

Electronic Companion to: *A preconditioned augmented Lagrangian method for solving semidefinite programming problems*

EC.1. Proof of Theorem 1

In the following analysis, we always assume $k \geq 2$. From (19) and (20), we have that

$$\partial_z g(z^k) - \mathcal{H}^*(\gamma^k) = \left(\partial_X \delta_{\mathbb{S}_+^n}(X^k) + \nabla f(X^k) - \mathcal{A}^*(\lambda^k) - \mathcal{B}^*(\mu^k), \partial_y \delta_{\mathcal{P}}(y^k) + \mu^k \right). \quad (\text{EC.1})$$

Because $X^k = R^k R^{k\top}$, $X^k \in \mathbb{S}_+^n$. From (14) and the dual update of μ in Algorithm 2, we know that

$$0 \in \partial_y \delta_{\mathcal{P}}(y^k) + \mu^k. \quad (\text{EC.2})$$

From Assumption 2 and the dual update in Algorithm 2, we have that

$$\text{dist} \left(\partial_X \delta_{\mathbb{S}_+^n}(X^k) + \nabla f(X^k) - \mathcal{A}^*(\lambda^k) - \mathcal{B}^*(\mu^k), 0 \right) < \epsilon_{k-1}. \quad (\text{EC.3})$$

Combining (EC.2) and (EC.3), we have that $\text{dist}(\partial_z g(z^k) - \mathcal{H}^*(\gamma^k), 0) \rightarrow 0$. Thus, we only have to prove that $\|\mathcal{H}(z^k) - b\|_2 \rightarrow 0$. The equations (EC.2) and (EC.3) imply that there exist $v^k \in \mathbb{X}$ such that

$$v^k \in \partial_z g(z^k) - \mathcal{H}^*(\gamma^k), \quad \|v^k\|_2 < \epsilon_{k-1}. \quad (\text{EC.4})$$

From (EC.4) and the convexity of g , we have that

$$g(z^{k+1}) \geq g(z^k) + \langle v^k + \mathcal{H}^*(\gamma^k), z^{k+1} - z^k \rangle. \quad (\text{EC.5})$$

Define

$$L_k := g(z^k) - \langle \gamma^k, \mathcal{H}(z^k) - c \rangle. \quad (\text{EC.6})$$

From (EC.5), we have that

$$\begin{aligned} L_{k+1} - L_k &\geq -\langle \gamma^{k+1}, \mathcal{H}(z^{k+1}) - c \rangle + \langle \gamma^k, \mathcal{H}(z^k) - c \rangle + \langle v^k + \mathcal{H}^*(\gamma^k), z^{k+1} - z^k \rangle \\ &= \langle \gamma^k - \gamma^{k+1}, \mathcal{H}(z^{k+1}) - c \rangle + \langle v^k, z^{k+1} - z^k \rangle. \end{aligned} \quad (\text{EC.7})$$

From the dual update of Algorithm 2, we have that

$$\begin{aligned} \langle \gamma^k - \gamma^{k+1}, \mathcal{H}(z^{k+1}) - c \rangle &= \langle \lambda^k - \lambda^{k+1}, \mathcal{A}(X^{k+1}) - b \rangle + \langle \mu^k - \mu^{k+1}, \mathcal{B}(X^{k+1}) - y^{k+1} \rangle \\ &= \beta_1 \|\mathcal{A}(X^{k+1}) - b\|_{W^k}^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2 \\ &\geq \beta_1 \sigma \|\mathcal{A}(X^{k+1}) - b\|_2^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2, \end{aligned} \quad (\text{EC.8})$$

where the last inequality follows from that $W^k \geq \sigma I_m$. Substituting (EC.8) into (EC.7), we get

$$L_{k+1} \geq L_k + \beta_1 \sigma \|\mathcal{A}(X^{k+1}) - b\|_2^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2 + \langle v^k, z^{k+1} - z^k \rangle. \quad (\text{EC.9})$$

By applying (EC.9) from $k = 2$ to $K \in \mathbb{N}^+$, we get

$$L_{K+1} \geq L_2 + \sum_{k=2}^K \left(\beta_1 \sigma \|\mathcal{A}(X^{k+1}) - b\|_2^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2 + \langle v^k, z^{k+1} - z^k \rangle \right). \quad (\text{EC.10})$$

From Assumption 1, we have that

$$g(z^*) = g(z^*) - \langle \gamma^{K+1}, \mathcal{H}(z^*) - c \rangle. \quad (\text{EC.11})$$

From (EC.4), we have that

$$g(z^*) - \langle \gamma^{K+1}, \mathcal{H}(z^*) - c \rangle \geq g(z^{K+1}) - \langle \gamma^{K+1}, \mathcal{H}(z^{K+1}) - c \rangle + \langle v^{K+1}, z^* - z^{K+1} \rangle. \quad (\text{EC.12})$$

Using the definition (EC.6) and (EC.11), (EC.12) can be simplified as

$$g(z^*) \geq L_{K+1} + \langle v^{K+1}, z^* - z^{K+1} \rangle. \quad (\text{EC.13})$$

Substituting (EC.10) into (EC.13), we get

$$\begin{aligned} & g(z^*) - \langle v^{K+1}, z^* - z^{K+1} \rangle \\ & \geq L_2 + \sum_{k=2}^K \left(\beta_1 \sigma \|\mathcal{A}(X^{k+1}) - b\|_2^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2 + \langle v^k, z^{k+1} - z^k \rangle \right). \end{aligned} \quad (\text{EC.14})$$

Because $\{z^k\}$ is bounded, there exists $M > 0$ such that $\|z^k\|_2 < M$ for any $k \in \mathbb{N}^+$. Using this as well as $\|v^k\|_2 \leq \epsilon_{k-1}$ in (EC.14), we get

$$\begin{aligned} & g(z^*) - L_2 + \epsilon_K (\|z^*\| + M) + 2M \sum_{k=1}^{K-1} \epsilon_k \\ & \geq \sum_{k=2}^K \left(\beta_1 \sigma \|\mathcal{A}(X^{k+1}) - b\|_2^2 + \beta_2 \|\mathcal{B}(X^{k+1}) - y^{k+1}\|_2^2 \right). \end{aligned} \quad (\text{EC.15})$$

From the summability of $\{\epsilon_k\}$ in Assumption 2, the left hand side of (EC.15) is bounded. Therefore, we have that $\|\mathcal{H}(z^k) - c\|_2 \rightarrow 0$. This completes the proof of Theorem 1.

