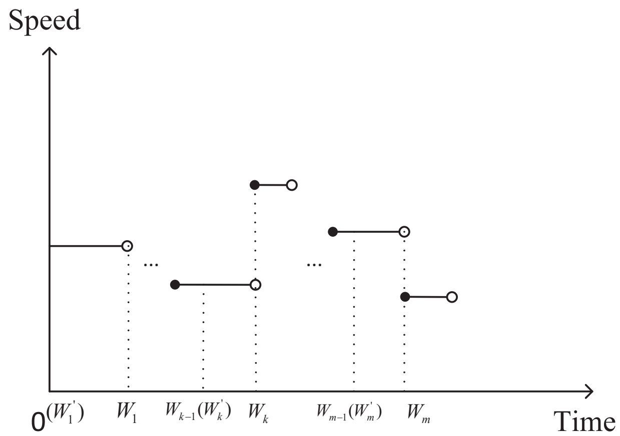

Figure EC.2 General case of vehicle speed changing over time

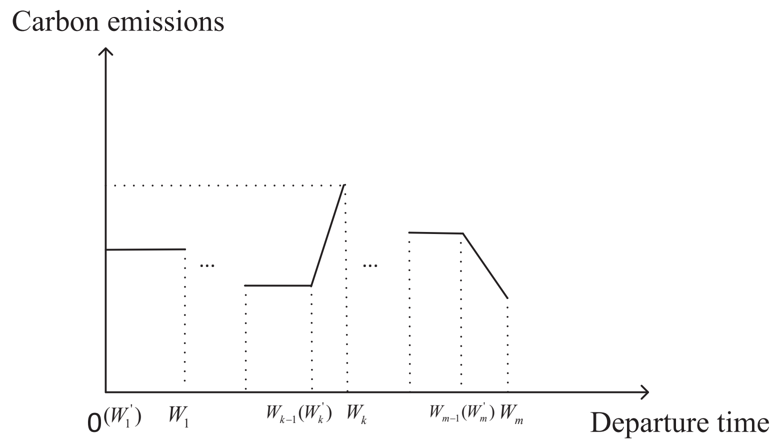

Figure EC.3 General case of carbon emissions function of arc (i,j)

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Appendix D: Proof of Theorem 2

Proof. Because the carbon emissions functions of $((i,t_i),(j,t_j^a))$ are independent of or increasing with the departure time, there are two cases for discussion:

Case 1: The carbon emissions functions of $((i,t_i),(j,t_j^a))$ are independent of the departure time. According to the definition of non-dominated TD arc (see Definition 5), the TD arc with the earliest departure time $t_i = ed_{ij}$ is the only non-dominated TD arc.

Case 2: The carbon emissions functions of $((i,t_i),(j,t_j^a))$ increase with the departure time. The TD arc with the earliest departure time has the lowest carbon emissions. According to the definition of non-dominated TD arc (see Definition 5), the TD arc with the earliest departure time ed_{ij} is the only non-dominated TD arc.

Case 1 and Case 2 contain all situations. Thus, for an arbitrary $(i,j) \in A$, the TD arc with the earliest departure time is the only non-dominated TD arc if the carbon emissions functions of $((i,t_i),(j,t_j^a))$ are independent of or increasing with the departure time (e.g.,$[ed_{ij},ld_{ij}] \cap [W_1^{'},W_1] = \emptyset$).

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Appendix E: Proof of Theorem 3

Proof. Because the carbon emissions functions of $((i,t_i),(j,t_j^a))$ decrease with the departure time, each TD arc in this type has an earlier departure time, but more carbon emissions. According to the definition of non-dominated TD arc (see Definition 5), each TD arc whose carbon emissions functions decrease with departure time (e.g., $t_i \in [ed_{ij},ld_{ij}] \cap [W_1^{'},W_1] \neq \emptyset$) is a non-dominated TD arc.

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References

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Ichoua S, Gendreau M, Potvin JY (2003) Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research* 144(2):379–396.