Proofs of statements, example, vehicle flow formulation and additional computational results

EC.1. Proofs of statements

This section gives the proofs of the different results of the main paper.

The proofs of Theorems 1 and 2 are based on the following lemma about the problem of minimizing a generic piecewise-linear convex function $f:\mathbb{R}_+\to\mathbb{R}$ expressed as $f(x)=\max_{h\in H}\{a_hx+b_h\}$, where $H=\{1,2,\dots,m\}$ is the index set of its breakpoints, over an interval $[l,u]$, with $l\le u$, $l,u\in\mathbb{R}_+$. Let $H^-=\{h\in H \mid a_h<0\}$, and let $H^+=\{h\in H \mid a_h\ge 0\}$. Based on sets H^- and H^+, let functions $f^-:\mathbb{R}_+\to\mathbb{R}$ and $f^+:\mathbb{R}_+\to\mathbb{R}$ defined as $f^-(x)=\max_{h\in H^-}\{a_hx+b_h\}$, if $H^-\ne\emptyset$, $f^-(x)=-\infty$ otherwise, and $f^+(x)=\max_{h\in H^+}\{a_hx+b_h\}$, if $H^+\ne\emptyset$, $f^+(x)=-\infty$ otherwise. The following lemma holds.

Lemma EC.1. *Let $z^*=\min_x\{f(x)\}$. Then $\min_{l\le x\le u}\{f(x)\}=\max\{f^-(u),f^+(l),z^*\}$.*

Proof. Let $x^*=\arg\min f(x)$. We have the following three cases.

(i) $u<x^*$. We have $f^-(u)\ge z^*\ge f^+(u)\ge f^+(l)$. Since $f(x)$ is a non-increasing function in $[l,u]$, we have $\min_{l\le x\le u} f(x)=f(u)=\max\{f^-(u),f^+(u)\}=f^-(u)=\max\{f^-(u),f^+(l),z^*\}$.

(ii) $l\le x^*\le u$. We have $z^*\ge f^-(u)$ and $z^*\ge f^+(l)$. Hence, $\min_{l\le x\le u} f(x)=z^*=\max\{f^-(u),f^+(l),z^*\}$.

(iii) $x^*<l$. We have $f^+(l)\ge z^*\ge f^-(l)\ge f^-(u)$. Since $f(x)$ is a non-decreasing function in $[l,u]$, we have $\min_{l\le x\le u} f(x)=f(l)=\max\{f^-(l),f^+(l)\}=f^+(l)=\max\{f^-(u),f^+(l),z^*\}$.

EC.1.1. Proof of Theorem 1

Given a forward path $P=(i_0=0,i_1,\dots,i_{s-1},i_s)$, the proof is by the induction principle on s. Let $W(P)\le Q$ be the maximum amount of product collected from the pickup customers along path P that can be delivered to the delivery customers to be appended to the path. If $s=0$, i.e., $P=(0)$, we have $W(P)=0$ and $\overline{c}^f_P(w)=-\mu_0$, hence the hypothesis holds. Assume that for path $P=(i_0=0,i_1,\dots,i_{s-1})$ the hypothesis holds, i.e., $\overline{c}^f_P(w)$ can be expressed in the form $\overline{c}^f_P(w)=\max_{h\in H}\{a_hw+b_h\}$, where H is the set of breakpoints and $a_h\ge 0$, $\forall h\in H$.

Consider a new vertex $i_s\notin V(P)$ appended at the end of path P and the corresponding path $P'=(i_0=0,i_1,\dots,i_{s-1},i_s)$. We have two cases:

(i) Vertex i_s is a pickup vertex, i.e., $q_{i_s}>0$. We have $W(P')=\min\{Q,W(P)+q_{i_s}\}$, and

$$\begin{aligned}\overline{c}^f_{P'}(w)&=\min_{\max\{0,w-q_{i_s}\}\le x\le\min\{W(P),w\}}\{\overline{c}^f_P(x)-\mu_{i_s}(w-x)+d_{i_{s-1}i_s}\},\\&=\min_{\max\{0,w-q_{i_s}\}\le x\le\min\{W(P),w\}}\left\{\max_{h\in H}\{a_hx+b_h\}-\mu_{i_s}(w-x)+d_{i_{s-1}i_s}\right\},\\&=\min_{\max\{0,w-q_{i_s}\}\le x\le\min\{W(P),w\}}\left\{\max_{h\in H}\{(a_h+\mu_{i_s})x+b_h\}-\mu_{i_s}w+d_{i_{s-1}i_s}\right\}.\end{aligned}$$

Let $H^+ = \{h : h \in H, a_h + \mu_{i_s} \geq 0\}$, $H^- = \{h : h \in H, a_h + \mu_{i_s} < 0\}$, and $z^* = \min_x\{\max_{h\in H}\{(a_h + \mu_{i_s})x + b_h\}\}$. From Lemma EC.1, we have:

$$\bar{c}^f_{P'}(w) = \max\left\{\begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\left\{(a_h+\mu_{i_s})\min\{W(P),w\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\}, \\ \max\limits_{h\in H^+}\left\{(a_h+\mu_{i_s})\max\{0, w-q_{i_s}\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\} \end{array}\right\},$$

$$= \max\left\{\begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\left\{\max\{(a_h+\mu_{i_s})W(P), (a_h+\mu_{i_s})w\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\}, \\ \max\limits_{h\in H^+}\left\{(a_h+\mu_{i_s})\max\{0, w-q_{i_s}\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\} \end{array}\right\},$$

$$= \max\left\{\begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\{(a_h+\mu_{i_s})W(P) + b_h\} - \mu_{i_s} w + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\{a_h w + b_h\} + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^+}\{b_h\} - \mu_{i_s} w + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^+}\{a_h w - (a_h+\mu_{i_s})q_{i_s} + b_h + d_{i_{s-1}i_s}\} \end{array}\right\},$$

$$= \max\left\{\begin{array}{l} -\mu_{i_s} w + \max\left\{z^*, \max\limits_{h\in H^-}\{(a_h+\mu_{i_s})W(P) + b_h\}, \max\limits_{h\in H^+}\{b_h\}\right\} + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\{a_h w + b_h\} + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^+}\{a_h w - (a_h+\mu_{i_s})q_{i_s} + b_h\} + d_{i_{s-1}i_s}, \end{array}\right\}.$$

Since $\mu_{i_s} \leq 0$, $\bar{c}^f_{P'}(w)$ is the maximum of piecewise-linear nondecreasing functions, hence it is a piecewise-linear convex nondecreasing function.

(ii) Vertex i_s is a delivery vertex, i.e., $q_{i_s} < 0$. We have $W(P') = W(P)$, and

$$\begin{aligned} \bar{c}^f_{P'}(w) &= \min_{w\leq x\leq \min\{W(P), w-q_{i_s}\}}\{\bar{c}^f_P(x) - \mu_{i_s}(w-x) + d_{i_{s-1}i_s}\} \\ &= \min_{w\leq x\leq \min\{W(P), w-q_{i_s}\}}\left\{\max_{h\in H}\{a_h x + b_h\} - \mu_{i_s}(w-x) + d_{i_{s-1}i_s}\right\} \\ &= \min_{w\leq x\leq \min\{W(P), w-q_{i_s}\}}\left\{\max_{h\in H}\{(a_h+\mu_{i_s})x + b_h\} - \mu_{i_s} w + d_{i_{s-1}i_s}\right\}. \end{aligned}$$

Let H^+, H^- and z^* be defined as for the previous case. We have:

$$\bar{c}^f_{P'}(w) = \max\left\{\begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\left\{(a_h+\mu_{i_s})\min\{W(P), w-q_{i_s}\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\}, \\ \max\limits_{h\in H^+}\left\{(a_h+\mu_{i_s})w + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\} \end{array}\right\},$$

$$= \max\left\{\begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1}i_s}, \\ \max\limits_{h\in H^-}\left\{\max\{(a_h+\mu_{i_s})W(P), (a_h+\mu_{i_s})(w-q_{i_s})\} + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\}, \\ \max\limits_{h\in H^+}\left\{(a_h+\mu_{i_s})w + b_h - \mu_{i_s} w + d_{i_{s-1}i_s}\right\} \end{array}\right\},$$

$$= \max \left\{ \begin{array}{l} -\mu_{i_s} w + z^* + d_{i_{s-1} i_s}, \\ -\mu_{i_s} w + \max\limits_{h \in H^-} \{(a_h + \mu_{i_s}) W(P) + b_h\} + d_{i_{s-1} i_s}, \\ \max\limits_{h \in H^-} \{a_h w - (a_h + \mu_{i_s}) q_{i_s} + b_h + d_{i_{s-1} i_s}\}, \\ \max\limits_{h \in H^+} \left\{(a_h + \mu_{i_s}) w + b_h - \mu_{i_s} w + d_{i_{s-1} i_s}\right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} -\mu_{i_s} w + \max\{z^*, \max\limits_{h \in H^-} \{(a_h + \mu_{i_s}) W(P) + b_h\}\} + d_{i_{s-1} i_s}, \\ \max\limits_{h \in H^-} \{a_h w - (a_h + \mu_{i_s}) q_{i_s} + b_h\} + d_{i_{s-1} i_s}, \\ \max\limits_{h \in H^+} \{a_h w + b_h\} + d_{i_{s-1} i_s} \end{array} \right\}.$$

Since $\mu_{i_s} \leq 0$, $\overline{c}^f_{P'}(w)$ is also a piecewise-linear convex nondecreasing function.

EC.1.2. Proof of Theorem 2

Given a backward path $\overline{P} = (i_s, i_{s-1}, \ldots, i_1, i_0 = n')$, the proof is by the induction principle on s. Let $W(\overline{P}) \leq Q$ be the maximum amount of the product required by the delivery customers along path $\overline{P}$ that can potentially be collected from the (pickup) customers, which is to be appended at the beginning of path $\overline{P}$.

If $s = 0$, i.e., $\overline{P} = (n')$, we have $W(\overline{P}) = 0$ and $\overline{c}^b_{\overline{P}}(w) = -\mu_0$, hence the hypothesis holds. Assume that for path $\overline{P} = (i_{s-1}, \ldots, i_1, i_0 = n')$ the hypothesis holds, that is, $\overline{c}^b_{\overline{P}}(w)$ can be expressed in the form $\overline{c}^b_{\overline{P}}(w) = \max_{h \in H}\{a_h w + b_h\}$, where H is the set of breakpoints and $a_h \leq 0$, $\forall h \in H$.

Consider a new vertex $i_s \notin V(\overline{P})$ appended at the beginning of path $\overline{P}$ and the corresponding path $\overline{P}' = (i_s, i_{s-1}, \ldots, i_1, i_0 = n')$. We have two cases:

(i) Vertex i_s is a delivery vertex, that is, $q_{i_s} < 0$. We have $W(\overline{P}) = \min\{Q, W(\overline{P}) - q_{i_s}\}$, and

$$\begin{aligned} \overline{c}^b_{\overline{P}}(w) &= \min_{\max\{0, w + q_{i_s}\} \leq x \leq \min\{W(\overline{P}), w\}} \{\overline{c}^b_{\overline{P}}(x) - \mu_{i_s}(x - w) + d_{i_s i_{s-1}}\} \\ &= \min_{\max\{0, w + q_{i_s}\} \leq x \leq \min\{W(\overline{P}), w\}} \left\{\max_{h \in H}\{a_h x + b_h\} - \mu_{i_s}(x - w) + d_{i_s i_{s-1}}\right\} \\ &= \min_{\max\{0, w + q_{i_s}\} \leq x \leq \min\{W(\overline{P}), w\}} \left\{\max_{h \in H}\{(a_h - \mu_{i_s}) x + b_h\} + \mu_{i_s} w + d_{i_s i_{s-1}}\right\}. \end{aligned}$$

Let $H^+ = \{h : h \in H, a_h - \mu_{i_s} \geq 0\}$, $H^- = \{h : h \in H, a_h - \mu_{i_s} < 0\}$ and $z^* = \min_x \max_{h \in H}\{(a_h - \mu_{i_s}) x + b_h\}$. From Lemma EC.1, we have:

$$\overline{c}^b_{\overline{P}}(w) = \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \max\limits_{h \in H^-} \left\{(a_h - \mu_{i_s}) \min\{W(\overline{P}), w\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}}\right\}, \\ \max\limits_{h \in H^+} \left\{(a_h - \mu_{i_s}) \max\{0, w + q_{i_s}\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}}\right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \max\limits_{h \in H^-} \left\{\max\{(a_h - \mu_{i_s}) W(\overline{P}), (a_h - \mu_{i_s}) w\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}}\right\}, \\ \max\limits_{h \in H^+} \left\{(a_h - \mu_{i_s}) \max\{0, w + q_{i_s}\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}}\right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \mu_{i_s} w + \max_{h \in H^-} \{(a_h - \mu_{i_s}) W(\overline{P}) + b_h\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \{a_h w + b_h + d_{i_s i_{s-1}}\}, \\ \mu_{i_s} w + \max_{h \in H^+} \{b_h\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^+} \{a_h w + (a_h - \mu_{i_s}) q_{i_s} + b_h + d_{i_s i_{s-1}}\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + \max \left\{ z^*, \max_{h \in H^-} \{(a_h - \mu_{i_s}) W(\overline{P}) + b_h\}, \max_{h \in H^+} \{b_h\} \right\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \{a_h w + b_h\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^+} \{a_h w + (a_h - \mu_{i_s}) q_{i_s} + b_h\} + d_{i_s i_{s-1}} \end{array} \right\}.$$

Since $\mu_{i_s} \le 0$, $\overline{c}^b_{\overline{P}'}(w)$ is the maximum of piecewise-linear nonincreasing functions, it is a piecewise-linear convex nonincreasing function.