EC.2. Usage and data structures of SDPF+

In this Section, we describe the usage and data structures of SDPF+. Although the constraint $\mathcal{H}(X) \in \mathcal{P}$ in (SDPG) encompasses many types of convex constraints, the implementation of SDPF+ includes several special cases that can be specified by users more conveniently.

Our solver can handle the following multi-block version of (SDPG):

$$\begin{aligned} & \min f(X) \\ & \text{s.t. } \mathcal{A}(X) = b, \quad l \leq \mathcal{B}(X) \leq u, \\ & \quad X \in \mathcal{P}, \quad \mathcal{H}(X) \in \mathcal{Q}, \quad X = (X_1, \dots, X_K) \in \mathcal{V}^{(1)} \times \dots \times \mathcal{V}^{(K)}, \end{aligned} \quad (\text{SDPG1})$$

where for each $i \in [K]$, $\mathcal{V}^{(i)} \subset \mathbb{V}^{(i)}$ is either $\mathbb{S}_+^{n_i} \subset \mathbb{S}^{n_i}$, $\mathbb{R}_+^{n_i} \subset \mathbb{R}^{n_i}$, or $\mathbb{R}^{n_i} \subset \mathbb{R}^{n_i}$. The mappings $\mathcal{A}(X)$, $\mathcal{B}(X)$ and $\mathcal{H}(X)$ are given by

$$\begin{aligned} \mathcal{A}(X) &= \mathcal{A}^{(1)}(X_1) + \dots + \mathcal{A}^{(K)}(X_K), \quad \mathcal{B}(X) = \mathcal{B}^{(1)}(X_1) + \dots + \mathcal{B}^{(K)}(X_K), \\ \mathcal{H}(X) &= \mathcal{H}^{(1)}(X_1) + \dots + \mathcal{H}^{(K)}(X_K), \end{aligned}$$

where for each $i \in [K]$, $\mathcal{A}^{(i)} : \mathbb{V}^{(i)} \rightarrow \mathbb{R}^m$, $\mathcal{B}^{(i)} : \mathbb{V}^{(i)} \rightarrow \mathbb{R}^q$ and $\mathcal{H}^{(i)} : \mathbb{V}^{(i)} \rightarrow \mathbb{R}^p$ are given linear mappings. Moreover, $\mathcal{P}$ is a closed convex subset of $\mathbb{V}^{(1)} \times \dots \times \mathbb{V}^{(K)}$, $\mathcal{Q}$ is a closed convex subset of $\mathbb{R}^p$, $b \in \mathbb{R}^m$, l, u are q -dimensional vectors whose entries could be $-\infty$ and ∞ , respectively.

The main routine of SDPF+ is `sdpfplus.m` with the following calling syntax:

```
runhist = sdpfplus(blk,At,C,b,Bt,l,u,par);
```

The input arguments of SDPF+ are described as follows:

1. `blk` is a $K \times 2$ cell array that specifies the block structure of the variables, where K is the number of blocks. For each $k \in [K]$, `blk{k,1}` is a character indicating the type of the k -th block, which can be one of the following:

- 's': symmetric positive semidefinite matrix variable,
- 'l': nonnegative vector variable,
- 'f': free vector variable.

`blk{k,2}` specifies the corresponding dimension of the k -th block.

2. `At` is a $K \times 1$ or $K \times 2$ cell array that describe the linear operator $\mathcal{A}$ in (SDPG1). It has two columns when the user wants to make use of the sparse plus low-rank structure of the constraint matrices A_i 's described in (Tang and Toh 2026). Suppose the k -th block is a matrix variable with dimension n and the constraint matrices $\{A_i\}_{i \in [m]}$ have sparse plus low-rank decomposition of the form: $A_i = A_i^s + U_i \text{diag}(d_i) U_i^\top$ such that $d_i \in \mathbb{R}^{r_i}$ for any $i \in [m]$. Then `At{k,1}` is a $n(n+1)/2 \times m$ matrix such that for any $i \in [m]$ and $1 \leq \alpha \leq \beta \leq n$:

$$\text{At}\{k,1\}[(\beta-1)\beta/2+\alpha,i] = \begin{cases} A_i^s[\alpha,\beta] & \text{if } \alpha = \beta \\ \sqrt{2}A_i^s[\alpha,\beta] & \text{if } \alpha < \beta, \end{cases} \quad (\text{EC.16})$$

`At{k,2}` is a $(n+2) \times \sum_{i=1}^m r_i$ matrix such that:

$$\text{At}\{k,2\} = \left[\begin{bmatrix} 1 \cdot \mathbf{1}_{1 \times r_1} \\ U_1 \\ d_1^\top \end{bmatrix} \begin{bmatrix} 2 \cdot \mathbf{1}_{1 \times r_2} \\ U_2 \\ d_2^\top \end{bmatrix} \cdots \begin{bmatrix} m \cdot \mathbf{1}_{1 \times r_m} \\ U_m \\ d_m^\top \end{bmatrix} \right]. \quad (\text{EC.17})$$

When the low-rank structure of the constraint matrices is not used, `At` will only have one column.

3. `C` is either a cell array or a function handle that describes the objective function f in (SDPG1). When the SDP problem has linear objective function $f(X) = \sum_{i=1}^K \langle C_i, X_i \rangle$ such that the matrix blocks have sparse plus low-rank decomposition $C_i = C_i^s + U_i \text{diag}(d_i) U_i^\top$, we have that

$$C\{i,1\} = \begin{cases} C_i & \text{if } \text{blk}\{i,1\} \neq \text{'s'} \\ C_i^s & \text{if } \text{blk}\{i,1\} = \text{'s'}. \end{cases} \quad (\text{EC.18})$$

$$C\{i,2\} = \begin{cases} [] & \text{if } \text{blk}\{i,1\} \neq \text{'s'} \\ \begin{bmatrix} U_i \\ d_i^\top \end{bmatrix} & \text{if } \text{blk}\{i,1\} = \text{'s'}. \end{cases} \quad (\text{EC.19})$$

When $f(X)$ is a nonlinear function, SDPF+ input $f(\cdot)$ using a function handle. The input to the function handle is a cell array representing the primal variable, i.e., $\{X_1; \dots; X_K\}$, and the output consists of the function value $f(X)$ and a cell array containing the gradients, i.e., $\{\nabla_{X_1} f(X); \dots; \nabla_{X_K} f(X)\}$. Consider the following example:

$$f(X) = \langle I_n, X_1 \rangle^2 + \gamma \exp(\mathbf{1}_\ell^\top X_2), \quad (\text{EC.20})$$

where $X_1 \in \mathbb{S}^n$, $X_2 \in \mathbb{R}^\ell$ and $\gamma > 0$ is a parameter. We can define `C = @Cfun(X,gama)`, where `Cfun` is defined in Listing 1.

