

Arcflow formulations and constraint generation frameworks for the two-bar charts packing problem – online supplement

A Pseudo-codes for graph construction

Algorithm 1 gives a pseudo-code for the graph construction procedure for the arcflow model for the 2D-VPP. This algorithm incorporates some of the symmetry reduction techniques introduced by Brandão and Pedroso (2016). The same graph is also used for the first steps of Eulerian-flow and link-flow.

Algorithm 1 Graph construction for the 2D-VPP arcflow model

Require: ordered w_i, q_i ($i = 1, 2, \dots, n$), c

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1:  $V \leftarrow \{(0, 0)\}; V' \leftarrow V; A \leftarrow \emptyset$ 
2: for  $i = 1, 2, \dots, n$  do ▷ for every bar chart type
3:   for all  $(v_1, v_2) \in V$  do ▷ for every potential tail (or active vertex)
4:      $(d_1, d_2) \leftarrow (v_1, v_2)$  ▷ define the arc tail
5:     for  $k = 1, 2, \dots, q_i$  do ▷ for every unit of demand
6:        $(e_1, e_2) \leftarrow (d_1, d_2) + (w_{i1}, w_{i2})$  ▷ define the arc head
7:       if  $e_1 \leq c$  and  $e_2 \leq c$  then ▷ if the head is feasible
8:          $V' \leftarrow V' \cup \{(e_1, e_2)\};$  ▷ save the new head
9:          $A \leftarrow A \cup \{((d_1, d_2), (e_1, e_2))\}$  ▷ create the arc
10:         $(d_1, d_2) \leftarrow (e_1, e_2)$  ▷ the new tail is now the old head
11:    $V \leftarrow V'$  ▷ add every head added for bar chart  $i$  to the set of active vertices

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Ensure: $G = (V, A)$

Algorithm 2 gives a pseudo-code for the graph construction procedure for Eulerian-flow. Instead of creating a graph from scratch for each vertex in V^h , it “shifts” the first dimension of both the tail and the head of every arc contained in A^α by v for every $v \in V^h$. Note that appropriate data structures (such as sets) should be used for vertex and arc sets to avoid duplicates.

Algorithm 2 Graph construction for Eulerian-flow

Require: ordered w_i, q_i ($i = 1, 2, \dots, n$), c

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1: apply Algorithm 1 to obtain graph  $G^\alpha = (V^\alpha, A^\alpha); V^\gamma \leftarrow V^\alpha; A^\gamma \leftarrow A^\alpha; V^h \leftarrow \emptyset$ 
2: for all  $(v_1, v_2) \in V^\alpha : v_2 \neq 0$  do ▷ for every vertex with non-zero second dimension
3:    $V^h \leftarrow V^h \cup \{v_2\}; V^\gamma \leftarrow V^\gamma \cup \{(v_2, 0)\}$  ▷ save distinct  $v_2$  values
4: for all  $v \in V^h$  do ▷ for every  $v_2$  value
5:   for all  $((d_1, d_2), (e_1, e_2)) \in A^\alpha$  do ▷ for every existing arc
6:     if  $e_1 + v \leq c$  then ▷ if the head of the “shifted” arc is feasible
7:        $V^\gamma \leftarrow V^\gamma \cup \{(d_1 + v, d_2)\} \cup \{(e_1 + v, e_2)\}$  ▷ add the new tail/head in the set of vertices
8:        $A^\gamma \leftarrow A^\gamma \cup \{((d_1 + v, d_2), (e_1 + v, e_2))\}$  ▷ add the new arc in the set of arcs
9: for all  $(v_1, v_2) \in V^\gamma$  do ▷ for every existing vertex
10:   $A^\gamma \leftarrow A^\gamma \cup \{((v_1, v_2), (v_2, 0))\}$  ▷ create a transition arc

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Ensure: $G^\gamma = (V^\gamma, A^\gamma)$

B Size and continuous relaxation values of the tested ILP models on data sets U-GEN, U-SMA, U-MED, and U-BIG

We report in Table 8 some supplementary information about the tested models on data set U-GEN. Column “# opt” indicates the number of times the continuous relaxation value of the model rounded up was equal to the optimal solution value. Column “diff” reports the difference between the optimal solution value and the continuous relaxation value of the model (which is itself reported in column “LP”). Columns “# var” and “# cons” indicate the average number of variables and constraints in the models.