(ii) Vertex i_s is a pickup vertex, i.e., $q_{i_s} > 0$. We have $W(\overline{P}') = W(\overline{P})$, and

$$\begin{aligned} \overline{c}^b_{\overline{P}}(w) &= \min_{w \le x \le \min\{W(\overline{P}), w + q_{i_s}\}} \{\overline{c}^b_{\overline{P}}(x) - \mu_{i_s}(x - w) + d_{i_s i_{s-1}}\} \\ &= \min_{w \le x \le \min\{W(\overline{P}), w + q_{i_s}\}} \left\{ \max_{h \in H} \{a_h x + b_h\} - \mu_{i_s}(x - w) + d_{i_s i_{s-1}} \right\} \\ &= \min_{w \le x \le \min\{W(\overline{P}), w + q_{i_s}\}} \left\{ \max_{h \in H} \{(a_h - \mu_{i_s}) x + b_h\} + \mu_{i_s} w + d_{i_s i_{s-1}} \right\}. \end{aligned}$$

Let H^+, H^- and z^* be defined as for the previous case. We have:

$$\overline{c}^b_{\overline{P}}(w) = \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \left\{ (a_h - \mu_{i_s}) \min\{W(\overline{P}), w + q_{i_s}\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}} \right\}, \\ \max_{h \in H^+} \left\{ (a_h - \mu_{i_s}) w + b_h + \mu_{i_s} w + d_{i_s i_{s-1}} \right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \left\{ \max\{(a_h - \mu_{i_s}) W(\overline{P}), (a_h - \mu_{i_s})(w + q_{i_s})\} + b_h + \mu_{i_s} w + d_{i_s i_{s-1}} \right\}, \\ \max_{h \in H^+} \left\{ (a_h - \mu_{i_s}) w + b_h + \mu_{i_s} w + d_{i_s i_{s-1}} \right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + z^* + d_{i_s i_{s-1}}, \\ \mu_{i_s} w + \max_{h \in H^-} \{(a_h - \mu_{i_s}) W(\overline{P}) + b_h\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \{a_h w + (a_h - \mu_{i_s}) q_{i_s} + b_h + d_{i_s i_{s-1}}\}, \\ \max_{h \in H^+} \left\{ a_h x + b_h + d_{i_s i_{s-1}} \right\} \end{array} \right\},$$

$$= \max \left\{ \begin{array}{l} \mu_{i_s} w + \max\{z^*, \max_{h \in H^-} \{(a_h - \mu_{i_s}) W(\overline{P}) + b_h\}\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^-} \{a_h w + (a_h - \mu_{i_s}) q_{i_s} + b_h\} + d_{i_s i_{s-1}}, \\ \max_{h \in H^+} \left\{ a_h w + b_h \right\} + d_{i_s i_{s-1}} \end{array} \right\}.$$

Since $\mu_{i_s} \leq 0$, $\overline{c}^b_{\overline{P}'}(w)$ is also a piecewise-linear convex nonincreasing function.

EC.1.3. Proof of Dominance 1

Let P be the forward path associated with label L^f_1, and we denote by $\{L^f_{i_1}, L^f_{i_2}, \ldots, L^f_{i_k}\}$ the set of labels $\mathscr{L}^f_1$, where a forward path P_{i_s} is associated with label $L^f_{i_s}$, $s = 1, \ldots, k$.

Consider the set $\mathcal{P}$ of all feasible extensions of path P, i.e., the set of backward paths starting at vertices different from vertex v^f_1 such that for each $\overline{P} \in \mathcal{P}$ route $R = (P, \overline{P})$ is feasible for the maximum duration constraint. Let $w(P)$, $w(P) \in [0, \min\{W^f_1, W(\overline{P})\}]$, be the load of the vehicle leaving the last vertex v^f_1 visited by route R where $W(\overline{P}) \leq Q$ is the maximum amount of the product required by the delivery customers along path $\overline{P}$, and let j be the first vertex visited by path $\overline{P}$. The reduced cost of route R and value $w(P)$ can be computed as follows:

$$\overline{c}(R, w(P)) = \overline{c}^f_P(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)).$$

Below, we show that for any path $\overline{P} \in \mathcal{P}$ and associated value $w(P)$, there exists a forward path $P' \in \{P_{i_s}\}^k_{s=1}$ such that route $R' = (P', \overline{P})$ is feasible and its reduced cost $\overline{c}(R', w(P))$ is less than or equal to the reduced cost $\overline{c}(R, w(P))$ of route R, and thus path P can be safely discarded.

Given a path $\overline{P} \in \mathcal{P}$, since $v^f = v^f_1$, $s^f \geq s^f_1$, and $V^f_1 \subseteq V^f$, for all $L^f \in \mathscr{L}^f_1$, each route $R' = (P', \overline{P})$ with $P' \in \{P_{i_s}\}^k_{s=1}$ is feasible for the maximum duration constraint. Let $i^* = \arg\min_{s=1,\ldots,k}\{\overline{c}^f_{P_{i_s}}(w(P)\}$ be the index of the labels in L^f_1 having minimum reduced cost computed with respect to value $w(P)$.

Since $w(P) \leq W^f_1$, we have $g_{L^f_1}(w(P)) = \overline{c}^f_P(w(P)) < +\infty$, and given the definition of function $\overline{g}_{\mathscr{L}^f_1}(w)$ we have $\overline{g}_{\mathscr{L}^f_1}(w(P)) = \overline{c}^f_{P_{i^*}}(w(P))$. Consider route $R' = (P_{i^*}, \overline{P})$. The reduced cost of route R' can be computed as

$$\overline{c}(R', w(P)) = \overline{c}^f_{P_{i^*}}(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)) = \overline{g}_{\mathscr{L}^f_1}(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)). \tag{EC.1}$$

From the hypothesis we have $g_{L^f_1}(w(P)) \geq \overline{g}_{\mathscr{L}^f_1}(w(P))$, and thus $\overline{g}_{\mathscr{L}^f_1}(w(P)) < +\infty$ and $w(P) \in [0, W^f_{i^*}]$, so from expression (EC.1) we obtain:

$$\begin{aligned} \overline{c}(R', w(P)) = \overline{g}_{\mathscr{L}^f_1}(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)) \leq \\ g_{L^f_1}(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)) = \\ \overline{c}^f_P(w(P)) + d_{v^f_1 j} + \overline{c}^b_{\overline{P}}(w(P)) = \\ \overline{c}(R, w(P)), \end{aligned}$$

and thus $\overline{c}(R', w(P)) \leq \overline{c}(R, w(P))$ and path P and the associated label L^f_1 can be safely discarded.

Table EC.1 Example of a fractional θ_{pr} solution satisfying the four branching rules

Variables θ_{pr}	Routes and demand patterns (in brackets)
0.5	0(0) 10(1) 3(-1) 8(3) 1(-3) 11(0)
1.0	0(0) 6(1) 9(-1) 4(6) 9(-4) 5(-2) 2(5) 5(-5) 1(0) 11(0)
0.5	0(0) 10(1) 3(-1) 8(6) 7(-6) 8(6.0) 7(-6.0) 8(3) 1(-3) 11(0)

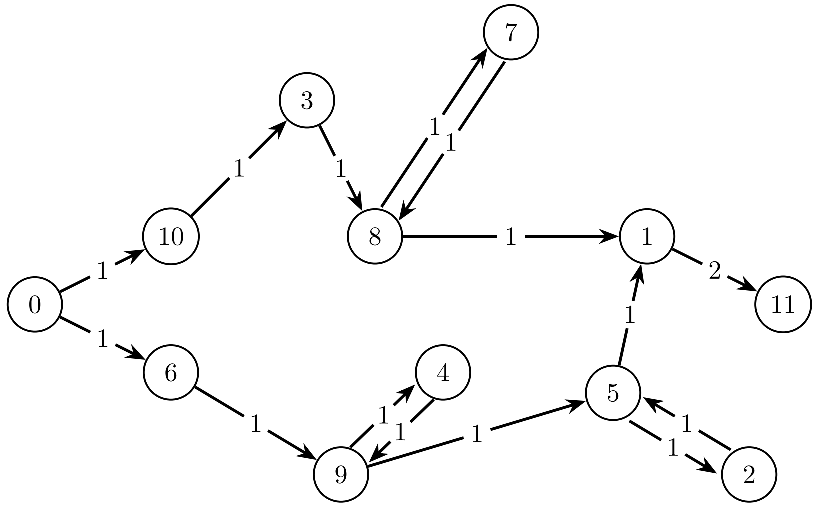

Figure EC.1 Support graph for the example of Table EC.1

EC.2. Example of a fractional RMP solution satisfying the four branching rules

Figure EC.1 shows an example of a fractional RMP solution satisfying the four branching rules. The example involves an $n = 10$ customers instance (numbered from 1 to 10) with initial and final depots indexed with numbers 0 and 11, respectively. The customers demands are $d_1 = -3$, $d_2 = 5$, $d_3 = -1$, $d_4 = 6$, $d_5 = -7$, $d_6 = 1$, $d_7 = -6$, $d_8 = 9$, $d_9 = -5$, $d_{10} = 1$, and the vehicle capacity is $Q = 6$. The routes and demand patterns of the solution are shown in Table EC.1, where the numbers in parentheses show the demand patterns. The support graph of the solution associated with values $\overline{\omega}_{ij}$ is depicted in Figure EC.1, where the number on each arc indicates the value of the corresponding variable $\overline{\omega}_{ij}$.

EC.3. A three-index (TI) vehicle flow formulation

The three-index (TI) vehicle flow formulation is based on three-index vehicle flow formulations proposed for the basic Capacitated VRP (CVRP) (see, for example, Toth and Vigo 2014) and on the single commodity flow formulation described in Salazar-González and Santos-Hernández (2015). In this section, we describe the formulation for the multiple-visit, different-vehicle case, and then we briefly observe how the formulation can be extended to deal with the multiple-visit case.

For a given $S \subseteq N$ we denote by $A(S)$ the set of arcs with both end-vertices in S, i.e., $A(S) = \{(i,j) \in A : i,j \in S\}$. Let x^k_{ij} be a binary variable that is equal to 1 if vehicle k traverses arc (i,j), and 0 otherwise, and y_i^k be a continuous variable representing the amount of products picked up (if positive) or delivered (if negative) to vertex i by vehicle k. In addition, let z_i^k be a nonnegative variable representing the amount of product carried by vehicle k when it leaves vertex i. The formulation is as follows.

$$(TI) \quad \min \sum_{k\in K}\sum_{(i,j)\in A} d_{ij}x_{ij}^k \tag{EC.2a}$$

$$\text{s.t.} \quad \sum_{(0,i)\in A} x_{0i}^k = \sum_{(i,n')\in A} x_{in'}^k, \quad \forall\, k\in K, \tag{EC.2b}$$

$$\sum_{(i,j)\in A} x_{ij}^k = \sum_{(j,i)\in A} x_{ji}^k, \quad \forall\, i\in V,\ k\in K, \tag{EC.2c}$$

$$\sum_{k\in K} y_i^k = q_i, \quad \forall\, i\in V, \tag{EC.2d}$$

$$y_i^k \le \min\{q_i, Q\} \sum_{(i,j)\in A} x_{ij}^k, \quad \forall\, i\in N^+,\ k\in K, \tag{EC.2e}$$

$$y_i^k \ge \max\{q_i, -Q\} \sum_{(i,j)\in A} x_{ij}^k, \quad \forall\, i\in N^-,\ k\in K, \tag{EC.2f}$$

$$z_j^k \ge z_i^k + y_j^k + (x_{ij}^k - 1)(Q+1), \quad \forall\, j\in N^+, (i,j)\in A,\ k\in K, \tag{EC.2g}$$

$$z_j^k \le z_i^k + y_j^k + (1 - x_{ij}^k)(Q+1), \quad \forall\, j\in N^-, (i,j)\in A,\ k\in K, \tag{EC.2h}$$

$$z_i^k \le Q, \quad \forall\, i\in V,\ k\in K, \tag{EC.2i}$$

$$\sum_{(i,j)\in A} t_{ij}x_{ij}^k \le T, \quad \forall\, k\in K, \tag{EC.2j}$$

$$\sum_{(i,j)\in A(S)} x_{ij}^k \le |S|-1, \quad \forall\, S\subseteq N, |S|>1,\ k\in K, \tag{EC.2k}$$

$$y_0^k = y_{n'}^k = 0, \quad \forall\, k\in K, \tag{EC.2l}$$

$$x_{ij}^k \in \{0,1\}, \quad \forall\, (i,j)\in A,\ k\in K, \tag{EC.2m}$$

$$y_i^k \ge 0, \quad \forall\, i\in N^+,\ k\in K, \tag{EC.2n}$$

$$y_i^k \le 0, \quad \forall\, i\in N^-,\ k\in K, \tag{EC.2o}$$

$$z_i^k \ge 0, \quad \forall\, i\in V, k\in K. \tag{EC.2p}$$

The objective function (EC.2a) aims at minimizing the total routing cost of the vehicles. Constraints (EC.2b) and (EC.2c) are the flow conservation constraints for each vehicle at the depot and the customers, respectively. The satisfaction of the demand of each customer is guaranteed by constraints (EC.2d). Constraints (EC.2e) and (EC.2f) state that a vehicle can pick up or deliver the product at a customer only if the customer is visited by the vehicle, respectively. Constraints (EC.2g) and (EC.2h) define the values of flow variables z for the pickup and delivery customers, respectively. Constraints (EC.2i) and (EC.2j) are the capacity and the maximum duration constraints, respectively. Constraints (EC.2k) are the *subtour elimination* constraints.

