

Results supplement for: Solving natural conic formulations with Hypatia.jl

Chris Coey¹, Lea Kapelevich¹, and Juan Pablo Vielma²

¹Operations Research Center, MIT, Cambridge, MA

²Google Research and MIT Sloan School of Management, Cambridge, MA

This file contains the results generated for “Solving natural conic formulations with Hypatia.jl”.

Table 1: Portfolio rebalancing solver statistics.

k	Hypatia-NF			Hypatia-EF			MOSEK-EF		
	st	it	time	st	it	time	st	it	time
1000	<u>co</u>	31	0.6	<u>co</u>	25	2.7	<u>co</u>	9	1.7
2000	<u>co</u>	36	2.9	<u>co</u>	28	16	<u>co</u>	10	7.0
4000	<u>co</u>	45	20	<u>co</u>	29	92	<u>co</u>	10	34
6000	<u>co</u>	49	60	<u>co</u>	34	292	<u>co</u>	10	83
8000	<u>co</u>	51	131	<u>co</u>	33	615	<u>co</u>	10	160
10000	<u>co</u>	55	244	<u>co</u>	36	1192	<u>co</u>	12	305
12000	<u>co</u>	62	421	<u>tl</u>	32	1805	<u>co</u>	10	433
14000	<u>co</u>	61	624	sk	*	*	rl	*	*
16000	<u>co</u>	63	924	sk	*	*	sk	*	*
18000	<u>co</u>	64	1327	sk	*	*	sk	*	*
20000	<u>co</u>	66	1810	sk	*	*	sk	*	*

Table 2: Portfolio rebalancing EF LP solver time comparisons.

k	ECOS	MOSEK		Gurobi			
	conic	conic	intpnt	simplex	barrier	P simplex	D simplex
1000	192	1.7	2.3	39	2.8	68	57
4000	*	34	66	*	55	*	*
10000	*	305	783	*	548	*	*
20000	*	*	*	*	*	*	*

Table 3: Matrix completion formulation statistics. Note $p = \bar{p} = |S|$.

m	k	NF				EF-exp			EF-sec		
		ν	n	p	q	$\bar{\nu}$	\bar{n}	\bar{q}	$\bar{\nu}$	\bar{n}	\bar{q}
10	5	57	251	200	302	207	302	1692	182	314	1730
	10	218	1001	794	1208	730	1208	6725	621	1256	6871
	15	472	2251	1795	2707	1532	2707	15062	1188	2762	15229
	20	846	4001	3176	4826	2694	4826	26784	2267	5024	27380
	25	1299	6251	4978	7524	4093	7524	41768	4370	8298	44092
	30	1858	9001	7174	10828	5810	10828	60095	4425	11048	60757
	35	2477	12251	9810	14692	7707	14692	81627	8576	16346	86591
	40	3256	16001	12786	19216	10084	19216	106664	8631	20096	109306
	45	4142	20251	16155	24347	12782	24347	135047	8686	24346	135046
20	5	114	501	393	609	428	609	5888	360	628	5947
	10	418	2001	1594	2408	1430	2408	23375	1233	2512	23689
	15	933	4501	3584	5418	3065	5418	52520	2362	5524	52840
	20	1663	8001	6359	9643	5345	9643	93335	4515	10048	94552
	25	2513	12501	10014	14988	7985	14988	145535	8716	16596	150361
	30	3643	18001	14389	21613	11465	21613	209600	8821	22096	211051

Table 4: Matrix completion solver statistics.