In some cases, the function value $f(RR^\top)$ and its gradient $\nabla f(RR^\top)$ can be computed more efficiently without explicitly forming the full matrix $RR^\top$ for each matrix block. To take advantage of this, users may define the

input X as a struct-type array with a field named R representing the low-rank factor. Then R is a $K \times 1$ cell array such that for any $i \in [K]$

$$R\{i\} = \begin{cases} U \text{ such that } UU^T = X_i & \text{if } \text{blk}\{i, 1\} = 's' \\ X_i & \text{if } \text{blk}\{i, 1\} \neq 's'. \end{cases} \quad (\text{EC.21})$$

We again use the example (EC.20) to present a computationally more efficient way to define the function in Listing 2.

```

1 function [f,g] = Cfun(X,gama)
2 n = size(X{1},1);
3 l = size(X{2},1);
4 f = (trace(X{1}))^2+gama*exp(sum(X{2}));
5 g = cell(2,1);
6 g{1} = 2*trace(X{1})*speye(n);
7 g{2} = gama*ones(1,1)*exp(sum(X{2}));
8 end

```

Listing 1: Example for defining a nonlinear objective function.

```

1 function [f,g] = Cfun(X,gama)
2 R = X.R;
3 n = size(R{1},1);
4 l = size(R{2},1);
5 f = R(:)'*R(:)+gama*exp(sum(R{2}));
6 g = cell(2,1);
7 g{1} = 2*R(:)'*R(:)*speye(n);
8 g{2} = gama*ones(1,1)*exp(sum(R{2}));
9 end

```

Listing 2: Example for defining a nonlinear objective function using the low-rank factor.

4. b is an m -dimensional vector which is exactly the vector b in (SDPG1).
5. Bt, l, u are used to denote the inequality constraint, $l \leq \mathcal{B}(X) \leq u$, in (SDPG1). The data structure of Bt is the same as At .
6. par is a struct-type array that contains the parameters and options for the solver. The user can refer to the main function `sdpfplus.m` for the complete list of available parameters. Although there are many parameters, most of them are intended for solver development and have been pre-tuned. Here, we highlight only a few important parameters that are commonly adjusted in numerical experiments.
 - `par.maxiter`: the maximum number of iterations allowed, default is 50000.
 - `par.maxtime`: the maximum running time allowed, default is 3600s.
 - `par.tol`: the tolerance on the KKT residual to terminate the algorithm, default is 10^{-6} .
 - `par.verbose`: a flag indicating whether to print the solver's progress information during the run. It can be set to 0 (silent mode) or 1 (verbose mode, default).

- `par.pfun`: the projection mapping for the constraint $X \in \mathcal{P}$ in (SDPG1). It is a function handle with the input being the cell array of the variable: $X = \{X_1; \dots; X_K\}$ and the output being the cell array of the variable after projection:

$$\{(\Pi_{\mathcal{P}}(X))_1; \dots; (\Pi_{\mathcal{P}}(X))_K\}.$$

- `par.Ht` and `par.Hpfun`: the linear operator and projection mapping used to model the convex constraint, $\mathcal{H}(X) \in \mathcal{Q}$, in (SDPG1). `Ht` has the same data structure as `At`. `Hpfun` is a function handle for the projection mapping onto the convex set $\mathcal{Q}$. Both the input and output of `Hpfun` are p -dimensional vectors.
- `par.Af`: a $K \times 1$ cell array containing the matrices $\{U_1; \dots; U_K\}$ that define “facial” constraints of the form $X_i U_i = 0$ for the matrix blocks. In certain problems, such as the SDP relaxations of non-convex quadratic programs (Burer 2009), they include constraints of the form $\langle X, U U^\top \rangle = 0$, which is equivalent to $XU = 0$. Motivated by the algorithms proposed in (Hou et al. 2025, 2026), this kind of constraint is further equivalent to $U^\top R = 0$ in the low-rank setting. In SDPF+, when such constraints are present, the algorithm eliminates them by restricting the iterates to lie in the orthogonal complement of the column space of U .

The output `runhist` contains the following information:

- `runhist.fval`: the primal objective function value.
- `runhist.dfval`: the dual objective function value.
- `runhist.ttime`: the total running time spent in SDPF+.
- `runhist.iter`: the total iteration number of SDPF+. This is the iteration number of the feasible method plus the outer iteration number the of PALM.
- `runhist.singular`: a flag indicating whether the problem is primal degenerate. It is set to 1 if the problem is detected to be degenerate, and 0 otherwise.
- `runhist.numChol`: the number of Cholesky decompositions computed in the SDPF+.
- `runhist.numCGiter`: the number of PCG iterations plus the number of ALM subiterations in SDPF+.
- `runhist.pfeas`, `runhist.dfeas`, `runhist.comp`: the KKT residuals defined in (25) and (26).
- `runhist.dfval`, `runhist.pdgap`: the dual objective function value and the duality gap, as defined in (27). These values are available only when $f(\cdot)$ is a linear function and the fields `pfun`, `Ht`, and `Hpfun` are empty. Otherwise, they are set to NaN.
- `runhist.X`: the primal solution. It is a struct-type array with a field `R`, which stores the matrix blocks in their low-rank form and the vector blocks in their original form.
- `runhist.lam`: the Lagrange multipliers of the constraints $\mathcal{A}(X) = b$, $l \leq \mathcal{B}(X) \leq u$, $X \in \mathcal{P}$ and $\mathcal{H}(X) \in \mathcal{Q}$ respectively.
- `runhist.lamk`: the Lagrange multiplier associated with the special constraint $X_{k,k} = I_k$. SDPF+ applies a specialized low-rank factorization to eliminate this constraint once it is detected in the problem. This technique has been employed in SDPF (Tang and Toh 2024).
- `runhist.S`: the dual slack matrix defined in (24).

EC.3. Tables for numerical results

In this section, we list tables of some numerical experiment results.

The following tables give the detailed residuals, objective values, and running times for the multi-instance experiments summarized in the main paper.