Equations (EC.2l) impose that each vehicle departs from and arrives at the depot empty. Finally, constraints (EC.2m), (EC.2n), (EC.2o) and (EC.2p) state the domains of the decision variables.

Due to the presence of constraints (EC.2k), formulation TI imposes that a customer can be visited by the same vehicle at most once. Nevertheless, the formulation can be extended to deal with the case where a customer can also be visited more than once by the same vehicle, thus modeling the multiple visits case. Indeed, as done for example by Salazar-González and Santos-Hernández (2015) and Bulhões et al. (2018), multiple visits by the same vehicle can be modeled on an extended network where each vertex is associated with a number of vertices (representing possible visits) and each vertex can be visited at most once. A drawback of the resulting mathematical formulation is the large number of variables required by the underlying extended formulation, and for this reason the maximum number of visits to each vertex is generally fixed to a small value, such as less than two or three (see, for example, Salazar-González and Santos-Hernández 2015, Bulhões et al. 2018). The corresponding details are omitted for sake of brevity.

EC.4. Additional computational details

EC.4.1. Branching rules

Figures EC.2 and EC.3 give an overview of the use of the different branching rules adopted in EXM (see §5.3) for S_A, S_B and S_C classes, respectively. The figures show the percentages of the nodes in which each rule is used over the total number of branching decisions. In Figure EC.2, the instances are grouped by the vehicle capacity Q and the maximum duration T, and the corresponding pairs are reported in the figures as $[Q,T]$. The corresponding detailed values can be found in the tables given in the e-companion.

The figures clearly show that branching on the number of vehicles visiting each customer (Rule 2) and branching on single arc (Rule 3) are among the most used rules. Rule 4, that is, branching on two consecutive arcs, is rarely used. Indeed, it is applied 16 and 14 times for classes S_A, S_B and S_C, respectively (the corresponding percentages cannot be seen from the figures). As discussed in Section 5.3, the four branching rules are not sufficient to fulfil the integrality requirements. Nevertheless, in our experiments, it never once occurred that none of the rules could be applied.

EC.4.2. Analysis of the SPDVRP solutions

In this section, we report some insights into the solutions computed by EXM regarding the number of routes and the type of routes generated.

Figures EC.4 and EC.5 reports the average number of routes in the optimal or best solutions found of classes S_A, S_B and S_C, respectively. The data are grouped by the values of Q and T and the number of customers. The figures show that the average number of routes ranges between 1 and about 8. The numbers for different values of T show that the maximum duration constraints

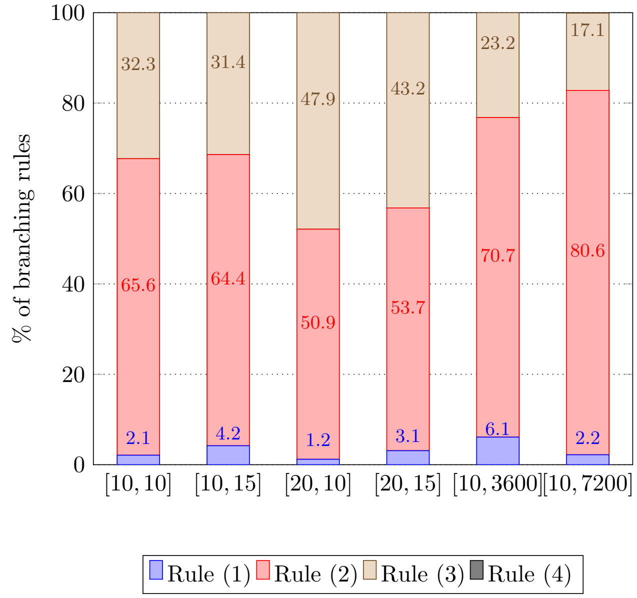

Figure EC.2 **Usage of branching decisions on S_A and S_B classes (values $[Q,T]$ under each bar)**

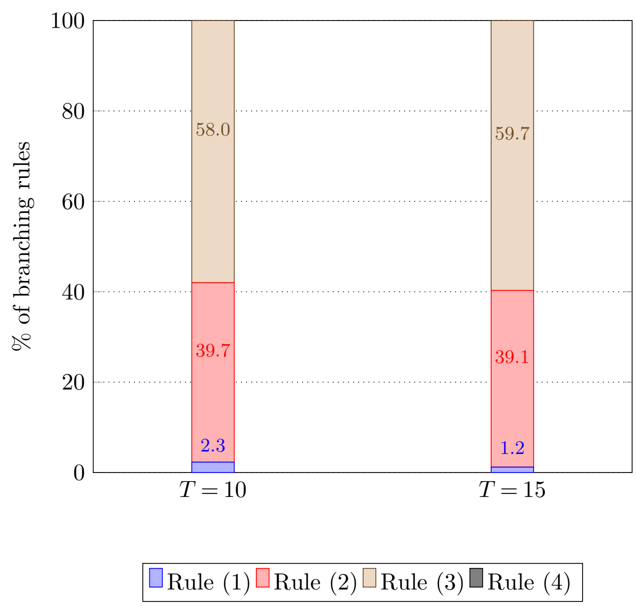

Figure EC.3 **Usage of branching decisions on S_C class**

(either as a cardinality or time constraint) are generally tight, as shown by the lower average numbers for increasing values of T.

Finally, Figures EC.6 and EC.7 give an overview of the structure of the solutions computed by EXM based on the way the customers are visited. More specifically, we denote by "sv" the total number of customers visited more than once by the same vehicle, and we denote by "dv" the total number of customers visited more than once by different vehicles. Parameters sv and dv can also be used as a measure of the difficulty of the instances. Figure EC.6 shows relevant data about

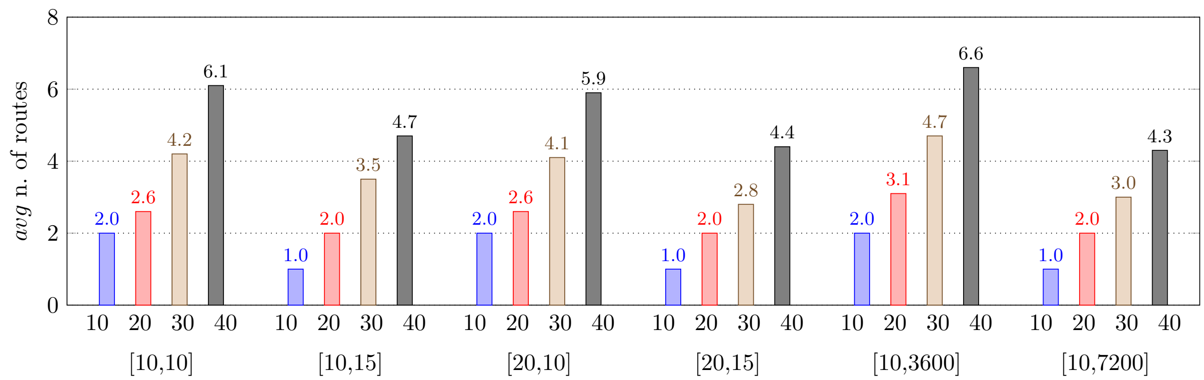

Figure EC.4 **Number of routes for the instances of classes S_A and S_B grouped by values $[Q,T]$ and number of customers**

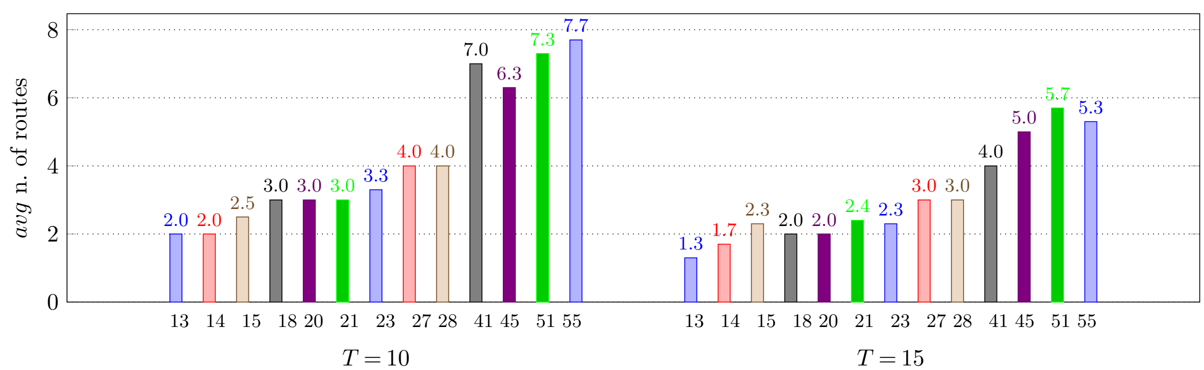

Figure EC.5 **Number of routes for the instances of class S_C grouped by the values of T and number of customers**

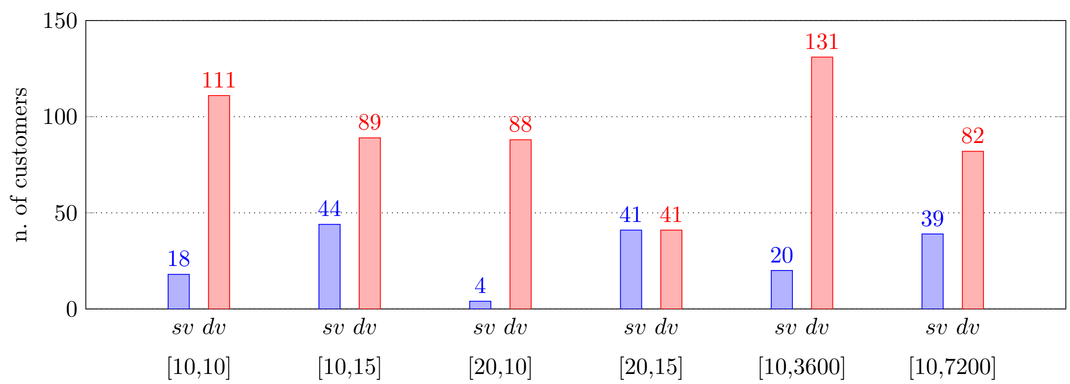

Figure EC.6 **Classes S_A and S_B: structure of the solutions grouped by values $[Q,T]$**

classes S_A and S_B, where the instances are grouped by values Q and T, whereas Figure EC.7 gives the details about class S_C, grouped by $T = 10$ and $T = 15$.

The figures clearly point out that the solutions computed (which include both optimal and heuristic solutions) feature a higher number of customers visited by different vehicles (measure dv) than the number of customers visited by the same vehicle (measure sv), and that instances with tight Q and T values generally involve more split customers. The comparison between classes S_A,

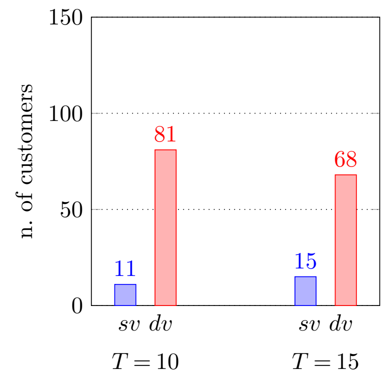

Figure EC.7 Class S_C: structure of the solutions grouped by values of T

S_B and S_C shows that class S_C features a reduced number of split customers with respect to classes S_A and S_B, that is to say that the routes forming the solutions of class S_C are generally elementary and disjoint. In this case, the literature shows that for classical (non-split) vehicle routing problems, the LP-relaxation of the set-partitioning formulation provides very tight lower bounds (see Poggi and Uchoa 2014). These differences could explain the fact that, according to Table 3, EXM shows tighter bounds for class S_C.

EC.5. Detailed computational results

This section reports detailed results about algorithm EXM on classes of instances S_A, S_B and S_C.