m	k	NF			EF-exp						EF-sec					
		Hypatia			Hypatia			MOSEK			Hypatia			MOSEK		
		st	it	time	st	it	time	st	it	time	st	it	time	st	it	time
10	5	<u>co</u>	14	0.0	<u>co</u>	19	0.1	<u>co</u>	15	0.9	<u>co</u>	18	0.2	<u>co</u>	11	0.7
	10	<u>co</u>	19	0.4	<u>co</u>	34	2.2	<u>co</u>	20	20	<u>co</u>	30	2.3	<u>co</u>	10	11
	15	<u>co</u>	23	2.6	<u>co</u>	42	16	<u>co</u>	21	120	<u>co</u>	41	18	<u>co</u>	9	58
	20	<u>co</u>	26	14	<u>co</u>	52	70	<u>co</u>	24	524	<u>co</u>	47	78	<u>co</u>	11	251
	25	<u>co</u>	30	52	<u>co</u>	59	225	<u>co</u>	26	1758	<u>co</u>	69	387	<u>co</u>	11	770
	30	<u>co</u>	34	166	<u>co</u>	61	556	tl	11	1817	<u>co</u>	55	587	<u>co</u>	10	1712
	35	<u>co</u>	39	402	<u>co</u>	61	1228	sk	*	*	tl	61	1817	rl	*	*
	40	<u>co</u>	48	949	tl	34	1820	sk	*	*	sk	*	*	sk	*	*
	45	<u>co</u>	47	1806	sk	*	*	sk	*	*	sk	*	*	sk	*	*
20	5	<u>co</u>	15	0.1	<u>co</u>	29	0.8	<u>co</u>	17	14	<u>co</u>	26	0.9	<u>co</u>	8	7.6
	10	<u>co</u>	22	2.2	<u>co</u>	48	25	<u>co</u>	25	448	<u>co</u>	45	27	<u>co</u>	10	203
	15	<u>co</u>	30	24	<u>co</u>	59	179	tl	14	1871	<u>co</u>	52	176	<u>co</u>	10	1375
	20	<u>co</u>	33	119	<u>co</u>	71	786	sk	*	*	<u>co</u>	70	924	rl	*	*
	25	<u>co</u>	41	448	tl	47	1822	sk	*	*	tl	26	1804	sk	*	*
	30	<u>co</u>	52	1305	sk	*	*	sk	*	*	sk	*	*	sk	*	*

Table 5: Multi-response regression formulation and solver statistics.

m	k	form. stats.		Hypatia-NF			Hypatia-EF			MOSEK-EF		
		\bar{n}	q	st	it	time	st	it	time	st	it	time
15	50	1622	977	<u>co</u>	11	0.1	<u>co</u>	12	1.1	<u>co</u>	4	0.6
	100	5397	1727	<u>co</u>	10	0.5	<u>co</u>	12	17	<u>co</u>	5	6.7
	150	11672	2477	<u>co</u>	10	1.2	<u>co</u>	13	98	<u>co</u>	5	36
	250	31722	3977	<u>co</u>	10	3.3	<u>co</u>	14	1331	<u>co</u>	5	308
	500	125597	7727	<u>co</u>	10	17	m	*	*	tl	*	*
	1000	*	15227	<u>co</u>	13	129	sk	*	*	sk	*	*
	1500	*	22727	<u>co</u>	10	209	sk	*	*	sk	*	*
	2000	*	30227	<u>co</u>	9	395	sk	*	*	sk	*	*
	2500	*	37727	<u>co</u>	11	949	sk	*	*	sk	*	*
	3000	*	45227	<u>co</u>	11	1375	sk	*	*	sk	*	*
30	50	2642	2402	<u>co</u>	13	1.4	<u>co</u>	11	3.7	<u>co</u>	5	1.6
	100	6417	3902	<u>co</u>	14	4.9	<u>co</u>	11	23	<u>co</u>	5	12
	150	12692	5402	<u>co</u>	12	7.0	<u>co</u>	12	123	<u>co</u>	5	47
	250	32742	8402	<u>co</u>	15	41	<u>co</u>	13	1412	<u>co</u>	5	409
	500	126617	15902	<u>co</u>	11	107	m	*	*	tl	*	*
	750	*	23402	<u>co</u>	11	232	sk	*	*	sk	*	*
	1000	*	30902	<u>co</u>	13	768	sk	*	*	sk	*	*
	1250	*	38402	<u>co</u>	12	1098	sk	*	*	sk	*	*
	1500	*	45902	<u>co</u>	12	1637	sk	*	*	sk	*	*

