Appendix
A. Additional Proofs

Restatement of Lemma 3. In every optimal solution of model (10)-(22), we have $xp_{h+1} + T_1^* = 1$ and $q_2\left((1-x)p_{h+1} + T_2^*\right) = 1$.

Proof of Lemma 3. We prove the claim by contradiction. First, it is straightforward to see that at least one of the constraints (11)-(12) must be tight in any optimal solution of the model. Therefore, appropriately increasing T_1 or T_2 (or both) always leads to an increase in the objective function C^{MLPT}. To satisfy constraint (13), assume that in an optimal solution either: (i) $xp_{h+1} + T_1^* = 1$ and $q_2\left((1-x)p_{h+1} + T_2^*\right) < 1$, or (ii) $xp_{h+1} + T_1^* < 1$ and $q_2\left((1-x)p_{h+1} + T_2^*\right) = 1$. In case (i), we can increase T_2^* since the second expression is strictly less than 1. In case (ii), we can increase T_1^* since the first expression is strictly less than 1. In both cases, constraint (19) allows us to increase T_1 or T_2 without violating any other constraints, leading to a strictly better solution with higher C^{MLPT}, thereby contradicting the optimality of the original solution. $\square$

Restatement of Lemma 4. In every optimal solution of model (10)-(22) both constraints (11)–(12) are tight, i.e. $C^{MLPT} = p_{h+1} + T_1$ and $C^{MLPT} = q_2\left(p_{h+1} + T_2\right)$.

Proof of Lemma 4. The lemma is again proved by contradiction. As at least one of the constraints (11)-(12) is tight in an optimal solution of the model, we first assume that constraint (11) is tight while (12) is not tight. The critical machine is therefore the first machine, and we have

$$C^{MLPT} = p_{h+1} + T_1 < q_2\left(p_{h+1} + T_2\right).$$

We show how such a solution can be improved by increasing C^{MLPT} by an arbitrarily small amount, while keeping the last inequality satisfied. We can increase C^{MLPT} by increasing T_1 and T_2^* by an arbitrarily small quantity $\theta > 0$ such that $C^{MLPT} = p_{h+1} + T_1 + \theta \leq q_2\left(p_{h+1} + T_2\right)$, where q_2 decreases and is equal to $\frac{1}{(1-x)p_{h+1} + T_2^* + \theta}$ in order to satisfy $q_2\left((1-x)p_{h+1} + T_2^*\right) = 1$, in accordance with Lemma 3. We note that all the remaining constraints in the model are satisfied after these changes, implying a feasible improvement to the considered solution. We now assume that only constraint (12) is tight in an optimal solution of the model. The second machine is critical, and we have

$$C^{MLPT} = q_2\left(p_{h+1} + T_2\right) = \frac{p_{h+1} + T_2}{(1-x)p_{h+1} + T_2^*} < p_{h+1} + T_1,$$

since $q_2 = \frac{1}{(1-x)p_{h+1}+T_2^*}$ from Lemma 3. If $x = 0$, we can increase C^{MLPT} by decreasing p_{h+1} by θ and increasing q_2, with $q_2 = \frac{1}{p_{h+1}+T_2^*-\theta}$. Indeed, for $\theta > 0$, we have

$$\frac{p_{h+1}+T_2-\theta}{p_{h+1}+T_2^*-\theta} > \frac{p_{h+1}+T_2}{p_{h+1}+T_2^*},$$

since $C^{MLPT} = q_2\left(p_{h+1}+T_2\right) > q_2\left(p_{h+1}+T_2^*\right) = C^* = 1 \implies p_{h+1}+T_2 > p_{h+1}+T_2^*$, or else MLPT would not yield a worse solution than the optimal schedule. As before, we can choose a proper positive value of θ in order to obtain a feasible improvement to the considered solution. Similarly, if $x = 1$, we can increase C^{MLPT} by decreasing p_{h+1} and increasing T_1^* by θ satisfying $xp_{h+1}+T_1^* = 1$, as indicated in Lemma 3. We also decrease T_2^* by θ in order to avoid altering the sum $T_1^*+T_2^*$ in constraint (19), and we have $q_2 = \frac{1}{T_2^*-\theta}$ from Lemma 3. The decrease of T_2^* is possible because $T_2^* \geq n_2^* p_{h+1}$ and $n_2^* \geq 1$. As in the previous case, all constraints are satisfied for an arbitrarily small value of θ such that