Table 8: Size and continuous relaxation values of the tested models on data set U-GEN

c	Ω	ERZIN						EUL-REL						LINK-REL					
		# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons
50	10	3	1.4	0	9.7	133	33	10	0.3	0	10.8	421	226	10	0.3	0	10.8	94	77
	50	0	3.0	0	50.5	2990	167	10	0.4	0.2	53.1	8002	1752	10	0.4	0	53.1	1657	746
	100	0	4.3	0.1	103.0	11 752	334	10	0.4	0.8	106.9	19 644	2239	10	0.4	0.1	106.9	6301	1521
	500	1	6.6	6.3	512.0	254 592	1572	10	0.3	6.8	518.3	129 492	2915	10	0.3	3	518.3	68 258	2773
	1000	0	14.7	38.4	1023.5	905 410	3025	10	0.4	13.9	1037.8	243 906	3320	10	0.4	6.7	1037.8	149 645	3263
100	10	2	1.5	0	10.1	141	35	9	0.3	0	11.3	371	205	9	0.3	0	11.3	84	74
	50	0	3.1	0	50.6	3114	171	10	0.1	1.3	53.6	19 833	5273	10	0.1	0	53.6	2500	1205
	100	0	4.0	0.1	101.6	11 751	332	10	0.2	7.9	105.4	57 676	7462	10	0.2	0.5	105.4	12 782	3734
	500	0	7.3	7.1	503.5	267 597	1581	10	0.4	82	510.4	468 857	10 120	10	0.4	33.2	510.4	208 137	9179
	1000	0	10.2	48.2	1008.0	1 026 480	3109	10	0.4	215.9	1017.8	1 022 159	10 754	10	0.4	79.8	1017.8	515 538	10 275

We gather in Tables 9 and 10 some supplementary information about the tested models on data sets U-SMA, U-MED, and U-BIG.

Table 9: Size and continuous relaxation values of the tested models on data set U-SMA

c	Ω	ERZIN						EUL-REL						LINK-REL					
		# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons
50	10	10	0.7	0	1.3	17	11	10	0	0.1	2.0	4550	1170	10	0	0	2.0	659	295
	50	10	0.4	0	6.0	164	35	10	0.4	1.6	6.0	32 474	2152	10	0.4	0.9	6.0	20 421	1786
	100	10	0.4	0	11.8	347	51	10	0.4	2	11.8	41 377	2298	10	0.4	1.2	11.8	29 738	2051
	500	10	0.6	0	60.5	1719	156	10	0.6	2.2	60.5	44 170	2352	10	0.6	1.2	60.5	33 626	2129
	1000	10	0.5	0	120.2	3463	290	10	0.5	2	120.2	44 170	2352	10	0.5	1.2	120.2	33 626	2129
100	10	9	0.8	0	1.2	21	13	10	0	0.5	2.0	15 384	3482	10	0	0	2.0	1429	647
	50	10	0.5	0	5.5	271	52	10	0.5	33.7	5.5	232 575	8733	10	0.5	15.0	5.5	148 691	7365
	100	10	0.5	0	10.9	780	87	10	0.5	68.2	10.9	428 992	9360	10	0.5	34.0	10.9	319 982	8531
	500	10	0.4	0.1	54.9	6044	219	10	0.4	128.8	54.9	722 866	9734	10	0.4	64.6	54.9	592 475	9266
	1000	10	0.3	0.2	110.0	12 120	339	10	0.3	135.2	110.0	734 934	9751	10	0.3	51.9	110.0	609 910	9301

Table 10: Size and continuous relaxation values of the tested models on data sets U-MED (top) and U-BIG (bottom)

c	Ω	ERZIN						EUL-REL						LINK-REL					
		# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons
50	10	0	2.1	0	11.3	162	38	10	0.1	0	13.3	135	85	10	0.1	0	13.3	53	51
	50	0	4.7	0	57.0	3508	189	10	0.2	0	61.5	2787	970	10	0.1	0	61.6	491	308
	100	0	7.6	0.1	113.1	13 571	369	10	0.2	0.2	120.6	9559	1641	10	0.2	0	120.6	2017	807
	500	0	30.4	8.8	575.1	302 301	1796	10	0.3	1.8	605.2	61 092	2593	10	0.3	0.6	605.2	21 053	2095
	1000	0	53.2	50.9	1147.8	1 071 532	3470	10	0.4	4.7	1200.6	130 781	3088	10	0.4	2.1	1200.6	61 558	2790
100	10	0	2.3	0	11.1	158	38	10	0.2	0	13.2	174	107	10	0.2	0	13.2	64	60
	50	0	4.2	0	55.5	3462	185	10	0.3	0.1	59.4	6451	2498	10	0.3	0	59.4	724	456
	100	0	7.1	0.1	111.4	13 465	367	10	0.3	0.3	118.2	19 557	4510	10	0.3	0	118.2	2387	1213
	500	0	29.5	10.5	565.7	322 551	1810	10	0.4	20.3	594.8	204 405	8401	10	0.4	3.8	594.8	47 117	5428
	1000	0	53.8	80.2	1136.6	1 243 855	3579	10	0.3	46.9	1190.1	475 345	9531	10	0.3	13.9	1190.1	141 764	7392
50	10	0	3.1	0	12.6	183	42	10	0	0	15.7	80	57	10	0	0	15.7	41	43
	50	0	12.8	0.1	63.4	4161	214	10	0	0	76.2	1065	524	10	0	0	76.2	215	183
	100	0	25.1	0.2	126.5	15 923	421	10	0	0	151.6	3182	1040	10	0	0	151.6	503	359
	500	0	120.2	9.3	638.2	347 154	2031	10	0	0.3	758.4	20 596	2114	10	0	0	758.4	4030	1238
	1000	0	243.9	67.1	1270.2	1 201 931	3923	10	0	0.7	1514.1	41 526	2577	10	0	0.2	1514.1	11 405	1818
100	10	0	2.6	0	12.3	182	42	10	0	0	14.9	93	64	10	0	0	14.9	45	47
	50	0	13.8	0.1	64.3	4311	218	10	0	0	78.1	1299	691	10	0	0	78.1	234	209
	100	0	22.9	0.2	123.6	16 019	417	10	0	0	146.5	5456	2216	10	0	0	146.5	640	466
	500	0	122.6	12.0	631.6	381 932	2066	10	0	1.0	754.2	56 728	6089	10	0	0.1	754.2	5908	2393
	1000	0	248.4	88.9	1262.5	1 453 501	4062	10	0	3.1	1510.9	138 577	7574	10	0	0.4	1510.9	16 726	3621