Table 6: D-optimal experiment design logdet variant formulation and solver statistics.

k	form. stats.			Hypatia-NF			Hypatia-EF			MOSEK-EF		
	\bar{n}	q	\bar{q}	st	it	time	st	it	time	st	it	time
50	1426	1378	5401	<u>co</u>	25	0.3	<u>co</u>	21	4.5	<u>co</u>	15	12
100	5351	5253	20801	<u>co</u>	26	0.9	<u>co</u>	25	91	<u>co</u>	15	277
150	11776	11628	46201	<u>co</u>	29	3.0	<u>co</u>	27	690	<u>tl</u>	14	1825
200	20701	20503	81601	<u>co</u>	28	7.2	tl	17	1849	sk	*	*
300	46051	45753	182401	<u>co</u>	36	36	sk	*	*	sk	*	*
400	81401	81003	323201	<u>co</u>	36	81	m	*	*	sk	*	*
500	126751	126253	504001	<u>co</u>	36	169	sk	*	*	sk	*	*
600	182101	181503	724801	<u>co</u>	36	298	sk	*	*	sk	*	*
700	*	246753	*	<u>co</u>	39	624	sk	*	*	m	*	*
800	*	322003	*	<u>co</u>	37	838	sk	*	*	sk	*	*
900	*	407253	*	<u>co</u>	37	1282	sk	*	*	sk	*	*
1000	*	502503	*	<u>tl</u>	37	1838	sk	*	*	sk	*	*

Table 7: D-optimal experiment design rt-det variant solver statistics.

k	NF			EF-exp						EF-sec					
	Hypatia			Hypatia			MOSEK			Hypatia			MOSEK		
	st	it	time	st	it	time	st	it	time	st	it	time	st	it	time
50	<u>co</u>	25	0.3	<u>co</u>	22	4.7	<u>co</u>	14	11	<u>co</u>	22	5.1	<u>co</u>	11	10
100	<u>co</u>	25	0.9	<u>co</u>	25	93	<u>co</u>	13	247	<u>co</u>	26	97	<u>co</u>	11	220
150	<u>co</u>	26	2.6	<u>co</u>	27	696	<u>co</u>	12	1580	<u>sp</u>	36	921	<u>co</u>	10	1432
200	<u>co</u>	23	5.8	tl	17	1821	tl	0	1821	tl	17	1848	tl	0	1868
300	<u>co</u>	31	31	sk	*	*	sk	*	*	sk	*	*	sk	*	*
400	<u>co</u>	29	67	m	*	*	sk	*	*	m	*	*	sk	*	*
500	<u>co</u>	32	152	sk	*	*	sk	*	*	sk	*	*	sk	*	*
600	<u>co</u>	33	281	sk	*	*	sk	*	*	sk	*	*	sk	*	*
700	<u>co</u>	32	530	sk	*	*	m	*	*	sk	*	*	sk	*	*
800	<u>co</u>	32	728	sk	*	*	sk	*	*	sk	*	*	sk	*	*
900	<u>co</u>	36	1253	sk	*	*	sk	*	*	sk	*	*	sk	*	*
1000	<u>co</u>	33	1729	sk	*	*	sk	*	*	sk	*	*	sk	*	*

Table 8: Polynomial minimization formulation and solver statistics.