$$\frac{p_{h+1}+T_2}{T_2^*} < \frac{p_{h+1}+T_2-\theta}{T_2^*-\theta} < p_{h+1}+T_1-\theta,$$

where the first inequality holds because $q_2\left(p_{h+1}+T_2\right) > q_2\left(T_2^*\right) \implies p_{h+1}+T_2 > T_2^*$. Therefore, we obtain again a feasible improvement to the initial solution. $\square$

B. Instances

B.1. LPT

The worst-case instances for $m = 2,3,4,5$ are those reported by Gonzalez et al. (1977) and Mitsunobu et al. (2024) ($m = 2$ and $m = 3,4,5$, respectively). Moreover, for $m = 6,7$, we confirm that the worst-case instances are those conjectured by Gonzalez et al. (1977). Let R_m be the unique positive root of the polynomial $P_m(x) = -2 - x - x^2 \ldots - x^{m-1} + 2x^m$. In the worst-case instances, for a given value of m, we have $p_j = R_m^j - \frac{1}{2}\sum_{k=0}^{j-1} R_m^k$ for $j = 1,\ldots,m+1$; $p_j = 0$ for $j = m+2,\ldots,\ell$; $q_1 = 1$ and $q_i = \frac{1}{p_{i-1}}$ for $i = 2,\ldots,m$.

For the remaining algorithms and for the considered values of m (and h for MLPT), we report only the strictly positive processing times and speed factors in the vectors $\vec{p}$ and $\vec{q}$, respectively. In general, all entries can be computed as the positive roots of specific polynomials. For conciseness, we report the corresponding approximate values below.

B.2. MLPT

$$m=2,h=3 \rightarrow \begin{cases} \vec{p} & = \{0.5917517, [0.4082483] \times 3\} \\ \vec{q} & = \{1, 1.22474483\} \end{cases}$$

$$m=2,h=4 \rightarrow \begin{cases} \vec{p} & = \{0.4069295, [0.29653525] \times 4\} \\ \vec{q} & = \{1, 1.68614017\} \end{cases}$$

$$m=2,h=5 \rightarrow \begin{cases} \vec{p} & = \{0.3050115, [0.23166412] \times 5\} \\ \vec{q} & = \{1, 2.15831509\} \end{cases}$$

$$m=2,h=6 \rightarrow \begin{cases} \vec{p} & = \{0.24169395, [0.18957651] \times 6\} \\ \vec{q} & = \{1, 2.63745754\} \end{cases}$$

$$m=2,h=7 \rightarrow \begin{cases} \vec{p} & = \{0.19905702, [0.1601886] \times 7\} \\ \vec{q} & = \{1, 3.12132083\} \end{cases}$$

$$m=3,h=4 \rightarrow \begin{cases} \vec{p} & = \{0.74741549, 0.55401839, [0.44598161] \times 3\} \\ \vec{q} & = \{1, 1.12112246, 1.3379439\} \end{cases}$$

$$m=3,h=5 \rightarrow \begin{cases} \vec{p} & = \{0.51950819, 0.35023387, [0.32488307] \times 4\} \\ \vec{q} & = \{1, 1.53901527, 1.92489747\} \end{cases}$$

$$m=3,h=6 \rightarrow \begin{cases} \vec{p} & = \{0.47872018, 0.37003906, [0.31498047] \times 5\} \\ \vec{q} & = \{1, 1.25992086, 1.58740001\} \end{cases}$$