Tables EC.2-EC.9 show the following details:

- Name of the instance ("Name").
- Number of vehicles available ("$|K|$").
- Number of customers ("n").
- Vehicle capacity ("Q").
- Cost of the best solution found ("z^*") by EXM.
- Number of routes composing the solution ("rt").
- Percentage deviation of the upper bound computed by the primal heuristic ("$\%UB$"), computed as $100.0 \times UB/z^*$ where UB is the value of the upper bound.
- Time in seconds spent by the primal heuristic ("t_{UB}").
- Percentage deviation of lower bound derived from the LP-relaxation of formulation $\overline{PB}$ ("$\%LB$"), computed as $100.0 \times LB/z^*$ where LB is the value of the lower bound.
- Percentage deviation of lower bound obtained from the column-and-row generation procedure ("$\%LB_r$"), computed as $100.0 \times LB_r/z^*$ where LB_r is the value of the lower bound.
- Number of cuts generated at the root node ("cut_r").
- Time in seconds spent by the pricing algorithm at the root node ("t_{PA}").
- Time in seconds spent to compute the lower bound at the root node ("t_r").

- Total number of cuts generated ("*cut*").
- Number of tree nodes explored ("*node*").
- Total computing time in seconds ("t").

EC.6. Detailed computational results about the pricing algorithms

This section reports detailed results about the comparison between the pricing algorithm of EXM and our implementation of the pricing algorithm proposed by Casazza et al. (2019).

Tables EC.10-EC.12 show the following details:

- Name of the instance ("Name").
- Number of customers ("n").
- Number of vehicles available ("$|K|$").
- Lower bound value ("LB").
- For each pricing algorithm:
  - —Time in seconds spent by the LP solver ("t_{LP}").
  - —Time in seconds spent by the pricing algorithm ("t_{PA}").
  - —Number of forward labels generated ("fw").
  - —Number of forward labels dominated ("fwd").
  - —Number of backward labels generated ("bw").
  - —Number of backward labels dominated ("bwd").

Table EC.2 Detailed results on set S_A with $Q = 10$ and $T = 10$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3719.0	2	105.2	0.8	86.3	95.2	63	0.2	0.4	67	27	1.5
n10A	10	5	3055.0	2	100.0	0.6	95.6	99.8	56	0.2	0.4	56	3	1.0
n10b	10	5	3192.0	2	100.0	0.8	84.2	99.8	24	0.3	0.4	24	3	1.3
n10B	10	5	3704.0	2	101.5	0.8	95.6	99.6	61	0.3	0.5	61	3	1.6
n10c	10	5	4239.0	2	100.0	0.4	98.7	100.0	8	0.2	0.3	8	1	0.9
n10C	10	5	3392.0	2	105.3	0.9	84.9	97.7	101	0.4	0.6	102	11	1.8
n10d	10	5	4497.0	2	102.4	0.7	86.7	96.5	28	0.3	0.4	34	35	2.0
n10D	10	5	3199.0	2	111.2	0.8	81.6	97.3	60	0.3	0.5	60	3	1.3
n10e	10	5	3823.0	2	100.0	0.5	89.0	100.0	87	0.4	0.6	87	1	1.2
n10E	10	5	4876.0	2	100.0	0.5	92.1	100.0	85	0.3	0.5	85	1	1.1
n10f	10	5	3468.0	2	126.4	1.0	76.6	86.0	65	0.4	0.6	109	1635	81.7
n10F	10	5	3796.0	2	100.0	0.6	96.5	99.8	53	0.2	0.4	53	3	1.3
n10g	10	5	4179.0	2	107.7	0.9	81.8	85.8	58	0.2	0.4	75	1953	102.8
n10G	10	5	3973.0	2	139.5	0.6	86.5	90.1	28	0.2	0.4	28	3	1.1
n10h	10	5	4168.0	2	100.0	0.6	88.4	96.3	66	0.2	0.4	67	13	1.3
n10H	10	5	3959.0	2	108.0	0.9	80.7	93.9	65	0.3	0.5	73	155	7.3
n10i	10	5	2645.0	2	145.0	0.7	91.7	95.6	56	0.2	0.4	58	9	1.3
n10I	10	5	3963.0	2	133.9	0.6	83.9	86.8	25	0.2	0.4	31	35	1.6
n10j	10	5	3453.0	2	115.0	0.6	94.0	94.6	19	0.2	0.4	21	17	1.2
n10J	10	5	3125.0	2	134.3	0.7	79.4	98.6	56	0.2	0.4	57	17	1.2
n20A	20	5	4826.0	2	115.7	2.5	94.0	99.3	172	0.6	1.0	176	13	4.2
n20B	20	5	5300.0	2	108.3	2.7	85.5	94.6	155	0.9	1.3	173	135	21.9
n20C	20	5	6508.0	3	115.3	4.9	92.0	96.2	486	1.3	1.9	604	8775	4591.5
n20D	20	5	6208.0	3	100.0	2.9	96.7	100.0	171	0.7	1.0	171	3	4.2
n20E	20	5	6491.0	3	114.2	4.1	88.6	98.2	367	1.1	1.7	401	261	85.7
n20F	20	5	5222.0	3	120.8	3.2	92.7	95.5	161	0.9	1.3	173	33	9.1
n20G	20	5	5795.0	3	122.6	3.3	86.0	94.3	190	1.0	1.3	277	1147	161.7
n20H	20	5	6207.0	2	127.8	4.1	85.8	93.1	340	1.0	1.5	358	547	77.8
n20I	20	5	5167.0	3	113.7	3.3	85.9	94.1	210	0.9	1.3	281	3521	789.5
n20J	20	5	4545.0	2	108.1	1.6	92.1	96.3	120	0.5	0.8	161	155	17.5
n30A	30	5	6865.0	4	105.0	15.0	92.5	97.3	1038	6.7	5.7	1252	5305	4637.3
n30B	30	5	6939.0	3	108.5	5.5	91.2	97.7	387	1.9	2.6	454	445	237.1
n30C	30	5	8613.0	5	100.0	16.6	73.0	79.3	594	8.5	3.2	1221	16991	10800.0
n30D	30	5	9314.0	5	100.0	8.2	66.6	71.4	534	2.3	2.8	923	16603	10800.0
n30E	30	5	6840.0	3	114.8	7.8	86.3	94.7	486	1.9	2.4	778	1039	1095.2
n30F	30	5	6715.0	4	106.2	7.5	88.4	95.4	319	2.2	2.9	502	2735	943.8
n30G	30	5	9462.0	4	105.3	12.8	90.7	97.0	778	3.3	4.5	1228	13401	10360.7
n30H	30	5	8182.0	5	100.0	7.3	75.1	79.8	452	2.4	3.0	943	17257	10800.0
n30I	30	5	6602.0	4	100.0	5.8	82.8	87.9	335	1.6	2.2	788	18217	10800.0
n30J	30	5	8753.0	5	100.0	9.0	70.7	73.2	365	3.2	2.1	1702	19701	10800.0
n40A	40	8	10430.0	6	100.0	18.7	71.5	74.4	789	3.4	4.2	1604	14157	10800.0
n40B	40	8	9805.0	7	100.0	18.9	69.1	72.4	736	3.1	3.7	1334	14171	10800.0
n40C	40	8	9097.0	5	100.0	12.4	81.4	85.9	818	3.3	4.4	1201	5011	10800.0
n40D	40	8	9837.0	5	100.0	19.7	80.9	86.2	964	3.9	5.0	1387	11993	10800.0
n40E	40	8	11882.0	7	100.0	13.9	55.8	61.2	792	2.4	3.0	1409	11447	10800.0
n40F	40	8	9275.0	5	100.0	15.8	79.5	85.2	711	3.4	4.2	1265	12583	10800.0
n40G	40	8	11017.0	7	100.0	16.5	73.7	76.7	878	3.6	4.5	1593	13383	10800.0
n40H	40	8	10129.0	7	100.0	15.3	72.9	75.0	869	3.0	3.6	1183	13475	10800.0
n40I	40	8	11222.0	6	100.0	14.4	66.1	70.2	798	3.1	3.9	1285	13119	10800.0
n40J	40	8	9265.0	6	100.0	11.8	73.8	77.4	574	2.5	3.1	1538	14155	10800.0

Table EC.3 Detailed results on set S_A with $Q=10$ and $T=15$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3484.0	1	148.6	1.2	90.3	100.0	52	0.5	0.7	52	3	2.0
n10A	10	5	2994.0	1	100.0	0.2	96.4	100.0	28	0.1	0.1	28	1	0.5
n10b	10	5	3122.0	1	100.0	0.4	84.7	100.0	24	0.1	0.2	24	1	0.6
n10B	10	5	3671.0	1	100.0	0.4	96.1	100.0	60	0.1	0.2	60	1	0.6
n10c	10	5	4226.0	1	100.0	0.1	98.0	100.0	16	0.0	0.0	16	1	0.2
n10C	10	5	3324.0	1	135.4	1.0	84.4	99.2	136	0.3	0.4	138	9	1.5
n10d	10	5	4238.0	1	100.0	0.3	89.0	100.0	28	0.1	0.1	28	1	0.5
n10D	10	5	3071.0	1	100.0	0.5	80.0	100.0	96	0.3	0.4	96	1	0.5
n10e	10	5	3816.0	1	100.0	0.5	88.3	100.0	77	0.2	0.2	77	1	0.2
n10E	10	5	4828.0	1	100.0	0.5	90.6	100.0	84	0.2	0.3	84	1	0.2
n10f	10	5	2996.0	1	170.4	0.6	83.4	98.0	67	0.2	0.3	68	13	2.7
n10F	10	5	3758.0	1	100.0	0.3	94.2	100.0	68	0.1	0.2	68	1	0.5
n10g	10	5	3685.0	1	134.9	0.3	85.8	93.1	61	0.1	0.1	71	359	43.9
n10G	10	5	3325.0	1	100.0	0.2	92.8	100.0	16	0.1	0.1	16	1	0.4
n10h	10	5	3960.0	1	126.3	0.3	87.8	99.8	58	0.1	0.1	58	3	0.7
n10H	10	5	3813.0	1	183.5	1.0	79.9	95.3	69	0.3	0.4	92	695	111.7
n10i	10	5	2390.0	1	100.0	0.2	96.4	100.0	23	0.1	0.1	23	1	0.4
n10I	10	5	3287.0	1	174.7	0.3	92.4	98.2	35	0.1	0.2	35	5	1.5
n10j	10	5	3249.0	1	122.3	0.2	97.5	98.9	44	0.1	0.1	44	11	1.6
n10J	10	5	3060.0	1	100.0	0.2	70.8	100.0	40	0.1	0.1	40	1	0.4
n20A	20	5	4713.0	2	148.2	2.9	90.7	99.2	178	1.0	1.3	179	9	8.9
n20B	20	5	4992.0	2	102.6	3.9	83.9	96.8	191	1.6	1.9	203	91	45.1
n20C	20	5	6110.0	2	133.4	12.9	93.6	99.0	680	5.0	5.8	702	371	2193.8
n20D	20	5	6146.0	2	103.4	6.8	93.0	99.3	372	2.0	2.5	396	135	155.4
n20E	20	5	6339.0	2	115.3	12.9	88.3	99.0	562	5.4	6.4	589	101	171.2
n20F	20	5	4851.0	2	157.2	3.3	93.0	97.3	258	1.1	1.3	272	25	19.9
n20G	20	5	5334.0	2	101.8	3.9	87.1	97.3	201	1.4	1.7	203	9	14.1
n20H	20	5	5847.0	2	140.1	5.2	84.7	95.1	327	1.7	2.1	405	1967	863.9
n20I	20	5	4797.0	2	125.4	7.3	85.8	97.3	483	1.5	2.1	529	307	263.7
n20J	20	5	4402.0	2	117.0	1.6	90.2	95.2	184	0.3	0.4	193	39	8.7
n30A	30	5	8367.0	4	100.0	54.2	70.1	75.5	1631	17.3	20.8	1796	2907	10800.0
n30B	30	5	6460.0	2	109.3	15.0	92.6	99.4	826	5.7	7.2	831	65	436.8
n30C	30	5	7676.0	4	100.0	42.7	74.7	83.0	1099	9.1	10.7	1361	6651	10800.0
n30D	30	5	7403.0	4	100.0	48.9	73.7	82.6	1242	10.5	11.3	1757	6345	10800.0
n30E	30	5	6286.0	2	126.3	19.2	83.9	94.9	632	4.5	5.3	1141	4065	10800.0
n30F	30	5	6205.0	3	124.6	19.3	86.4	94.4	556	7.4	9.1	1012	8487	10800.0
n30G	30	5	9163.0	3	117.6	64.1	88.8	97.1	1270	15.8	18.8	1738	3347	10800.0
n30H	30	5	9191.0	5	100.0	16.0	60.7	65.0	617	4.5	5.4	1249	6897	10800.0
n30I	30	8	7510.0	5	100.0	12.4	65.2	71.6	713	3.5	4.9	868	4565	10800.0
n30J	30	8	6204.0	3	113.6	11.4	89.7	95.3	638	2.7	3.4	1000	3351	4142.6
n40A	40	8	12046.0	6	100.0	44.5	56.2	59.2	1204	10.5	12.7	1566	3811	10800.0
n40B	40	8	7989.0	4	100.0	37.9	73.2	78.9	780	6.3	7.1	1193	5771	10800.0
n40C	40	8	9431.0	5	100.0	71.8	72.1	78.1	1395	26.9	31.5	1525	1895	10800.0
n40D	40	8	8354.0	3	100.0	99.2	87.1	95.7	991	35.0	38.3	1187	1511	10800.0
n40E	40	8	11064.0	6	100.0	34.1	51.9	59.4	805	7.4	8.8	1166	4267	10800.0
n40F	40	8	9281.0	5	100.0	71.1	71.4	78.3	1395	17.5	19.7	1788	2557	10800.0
n40G	40	8	8150.0	4	104.4	31.1	88.8	93.9	800	9.5	11.1	1263	2953	10800.0
n40H	40	8	9224.0	5	100.0	34.5	70.3	73.2	945	9.4	11.5	1157	4329	10800.0
n40I	40	8	10135.0	4	100.0	45.5	64.8	71.2	879	10.7	11.9	1561	3809	10800.0
n40J	40	8	10608.0	5	100.0	35.7	56.1	61.5	1011	5.0	5.9	1313	5747	10800.0