m	k	form. stats.			Hypatia-NF			Hypatia-EF			MOSEK-EF		
		ν	n	\bar{q}	st	it	time	st	it	time	st	it	time
1	100	201	201	10201	<u>co</u>	12	0.1	<u>co</u>	34	1.2	<u>co</u>	15	27
1	200	401	401	40401	<u>co</u>	14	0.3	<u>co</u>	39	13	<u>co</u>	11	409
1	500	1001	1001	251001	<u>co</u>	18	2.4	<u>co</u>	57	329	rl	*	*
1	1000	2001	2001	*	<u>co</u>	19	11	m	*	*	sk	*	*
1	2000	4001	4001	*	<u>co</u>	21	73	sk	*	*	sk	*	*
1	3000	6001	6001	*	<u>co</u>	24	235	sk	*	*	sk	*	*
1	4000	8001	8001	*	<u>co</u>	24	508	sk	*	*	sk	*	*
1	5000	10001	10001	*	<u>co</u>	24	916	sk	*	*	sk	*	*
2	15	376	496	23836	<u>co</u>	15	0.4	<u>co</u>	21	5.0	<u>co</u>	10	87
2	30	1426	1891	339946	<u>co</u>	25	10	<u>co</u>	49	751	rl	*	*
2	45	3151	4186	*	<u>co</u>	22	58	m	*	*	sk	*	*
2	60	5551	7381	*	<u>co</u>	28	300	sk	*	*	sk	*	*
2	75	8626	11476	*	<u>co</u>	30	1019	sk	*	*	sk	*	*
3	6	252	455	8358	<u>co</u>	17	0.3	<u>co</u>	17	1.6	<u>co</u>	9	9.1
3	9	715	1330	65395	<u>co</u>	20	3.1	<u>co</u>	24	104	<u>co</u>	9	799
3	12	1547	2925	303030	<u>co</u>	23	20	<u>co</u>	33	1775	rl	*	*
3	15	2856	5456	*	<u>co</u>	26	89	m	*	*	sk	*	*
3	18	4750	9139	*	er	34	1340	sk	*	*	sk	*	*
4	4	210	495	5005	<u>co</u>	18	0.4	<u>co</u>	16	1.7	<u>co</u>	8	3.9
4	6	714	1820	54159	<u>co</u>	15	4.8	<u>co</u>	18	222	<u>co</u>	10	579
4	8	1815	4845	*	<u>co</u>	20	63	m	*	*	m	*	*
4	10	3861	10626	*	<u>co</u>	22	458	sk	*	*	sk	*	*
8	2	117	495	1395	<u>co</u>	26	0.5	<u>co</u>	21	0.7	<u>co</u>	11	0.9
8	3	525	3003	21975	<u>co</u>	18	15	<u>co</u>	16	148	<u>co</u>	8	125
8	4	1815	12870	*	<u>co</u>	27	633	m	*	*	m	*	*
16	1	33	153	169	<u>co</u>	13	0.1	<u>co</u>	12	0.7	<u>co</u>	7	0.0
16	2	425	4845	14229	<u>co</u>	27	86	<u>co</u>	22	174	<u>sp</u>	10	192
32	1	65	561	593	<u>co</u>	15	0.7	<u>co</u>	12	1.0	<u>co</u>	7	0.2
64	1	129	2145	2209	<u>co</u>	15	14	<u>co</u>	12	3.1	<u>co</u>	9	3.0

Table 9: Smooth density estimation formulation and solver statistics.

m	$2k$	form. stats.			Hypatia-NF			Hypatia-EF			MOSEK-EF		
		ν	n	\bar{n}	st	it	time	st	it	time	st	it	time
1	250	753	252	16628	<u>co</u>	35	0.2	sp	43	1040	<u>co</u>	25	112
1	500	1003	502	64003	<u>co</u>	42	1.1	rl	*	*	tl	18	1814
1	1000	1503	1002	*	<u>co</u>	41	5.1	m	*	*	m	*	*
1	2000	2503	2002	*	<u>co</u>	56	23	sk	*	*	sk	*	*
1	4000	4503	4002	*	<u>co</u>	82	185	sk	*	*	sk	*	*
1	6000	6503	6002	*	<u>co</u>	106	663	sk	*	*	sk	*	*
2	20	678	232	6023	<u>co</u>	50	0.3	<u>co</u>	29	73	<u>co</u>	19	7.4
2	40	1153	862	72468	<u>co</u>	34	2.9	rl	*	*	sp	22	1522
2	60	1928	1892	*	<u>co</u>	36	11	m	*	*	m	*	*
2	80	3003	3322	*	<u>co</u>	53	64	sk	*	*	sk	*	*
2	100	4378	5152	*	<u>co</u>	64	247	sk	*	*	sk	*	*
3	12	754	456	9314	<u>co</u>	57	0.9	<u>co</u>	25	293	sp	19	20
3	18	1217	1331	67226	<u>co</u>	55	6.6	rl	*	*	sp	17	1216
3	24	2049	2926	*	<u>co</u>	46	35	m	*	*	m	*	*
3	30	3358	5457	*	<u>co</u>	63	348	sk	*	*	sk	*	*
4	8	712	496	6001	<u>co</u>	57	1.0	<u>co</u>	25	133	sp	22	12
4	12	1216	1821	56480	<u>co</u>	72	16	tl	*	*	sp	20	934
4	16	2317	4846	*	<u>co</u>	70	192	m	*	*	m	*	*
8	4	619	496	2391	<u>co</u>	96	2.1	<u>co</u>	30	9.8	sp	17	2.1
8	6	1027	3004	25479	<u>co</u>	90	62	tl	*	*	sp	17	379