C Size and continuous relaxation values of the tested ILP models on data sets TRIPLETS and DONUTS

We gather in Tables 11 and 12 some supplementary information about the tested models on data sets TRIPLETS and DONUTS, respectively.

Table 11: Size and continuous relaxation values of the tested models on data set TRIPLETS

c	\bar{z}	ERZIN					EUL-REL					LINK-REL							
		# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons
80	20	10	0	0	20	727	77	10	0	0.1	20	5871	1690	10	0	0	20	1597	713
	50	10	0	0	50	3951	185	10	0	0.2	50	19974	2438	10	0	0.1	50	7224	1427
	100	10	0	0.2	100	13826	355	10	0	0.7	100	40870	2731	10	0	0.4	100	15569	1758
	250	10	0	1	250	62973	809	10	0	1.7	250	81472	2975	10	0	1.2	250	36008	2075
240	20	10	0	0	20	736	77	10	0	0.1	20	12044	5271	10	0	0	20	2161	1202
	50	10	0	0.1	50	4482	193	10	0	1.1	50	79751	13271	10	0	0.4	50	19134	5366
	100	10	0	0.2	100	17461	384	10	0	7.9	100	264103	18184	10	0	5.3	100	81688	9204
	250	10	0	2	250	103207	943	10	0	49.1	250	874338	22012	10	0	27.3	250	292389	12989
400	20	10	0	0	20	748	78	10	0	0.1	20	14448	6834	10	0	0	20	2215	1310
	50	10	0	0.1	50	4559	195	10	0	2	50	114947	27394	10	0	0.3	50	24055	9018
	100	10	0	0.2	100	17925	388	10	0	17.3	100	475069	40289	10	0	11.3	100	120840	17747
	250	10	0	2.1	250	108130	959	10	0	155.7	250	2138746	56063	10	0	143.5	250	662809	30044

Table 12: Size and continuous relaxation values of the tested models on data set DONUTS

c	\bar{z}	ERZIN					EUL-REL					LINK-REL							
		# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons	# opt	diff	time	LP	# var	# cons
80	20	0	1	0	26	865	88	0	1	0	26	3309	978	0	1	0	26	1041	421
	50	0	1	0	62	2985	185	0	1	0.1	62	5308	1158	0	1	0	62	1797	535
	100	0	1	0.1	122	7235	335	0	1	0.1	122	6716	1251	0	1	0	122	2404	602
	250	0	1	0.3	302	20850	767	0	1	0.1	302	7918	1306	0	1	0	302	3007	657
240	20	0	1	0	26	1162	97	0	1	0.1	26	12723	4141	0	1	0	26	3018	1305
	50	0	1	0.1	62	5982	226	0	1	0.7	62	46597	7095	0	1	0.3	62	12313	2600
	100	0	1	0.2	122	19164	417	0	1	1.9	122	91755	8441	0	1	0.9	122	25698	3399
	250	0	1	1.6	302	81096	934	0	1	5	302	175471	9449	0	1	2	302	53536	4181
400	20	0	1	0	26	1248	100	0	1	0.2	26	19897	7479	0	1	0	26	3977	1917
	50	0	1	0.1	62	6707	235	0	1	1.7	62	92399	15477	0	1	0.6	62	22918	5107
	100	0	1	0.3	122	23700	448	0	1	7.3	122	240432	19947	0	1	3.5	122	61989	7356
	250	0	1	2.7	302	118046	1034	0	1	31.1	302	618861	23808	0	1	11.8	302	169696	9806