Table EC.1: Detailed test results for Lovász Theta SDP problems.

problem	algorithm	pfeas	dfeas	comp	fval	time
1dc.1024	SDPF+	6.78e-07	4.74e-07	9.90e-08	-9.5986179e+01	4.62e+01
n=1024	SDPNAL+	9.52e-07	7.33e-08	8.52e-09	-9.5986460e+01	9.67e+01
m=24064	SDPT3	5.86e-12	0.00e+00	8.67e-08	-9.5984581e+01	6.25e+02
rank=163	ManiSDP	9.95e-07	2.35e-07	3.30e-18	-9.5987258e+01	5.84e+02
1dc.2048	SDPF+	5.33e-07	8.42e-07	2.07e-07	-1.7473140e+02	1.94e+02
n=2048	SDPNAL+	3.12e-07	7.72e-08	1.03e-09	-1.7473104e+02	9.32e+02
m=58368	SDPT3	-	-	-	-	-
rank=256	ManiSDP	3.36e-06	2.93e-07	1.68e-17	-1.7475532e+02	3.61e+03
1et.1024	SDPF+	6.65e-07	7.99e-07	9.48e-14	-1.8422910e+02	1.04e+02
n=1024	SDPNAL+	6.28e-07	6.36e-09	2.64e-09	-1.8422783e+02	3.71e+02
m=9601	SDPT3	1.47e-12	0.00e+00	1.06e-07	-1.8422591e+02	7.86e+01
rank=226	ManiSDP	1.68e-06	4.66e-05	4.85e-17	-1.8422889e+02	3.61e+03
1et.2048	SDPF+	9.98e-07	3.07e-07	3.20e-11	-3.4203828e+02	2.16e+02
n=2048	SDPNAL+	6.51e-07	1.18e-08	1.04e-08	-3.4203305e+02	1.91e+03
m=22529	SDPT3	4.74e-13	0.00e+00	2.37e-07	-3.4202835e+02	5.50e+02
rank=296	ManiSDP	2.21e-05	4.23e-06	2.97e-17	-3.4210737e+02	3.60e+03
1tc.1024	SDPF+	9.89e-07	7.92e-07	3.35e-10	-2.0630739e+02	1.29e+02
n=1024	SDPNAL+	6.34e-07	8.30e-09	4.54e-06	-2.0631074e+02	5.37e+02
m=7937	SDPT3	2.75e-14	0.00e+00	2.08e-07	-2.0630400e+02	5.30e+01
rank=200	ManiSDP	4.04e-06	7.34e-07	1.02e-16	-2.0632068e+02	3.60e+03
1tc.2048	SDPF+	9.94e-07	2.37e-07	1.12e-14	-3.7465327e+02	2.54e+02
n=2048	SDPNAL+	4.69e-05	2.71e-07	1.44e-09	-3.7465245e+02	3.64e+03
m=18945	SDPT3	9.22e-13	0.00e+00	1.61e-07	-3.7464286e+02	3.96e+02
rank=281	ManiSDP	1.28e-05	2.15e-06	2.44e-17	-3.7482453e+02	3.61e+03
1zc.1024	SDPF+	1.07e-07	5.81e-07	4.11e-12	-1.2866665e+02	5.40e+01
n=1024	SDPNAL+	2.37e-07	5.06e-08	1.48e-08	-1.2866636e+02	4.23e+01
m=16641	SDPT3	2.28e-15	0.00e+00	1.24e-07	-1.2866657e+02	1.83e+02
rank=294	ManiSDP	6.08e-07	1.85e-07	2.02e-17	-1.2866718e+02	2.69e+02
1zc.2048	SDPF+	1.67e-07	4.33e-07	8.93e-08	-2.3740040e+02	6.29e+01
n=2048	SDPNAL+	6.05e-07	1.59e-09	6.11e-11	-2.3740044e+02	4.06e+02
m=39425	SDPT3	-	-	-	-	-
rank=396	ManiSDP	8.65e-07	3.08e-07	1.58e-18	-2.3740255e+02	4.72e+02
1zc.4096	SDPF+	4.67e-07	9.85e-07	2.67e-09	-4.4916594e+02	1.19e+03
n=4096	SDPNAL+	6.55e-07	1.73e-08	3.70e-09	-4.4916755e+02	2.91e+03
m=92161	SDPT3	-	-	-	-	-
rank=799	ManiSDP	2.40e-05	1.11e-06	2.52e-17	-4.4944055e+02	3.64e+03

Table EC.2: Detailed test results for Theta+ problem.

problem	algorithm	pfeas	dfeas	comp	fval	time
1dc.1024	SDPF+	7.89e-07	9.17e-07	4.28e-11	-9.5552604e+01	4.46e+02
rank=386	SDPNAL+	5.42e-07	2.84e-07	2.92e-08	-9.5551160e+01	1.76e+02
1dc.2048	SDPF+	7.01e-07	7.51e-07	4.99e-10	-1.7426269e+02	2.38e+03
rank=670	SDPNAL+	9.70e-07	5.37e-07	8.77e-07	-1.7425908e+02	1.32e+03
1et.1024	SDPF+	5.67e-07	8.72e-07	1.08e-08	-1.8207581e+02	3.96e+02
rank=365	SDPNAL+	9.33e-07	4.62e-07	3.66e-06	-1.8207513e+02	1.44e+02
1et.2048	SDPF+	1.77e-06	6.05e-07	2.30e-08	-3.3819558e+02	3.60e+03
rank=640	SDPNAL+	1.31e-06	4.68e-07	4.57e-06	-3.3817316e+02	1.64e+03
1tc.1024	SDPF+	1.00e-06	4.63e-07	2.43e-09	-2.0420941e+02	1.10e+03
rank=346	SDPNAL+	1.40e-06	5.90e-07	5.51e-06	-2.0420916e+02	4.00e+02
1tc.2048	SDPF+	4.50e-07	9.78e-07	3.34e-10	-3.7049547e+02	2.69e+03
rank=603	SDPNAL+	9.11e-07	6.90e-07	1.22e-06	-3.7049077e+02	2.05e+03
1zc.1024	SDPF+	9.38e-07	3.97e-07	3.07e-09	-1.2800306e+02	3.51e+02
rank=584	SDPNAL+	9.56e-07	8.08e-07	2.56e-06	-1.2799771e+02	5.31e+01
1zc.2048	SDPF+	1.98e-07	9.94e-07	5.04e-10	-2.3740062e+02	5.23e+02
rank=505	SDPNAL+	5.07e-07	2.36e-07	5.68e-11	-2.3739990e+02	1.45e+02
1zc.4096	SDPF+	7.07e-06	4.34e-06	2.60e-07	-4.4377186e+02	3.60e+03
rank=1326	SDPNAL+	7.80e-06	5.22e-06	4.74e-08	-4.4373543e+02	3.61e+03