Table EC.4 Detailed results on set S_A with $Q = 20$ and $T = 10$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3524.0	2	111.6	0.7	86.4	94.5	31	0.3	0.4	33	13	1.4
n10A	10	5	2936.0	2	117.6	0.6	84.7	87.3	24	0.3	0.4	24	5	1.1
n10b	10	5	3192.0	2	115.8	0.7	82.4	92.6	13	0.3	0.4	13	3	2.8
n10B	10	5	3587.0	2	100.8	0.8	86.4	88.5	36	0.3	0.4	36	3	1.2
n10c	10	5	3324.0	2	115.3	0.7	94.9	94.9	0	0.3	0.5	0	3	1.9
n10C	10	5	3013.0	2	118.2	1.0	82.0	91.1	32	0.3	0.5	32	3	2.4
n10d	10	5	4043.0	2	109.5	0.7	86.3	88.5	24	0.3	0.5	24	43	3.8
n10D	10	5	2988.0	2	104.0	0.7	84.6	97.9	23	0.3	0.4	23	3	1.3
n10e	10	5	3130.0	2	117.7	1.5	87.8	92.4	38	0.4	0.7	39	25	3.0
n10E	10	5	3617.0	2	100.0	0.8	95.1	99.6	57	0.3	0.5	57	3	1.4
n10f	10	5	3468.0	2	127.1	1.5	74.7	83.6	42	0.4	0.5	148	2613	160.1
n10F	10	5	3601.0	2	102.3	0.5	94.2	96.1	12	0.2	0.4	12	3	1.5
n10g	10	5	4179.0	2	108.1	0.9	77.2	83.1	32	0.3	0.4	90	2011	126.4
n10G	10	5	3763.0	2	129.8	0.6	84.3	88.6	12	0.3	0.4	12	3	0.8
n10h	10	5	3885.0	2	127.6	0.8	89.8	91.8	28	0.3	0.5	30	17	2.2
n10H	10	5	3600.0	2	114.3	1.0	78.9	85.5	34	0.3	0.5	37	89	4.9
n10i	10	5	2548.0	2	123.2	0.5	84.7	93.8	18	0.2	0.3	19	9	2.0
n10I	10	5	3963.0	2	133.9	0.9	82.8	86.3	24	0.3	0.5	25	49	4.4
n10j	10	5	3172.0	2	121.9	0.9	89.6	95.3	26	0.3	0.5	27	15	2.9
n10J	10	5	2992.0	1	100.0	0.5	80.6	100.0	20	0.2	0.4	20	1	0.9
n20A	20	5	4380.0	2	124.4	1.3	87.8	89.9	69	0.5	0.7	466	4431	1513.9
n20B	20	5	4980.0	3	110.1	2.8	86.0	92.8	76	1.0	1.2	177	1245	435.0
n20C	20	5	5453.0	3	122.1	3.4	89.6	91.1	165	0.8	1.1	1565	22505	9362.2
n20D	20	5	5021.0	3	108.6	2.3	91.9	96.4	118	0.9	1.1	127	433	73.8
n20E	20	5	4998.0	2	100.0	1.8	94.6	97.1	64	0.7	0.9	67	23	4.0
n20F	20	5	5031.0	3	124.5	4.6	88.5	93.5	111	1.1	1.4	114	87	16.4
n20G	20	5	5304.0	3	127.9	3.2	83.9	90.9	99	0.9	1.1	151	609	60.6
n20H	20	5	5312.0	2	113.9	2.2	91.5	92.0	41	0.6	0.7	105	143	148.3
n20I	20	5	4793.0	3	128.9	1.9	84.9	89.8	84	0.6	0.8	604	5179	1034.4
n20J	20	5	4139.0	2	103.6	1.2	91.3	98.6	52	0.4	0.6	55	21	5.1
n30A	30	5	6131.0	4	115.2	11.3	92.7	93.8	96	7.5	5.3	372	10261	5259.8
n30B	30	5	7278.0	5	100.0	4.3	74.3	76.5	88	1.4	1.6	959	10923	10800.0
n30C	30	5	6311.0	4	100.0	6.8	86.4	90.4	140	2.3	2.7	1127	12551	10800.0
n30D	30	5	6156.0	4	125.8	7.1	88.3	92.7	173	1.9	2.2	673	16181	10800.0
n30E	30	5	7884.0	4	100.0	5.7	65.9	70.3	149	1.6	1.9	678	4241	10800.0
n30F	30	5	7374.0	4	100.0	7.6	71.1	75.9	181	2.8	2.7	1394	16959	10800.0
n30G	30	5	7273.0	3	109.4	6.9	86.7	95.1	219	2.6	3.1	506	929	1865.2
n30H	30	5	6058.0	4	100.0	5.3	83.6	85.6	137	1.8	2.2	1284	15379	10800.0
n30I	30	5	8452.0	5	100.0	10.1	59.6	61.1	127	5.8	2.0	1223	16985	10800.0
n30J	30	5	7173.0	4	100.0	6.2	74.3	78.9	144	2.0	2.3	1247	14089	10800.0
n40A	40	8	8575.0	5	100.0	12.7	76.8	77.6	215	3.4	3.7	1155	10313	10800.0
n40B	40	8	9883.0	6	100.0	17.9	66.5	67.7	229	3.4	3.6	1221	10081	10800.0
n40C	40	8	6390.0	4	128.2	10.7	94.8	97.9	316	2.4	2.6	362	629	1570.4
n40D	40	8	10531.0	7	100.0	14.3	64.9	66.7	311	2.1	2.2	775	8337	10800.0
n40E	40	8	10573.0	8	100.0	12.6	59.1	62.3	286	2.2	2.5	643	10033	10800.0
n40F	40	8	10398.0	6	100.0	14.9	61.4	62.6	249	2.3	2.4	1077	8603	10800.0
n40G	40	8	10981.0	7	100.0	21.3	64.6	66.5	455	3.9	4.3	921	10263	10800.0
n40H	40	8	9871.0	7	100.0	17.8	66.7	67.6	330	2.9	3.1	1096	8609	10800.0
n40I	40	8	8394.0	5	100.0	15.3	75.9	78.2	319	2.3	2.5	663	6897	10800.0
n40J	40	8	6739.0	4	158.0	18.6	90.9	94.8	330	2.2	2.3	398	431	1441.6

Table EC.5 Detailed results on set S_A with $Q=20$ and $T=15$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3292.0	1	146.2	0.8	86.3	98.4	34	0.4	0.5	35	17	3.5
n10A	10	5	2471.0	1	146.3	0.3	94.8	99.3	35	0.1	0.2	35	3	3.3
n10b	10	5	2876.0	1	100.0	0.5	86.8	100.0	32	0.2	0.3	32	1	0.8
n10B	10	5	3066.0	1	100.0	0.6	93.1	100.0	44	0.2	0.3	44	1	0.9
n10c	10	5	2948.0	1	100.0	0.1	99.7	99.7	0	0.1	0.1	0	3	0.2
n10C	10	5	2674.0	1	142.9	0.7	85.5	99.5	60	0.3	0.4	60	3	1.8
n10d	10	5	3438.0	1	100.0	0.2	98.3	100.0	24	0.1	0.1	24	1	0.5
n10D	10	5	2885.0	1	134.0	0.6	79.4	98.8	34	0.2	0.2	34	9	2.8
n10e	10	5	2823.0	1	100.0	0.5	90.9	100.0	76	0.2	0.2	76	1	0.8
n10E	10	5	3569.0	1	100.0	0.3	94.4	100.0	56	0.1	0.2	56	1	0.5
n10f	10	5	2996.0	1	163.1	0.4	78.9	94.8	32	0.2	0.2	45	91	10.2
n10F	10	5	3315.0	1	100.0	0.2	94.8	100.0	18	0.1	0.1	18	1	0.4
n10g	10	5	3685.0	1	122.1	0.6	80.2	89.4	30	0.2	0.2	77	851	144.7
n10G	10	5	3115.0	1	100.0	0.3	86.9	100.0	25	0.1	0.1	25	1	0.5
n10h	10	5	3388.0	1	100.0	0.2	97.1	100.0	15	0.1	0.1	15	1	0.4
n10H	10	5	3104.0	1	255.3	0.4	82.6	92.3	34	0.1	0.2	38	119	12.1
n10i	10	5	2279.0	1	100.0	0.3	85.3	100.0	26	0.1	0.1	26	1	0.5
n10I	10	5	3287.0	1	174.7	0.4	89.9	98.0	27	0.2	0.2	29	21	3.7
n10j	10	5	2932.0	1	100.0	0.3	92.6	100.0	28	0.1	0.2	28	1	0.5
n10J	10	5	2992.0	1	100.0	0.2	69.0	100.0	25	0.1	0.1	25	1	0.4
n20A	20	5	4038.0	2	107.9	1.0	85.8	89.4	46	0.5	0.5	590	9653	3839.7
n20B	20	5	4662.0	2	138.5	3.9	80.1	90.8	92	1.1	1.2	1423	13979	10800.0
n20C	20	5	4845.0	2	147.3	5.0	90.8	94.9	156	1.8	2.1	189	1069	955.3
n20D	20	5	4801.0	2	108.1	3.1	88.4	98.5	182	1.2	1.4	185	67	93.5
n20E	20	5	4755.0	2	126.6	2.8	90.7	98.3	159	1.1	1.3	159	3	19.7
n20F	20	5	4620.0	2	153.3	3.5	88.0	93.8	81	1.3	1.4	100	241	134.3
n20G	20	5	4814.0	2	116.5	3.2	82.1	92.6	108	1.2	1.3	186	1973	1084.2
n20H	20	5	4869.0	2	179.4	3.3	87.9	89.7	123	0.5	0.6	201	521	244.4
n20I	20	5	4327.0	2	117.4	3.2	83.9	94.5	119	1.1	1.3	149	345	257.6
n20J	20	5	4098.0	2	112.0	2.2	82.3	92.8	61	1.0	1.3	70	127	25.8
n30A	30	5	5759.0	3	105.8	10.0	85.7	87.3	297	3.8	4.4	1118	10381	10800.0
n30B	30	5	5145.0	2	150.5	11.2	91.2	96.9	228	3.1	3.3	293	891	1515.4
n30C	30	5	6024.0	3	100.0	12.1	79.1	85.3	185	3.5	3.8	470	1061	10800.0
n30D	30	5	6953.0	4	100.0	12.3	66.4	73.2	265	3.0	3.4	523	1559	10800.0
n30E	30	5	5192.0	2	181.4	12.2	86.1	94.3	251	2.4	2.6	1014	6973	10800.0
n30F	30	5	8206.0	4	100.0	15.8	53.9	59.8	261	2.9	3.1	997	11295	10800.0
n30G	30	5	9029.0	4	100.0	12.9	63.9	72.1	384	4.2	4.8	798	3259	10800.0
n30H	30	5	4569.0	2	108.6	6.4	94.0	96.5	214	2.4	2.7	254	173	988.5
n30I	30	5	4847.0	2	126.1	5.3	89.1	92.2	163	1.8	2.0	970	3743	10800.0
n30J	30	5	5041.0	2	115.4	10.9	92.2	98.9	141	3.8	4.1	146	41	159.9
n40A	40	8	8620.0	5	100.0	26.9	65.5	66.8	311	6.9	7.6	1058	6943	10800.0
n40B	40	8	8349.0	5	130.1	33.7	65.8	68.9	248	7.4	7.7	976	5967	10800.0
n40C	40	8	5812.0	3	124.0	27.9	91.1	96.8	431	9.4	10.4	633	5581	10800.0
n40D	40	8	7757.0	5	122.5	36.7	75.1	81.8	325	7.8	8.4	600	5337	10800.0
n40E	40	8	8995.0	4	100.0	24.1	57.8	63.2	231	5.5	5.8	444	7159	10800.0
n40F	40	8	5914.0	3	127.5	28.1	92.2	97.0	273	6.2	6.5	448	1387	2652.4
n40G	40	8	9918.0	5	100.0	55.7	59.0	62.6	528	7.8	8.3	739	4379	10800.0
n40H	40	8	8879.0	5	100.0	40.0	61.4	63.3	309	8.7	9.3	753	5883	10800.0
n40I	40	8	11188.0	5	100.0	34.4	47.8	50.6	464	6.3	7.0	811	8427	10800.0
n40J	40	8	7330.0	4	100.0	37.8	30.7	74.3	194	12.1	13.1	237	1543	10800.0

