Appendix A: Missing Definitions, Lemmas and Proofs

A.1. Proof of Theorem 3

Proof: The proof of the running time is trivial. Because π^f takes k rounds to select k items, and the running time of each round is bounded by $\frac{n}{k}\log\frac{1}{\epsilon}$ which is the size of the random set H. Thus, the total running time is bounded by $O(k\times\frac{n}{k}\log\frac{1}{\epsilon})=O(n\log\frac{1}{\epsilon})$.

We next prove the approximation ratio of π^f. Let ϕ' denote the worst-case realization for π^f, i.e., $\phi'=\arg\min_\phi \mathbb{E}[f(E(\pi^f,\phi),\phi)]$. Let random variables Ψ'_t denote the partial realization after selecting the first t items $\mathbf{S}_t$, which is also a random variable, conditioned on ϕ'. We can represent the expected worst-case utility $\tilde{f}_{wc}(\pi^f)$ of π^f as follows:

$$\begin{aligned}\tilde{f}_{wc}(\pi^f)&=\mathbb{E}_\Pi[f(E(\pi^f,\phi'),\phi')]\\&=\sum_{t\in[k]}\{\mathbb{E}_\Pi[f(E(\pi^f_t,\phi'),\phi')]-\mathbb{E}_\Pi[f(E(\pi^f_{t-1},\phi'),\phi')]\}\\&=\sum_{t\in[k]}\{\mathbb{E}_{\mathbf{S}_t,\Psi'_t}[f(\mathbf{S}_t,\Psi'_t)]-\mathbb{E}_{\mathbf{S}_{t-1},\Psi'_{t-1}}[f(\mathbf{S}_{t-1},\Psi'_{t-1})]\}\end{aligned}\tag{33}$$

To prove this theorem, it suffices to show that for all $t\in[k]$,

$$\mathbb{E}_{\mathbf{S}_t,\Psi'_t}[f(\mathbf{S}_t,\Psi'_t)]-\mathbb{E}_{\mathbf{S}_{t-1},\Psi'_{t-1}}[f(\mathbf{S}_{t-1},\Psi'_{t-1})]\geq(1-\epsilon)\frac{f_{wc}(\pi^*_{wc})-\mathbb{E}_{\mathbf{S}_{t-1},\Psi'_{t-1}}[f(\mathbf{S}_{t-1},\Psi'_{t-1})]}{k}\tag{34}$$

This is because by induction, (34) and (33) imply that $\tilde{f}_{wc}(\pi^f)\geq(1-(1-\frac{1-\epsilon}{k})^k)f_{wc}(\pi^*_{wc})\geq(1-e^{-(1-\epsilon)})f_{wc}(\pi^*_{wc})\geq(1-1/e-\epsilon)f_{wc}(\pi^*_{wc})$.

The rest of the proof is devoted to proving (34). Consider a given partial realization ψ'_{t-1} after selecting the first $t-1$ items S_{t-1}. Let $W\in\arg\max_{S\subseteq E:|S|=k}\sum_{e\in S}f_{wc}(e\mid\psi'_{t-1})$ denote the top k items that have the largest worst-case marginal utility on top of ψ'_{t-1}. Recall that H is a random set of size $\frac{n}{k}\log\frac{1}{\epsilon}$ sampled uniformly at random from E, we first show that the probability that $H\cap W=\emptyset$ is upper bounded by $e^{-|H|\frac{k}{n}}$.

$$\Pr[H\cap W=\emptyset]\leq(1-\frac{|W|}{|E|})^{|H|}=(1-\frac{k}{n})^{|H|}\leq e^{-|H|\frac{k}{n}}$$

It follows that $\Pr[H\cap W\neq\emptyset]\geq 1-e^{-\frac{|H|}{n}k}$. Because we assume $|H|=\frac{n}{k}\log\frac{1}{\epsilon}$, it follows that

$$\Pr[H\cap W\neq\emptyset]\geq 1-e^{-|H|\frac{k}{n}}\geq 1-e^{-\frac{\frac{n}{k}\log\frac{1}{\epsilon}}{n}k}\geq 1-\epsilon\tag{35}$$