Table EC.3: Detailed test results for convex relaxation of quadratic assignment problem (30). The coefficient matrices A, B are normalized by their maximum entries.

problem	algorithm	pfeas	dfeas	comp	fval	time
esc32a	SDPF+	3.31e-07	9.71e-07	1.03e-09	6.7789147e-02	2.52e+03
rank=827	SDPNAL+	1.01e-06	1.92e-07	1.47e-06	6.7789339e-02	2.49e+02
esc32b	SDPF+	6.29e-07	9.18e-07	1.44e-08	1.0646688e-01	5.51e+02
rank=730	SDPNAL+	1.00e-06	1.74e-07	2.24e-06	1.0642686e-01	2.74e+02
esc32c	SDPF+	4.41e-07	6.97e-07	1.03e-08	2.5019981e-01	1.20e+02
rank=565	SDPNAL+	7.40e-07	4.27e-07	6.82e-06	2.5018622e-01	2.16e+02
esc32d	SDPF+	5.62e-07	7.91e-07	4.57e-09	1.7517284e-01	3.55e+02
rank=660	SDPNAL+	6.14e-07	3.06e-07	3.02e-06	1.7517317e-01	4.64e+01
esc32e	SDPF+	8.70e-08	6.87e-07	5.46e-08	2.9102252e-03	8.65e+01
rank=540	SDPNAL+	2.38e-07	5.60e-07	1.96e-08	2.9101854e-03	8.33e+01
esc32g	SDPF+	3.34e-07	6.32e-07	9.23e-10	1.3079328e-02	1.08e+02
rank=560	SDPNAL+	4.24e-07	5.88e-07	1.82e-08	1.3079318e-02	4.12e+01
esc32h	SDPF+	6.63e-07	9.37e-07	3.72e-08	2.5145117e-01	1.01e+03
rank=843	SDPNAL+	8.34e-07	4.04e-07	2.25e-05	2.5139278e-01	8.23e+02
kra32	SDPF+	4.48e-07	9.73e-07	2.39e-09	3.0164203e-01	1.61e+03
rank=596	SDPNAL+	1.21e-06	2.01e-07	2.05e-05	3.0147487e-01	6.66e+02
lipa40a	SDPF+	6.42e-07	7.14e-07	5.82e-07	8.7819603e-01	1.99e+03
rank=840	SDPNAL+	3.32e-07	1.65e-07	2.02e-09	8.7819411e-01	1.27e+03
lipa40b	SDPF+	6.99e-07	0.00e+00	1.53e-08	6.1038000e-01	5.52e+02
rank=832	SDPNAL+	1.17e-07	5.53e-07	3.60e-09	6.1037986e-01	5.28e+02
sko42	SDPF+	4.65e-06	6.86e-06	8.61e-07	4.7098467e-01	3.60e+03
rank=1113	SDPNAL+	1.52e-06	3.63e-07	2.59e-06	4.7094929e-01	1.54e+03
ste36a	SDPF+	2.65e-05	1.18e-05	1.34e-07	8.8679914e-02	3.60e+03
rank=826	SDPNAL+	1.60e-06	4.30e-07	3.09e-06	8.8669185e-02	1.06e+03
ste36b	SDPF+	1.81e-05	1.02e-05	7.35e-08	3.2160132e-02	3.60e+03
rank=828	SDPNAL+	9.03e-07	5.12e-07	3.19e-06	3.2181769e-02	2.41e+03
ste36c	SDPF+	1.40e-05	7.08e-06	1.98e-07	9.4396324e-02	3.60e+03
rank=828	SDPNAL+	1.56e-06	4.07e-07	3.30e-06	9.4389140e-02	1.33e+03
tai35a	SDPF+	6.69e-07	1.99e-07	6.88e-10	5.6398422e-01	4.31e+02
rank=650	SDPNAL+	1.06e-06	1.62e-07	2.70e-05	5.6395534e-01	3.80e+02
tai35b	SDPF+	1.32e-05	1.98e-05	4.46e-07	1.3004610e-01	3.60e+03
rank=833	SDPNAL+	6.82e-07	6.84e-07	7.30e-06	1.3000286e-01	2.71e+03
tai40a	SDPF+	9.71e-07	2.63e-07	8.26e-11	5.5824653e-01	9.06e+02
rank=849	SDPNAL+	1.20e-06	1.48e-07	1.60e-05	5.5822841e-01	8.90e+02
tai40b	SDPF+	1.31e-05	5.33e-05	5.80e-07	1.3799361e-01	3.60e+03
rank=1222	SDPNAL+	1.56e-06	6.94e-07	1.95e-05	1.3794748e-01	3.34e+03
tho40	SDPF+	3.49e-06	1.43e-06	9.34e-11	3.2487741e-01	3.60e+03
rank=946	SDPNAL+	8.91e-07	2.37e-07	1.76e-05	3.2480955e-01	1.32e+03

Table EC.4: Detailed test results for DNN relaxations of graph equipartition problems.