Table EC.6 Detailed results on set S_B with $Q = 10$ and $T = 3600$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3670.0	1	102.3	0.5	88.7	97.0	46	0.2	0.4	46	3	1.1
n10A	10	5	3055.0	2	100.0	0.6	97.1	100.0	36	0.2	0.4	36	1	1.1
n10b	10	5	3538.0	2	111.0	0.9	91.1	97.4	26	0.3	0.5	27	29	3.5
n10B	10	5	4256.0	2	110.9	0.4	89.2	91.4	56	0.1	0.3	110	2161	92.7
n10c	10	5	4408.0	2	100.0	0.3	98.7	100.0	8	0.1	0.3	8	1	0.7
n10C	10	5	3492.0	2	100.0	0.5	91.0	99.1	30	0.2	0.4	39	3	0.8
n10d	10	5	4593.0	2	100.0	0.6	88.5	97.8	27	0.2	0.4	28	35	3.1
n10D	10	5	3273.0	2	147.3	0.8	82.0	97.3	68	0.2	0.4	76	13	1.3
n10e	10	5	4055.0	2	101.5	0.4	93.8	97.7	41	0.1	0.3	46	29	1.3
n10E	10	5	4876.0	2	100.0	0.4	96.3	100.0	81	0.2	0.5	81	1	0.9
n10f	10	5	3468.0	2	106.9	0.6	85.9	90.7	38	0.2	0.4	40	239	10.7
n10F	10	5	4320.0	2	114.7	0.7	89.7	96.0	60	0.2	0.4	67	83	3.2
n10g	10	5	4431.0	2	106.7	0.5	87.1	89.1	14	0.1	0.3	35	423	14.1
n10G	10	5	4790.0	2	104.3	0.4	89.5	92.0	27	0.2	0.3	38	185	5.2
n10h	10	5	4194.0	2	100.0	0.5	97.1	98.7	40	0.1	0.3	41	3	2.9
n10H	10	5	4037.0	2	120.5	0.9	81.4	93.7	63	0.3	0.5	73	167	15.1
n10i	10	5	2677.0	2	100.0	0.4	99.9	100.0	15	0.1	0.3	15	1	0.7
n10I	10	5	3783.0	1	163.5	0.7	90.1	93.1	24	0.3	0.5	24	3	1.3
n10j	10	5	3580.0	2	100.0	0.5	97.3	99.2	54	0.2	0.4	54	3	1.5
n10J	10	5	3604.0	3	100.0	0.7	82.8	93.8	51	0.2	0.4	62	113	4.3
n20A	20	5	5200.0	3	100.0	2.0	96.9	99.5	97	0.8	1.1	97	5	7.1
n20B	20	5	5552.0	3	117.5	3.5	88.0	94.8	100	1.2	1.6	233	1723	325.4
n20C	20	5	6740.0	3	122.7	5.5	93.4	97.2	352	1.7	2.1	476	1339	332.8
n20D	20	5	6745.0	3	100.8	2.3	95.8	97.2	186	0.7	1.2	296	465	82.1
n20E	20	5	6691.0	3	108.1	3.7	90.9	98.4	299	1.4	2.0	366	627	151.6
n20F	20	5	5930.0	4	103.1	3.9	92.9	94.4	102	1.7	1.7	158	373	71.6
n20G	20	5	6070.0	3	108.5	3.5	90.2	96.7	117	1.3	1.6	188	729	130.6
n20H	20	5	6886.0	3	134.6	7.7	89.3	91.3	240	2.2	1.4	522	3671	919.9
n20I	20	5	5724.0	3	113.6	3.9	88.9	94.0	202	1.2	1.3	413	3545	1512.3
n20J	20	5	5290.0	3	151.6	3.3	87.8	91.2	340	0.7	0.9	471	3223	467.0
n30A	30	5	7491.0	4	119.1	40.5	95.7	97.9	344	27.1	8.5	514	2437	1951.7
n30B	30	5	7446.0	4	136.8	18.1	89.4	96.3	575	8.0	4.6	821	1289	704.0
n30C	30	5	9096.0	5	100.0	33.9	77.2	81.3	431	18.0	6.9	1492	13481	10800.0
n30D	30	5	9114.0	5	100.0	29.1	74.3	77.3	654	15.5	6.9	1160	15117	10800.0
n30E	30	5	7454.0	4	101.6	12.7	88.2	94.1	240	6.4	4.7	531	14787	10730.9
n30F	30	5	8541.0	5	100.0	73.6	80.5	84.5	366	53.7	6.8	1308	11799	10800.0
n30G	30	5	10209.0	5	100.0	29.2	88.8	93.4	961	13.8	6.0	2029	11503	10800.0
n30H	30	5	8277.0	4	100.0	26.1	82.1	84.9	408	12.8	7.0	718	12059	10800.0
n30I	30	6	8587.0	6	100.0	10.3	75.5	76.9	197	4.6	3.8	665	17733	10800.0
n30J	30	5	9116.0	5	100.0	52.7	71.7	74.4	507	24.8	5.2	1339	15049	10800.0
n40A	40	8	11640.0	8	100.0	59.7	70.2	74.1	764	24.6	10.3	1402	11369	10800.0
n40B	40	8	10456.0	6	100.0	66.1	70.1	73.1	587	20.3	12.3	1452	7783	10800.0
n40C	40	8	8588.0	5	123.6	32.6	94.6	98.2	520	12.1	8.1	825	11301	10800.0
n40D	40	8	10174.0	5	100.0	43.7	83.1	87.9	581	12.8	12.0	984	5615	10800.0
n40E	40	8	12328.0	8	100.0	41.0	59.4	63.6	783	8.7	7.5	1585	8969	10800.0
n40F	40	8	10972.0	7	100.0	47.9	76.9	80.9	623	19.0	11.3	1529	7269	10800.0
n40G	40	8	12933.0	7	100.0	87.1	67.3	69.3	705	26.0	14.3	1203	6721	10800.0
n40H	40	8	11816.0	7	100.0	110.4	69.1	70.6	708	48.8	12.7	1282	9221	10800.0
n40I	40	8	11072.0	7	100.0	75.0	78.3	83.1	569	30.7	7.8	1413	11667	10800.0
n40J	40	8	10418.0	6	100.0	24.7	70.6	73.9	424	8.0	6.9	1053	9539	10800.0

Table EC.7 Detailed results on set S_B with $Q = 10$ and $T = 7200$

Name	n	$\|K\|$	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
n10a	10	5	3484.0	1	110.2	0.3	90.0	100.0	58	0.7	1.0	58	3	1.4
n10A	10	5	2994.0	1	100.0	0.4	96.2	100.0	28	0.3	0.6	28	1	1.4
n10b	10	5	3122.0	1	100.0	0.4	86.4	100.0	24	0.7	1.0	24	1	1.4
n10B	10	5	3671.0	1	100.0	1.1	96.2	100.0	63	0.5	0.8	63	1	2.1
n10c	10	5	4226.0	1	100.0	0.6	98.0	100.0	16	0.2	0.5	16	1	1.2
n10C	10	5	3380.0	1	122.5	1.6	83.9	97.8	122	0.8	1.2	133	29	16.8
n10d	10	5	4238.0	1	100.0	0.9	89.0	100.0	24	0.4	0.7	24	1	1.8
n10D	10	5	3071.0	1	100.0	1.0	79.3	100.0	80	0.9	1.3	80	1	2.5
n10e	10	5	3816.0	1	100.0	0.1	88.8	100.0	60	0.4	0.7	60	1	0.8
n10E	10	5	4828.0	1	100.0	0.7	90.7	99.9	88	0.6	1.0	88	3	1.6
n10f	10	5	2996.0	1	166.3	2.1	83.9	98.0	65	0.9	1.2	66	13	20.7
n10F	10	5	3758.0	1	100.0	0.9	95.1	100.0	44	0.4	0.7	44	1	1.7
n10g	10	5	3685.0	1	113.4	1.3	87.1	94.2	54	0.5	0.8	58	61	5.0
n10G	10	5	3325.0	1	100.0	0.7	96.0	100.0	12	0.3	0.5	12	1	1.3
n10h	10	5	3960.0	1	117.8	0.5	91.7	99.9	48	0.4	0.7	48	3	1.5
n10H	10	5	3813.0	1	100.8	2.0	79.8	95.1	48	1.5	2.0	69	235	27.2
n10i	10	5	2390.0	1	100.0	0.9	96.5	100.0	25	0.4	0.7	25	1	1.9
n10I	10	5	3287.0	1	250.7	1.6	90.7	97.4	30	0.8	1.1	31	5	3.0
n10j	10	5	3249.0	1	119.9	1.0	98.0	99.1	20	0.5	0.7	20	9	1.9
n10J	10	5	3060.0	1	100.0	0.6	71.6	100.0	31	0.2	0.5	31	1	1.4
n20A	20	5	4815.0	2	111.4	7.6	90.1	97.6	240	2.0	2.9	278	339	113.0
n20B	20	5	4992.0	2	111.2	9.8	83.1	96.5	188	4.7	5.7	194	85	41.1
n20C	20	5	6089.0	2	100.4	5.3	93.8	99.5	699	12.0	14.3	702	5	21.5
n20D	20	5	6149.0	2	105.2	10.0	94.1	99.2	316	3.9	5.1	321	23	31.2
n20E	20	5	6343.0	2	104.1	37.9	88.8	99.1	683	11.1	13.2	692	75	196.6
n20F	20	5	4973.0	2	126.7	9.4	93.7	97.1	407	2.5	3.6	421	141	78.0
n20G	20	5	5364.0	2	139.9	18.6	87.1	97.6	303	5.4	6.4	311	17	110.7
n20H	20	5	5847.0	2	117.2	19.4	87.1	95.5	426	4.9	6.1	533	933	924.3
n20I	20	5	4818.0	2	116.1	15.4	86.6	97.2	386	4.4	5.6	442	283	375.7
n20J	20	5	4402.0	2	134.6	3.7	91.5	96.6	140	1.3	2.0	203	49	121.0
n30A	30	5	8069.0	4	100.0	91.1	73.7	79.0	1559	29.7	33.1	1751	2157	10800.0
n30B	30	5	7645.0	4	100.0	83.8	78.0	84.1	1384	28.6	33.7	1419	519	10800.0
n30C	30	5	8326.0	4	100.0	122.8	70.2	77.1	867	21.3	24.3	1122	1127	10800.0
n30D	30	5	7654.0	3	100.0	108.0	71.5	79.9	973	22.3	25.4	1346	841	10800.0
n30E	30	5	7216.0	3	100.0	53.5	73.3	82.5	534	19.2	21.8	726	1353	10800.0
n30F	30	5	6310.0	3	117.1	29.6	88.4	95.5	398	11.6	13.1	692	4099	10800.0
n30G	30	5	9037.0	2	116.4	111.6	90.4	98.5	1318	41.4	47.1	1764	357	10800.0
n30H	30	5	6194.0	2	119.5	76.4	91.3	96.6	531	28.5	30.3	669	359	10800.0
n30I	30	5	5975.0	3	100.0	43.6	82.4	90.0	551	10.6	12.7	1022	3143	10800.0
n30J	30	5	5984.0	2	103.1	63.4	92.5	98.6	502	17.7	19.9	549	133	3383.8
n40A	40	8	9413.0	5	100.0	134.0	72.4	76.6	1192	29.7	34.2	1375	441	10800.0
n40B	40	8	7295.0	3	100.0	147.7	79.3	85.5	664	43.7	47.3	940	1629	10800.0
n40C	40	8	7901.0	4	100.0	138.5	87.0	93.5	1149	59.8	65.2	1559	1265	10800.0
n40D	40	9	10391.0	4	100.0	358.0	70.2	77.2	1093	117.1	125.7	1159	551	10800.0
n40E	40	8	9677.0	5	100.0	193.7	59.5	68.0	990	42.5	46.4	1361	1591	10800.0
n40F	40	8	9218.0	4	100.0	148.7	73.3	80.0	1101	47.7	51.4	1356	707	10800.0
n40G	40	8	8415.0	4	100.0	198.8	86.2	90.3	876	55.4	57.6	1031	817	10800.0
n40H	40	8	9239.0	5	100.0	124.6	71.0	73.9	1249	26.5	29.8	1436	1539	10800.0
n40I	40	8	9089.0	4	100.0	100.9	74.2	81.3	699	32.5	34.8	1212	761	10800.0
n40J	40	8	10075.0	5	100.0	202.2	57.8	64.3	866	55.8	58.6	1254	1963	10800.0