Now we are ready to prove (34). Again, consider a fixed partial realization ψ'_{t-1} after selecting the first $t-1$ items S_{t-1}, we have

$$\begin{aligned}&\mathbb{E}_{\mathbf{S}_t,\Psi'_t}[f(\mathbf{S}_t,\Psi'_t)\mid S_{t-1},\psi'_{t-1}]-f(S_{t-1},\psi'_{t-1})\\&=\mathbb{E}_{\mathbf{e}'_t}[f(S_{t-1}\cup\{\mathbf{e}'_t\},\psi'_{t-1}\cup\{(\mathbf{e}'_t,\phi'(\mathbf{e}'_t))\})-f(S_{t-1},\psi'_{t-1})\mid S_{t-1},\psi'_{t-1}]\\&\geq\mathbb{E}_{\mathbf{e}'_t}[f_{wc}(\mathbf{e}'_t\mid\psi'_{t-1})\mid S_{t-1},\psi'_{t-1}]\geq\mathbb{E}_H[\max_{e\in H}f_{wc}(e\mid\psi'_{t-1})\mid S_{t-1},\psi'_{t-1}]\\&\geq\Pr[H\cap W\neq\emptyset]\frac{\sum_{e\in H}f_{wc}(e\mid\psi'_{t-1})}{k}\geq(1-\epsilon)\frac{\sum_{e\in W}f_{wc}(e\mid\psi'_{t-1})}{k}\geq(1-\epsilon)\frac{\sum_{i\in[k]}f_{wc}(e^*_i\mid\psi'_{t-1})}{k}\\&\geq(1-\epsilon)\frac{\sum_{i\in[k]}f_{wc}(e^*_i\mid\psi'_{t-1}\cup\psi^*_{i-1})}{k}=(1-\epsilon)\frac{f(S_{t-1}\cup S^*_k,\psi'_{t-1}\cup\psi^*_{k-1})-f(S_{t-1},\psi'_{t-1})}{k}\\&\geq(1-\epsilon)\frac{f(S^*_k,\psi^*_k)-f(S_{t-1},\psi'_{t-1})}{k}\\&=(1-\epsilon)\frac{f_{wc}(\pi^*_{wc})-f(S_{t-1},\psi'_{t-1})}{k}\end{aligned}\tag{36}$$

The third inequality is due to each item of W has equal probability of being included in $H \cap W$. The forth inequality is due to (35). The fifth inequality is due to $W \in \arg\max_{S\subseteq E:|S|=k} \sum_{e\in S} f_{wc}(e \mid \psi'_{t-1})$. The sixth inequality is due to (5). Taking the expectation of (36) over $(\mathbf{S}_{t-1}, \Psi'_t)$, we have (34). $\Box$

A.2. Proof of Proposition 1

Proof: Consider any two partial realizations ψ and ψ' such that $\psi \subseteq \psi'$ and any item $e \in E \setminus \mathrm{dom}(\psi')$, we have

$$\begin{aligned} f_{wc}(e \mid \psi) &= \min_{o\in O(e,\psi)} f(\mathrm{dom}(\psi)\cup\{e\}, \psi\cup\{(e,o)\}) - f(\mathrm{dom}(\psi), \psi) \\ &= \min_{o\in O(e,\psi)} 1 - p_H(\mathcal{H}(\psi\cup\{(e,o)\})) - (1 - p_H(\mathcal{H}(\psi))) \\ &= \min_{o\in O(e,\psi)} p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o)\})) \end{aligned}$$

Similarly, we have

$$f_{wc}(e \mid \psi') = \min_{o\in O(e,\psi')} p_H(\mathcal{H}(\psi')) - p_H(\mathcal{H}(\psi'\cup\{(e,o)\}))$$