problem	algorithm	pfeas	dfeas	comp	fval	time
n=300	SDPF+	3.97e-07	6.37e-07	3.20e-07	8.5204105e+03	1.87e+01
K=3,rank=35	SDPNAL+	2.51e-08	7.26e-07	7.25e-06	8.5204005e+03	2.68e+01
n=500	SDPF+	8.66e-07	5.69e-07	7.15e-08	2.3837915e+04	3.61e+01
K=5,rank=89	SDPNAL+	3.65e-07	4.96e-07	3.25e-07	2.3837913e+04	1.84e+02
n=1000	SDPF+	7.38e-07	6.25e-07	3.70e-10	9.3643421e+04	1.19e+02
K=10,rank=228	SDPNAL+	3.40e-07	2.74e-07	1.93e-06	9.3643408e+04	1.01e+03
n=600	SDPF+	5.21e-07	5.12e-07	7.70e-09	3.5763287e+04	1.16e+02
K=3,rank=49	SDPNAL+	2.52e-09	1.46e-07	3.55e-06	3.5763274e+04	1.22e+02
n=1000	SDPF+	5.00e-07	3.64e-07	6.43e-07	9.8624195e+04	2.45e+02
K=5,rank=148	SDPNAL+	7.77e-07	5.26e-07	4.08e-06	9.8624177e+04	3.27e+02
n=2000	SDPF+	2.21e-07	6.90e-07	8.03e-09	3.9223713e+05	5.65e+02
K=10,rank=318	SDPNAL+	1.48e-07	4.05e-07	1.68e-06	3.9223710e+05	3.33e+03
n=1500	SDPF+	9.34e-07	0.00e+00	3.16e-08	2.2561800e+05	3.59e+02
K=3,rank=7	SDPNAL+	1.60e-07	2.76e-07	7.74e-07	2.2561797e+05	2.81e+02
n=2500	SDPF+	3.67e-07	1.18e-09	9.22e-07	6.2291001e+05	1.03e+03
K=5,rank=7	SDPNAL+	2.27e-08	1.12e-05	1.15e-06	6.2291036e+05	3.60e+03
n=5000	SDPF+	5.28e-05	8.41e-04	2.50e-04	2.4990046e+06	3.61e+03
K=10,rank=523	SDPNAL+	6.40e-05	3.61e-04	2.63e-04	2.5388198e+06	3.60e+03

Table EC.5: Detailed test results for rank-1 tensor approximation SDP problems.

problem	algorithm	pfeas	dfeas	comp	fval	time
nonsym(10,4)	SDPF+	9.56e-07	0.00e+00	3.03e-11	1.6946676e+00	3.26e+01
n=1000,rank=999	SDPNAL+	4.85e-07	7.29e-07	3.85e-05	1.6947107e+00	6.16e+01
m=166374	ManiSDP	3.37e-07	0.00e+00	2.72e-15	1.6947151e+00	4.70e+01
nonsym(11,4)	SDPF+	2.16e-08	0.00e+00	4.42e-10	2.9134832e+00	3.41e+01
n=1331,rank=1330	SDPNAL+	4.66e-07	3.38e-07	3.55e-05	2.9134838e+00	1.14e+02
m=287495	ManiSDP	2.55e-07	0.00e+00	1.74e-15	2.9134851e+00	5.34e+01
nonsym(12,4)	SDPF+	2.87e-07	0.00e+00	3.12e-11	5.9216048e+00	6.29e+01
n=1728,rank=1727	SDPNAL+	2.34e-07	0.00e+00	2.84e-06	5.9216162e+00	2.03e+02
m=474551	ManiSDP	4.28e-07	0.00e+00	4.52e-15	5.9216201e+00	1.53e+02

Table EC.5: Detailed test results for rank-1 tensor approximation SDP problems.

problem	algorithm	pfeas	dfeas	comp	fval	time
nonsym(13,4) n=2197,rank=2196 m=753570	SDPF+	1.92e-07	0.00e+00	3.54e-10	7.2745074e+00	1.51e+02
	SDPNAL+	8.59e-07	2.75e-08	9.28e-06	7.2745053e+00	3.88e+02
	ManiSDP	4.60e-07	0.00e+00	2.23e-15	7.2745076e+00	2.89e+02
nonsym(14,4) n=2744,rank=2743 m=1157624	SDPF+	1.81e-08	0.00e+00	1.68e-08	9.6880589e+00	2.83e+02
	SDPNAL+	6.47e-07	8.27e-08	4.16e-05	9.6880401e+00	6.72e+02
	ManiSDP	3.70e-07	0.00e+00	9.10e-16	9.6880544e+00	8.16e+02
nonsym(15,4) n=3375,rank=3374 m=1727999	SDPF+	2.68e-07	0.00e+00	1.32e-08	1.3382864e+01	6.14e+02
	SDPNAL+	1.28e-07	3.06e-08	2.37e-05	1.3382918e+01	1.21e+03
	ManiSDP	2.58e-07	4.28e-17	3.79e-16	1.3382916e+01	1.06e+03
nonsym(16,4) n=4096,rank=4095 m=2515455	SDPF+	9.75e-08	0.00e+00	1.23e-07	1.5151446e+01	1.05e+03
	SDPNAL+	6.87e-07	3.71e-07	3.58e-04	1.5151349e+01	2.11e+03
	ManiSDP	5.58e-03	0.00e+00	5.95e-15	1.1588777e+01	3.62e+03
sym_rd(3,45) n=1081,rank=1080 m=211875	SDPF+	3.52e-07	7.89e-08	1.77e-08	2.1407698e+00	1.95e+02
	SDPNAL+	2.60e-07	4.17e-07	1.31e-05	2.1407744e+00	6.84e+01
	ManiSDP	8.12e-08	0.00e+00	2.55e-12	2.1407693e+00	1.55e+03
sym_rd(3,50) n=1326,rank=1325 m=316250	SDPF+	6.94e-07	2.10e-07	4.74e-09	2.0695003e+00	2.56e+02
	SDPNAL+	9.90e-07	5.78e-09	2.74e-06	2.0694971e+00	1.38e+02
	ManiSDP	-	-	-	-	-
sym_rd(3,55) n=1596,rank=1595 m=455125	SDPF+	5.79e-07	1.19e-07	1.80e-08	2.1562463e+00	4.31e+02
	SDPNAL+	8.90e-07	3.92e-08	2.98e-06	2.1562784e+00	3.70e+02
	ManiSDP	-	-	-	-	-
sym_rd(3,60) n=1891,rank=1890 m=635375	SDPF+	4.97e-07	0.00e+00	5.30e-08	2.3910249e+00	6.35e+02
	SDPNAL+	6.63e-07	8.73e-09	7.81e-07	2.3910013e+00	4.15e+02
	ManiSDP	-	-	-	-	-
nsym_rd((35,35,35)) n=1225,rank=1224 m=396899	SDPF+	4.48e-07	0.00e+00	1.77e-08	3.0704739e+00	4.20e+02
	SDPNAL+	7.33e-07	2.49e-09	1.80e-06	3.0704742e+00	1.25e+02
	ManiSDP	2.75e-09	0.00e+00	1.45e-12	3.0704742e+00	1.55e+03
nsym_rd((40,40,40)) n=1600,rank=1599 m=672399	SDPF+	4.56e-08	0.00e+00	7.28e-10	3.8787321e+00	1.42e+03
	SDPNAL+	2.17e-07	1.43e-08	8.73e-06	3.8787395e+00	1.62e+02
	ManiSDP	2.06e-05	1.44e-05	2.28e-08	3.8785974e+00	3.60e+03