Table EC.8 Detailed results on set S_C with $|K| = 8$ and $T = 10$

Name	n	Q	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
1Bari30	13	30	13000.0	2	100.0	1.4	92.5	95.8	22	0.6	0.8	27	55	16.3
2Bari20	13	20	13000.0	2	100.0	1.6	92.5	98.4	28	0.8	1.0	28	9	4.3
3Bari10	13	10	14200.0	2	100.0	1.0	95.4	100.0	52	0.3	0.6	52	1	2.2
4ReggioEmilia30	14	30	15800.0	2	107.0	1.2	91.7	92.2	21	0.6	0.8	24	93	13.0
5ReggioEmilia20	14	20	15800.0	2	100.0	1.3	95.3	99.9	61	0.5	0.8	61	1	2.2
6ReggioEmilia10	14	10	20600.0	2	104.9	1.5	93.8	99.0	185	0.4	0.8	190	27	2.6
7Bergamo30	15	30	11300.0	3	100.0	1.5	95.4	100.0	32	0.6	0.8	32	1	2.6
8Bergamo20	15	20	11300.0	3	100.0	0.8	97.8	100.0	44	0.3	0.4	44	1	1.6
9Bergamo12	15	12	11600.0	3	100.0	0.7	98.6	100.0	15	0.1	0.3	15	1	1.1
10Parma30	15	30	24100.0	2	100.0	1.3	98.6	100.0	6	0.6	0.9	6	1	2.3
11Parma20	15	20	24100.0	2	100.0	1.1	98.7	100.0	4	0.6	0.8	4	1	2.2
12Parma10	15	10	24100.0	2	100.0	0.9	100.0	100.0	0	0.3	0.6	0	1	1.5
13Treviso30	18	30	24138.0	3	107.9	1.7	91.5	96.9	22	0.7	0.8	33	99	13.8
14Treviso20	18	20	24138.0	3	107.9	2.4	91.5	96.9	21	0.9	1.2	32	93	16.8
15Treviso10	18	10	24138.0	3	117.8	1.6	92.4	96.9	19	0.6	0.8	37	105	9.5
16LaSpezia30	20	30	19298.0	3	107.5	2.8	92.1	93.5	40	1.0	1.1	205	5441	2148.3
17LaSpezia20	20	20	19298.0	3	107.5	2.5	92.1	93.5	40	0.8	1.0	201	5371	1970.6
18LaSpezia10	20	10	19409.0	3	114.3	2.5	93.1	94.5	55	0.8	1.1	191	3487	539.7
19BuenosAires30	21	30	57020.0	3	103.5	3.9	95.9	98.3	302	0.8	1.2	517	1001	227.6
20BuenosAires20	21	20	64641.0	3	103.0	3.2	97.5	99.1	996	0.5	1.0	2570	585	332.2
21Ottawa30	21	30	17475.0	3	106.5	2.7	97.5	97.5	22	1.2	1.3	22	13	5.7
22Ottawa20	21	20	17475.0	3	106.5	2.4	97.5	97.5	22	0.9	1.1	22	13	5.4
23Ottawa10	21	10	17475.0	3	102.8	1.8	97.5	97.6	29	0.6	0.8	29	13	3.1
24SanAntonio30	23	30	19821.0	3	100.0	3.0	93.3	95.8	82	1.0	1.1	103	369	231.7
25SanAntonio20	23	20	19877.0	3	105.8	1.9	95.5	97.4	98	0.6	0.6	123	97	30.1
26SanAntonio10	23	10	24743.0	4	100.7	2.3	94.5	97.5	371	0.4	0.5	462	1299	449.7
27Brescia30	27	30	24300.0	4	105.3	3.3	93.0	96.9	84	0.8	0.8	551	7989	3210.2
28Brescia20	27	20	24600.0	4	106.1	3.1	92.7	96.1	82	0.6	0.7	931	15875	10800.0
29Brescia11	27	11	25700.0	4	102.3	2.5	94.5	98.1	214	0.4	0.5	510	9283	5546.7
30Roma30	28	30	52400.0	4	144.8	3.4	83.2	99.2	279	0.7	0.8	314	297	366.5
31Roma20	28	20	54200.0	4	100.0	5.0	86.3	100.0	477	1.2	1.7	477	1	7.1
32Roma18	28	18	55100.0	4	101.5	3.1	88.6	99.1	487	0.5	0.7	1335	2201	1574.0
33Madison30	28	30	36204.0	4	105.1	9.6	94.0	96.4	44	2.7	2.7	259	3749	3750.6
34Madison20	28	20	36204.0	4	105.1	13.1	94.0	96.4	44	4.3	4.7	171	2289	2202.6
35Madison10	28	10	38358.0	4	115.8	4.1	92.8	96.9	84	0.9	1.0	298	2189	942.6
36Guadalajara30	41	30	52083.0	8	100.0	59.3	82.3	86.9	53	48.0	20.1	472	2669	10800.0
37Guadalajara20	41	20	49196.0	7	100.0	16.0	87.2	92.0	80	6.0	3.9	191	615	10800.0
38Guadalajara11	41	11	50075.0	6	100.0	19.7	87.8	92.1	352	8.3	7.5	477	573	10800.0
39Dublin30	45	30	37357.0	6	100.0	45.1	84.2	85.9	210	21.4	12.4	1032	7789	10800.0
40Dublin20	45	20	36806.0	6	100.0	38.0	86.9	88.4	233	7.7	8.4	747	5989	10800.0
41Dublin11	45	11	41989.0	7	100.0	34.3	86.4	88.0	1890	6.3	8.4	2408	2367	10800.4
42Denver30	51	30	55212.0	7	100.0	320.7	84.9	89.5	412	160.2	42.2	536	1729	10800.0
43Denver20	51	20	52669.0	7	100.0	167.2	89.9	94.1	321	20.5	21.2	396	319	10800.0
44Denver10	51	10	57707.0	8	100.0	323.3	89.9	96.2	881	225.1	78.4	985	1323	10800.0
45RioDeJaneiro30	55	30	136797.0	7	100.0	1864.2	91.3	92.1	331	1641.7	508.8	718	3171	10800.0
46RioDeJaneiro20	55	20	145392.0	8	100.0	1594.6	87.2	88.0	1493	1312.4	290.2	1831	4349	10800.0
47RioDeJaneiro10	55	10	162198.0	8	100.0	690.4	94.3	97.0	9328	414.3	150.5	9546	227	10800.0

Table EC.9 Detailed results on set S_C with $|K| = 8$ and $T = 15$

Name	n	Q	z^*	rt	$\%UB$	t_{UB}	$\%LB$	$\%LB_r$	cut_r	t_{PA}	t_r	cut	$node$	t
1Bari30	13	30	12000.0	1	100.0	2.2	94.7	100.0	12	1.3	1.4	12	1	3.7
2Bari20	13	20	12700.0	1	100.0	0.9	90.8	100.0	24	0.5	0.5	24	1	1.5
3Bari10	13	10	14200.0	2	100.0	0.6	95.1	100.0	57	0.1	0.1	57	1	1.1
4ReggioEmilia30	14	30	13100.0	1	100.0	0.9	100.0	100.0	5	0.4	0.4	5	1	1.4
5ReggioEmilia20	14	20	15800.0	2	131.6	0.6	91.6	99.6	81	0.2	0.2	81	9	2.5
6ReggioEmilia10	14	10	20300.0	2	100.0	0.8	94.9	100.0	135	0.3	0.4	135	1	1.3
7Bergamo30	15	30	11300.0	2	100.0	1.1	94.5	100.0	42	0.2	0.2	42	1	1.6
8Bergamo20	15	20	11300.0	3	100.0	0.4	97.3	100.0	38	0.1	0.1	38	1	0.9
9Bergamo12	15	12	11600.0	3	100.0	0.3	98.6	100.0	21	0.0	0.1	21	1	0.7
10Parma30	15	30	24100.0	2	100.0	1.0	97.2	100.0	16	0.3	0.3	16	1	1.4
11Parma20	15	20	24100.0	2	100.0	0.7	97.7	100.0	16	0.2	0.2	16	1	0.9
12Parma10	15	10	24100.0	2	100.0	0.2	100.0	100.0	1	0.0	0.1	1	1	0.3
13Treviso30	18	30	22837.0	2	100.0	4.5	91.2	100.0	31	1.4	1.5	31	1	1.5
14Treviso20	18	20	22837.0	2	100.0	2.6	91.2	100.0	27	1.1	1.2	27	1	3.8
15Treviso10	18	10	22837.0	2	100.0	0.8	93.1	100.0	24	0.3	0.4	24	1	1.3
16LaSpezia30	20	30	17882.0	2	107.1	7.8	91.1	93.7	38	3.9	4.1	622	10583	10800.0
17LaSpezia20	20	20	17874.0	2	107.1	4.3	91.2	93.7	42	1.7	1.8	644	11873	10800.0
18LaSpezia10	20	10	17863.0	2	116.9	2.2	94.3	96.6	58	0.9	1.0	100	809	443.3
19BuenosAires30	21	30	56898.0	3	105.4	2.9	95.2	98.0	309	0.6	0.7	937	3737	2720.6
20BuenosAires20	21	20	64087.0	3	100.0	1.9	98.2	99.9	577	0.5	0.8	598	5	4.1
21Ottawa30	21	30	16198.0	2	107.9	6.7	93.8	94.3	32	2.2	2.3	116	617	702.2
22Ottawa20	21	20	16198.0	2	109.3	5.0	93.8	94.3	32	1.7	1.7	105	487	491.2
23Ottawa10	21	10	16196.0	2	100.0	2.6	94.2	95.9	67	1.0	1.1	88	187	148.2
24SanAntonio30	23	30	17279.0	2	100.0	10.0	92.9	99.2	139	4.1	4.3	140	9	29.7
25SanAntonio20	23	20	17581.0	2	100.0	3.9	97.1	100.0	151	1.5	1.6	151	1	5.8
26SanAntonio10	23	10	23893.0	3	100.0	3.5	97.0	99.9	370	0.9	1.0	370	1	4.7
27Brescia30	27	30	22900.0	3	112.7	11.2	93.3	98.4	78	3.0	3.1	199	491	3061.8
28Brescia20	27	20	23300.0	3	105.2	9.0	93.2	97.6	104	3.3	3.8	343	6221	6753.2
29Brescia11	27	11	25100.0	3	102.0	5.1	93.8	99.0	293	0.9	1.1	471	2813	2535.5
30Roma30	28	30	51800.0	3	106.4	8.5	77.6	99.6	278	2.3	2.7	305	125	464.8
31Roma20	28	20	54200.0	4	100.0	5.3	84.4	100.0	371	1.1	1.4	371	1	7.1
32Roma18	28	18	54800.0	3	103.5	6.3	87.2	99.4	532	0.7	0.9	1627	4741	10800.0
33Madison30	28	30	32407.0	3	107.8	27.4	92.3	95.3	92	8.8	8.9	347	1967	10800.0
34Madison20	28	20	32407.0	3	107.6	15.8	92.3	95.4	91	5.2	5.3	339	2053	7695.5
35Madison10	28	10	34815.0	2	122.7	8.6	90.6	95.6	103	2.5	2.7	525	2197	3579.2
36Guadalajara30	41	30	43867.0	4	101.1	39.9	90.5	96.9	59	14.2	14.2	199	1563	10800.0
37Guadalajara20	41	20	43963.0	4	100.9	30.0	90.8	96.7	70	9.4	9.5	408	2987	10800.0
38Guadalajara11	41	11	44411.0	4	108.5	34.7	93.2	98.6	738	7.3	8.8	856	451	3364.2
39Dublin30	45	30	36032.0	5	100.0	74.2	75.6	77.5	259	10.7	10.9	687	3721	10800.0
40Dublin20	45	20	36841.0	5	100.0	46.0	77.2	79.4	661	7.9	8.7	994	4509	10800.0
41Dublin11	45	11	39408.0	5	100.0	63.5	87.8	90.6	2055	8.8	13.6	2960	1871	10800.0
42Denver30	51	30	50270.0	5	100.0	193.2	84.6	91.6	340	37.8	39.0	426	499	10800.0
43Denver20	51	20	48122.0	5	100.0	127.4	90.5	96.7	486	24.0	25.0	687	1553	10800.0
44Denver10	51	10	53320.0	7	100.0	61.4	90.9	99.8	2276	13.1	17.3	2714	541	10800.0
45RioDeJaneiro30	55	30	119839.0	5	100.0	608.8	85.6	87.6	618	88.7	90.6	751	1013	10800.0
46RioDeJaneiro20	55	20	124209.0	5	100.0	345.4	86.7	89.0	2256	61.3	66.8	2632	1483	10800.0
47RioDeJaneiro10	55	10	156826.0	6	100.0	1495.8	96.1	99.6	15515	191.8	313.7	15528	79	10800.0

