Online Supplement for "Network Migration Problem: A Hybrid Logic-based Benders Decomposition Approach"

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Appendix A: Detailed Models

A.1. Compact Formulation For the NMP

With the parameters and decision variables described in Sections 2.2 and 3.3, the compact formulation for the NMP modeled as a CP is as follows:

$$
\begin{aligned}
\min\quad & \textsc{cost} \sum_{w\in\mathcal{W}} \sum_{r\in\mathcal{R}} \sum_{t\in\mathcal{T}_{rw}} \Delta^{rt}_{\text{SHIFT}} & \\
\text{s.t.}\quad & \sum_{w\in\mathcal{W}} m_{ss'w} \geq \phi_{ss'} & \{s,s'\}\in\mathcal{S}_p, s<s' \\
& m_{ss'w} = m_{s'sw} & \{s,s'\}\in\mathcal{S}_p, s<s', w\in\mathcal{W} \\
& \sum_{\{s,s'\}\in\mathcal{S}_p} m_{ss'w} \leq 2\eta^{\text{CIR}} & w\in\mathcal{W} \\
& \text{NoOverlap}(\textsc{seq}_{rt}, T) & t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W} \\
& \text{Span}(\textsc{wtime}_{rt}, \{x_{rts} : s\in\mathcal{S}_r\}) & t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W} \\
& \text{IfThen}(\text{PresenceOf}(\textsc{wtime}_{rt}), \text{EndOf}(\textsc{wtime}_{rt}) \leq \Delta^{rt}_{\text{SHIFT}}) & t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W} \\
& \text{PresenceOf}(x_{rts}) = \text{PresenceOf}(n^{\text{S}}_{rts}) & s\in\mathcal{S}_r, t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W} \\
& \text{IfThen}(\text{PresenceOf}(x_{rts}), \text{LengthOf}(x_{rts}) = \theta n^{\text{S}}_{rts}) & s\in\mathcal{S}_r, t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W} \\
& \sum_{s':\{s,s'\}\in\mathcal{S}_p} n^{\text{SP}}_{rtss'} = n^{\text{S}}_{rts} & s\in\mathcal{S}_r, t\in\mathcal{T}_{rw}, r\in\mathcal{R}, w\in\mathcal{W}
\end{aligned}
$$

$$n^{\text{SP}}_{rtss'} = n^{\text{SP}}_{rts's} \qquad \{s,s'\} \in \mathcal{S}_p, t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W}$$

$$\sum_{r\in\mathcal{R}} \sum_{t\in\mathcal{T}_{rw}} n^{\text{SP}}_{rtss'} = \overline{m}_{ss'w} \qquad \{s,s'\} \in \mathcal{S}_p, w \in \mathcal{W}$$

$$\sum_{r\in\mathcal{R}} \sum_{t\in\mathcal{T}_{rw}} \text{PresenceOf}(\text{WTIME}_{rt}) \le \alpha^{\text{ENG}} \eta^{\text{ENG}} \qquad w \in \mathcal{W}.$$

As stated in the main text, the problem can also be formulated as a MIP, which includes a large number of loop elimination constraints.

A.2. Auxiliary MIP Pricing Problem

In addition to $n_{ss'}$ and n_{CIR} described in Section 2.2, we define the new decision variables in Table 1. Denote

Table 1 New decision variables for the CG subproblem

Variable	Type	Description
$h_{ss'}$	Binary	$=1$ if the technician migrates at least one circuit endpoint in site s with the other endpoint in site s'.
h_s	Binary	$=1$ if the technician works in site s in this shift, 0 otherwise.
$t_{ss'}$	Binary	$=1$ if a travel from site s to site s' occurs in the shift under construction, 0 otherwise.
x_δ	Binary	$=1$ if the length of the shift under construction is equal to Δ_δ, 0 otherwise.