Table EC.6: Test results for SDP problems of second-order reduced density matrices.

problem	algorithm	pfeas	dfeas	comp	fval	time
AIH.1Sigma+.STO6G m=7230	SDPF+	6.82e-06	8.71e-06	2.88e-06	2.4568419e+02	1.32e+03
	SDPNAL+	8.66e-06	9.35e-06	1.65e-05	2.4568369e+02	6.78e+02
B.2P.DZ m=7230	SDPF+	9.90e-06	9.52e-06	1.81e-06	2.4573770e+01	1.46e+03
	SDPNAL+	9.30e-06	5.56e-06	2.96e-06	2.4573585e+01	9.96e+02
B.2P.SV m=4743	SDPF+	6.57e-06	8.23e-06	2.66e-07	2.4559680e+01	4.67e+02
	SDPNAL+	6.81e-06	2.26e-06	4.94e-06	2.4559466e+01	4.52e+02
B2.3Sigma+.STO6G m=7230	SDPF+	4.48e-06	8.84e-06	1.60e-06	5.7336756e+01	9.24e+02
	SDPNAL+	9.85e-06	3.59e-06	5.69e-06	5.7335793e+01	1.24e+03
BF.1Sigma+.STO6G m=7230	SDPF+	4.43e-06	8.62e-06	1.81e-06	1.4247491e+02	1.21e+03
	SDPNAL+	9.80e-06	3.90e-06	6.02e-06	1.4247334e+02	9.75e+02
BH+.2Sigma+.STO6G m=948	SDPF+	3.86e-06	7.98e-06	2.71e-06	2.6980053e+01	2.00e+01
	SDPNAL+	1.40e-06	3.58e-06	2.60e-06	2.6979873e+01	1.99e+01
BH.1Sigma+.DZ m=15018	SDPF+	1.51e-05	1.47e-05	1.04e-06	2.7336753e+01	3.62e+03
	SDPNAL+	5.76e-06	5.70e-06	1.33e-05	2.7335536e+01	3.03e+03
BN.3Pi.STO6G m=7230	SDPF+	2.71e-06	9.80e-06	8.51e-07	9.3285740e+01	1.51e+03
	SDPNAL+	8.81e-06	8.93e-06	8.04e-06	9.3285999e+01	1.62e+03
BO.2Sigma+.STO6G m=7230	SDPF+	6.78e-06	7.18e-06	4.08e-06	1.1683343e+02	1.33e+03
	SDPNAL+	9.84e-06	4.12e-06	8.37e-06	1.1683227e+02	1.37e+03
Be.1S.STO6G m=465	SDPF+	9.42e-06	4.71e-06	7.55e-07	1.4556158e+01	9.84e+00
	SDPNAL+	1.35e-06	4.02e-06	1.38e-06	1.4556168e+01	6.01e+00
Be.1S.SV m=4743	SDPF+	8.40e-06	6.57e-06	1.43e-07	1.4615922e+01	5.19e+02
	SDPNAL+	6.78e-06	6.67e-06	6.58e-06	1.4615872e+01	2.66e+02
Be2.1Sigma+.infy.STO6G m=7230	SDPF+	4.77e-06	8.42e-06	9.96e-07	2.9112395e+01	7.02e+02
	SDPNAL+	1.37e-06	4.71e-06	4.85e-06	2.9112189e+01	3.65e+02
BeF.2Sigma+.STO6G m=7230	SDPF+	4.22e-06	8.63e-06	2.78e-06	1.2763975e+02	1.16e+03
	SDPNAL+	9.99e-06	2.91e-06	1.23e-05	1.2763809e+02	1.20e+03
BeH+.1Sigma+.STO6G m=948	SDPF+	5.71e-06	9.00e-06	5.15e-06	1.6456779e+01	2.57e+01
	SDPNAL+	2.70e-06	3.35e-06	2.92e-06	1.6456520e+01	2.63e+01
BeH.2Sigma+.STO6G m=948	SDPF+	3.67e-06	8.70e-06	6.96e-07	1.6693095e+01	2.49e+01
	SDPNAL+	1.34e-06	3.99e-06	2.60e-06	1.6692939e+01	2.45e+01
BeO.1Sigma+.STO6G m=7230	SDPF+	4.89e-06	9.64e-06	7.54e-06	1.0192671e+02	1.31e+03
	SDPNAL+	8.36e-06	8.52e-06	1.64e-05	1.0192578e+02	1.08e+03
C.1S.STO6G m=465	SDPF+	2.66e-06	9.80e-06	3.83e-06	3.7520370e+01	1.77e+01
	SDPNAL+	6.45e-06	6.82e-06	1.08e-06	3.7520575e+01	6.29e+00
C.3P.DZ m=7230	SDPF+	4.97e-06	9.09e-06	1.07e-05	3.7738361e+01	1.51e+03
	SDPNAL+	2.84e-13	3.36e-06	3.45e-05	3.7739141e+01	1.74e+03
C.3P.STO6G m=465	SDPF+	2.32e-06	4.84e-06	2.58e-06	3.7592869e+01	9.22e+00
	SDPNAL+	2.40e-07	8.19e-06	8.74e-06	3.7592867e+01	5.89e+00
C.3PSZ0.DZ m=7230	SDPF+	4.08e-06	9.40e-06	3.26e-06	3.7739716e+01	1.91e+03
	SDPNAL+	9.91e-06	3.57e-06	8.54e-07	3.7739725e+01	1.24e+03
C2-.2Sigma+.STO6G m=7230	SDPF+	7.31e-06	9.05e-06	1.03e-06	9.0338358e+01	9.62e+02
	SDPNAL+	7.64e-06	9.06e-06	2.99e-06	9.0336715e+01	7.72e+02
C2.1Sigma+.STO6G m=7230	SDPF+	6.39e-06	9.25e-06	1.77e-06	9.0771933e+01	1.40e+03
	SDPNAL+	9.73e-06	5.69e-06	1.02e-06	9.0769770e+01	1.01e+03
CF.2Pir.STO6G m=7230	SDPF+	8.64e-06	9.98e-06	2.98e-06	1.5914929e+02	1.47e+03
	SDPNAL+	1.17e-12	4.13e-06	1.27e-05	1.5914776e+02	1.39e+03