We next show that $f_{wc}(e \mid \psi) \geq f_{wc}(e \mid \psi')$. We first consider the case when there exists a state $o' \in \arg\min_{o\in O(e,\psi)} p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o)\}))$ such that $o' \in O(e,\psi')$. In this case, we have

$$f_{wc}(e \mid \psi) = p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o')\})) \tag{37}$$

and

$$\begin{aligned} f_{wc}(e \mid \psi') &= \min_{o\in O(e,\psi')} p_H(\mathcal{H}(\psi')) - p_H(\mathcal{H}(\psi'\cup\{(e,o)\})) \\ &\leq p_H(\mathcal{H}(\psi')) - p_H(\mathcal{H}(\psi'\cup\{(e,o')\})) \end{aligned} \tag{38}$$

Because $\psi \subseteq \psi'$, any hypotheses $h \in \mathcal{H}(\psi')$ that is not consistent with $\psi'\cup\{(e,o')\}$ must satisfy that $h \in \mathcal{H}(\psi)$ and h is not consistent with $\psi\cup\{(e,o')\}$. Thus, we have $\mathcal{H}(\psi')\setminus\mathcal{H}(\psi'\cup\{(e,o')\}) \subseteq \mathcal{H}(\psi)\setminus\mathcal{H}(\psi\cup\{(e,o')\})$. Thus, $p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o')\})) \leq p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o')\}))$. This together with (37) and (38) implies that $f_{wc}(e \mid \psi) \geq f_{wc}(e \mid \psi')$.

We next consider the case when there does not exist a state $o' \in \arg\min_{o\in O(e,\psi)} p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o)\}))$ such that $o' \in O(e,\psi')$. In this case, for any state $o'' \in \arg\min_{o\in O(e,\psi)} p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o)\}))$, we have

$$\mathcal{H}(\psi') \subseteq \mathcal{H}(\psi)\setminus\mathcal{H}(\psi\cup\{(e,o'')\}) \tag{39}$$

This is because none of the hypotheses from $\mathcal{H}(\psi')$ is consistent with $\{(e,o'')\}$. It follows that

$$\begin{aligned} f_{wc}(e \mid \psi) &= p_H(\mathcal{H}(\psi)) - p_H(\mathcal{H}(\psi\cup\{(e,o'')\})) \\ &\geq p_H(\mathcal{H}(\psi')) \\ &\geq \min_{o\in O(e,\psi')} p_H(\mathcal{H}(\psi')) - p_H(\mathcal{H}(\psi'\cup\{(e,o)\})) \\ &= f_{wc}(e \mid \psi') \end{aligned}$$

The first equality is due to the definition of o'' and the first inequality is due to (39). This finishes the proof of this proposition. $\Box$

A.3. Proof of Proposition 2

Proof: The proof of worst-case monotonicity is trivial. For any $e \in E$ and ψ such that $e \notin \mathrm{dom}(\psi)$, let $o' \in \arg\min_{o \in O(e,\psi)} f(\mathrm{dom}(\psi) \cup \{e\}, \psi \cup \{(e,o)\}) - f(\mathrm{dom}(\psi)$. Consider any realization ϕ' that is consistent with $\psi \cup \{(e,o')\}$, i.e., $\phi' \sim \psi \cup \{(e,o')\}$, we have

$$\begin{aligned} f_{wc}(e \mid \psi) &= f(\mathrm{dom}(\psi) \cup \{e\}, \psi \cup \{(e,o')\}) - f(\mathrm{dom}(\psi), \psi) \\ &= f(\mathrm{dom}(\psi) \cup \{e\}, \phi'\}) - f(\mathrm{dom}(\psi), \phi') \\ &\geq 0 \end{aligned}$$

The second equality is due to $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ satisfies the property of minimal dependency and the inequality is due to $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ is pointwise monotone with respect to $p(\phi)$.