by $\boldsymbol{\pi}^{(5b)}, \overline{\boldsymbol{\pi}}^{(5c)}, \boldsymbol{\pi}^{(5d)}$, the optimal dual solutions associated with constraints (5b), (5c), (5d), respectively. The pricing problem generating a shift for (r,w) is:

$$\text{SP}'^{\text{CG}}_{rw} = \min \ \text{COST}\Delta_{\text{SHIFT}} - \sum_{s\in\mathcal{S}_r} \sum_{s'\in\mathcal{S}} n_{ss'} \overline{\pi}^{(5b)}_{ss'} - \overline{\pi}^{(5c)}_{rw} - \overline{\pi}^{(5d)}_{w} \tag{1a}$$

$$\text{s.t.} \quad h_{ss'} + h_{s's} \le 1 \qquad s,s' \in \mathcal{S}_r, s \ne s' \tag{1b}$$

$$h_s \le \sum_{s'\in S} h_{ss'} \le M h_s \qquad s \in \mathcal{S}_r \tag{1c}$$

$$h_{ss'} \le n_{ss'} \le M h_{ss'} \qquad s \in \mathcal{S}_r, s' \in S, s \ne s' \tag{1d}$$

$$\sum_{s\in\mathcal{S}_r} t_{s_{\text{SRC}},s} = 1, \qquad \sum_{s\in\mathcal{S}_r} t_{s,s_{\text{DST}}} = 1 \tag{1e}$$

$$t_{ss'} \le \frac{1}{2}(h_s + h_{s'}) \qquad s,s' \in \mathcal{S}_r, s' > s \tag{1f}$$

$$\sum_{\substack{s'\in\mathcal{S}_r^+ \\ s'>s}} t_{ss'} + \sum_{\substack{s'\in\mathcal{S}_r^+ \\ s'<s}} t_{s's} = 2h_s \qquad s \in \mathcal{S}_r \tag{1g}$$

$$\sum_{s\in\mathcal{S}_r} \sum_{\substack{s'\in\mathcal{S}_r \\ s'>s}} t_{ss'} = \sum_{s\in\mathcal{S}_r} h_s - 1 \tag{1h}$$

$$\theta \sum_{s\in S_r} \sum_{\substack{s'\in S \\ s\ne s'}} n_{ss'} + \sum_{s\in S_r} \sum_{\substack{s'\in S_r \\ s'>s}} T_{ss'} t_{ss'} \le \Delta_{\text{SHIFT}} \tag{1i}$$

$$\sum_{\delta\in\Delta} x_\delta = 1 \tag{1j}$$

$$\sum_{\delta\in\Delta} \Delta_\delta x_\delta = \Delta_{\text{SHIFT}} \tag{1k}$$

$$n_{ss'}, h_{ss'} \in \mathbb{Z}_+, h_s \in \{0,1\} \qquad s \in \mathcal{S}_r, s' \in \mathcal{S}, s \ne s' \tag{1l}$$

where $M \geq 0$ is a large number and $\mathcal{S}^+ = \mathcal{S} \cup \{s_{\text{SRC}}, s_{\text{DST}}\}$, $\mathcal{S}_r^+ = S_r \cup \{s_{\text{SRC}}, s_{\text{DST}}\}$. s_{SRC} and s_{DST} are two dummy sites introduced as the beginning and end of a path. Objective function (1a) is the reduced-cost. Constraints (1b) ensure that at most one of the endpoints of every circuit $c \in \mathcal{C}_{ss'}$ can be migrated in every shift. Constraints (1c) determine the sites where a technician works in the current shift. Constraints (1d) assure that all migrated circuit endpoints are from the sites where the technician works in the current shift. Thanks to constraints (1e), generated path in this configuration starts form dummy site s_{SRC} and ends in dummy site s_{DST}. Constraints (1f) guarantee that if a travel occurs between the two sites s and s', the technician works in both. Constraints (1g) determine the sites visited before and after every site $s \in \mathcal{S}_r$. Constraint (1h) ensures that we have a path linking all visited sites. Constraint (1i) makes sure that the shift duration does not exceed the maximum predefined duration. Constraints (1j) and (1k) together determine the duration of the shift. The rest of the constraints determine the variables domain.