$$m=3,h=7 \rightarrow \begin{cases} \vec{p} & = \{0.3629109, 0.25913569, [0.24695477] \times 6\} \\ \vec{q} & = \{1, 1.63970535, 2.02466225\} \end{cases}$$

$$m=4, h=5 \rightarrow \begin{cases} \vec{p} & = \{0.84239814, 0.71348973, 0.53281178, [0.46718822] \times 3\} \\ \vec{q} & = \{1, 1.07023246, 1.18708444, 1.40156186\} \end{cases}$$

$$m=4, h=6 \rightarrow \begin{cases} \vec{p} & = \{0.73397077, 0.57448677, 0.51142773, [0.42551335] \times 4\} \\ \vec{q} & = \{1, 1.06730172, 1.17505159, 1.36245345\} \end{cases}$$

We note that, for $m=2$, the instances correspond to those indicated in Section 3.3.

B.3. FFD

$$m=2 \rightarrow \begin{cases} \vec{p} & = \{0.59175168, [0.40824832] \times 3\} \\ \vec{q} & = \{1, 1.22474478\} \end{cases}$$

$$m=3 \rightarrow \begin{cases} \vec{p} & = \{[0.5] \times 2, [0.3201943] \times 4\} \\ \vec{q} & = \{1, [1.56155184] \times 2\} \end{cases}$$

$$m=4 \rightarrow \begin{cases} \vec{p} & = \{[0.53068686] \times 2, 0.46931314, [0.32753248] \times 5\} \\ \vec{q} & = \{1, 1.16520329, [1.52656615] \times 2\} \end{cases}$$

B.4. Reversed-FFD

$$m=2 \rightarrow \begin{cases} \vec{p} & = \{0.78077641, [0.5] \times 2\} \\ \vec{q} & = \{1, 1.28077641\} \end{cases}$$

$$m=3 \rightarrow \begin{cases} \vec{p} & = \{0.88367296, 0.72271429, [0.5] \times 2\} \\ \vec{q} & = \{1, 1.13164037, 1.38367266\} \end{cases}$$

$$m=4 \rightarrow \begin{cases} \vec{p} & = \{0.93266724, 0.83620178, 0.69799885, [0.5] \times 2\} \\ \vec{q} & = \{1, 1.07219376, 1.19588360, 1.43266712\} \end{cases}$$

B.5. Bidirectional-FFD

$$m = 2 \rightarrow \begin{cases} \vec{p} & = \{0.5, 0.40138809, [0.25] \times 3\} \\ \vec{q} & = \{1, 1.53518264\} \end{cases}$$

$$m = 3 \rightarrow \begin{cases} \vec{p} & = \{0.8164963, 0.59175185, [0.40824815] \times 3\} \\ \vec{q} & = \{1, [1.22474529] \times 2\} \end{cases}$$

$$m = 4 \rightarrow \begin{cases} \vec{p} & = \{[0.5] \times 3, [0.32019412] \times 4\} \\ \vec{q} & = \{1, [1.56155271] \times 2, 2\} \end{cases}$$

B.6. FFD + LPT

$$m = 2 \rightarrow \begin{cases} \vec{p} & = \{[0.5] \times 2, [0.35355357] \times 2\} \\ \vec{q} & = \{1, 1.41421284\} \end{cases}$$

$$m = 3 \rightarrow \begin{cases} \vec{p} & = \{0.69564243, [0.39128218] \times 2, 0.30435757, [0.26765418] \times 2\} \\ \vec{q} & = \{1, 1.27784977, 1.86808217\} \end{cases}$$

B.7. Bidirectional-FFD + LPT

$$m = 2 \rightarrow \begin{cases} \vec{p} & = \{0.56433304, [0.34650059] \times 2, [0.21783348] \times 2\} \\ \vec{q} & = \{1, 1.4430004\} \end{cases}$$

$$m = 3 \rightarrow \begin{cases} \vec{p} & = \{0.81649665, 0.59175168, [0.40824832] \times 3\} \\ \vec{q} & = \{1, [1.22474477] \times 2\} \end{cases}$$