Table EC.10 Detailed results of the pricing algorithms on set S_A

				EXM pricing algorithm						Pricing algorithm based on Casazza et al. (2019)					
Name	n	$\|K\|$	LB	t_{LP}	t_{PA}	fw	fwd	bw	bwd	t_{LP}	t_{PA}	fw	fwd	bw	bwd
n10a	10	5	3210.46	0.22	0.12	1295	7699	1153	7182	0.25	0.12	10116	179826	3841	68628
n10A	10	5	2920.76	0.07	0.06	1289	7886	800	4615	0.09	0.08	13319	248455	5481	106743
n10b	10	5	2687.58	0.06	0.05	1611	10768	708	4274	0.06	0.05	7421	95630	4787	60123
n10B	10	5	3540.84	0.04	0.03	1096	6971	862	5552	0.08	0.07	13907	295130	6385	130118
n10c	10	5	4183.67	0.04	0.04	1153	7460	716	4586	0.06	0.05	9525	181410	3767	67273
n10C	10	5	2878.65	0.04	0.04	1242	7641	895	5536	0.07	0.06	13383	276098	5247	103073
n10d	10	5	3897.11	0.04	0.03	994	5904	837	5400	0.05	0.04	9599	165563	4193	67251
n10D	10	5	2609.49	0.03	0.03	862	5677	533	3309	0.07	0.06	13227	252539	5442	99873
n10e	10	5	3403.77	0.04	0.03	1300	7867	847	4962	0.07	0.06	13018	279380	5027	114377
n10E	10	5	4489.26	0.03	0.02	1253	7763	829	5005	0.09	0.08	13725	266647	5237	108301
n10f	10	5	2656.90	0.04	0.04	1380	8916	1154	7203	0.04	0.04	10434	147948	4935	63680
n10F	10	5	3662.62	0.05	0.04	1055	6473	1046	6358	0.08	0.07	21457	418085	8588	157857
n10g	10	5	3417.47	0.04	0.04	1336	8495	1026	6297	0.03	0.03	10652	148853	4986	66678
n10G	10	5	3435.92	0.05	0.04	1588	10325	1082	6433	0.06	0.05	14029	255705	8463	145785
n10h	10	5	3683.04	0.02	0.02	728	4769	571	3674	0.06	0.04	11629	256757	5990	129142
n10H	10	5	3196.66	0.04	0.04	1797	12068	922	5400	0.06	0.05	12486	210277	6164	93828
n10i	10	5	2424.54	0.03	0.03	1128	7155	843	5213	0.05	0.04	11756	248329	5727	114724
n10I	10	5	3325.31	0.04	0.04	1423	8685	936	5687	0.06	0.05	15879	270575	8381	120506
n10j	10	5	3247.21	0.05	0.03	1422	8486	1350	8644	0.04	0.03	11742	197631	5351	79767
n10J	10	5	2481.71	0.02	0.02	1032	5650	715	3775	0.06	0.06	14687	276466	7171	132925
n20A	20	5	4536.44	0.08	0.08	2517	30782	1929	22382	0.19	0.17	44854	1484500	13808	454721
n20B	20	5	4531.57	0.13	0.10	3702	48873	2588	31230	0.19	0.17	35617	1029506	20913	595797
n20C	20	5	5984.86	0.14	0.13	3813	56161	2887	41968	0.29	0.26	56568	2245901	17478	671402
n20D	20	5	6003.32	0.10	0.09	2623	36310	2003	27690	0.23	0.21	54882	2262819	16502	637700
n20E	20	5	5753.30	0.08	0.07	2668	33141	1886	23269	0.31	0.29	70617	2881158	17137	702416
n20F	20	5	4839.56	0.18	0.17	4108	59754	3143	45840	0.27	0.25	47092	1588739	16122	531393
n20G	20	5	4982.84	0.11	0.10	3753	51035	2844	37447	0.31	0.30	53745	1866876	20101	679978
n20H	20	5	5325.79	0.12	0.11	3973	52095	2260	27363	0.23	0.21	43700	1563580	19655	666687
n20I	20	5	4440.42	0.11	0.10	3398	45527	2909	37963	0.28	0.26	57772	2295937	22516	810477
n20J	20	5	4184.48	0.05	0.05	2273	26140	1698	18294	0.16	0.15	39773	1245442	14243	418189
n30A	30	5	6349.58	1.71	1.62	56385	1426499	6406	134728	3.53	3.13	674153	30257204	67189	3300935
n30B	30	5	6330.89	0.27	0.22	7745	146530	4611	82087	0.89	0.80	130008	5974514	38371	1680991
n30C	30	5	6286.25	0.37	0.35	10154	218637	6581	132497	1.56	1.49	240993	12669772	62448	3285250
n30D	30	5	6205.94	0.36	0.33	10744	239889	5613	116114	1.29	1.23	173304	9847364	60720	2959403
n30E	30	5	5904.85	0.26	0.24	8251	160423	6124	112176	1.10	1.06	163593	8165108	55175	2530636
n30F	30	5	5934.98	0.48	0.45	14419	337181	6904	143155	1.33	1.26	194338	9641139	63077	2903879
n30G	30	5	8583.22	0.29	0.27	9805	209045	5063	99174	1.57	1.47	229395	12397868	52668	2784856
n30H	30	5	6144.99	0.30	0.28	10497	224716	6127	117241	1.14	1.09	169089	8154504	54269	2490230
n30I	30	5	5463.43	0.30	0.27	10430	219272	6274	117921	1.11	1.06	176453	9357450	58018	2964312
n30J	30	5	6188.78	0.31	0.29	10456	223443	6168	119002	0.82	0.77	148975	6980344	50911	2440887
n40A	40	8	7461.64	0.68	0.65	17652	527935	12711	358630	3.80	3.49	366716	27379934	116736	7955340
n40B	40	8	6776.35	0.74	0.71	20899	598727	14351	386624	2.76	2.63	279112	17943842	107500	6347114
n40C	40	8	7406.27	0.51	0.48	14760	399686	9593	248351	2.35	2.23	264213	18144923	70780	4709741
n40D	40	8	7957.55	0.54	0.51	14140	395622	10712	281881	2.11	1.97	237273	16154355	81492	5157916
n40E	40	8	6633.23	0.25	0.24	8010	208214	5212	125564	1.62	1.54	207328	13982110	81558	4851519
n40F	40	8	7375.05	0.65	0.62	16476	477856	12356	329648	2.30	2.19	272667	18757723	85861	5394279
n40G	40	8	8118.96	0.66	0.64	17376	531241	12127	345382	2.23	2.13	257001	16443458	86367	5484495
n40H	40	8	7386.82	0.69	0.66	18667	570597	11026	306135	2.35	2.23	268078	17299706	101216	6273152
n40I	40	8	7417.07	0.50	0.47	12483	360401	8509	223161	2.17	2.08	281657	19028251	77194	4929908
n40J	40	8	6842.06	0.45	0.43	13259	374194	8015	212394	1.94	1.85	261514	16575702	80910	5112481

Table EC.11 Detailed results of the pricing algorithms on set S_A-3

				EXM pricing algorithm						Pricing algorithm based on Casazza et al. (2019)					
Name	n	$\|K\|$	LB	t_{LP}	t_{PA}	fw	fwd	bw	bwd	t_{LP}	t_{PA}	fw	fwd	bw	bwd
n10a	10	5	3210.46	0.02	0.02	1199	7386	990	6133	0.66	0.51	75532	4059582	24419	1256555
n10A	10	5	2920.76	0.02	0.01	1060	6708	648	3856	0.43	0.41	81000	4058653	30545	1897964
n10b	10	5	2687.58	0.02	0.02	1448	9340	733	4431	0.24	0.22	44033	1678240	26549	984305
n10B	10	5	3540.84	0.02	0.01	970	6308	702	4417	0.48	0.47	95236	5825666	41190	2586838
n10c	10	5	4183.67	0.02	0.01	967	6532	639	4027	0.47	0.45	93910	5146221	20951	1356481
n10C	10	5	2878.65	0.01	0.01	905	5602	623	4032	0.53	0.51	90407	5247109	34566	1975468
n10d	10	5	3897.11	0.02	0.01	1061	6439	920	6052	0.38	0.37	77417	3945299	13157	710748
n10D	10	5	2609.49	0.02	0.01	1082	6863	745	4552	0.47	0.46	102084	5496797	31631	1716018
n10e	10	5	3403.77	0.02	0.02	1211	7523	815	4822	0.45	0.44	79163	4648446	27979	1912797
n10E	10	5	4489.26	0.02	0.01	1149	6955	736	4398	0.38	0.37	82734	4526282	25140	1465485
n10f	10	5	2656.90	0.03	0.02	1632	10352	1429	8673	0.35	0.34	88298	3493301	28483	1038123
n10F	10	5	3662.62	0.02	0.02	1151	7080	1185	7314	0.72	0.71	138380	8332747	46622	2396069
n10g	10	5	3417.47	0.02	0.02	1301	8050	831	5246	0.26	0.23	72538	2929255	31682	1252324
n10G	10	5	3435.92	0.02	0.02	1400	9066	981	5769	0.45	0.44	95400	4584860	48731	2340879
n10h	10	5	3683.04	0.01	0.01	818	5200	626	4048	0.55	0.54	94824	6436342	43833	2825061
n10H	10	5	3196.66	0.02	0.02	1881	12641	936	5511	0.33	0.32	87921	3880680	45893	2026163
n10i	10	5	2424.54	0.02	0.01	1183	7595	864	5261	0.45	0.44	87300	5057647	39376	2090985
n10I	10	5	3325.31	0.02	0.02	1603	10212	979	6196	0.52	0.51	123629	6148698	49886	1994902
n10j	10	5	3247.21	0.02	0.02	1321	8176	1345	8720	0.26	0.25	73333	3599945	23962	1053735
n10J	10	5	2481.71	0.01	0.01	796	4306	562	3021	0.44	0.43	107725	5591096	44922	2548576
n20A	20	5	4536.44	0.06	0.05	2737	32882	2145	25037	3.32	3.28	443912	43121024	124897	11695197
n20B	20	5	4531.57	0.09	0.08	3922	51239	2709	32940	2.64	2.58	307507	25273854	168025	13241426
n20C	20	5	5984.86	0.13	0.12	3989	58026	3020	43086	4.95	4.87	565810	63771825	114221	13001572
n20D	20	5	6003.32	0.07	0.06	2689	37956	2163	29695	5.92	5.84	578622	69047130	135188	14708974
n20E	20	5	5753.30	0.07	0.06	2923	36847	1903	23505	5.21	5.15	686398	83361354	125350	14754613
n20F	20	5	4839.56	0.10	0.09	4155	59390	3658	52231	3.81	3.76	443544	44722604	152581	14203382
n20G	20	5	4982.84	0.08	0.07	4047	54324	3217	42035	4.17	4.13	536860	52707442	177177	17278303
n20H	20	5	5325.79	0.07	0.06	3532	45894	2072	25121	3.24	3.19	374961	38549588	156528	14992064
n20I	20	5	4440.42	0.07	0.07	3328	44704	2607	34185	5.14	5.10	572084	68727871	201839	20558300
n20J	20	5	4184.48	0.04	0.04	2143	24359	1489	16151	2.07	2.04	361871	32684808	97298	7832545
n30A	30	5	6349.58	1.19	1.13	46855	1179426	6052	132026	42.08	41.11	7518902	964230906	439074	62481993
n30B	30	5	6330.89	0.21	0.18	7002	132197	4389	78259	8.41	8.26	1469045	182658863	319257	38852489
n30C	30	5	6286.25	0.32	0.29	9675	212354	5699	114102	15.29	15.15	2355257	355641230	498665	75243238
n30D	30	5	6205.94	0.32	0.30	10316	229184	5475	114658	11.86	11.74	1607706	269818564	515406	71030674
n30E	30	5	5904.85	0.25	0.23	7987	156519	5355	97746	10.13	10.05	1596974	235981030	431013	57006355
n30F	30	5	5934.98	0.46	0.43	13708	320136	6365	133788	11.68	11.56	1701316	258495307	511701	67573123
n30G	30	5	8583.22	0.29	0.27	9517	202617	5031	98396	12.72	12.59	1932139	300326473	371877	56362814
n30H	30	5	6144.99	0.32	0.30	10801	230428	5794	113199	9.71	9.62	1547122	217369106	459550	59098578
n30I	30	5	5463.43	0.30	0.27	10299	217349	6051	113593	11.37	11.26	1685890	254942114	486980	71570948
n30J	30	5	6188.78	0.29	0.27	9814	207544	6262	120676	8.62	8.53	1421050	188666310	404595	52421910
n40A	40	8	7461.64	0.73	0.70	17656	533450	12360	348809	32.09	31.90	3514200	782776318	1063877	206802523
n40B	40	8	6776.35	0.73	0.70	19495	563717	13126	351033	22.18	21.98	2601061	489332814	908405	146955703
n40C	40	8	7406.27	0.59	0.56	16273	434869	11233	292509	21.63	21.43	2585768	499430889	597004	113821299
n40D	40	8	7957.55	0.60	0.57	15459	428859	11699	316665	20.91	20.72	2387709	485151456	659014	117801454
n40E	40	8	6633.23	0.32	0.30	9851	252473	6772	163729	17.82	17.70	2119375	413324121	746104	125654665
n40F	40	8	7375.05	0.68	0.65	16628	482509	12205	329692	25.17	24.96	2940877	596251755	835060	149749272
n40G	40	8	8118.96	0.72	0.69	18442	554633	13638	386288	22.20	21.99	2557231	483219065	691324	128098557
n40H	40	8	7386.82	0.66	0.64	18278	555759	11512	324029	20.42	20.23	2255555	430892215	803073	141456285
n40I	40	8	7417.07	0.56	0.53	13459	385454	8880	235315	23.79	23.61	2809435	568052227	795548	142507998
n40J	40	8	6842.06	0.54	0.51	14167	397045	9328	245420	20.76	20.60	2683575	490000582	718190	128810720