Appendix B: Detailed Numerical Results

Table 2 Characteristics of instances

Dataset	Instance 1 Endpoints	1 $\lvert\mathcal{S}_p\rvert$	2 Endpoints	2 $\lvert\mathcal{S}_p\rvert$	3 Endpoints	3 $\lvert\mathcal{S}_p\rvert$	4 Endpoints	4 $\lvert\mathcal{S}_p\rvert$	5 Endpoints	5 $\lvert\mathcal{S}_p\rvert$	6 Endpoints	6 $\lvert\mathcal{S}_p\rvert$
EUNetworks5	66	27	60	26	46	22	66	26	66	30	66	32
EUNetworks6	70	30	72	32	84	31	88	34	80	33	74	28
EUNetworks7	98	40	84	36	94	34	80	35	90	39	86	35
EUNetworks8	116	41	94	40	110	43	102	38	106	42	108	39
EUNetworks9	110	46	150	44	112	45	112	41	104	42	104	42
NextGen5	54	26	52	23	56	26	74	31	66	31	56	26
NextGen6	86	39	62	26	88	34	80	34	96	39	74	35
NextGen7	82	36	86	35	90	35	92	37	92	35	78	31
NextGen8	122	46	126	47	108	44	92	35	118	44	98	40
NextGen9	134	45	126	51	106	41	124	45	114	45	98	44
Pionier5	78	38	82	39	70	35	90	39	98	43	74	36
Pionier6	112	48	112	48	106	51	106	47	96	40	104	42
Sago5	76	35	64	29	88	39	74	33	82	35	90	39
Sago6	86	37	94	40	94	40	80	32	86	38	90	40
Savvis5	72	35	98	42	102	41	80	36	54	25	86	40
Savvis6	104	40	90	40	84	40	106	41	88	38	94	40
Savvis7	120	46	106	44	88	37	114	47	114	45	92	39
VisionNet5	86	40	100	47	104	45	88	38	102	44	82	36
VisionNet6	116	52	106	51	120	52	120	47	126	52	110	49

Table 3 Impact of the CG subproblem formulation on the overall performance of the method

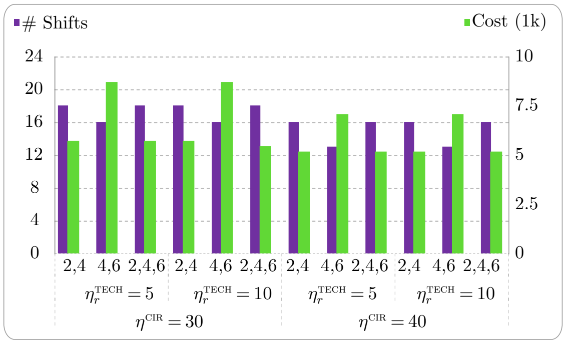
Dataset	CP-sub Time (s) CG	CP-sub Time (s) MP^{LBBD}	CP-sub Gap (%)	MIP-sub Time (s) CG	MIP-sub Time (s) MP^{LBBD}	MIP-sub Gap (%)
EUNetworks5	4184.5	4192.2	0.0%	Timeout	Timeout	18.0%
NextGen5	304.2	305.8	0.0%	Timeout	Timeout	21.0%
Pionier5	Timeout	Timeout	47.4%	Timeout	Timeout	47.4%
Sago5	Timeout	Timeout	31.6%	Timeout	Timeout	52.5%
Savvis5	1219.7	1257.3	0.0%	Timeout	Timeout	26.0%
VisionNet5	Timeout	Timeout	50.0%	Timeout	Timeout	23.1%