We next prove that $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ is worst-case submodular with respect to $p(\phi)$. Consider any two partial realizations ψ and ψ' such that $\psi \subseteq \psi'$ and any item $e \in E \setminus \mathrm{dom}(\psi')$. Because of the first condition, we have $O(e,\psi) = O(e,\psi')$. Then for any $o \in O(e,\psi)$, $\phi \sim \psi \cup \{(e,o)\}$ and $\phi' \sim \psi' \cup \{(e,o)\}$, we have

$$\begin{aligned} & f(\mathrm{dom}(\psi) \cup \{e\}, \psi \cup \{(e,o)\}) - f(\mathrm{dom}(\psi), \psi) \\ &= f(\mathrm{dom}(\psi) \cup \{e\}, \phi) - f(\mathrm{dom}(\psi), \phi) \\ &= f(\mathrm{dom}(\psi) \cup \{e\}, \phi') - f(\mathrm{dom}(\psi), \phi') \\ &\geq f(\mathrm{dom}(\psi') \cup \{e\}, \phi') - f(\mathrm{dom}(\psi'), \phi') \\ &= f(\mathrm{dom}(\psi') \cup \{e\}, \psi' \cup \{(e,o)\}) - f(\mathrm{dom}(\psi'), \psi') \end{aligned} \tag{40}$$

All equalities are due to $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ satisfies the property of minimal dependency and the inequality is due to $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ is pointwise submodular.

Let $o' \in \arg\min_{o \in O(e,\psi)} f(\mathrm{dom}(\psi) \cup \{e\}, \psi \cup \{(e,o)\}) - f(\mathrm{dom}(\psi), \psi)$, we have

$$\begin{aligned} f_{wc}(e \mid \psi) &= f(\mathrm{dom}(\psi) \cup \{e\}, \psi \cup \{(e,o')\}) - f(\mathrm{dom}(\psi), \psi) \\ &\geq f(\mathrm{dom}(\psi') \cup \{e\}, \psi \cup \{(e,o')\}) - f(\mathrm{dom}(\psi'), \psi') \\ &\geq \min_{o \in O(e,\psi')} f(\mathrm{dom}(\psi') \cup \{e\}, \psi \cup \{(e,o)\}) - f(\mathrm{dom}(\psi'), \psi') \\ &= f_{wc}(e \mid \psi') \end{aligned} \tag{41}$$

The first inequality is due to (40). This finishes the proof of this proposition. □

A.4. Proof of Proposition 3

Proof: It is easy to verify that $f: 2^E \times O^E \to \mathbb{R}_{\geq 0}$ is worst-case monotone and it satisfies the property of minimal dependency. We next show that it is also worst-case submodular with respect to $p(\phi)$.

Consider any two partial realizations ψ and ψ' such that $\psi \subseteq \psi'$ and any node $e \in E \setminus \mathrm{dom}(\psi')$. Let $Z(\psi)$ denote the set of edges whose status is observed under ψ and let $E(\psi)$ denote the set of nodes that are influenced under ψ. Clearly, $Z(\psi) \subseteq Z(\psi')$ and $E(\psi) \subseteq E(\psi')$. Now consider the worst-case marginal utility $f_{wc}(e \mid \psi)$ of e on top of ψ, the worst-case realization of e, say o, occurs when all edges from $\{(u,v) \in Z \setminus Z(\psi) \mid p(u,v) < 1\}$ are blocked. Similarly, for the worst-case marginal utility $f_{wc}(e \mid \psi')$ of e on top of ψ', the worst-case realization of e, say o', occurs when all edges from $\{(u,v) \in Z \setminus Z(\psi') \mid p(u,v) < 1\}$ are blocked. Because $Z(\psi) \subseteq Z(\psi')$, we have $Z \setminus Z(\psi') \subseteq Z \setminus Z(\psi)$. This together with $E(\psi) \subseteq E(\psi')$ implies that if a node $w \in E \setminus E(\psi')$ can be reached by some live edge under (e, o'), it must be the case that $w \in E \setminus E(\psi)$ and w can be reached by some live edge under (e, o). We conclude that the additional nodes influenced by e on top of ψ under the worst-case realization is a superset of the additional nodes influenced by e on top of ψ' under the worst-case realization. It follows that $f_{wc}(e \mid \psi) \geq f_{wc}(e \mid \psi')$. This finishes the proof of this proposition. $\square$