Table 10: Shape constrained regression formulation and solver statistics.

m	$2k$	form. stats.				Hypatia-NF			Hypatia-EF			MOSEK-EF		
		ν	n	\bar{n}	q	st	it	time	st	it	time	st	it	time
2	10	72	67	952	203	<u>co</u>	18	0.0	<u>co</u>	24	0.6	<u>sp</u>	19	0.4
2	20	292	232	14527	803	<u>co</u>	37	0.5	<u>sp</u>	68	1348	<u>sp</u>	20	60
2	30	662	497	73727	1803	<u>co</u>	58	5.6	rl	*	*	tl	12	1845
2	40	1182	862	*	3203	<u>co</u>	84	34	m	*	*	sk	*	*
2	50	1852	1327	*	5003	er	52	161	sk	*	*	sk	*	*
2	60	2672	1892	*	7203	<u>er</u>	120	939	sk	*	*	sk	*	*
3	8	152	166	3391	671	<u>co</u>	19	0.1	<u>co</u>	27	14	<u>sp</u>	23	4.3
3	12	485	456	31347	2173	<u>co</u>	38	3.5	tl	*	*	<u>sp</u>	15	244
3	16	1118	970	161584	5051	<u>co</u>	61	44	m	*	*	rl	*	*
3	20	2147	1772	*	9753	<u>co</u>	87	325	sk	*	*	sk	*	*
3	24	3668	2926	*	16727	<u>co</u>	111	1605	sk	*	*	sk	*	*
4	6	142	211	2881	912	<u>co</u>	17	0.4	<u>co</u>	23	7.8	<u>sp</u>	18	3.7
4	8	382	496	17686	2597	<u>co</u>	24	3.1	<u>co</u>	38	1621	<u>sp</u>	14	94
4	10	842	1002	79822	5953	<u>co</u>	38	35	rl	*	*	tl	12	2068
4	12	1626	1821	*	11832	<u>co</u>	58	283	m	*	*	sk	*	*
4	14	2858	3061	*	21262	<u>co</u>	72	1430	sk	*	*	sk	*	*
6	4	80	211	1240	800	<u>co</u>	13	0.3	<u>co</u>	15	0.7	<u>co</u>	9	0.6
6	6	422	925	20539	5336	<u>co</u>	22	15	<u>co</u>	32	1630	<u>sp</u>	17	260
6	8	1514	3004	215440	22409	<u>co</u>	29	638	m	*	*	rl	*	*
8	4	138	496	3412	2117	<u>co</u>	19	2.1	<u>co</u>	20	8.0	<u>co</u>	11	4.1
8	6	938	3004	89008	20825	<u>co</u>	31	515	rl	*	*	rl	*	*
10	4	212	1002	7657	4633	<u>co</u>	26	13	<u>co</u>	24	87	<u>co</u>	13	25
12	4	302	1821	15003	8920	<u>co</u>	29	73	<u>co</u>	28	504	<u>co</u>	11	125
14	4	408	3061	26686	15662	<u>co</u>	33	346	tl	0	1884	<u>sp</u>	21	767

Figure 1: Solve times (in seconds) for solve runs satisfying the convergence check in (19).