Table 4 Breakdown of solution times

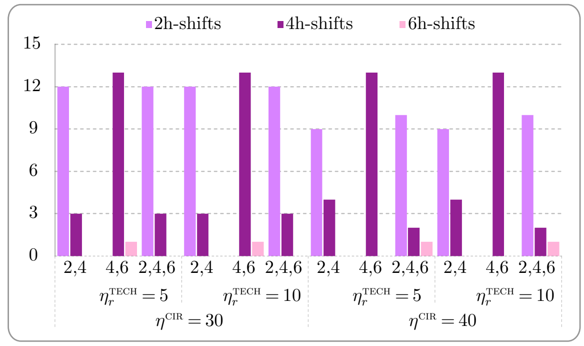
Dataset	Iterations		Sol Time		LBBD MP Time		CG Time		CP Time	
	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width
EUNetworks5	3.5	0.6	1124.1	268.6	548.2	198.3	540.1	196.0	575.8	147.6
EUNetworks6	10.5	4.3	4607.1	2309.7	2953.3	1929.8	2939.2	1928.9	1653.5	680.5
EUNetworks7	6.5	2.0	1459.0	509.7	45.7	23.4	27.6	13.7	1413.2	497.5
EUNetworks8	7.3	1.6	2582.6	539.9	693.8	271.9	531.6	225.2	1888.4	563.5
EUNetworks9	10.8	6.1	5109.8	3376.7	2201.9	1889.0	1384.5	1000.8	2907.4	1817.8
NextGen5	6.0	3.0	699.2	560.3	18.0	16.1	4.6	2.7	680.9	561.0
NextGen6	4.8	2.3	675.7	374.2	94.8	42.8	17.6	6.3	580.8	361.1
NextGen7	5.3	2.0	997.9	415.8	177.6	63.9	32.9	15.0	820.1	381.1
NextGen8	8.2	4.1	2445.6	399.6	1182.4	626.9	135.9	66.9	1262.8	433.9
NextGen9	8.0	2.9	3120.4	1939.1	1980.7	1546.1	67.1	42.6	1139.5	543.4
Pionier5	5.7	1.6	1817.2	796.4	812.9	505.1	726.2	475.5	1004.1	322.2
Pionier6	9.2	2.0	4929.4	1385.8	3267.2	1306.1	2961.9	1167.6	1661.8	272.9
Sago5	8.3	2.3	2387.6	697.1	836.1	293.6	748.0	319.1	1551.2	480.3
Sago6	5.7	2.3	3635.1	2640.4	2630.0	2194.2	2473.5	2132.5	1005.0	490.1
Savvis5	4.5	1.4	619.6	475.2	400.5	408.2	27.1	12.1	218.8	94.4
Savvis6	6.0	1.6	855.6	460.3	608.2	393.2	36.1	15.3	247.1	108.7
Savvis7	6.0	3.7	2472.6	2731.3	2184.8	2503.3	96.2	108.1	287.3	251.1
VisionNet5	5.8	3.5	2259.2	899.9	1217.2	523.3	1112.4	507.2	1041.8	541.1
VisionNet6	7.3	3.1	3308.0	769.4	1759.4	718.7	139.2	36.3	1548.3	996.6

Table 5 The number of cuts and columns generated during the solution process

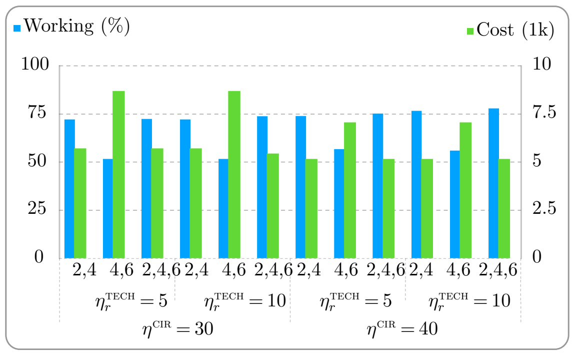
Dataset	$\#\text{Cut}^{\text{LBBD}}_{\text{OPT}}$		$\#\text{Cut}^{\text{LBBD}}_{\text{FEAS}}$		$\#\text{Cut}^{\text{BD}}_{\text{OPT}}$		#Columns	
	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width
EUNetworks5	121.8	35.3	27.3	11.2	33.0	11.6	408.8	180.2
EUNetworks6	566.8	273.0	84.0	30.5	79.2	43.0	951.2	375.3
EUNetworks7	361.2	101.1	66.3	44.1	76.8	36.8	1350.8	565.1
EUNetworks8	463.7	114.1	88.3	29.0	170.8	62.0	3693.8	1120.1
EUNetworks9	919.2	605.7	119.3	104.5	480.7	464.9	7318.7	4824.6
NextGen5	218.3	136.2	59.2	23.6	35.3	28.4	45.5	0.9
NextGen6	237.5	167.3	42.7	25.4	115.8	43.8	159.8	162.7
NextGen7	267.7	130.6	61.5	40.4	220.5	50.3	274.8	411.2
NextGen8	434.3	148.6	66.0	31.4	386.2	164.1	1405.5	646.1
NextGen9	622.3	317.5	103.7	32.8	666.5	405.6	1456.2	991.8
Pionier5	289.0	96.7	76.0	20.5	110.0	73.0	741.5	462.9
Pionier6	666.8	159.0	124.8	36.9	308.2	70.9	2101.8	1108.5
Sago5	476.8	185.9	78.3	36.1	232.7	165.6	1026.8	691.0
Sago6	307.7	161.8	37.0	17.9	365.5	260.5	1024.3	582.1
Savvis5	215.8	83.0	50.0	29.9	176.0	107.9	57.0	0.0
Savvis6	343.3	137.5	72.2	13.6	219.0	83.0	57.0	0.0
Savvis7	375.7	320.2	65.2	42.6	661.0	728.0	57.0	0.0
VisionNet5	385.5	322.0	63.3	43.9	136.7	53.6	1772.0	1091.4
VisionNet6	571.3	328.8	115.8	53.0	411.5	102.8	4607.8	1571.0



(a) # Shifts vs. cost

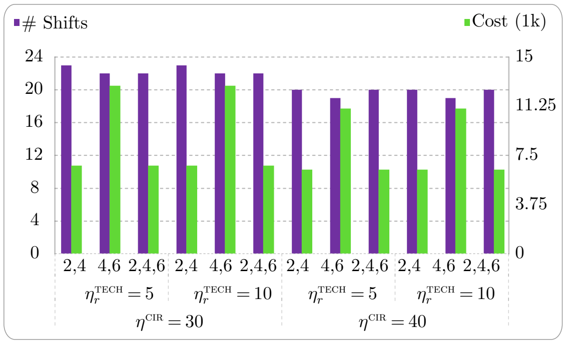


(b) Distribution of shifts

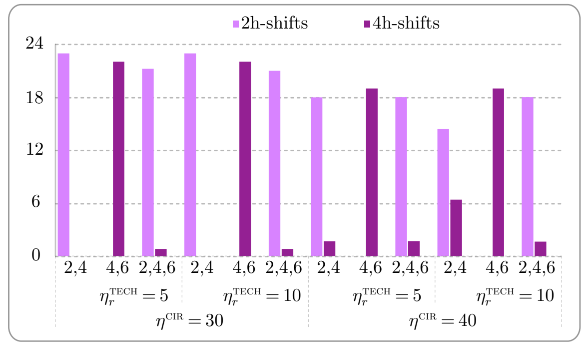


(c) Cost vs. efficient working time

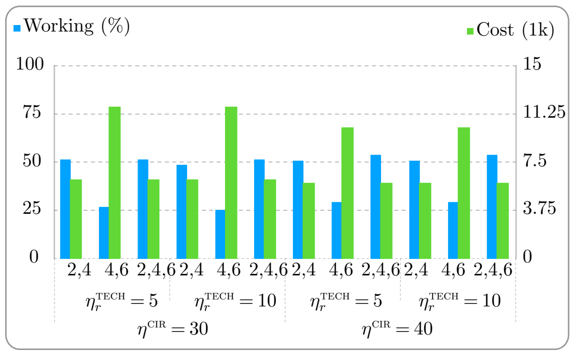
Figure 1 **Cost analysis for Sago**



(a) # Shifts vs. cost



(b) Distribution of shifts



(c) Cost vs. efficient working time

Figure 2 **Cost analysis for Savvis**