Table EC.6: Test results for SDP problems of second-order reduced density matrices.

problem	algorithm	pfeas	dfeas	comp	fval	time
CH.2Pir.DZ m=15018	SDPF+	2.18e-05	1.12e-05	1.43e-05	4.1176874e+01	3.63e+03
	SDPNAL+	6.45e-06	8.74e-06	9.13e-06	4.1175943e+01	4.06e+03
CH3+.1E".STO6G m=2964	SDPF+	4.94e-06	9.31e-06	7.70e-07	4.8751823e+01	2.28e+02
	SDPNAL+	7.72e-06	7.54e-06	4.34e-06	4.8751264e+01	1.06e+02
CH4.1A1.STO6G m=4743	SDPF+	4.90e-06	6.68e-06	1.18e-06	5.3663795e+01	6.18e+02
	SDPNAL+	5.59e-06	5.47e-06	1.70e-06	5.3663329e+01	2.11e+02
CN.2Sigma+.STO6G m=7230	SDPF+	5.20e-06	8.96e-06	6.21e-06	1.1104020e+02	1.32e+03
	SDPNAL+	9.86e-06	1.12e-05	1.09e-05	1.1103988e+02	7.89e+02
CO+.2Sigma+.STO6G m=7230	SDPF+	7.86e-06	8.43e-06	2.14e-06	1.3481693e+02	1.35e+03
	SDPNAL+	9.97e-06	6.84e-06	8.38e-06	1.3481557e+02	1.08e+03
CO.1Sigma+.STO6G m=7230	SDPF+	7.79e-06	8.55e-06	4.52e-06	1.3495777e+02	1.28e+03
	SDPNAL+	3.92e-06	9.81e-06	4.25e-06	1.3495644e+02	7.69e+02
FH2+.1A1.STO6G m=1743	SDPF+	2.65e-06	9.71e-06	5.23e-06	1.0999157e+02	1.28e+02
	SDPNAL+	2.30e-06	2.26e-06	1.18e-06	1.0999055e+02	6.34e+01
H3.2A1".DZ m=948	SDPF+	3.77e-06	9.51e-06	8.17e-07	3.3648050e+00	4.42e+01
	SDPNAL+	6.62e-07	3.07e-06	3.38e-07	3.3646782e+00	2.23e+01
HF.1Sigma+.DZ m=15018	SDPF+	3.71e-05	1.95e-05	8.92e-05	1.0536495e+02	3.60e+03
	SDPNAL+	2.20e-05	7.38e-06	4.62e-05	1.0535898e+02	3.88e+03
HLi2.2A1.STO6G m=10593	SDPF+	6.87e-06	7.91e-06	1.68e-06	1.9145482e+01	1.84e+03
	SDPNAL+	9.25e-08	1.60e-06	7.57e-06	1.9145128e+01	3.56e+03
HN2+.1Sigma+.STO6G m=10593	SDPF+	4.00e-06	9.55e-06	6.81e-06	1.3798216e+02	2.98e+03
	SDPNAL+	9.36e-06	1.05e-05	3.17e-06	1.3797971e+02	1.37e+03
HNO.1A".STO6G m=10593	SDPF+	2.27e-06	8.73e-06	5.24e-06	1.5967301e+02	2.86e+03
	SDPNAL+	5.30e-06	1.20e-05	6.78e-06	1.5967075e+02	1.64e+03
Li.2S.STO6G m=465	SDPF+	6.85e-06	8.30e-06	1.83e-06	7.4003136e+00	1.59e+01
	SDPNAL+	1.63e-06	9.82e-07	1.03e-06	7.4002434e+00	9.51e+00
Li2.1Sigma+.STO6G m=7230	SDPF+	9.00e-06	4.15e-06	1.60e-06	1.6619854e+01	1.30e+03
	SDPNAL+	5.04e-06	5.17e-06	6.71e-06	1.6619566e+01	5.82e+02
LiF.1Sigma+.STO6G m=7230	SDPF+	7.11e-06	7.45e-06	7.55e-07	1.1558150e+02	1.07e+03
	SDPNAL+	9.86e-06	7.86e-06	5.58e-06	1.1558042e+02	8.99e+02
LiH.1Sigma+.DZ m=15018	SDPF+	5.50e-06	7.18e-06	4.11e-07	9.0038110e+00	3.56e+03
	SDPNAL+	3.39e-06	6.01e-06	3.45e-06	9.0037725e+00	2.31e+03
LiH.1Sigma+.STO6G m=948	SDPF+	8.57e-06	8.40e-06	4.31e-06	8.9673644e+00	3.75e+01
	SDPNAL+	4.73e-06	2.67e-06	1.71e-06	8.9672695e+00	2.95e+01
LiOH.1Sigma+.STO6G m=10593	SDPF+	3.38e-06	8.44e-06	6.02e-06	9.5671183e+01	2.29e+03
	SDPNAL+	5.68e-06	6.45e-06	1.18e-05	9.5669720e+01	1.81e+03
N.4S.DZ m=7230	SDPF+	8.35e-06	4.18e-06	1.19e-06	5.4443861e+01	2.70e+03
	SDPNAL+	7.37e-06	4.37e-06	1.64e-06	5.4444622e+01	7.88e+02
N2+.2Sigma+.STO6G m=7230	SDPF+	6.70e-06	8.27e-06	8.60e-07	1.3145110e+02	1.23e+03
	SDPNAL+	7.79e-06	6.41e-06	2.52e-06	1.3144894e+02	8.34e+02
N2.1Sigma+.STO6G m=7230	SDPF+	6.93e-06	8.63e-06	6.62e-06	1.3232650e+02	1.39e+03
	SDPNAL+	6.72e-07	1.11e-05	4.02e-07	1.3232514e+02	5.71e+02
NH.1Delta.DZ m=15018	SDPF+	7.39e-06	1.91e-05	9.85e-06	5.8555132e+01	3.60e+03
	SDPNAL+	1.42e-05	7.61e-06	3.50e-05	5.8548842e+01	3.84e+03
NH.3Sigma+.DZ m=15018	SDPF+	4.45e-05	1.75e-04	4.35e-04	5.8617473e+01	3.65e+03
	SDPNAL+	1.43e-05	5.96e-06	1.83e-05	5.8614018e+01	3.93e+03
NH2-.1A1.STO6G m=1743	SDPF+	6.56e-06	9.67e-06	1.16e-06	6.2707265e+01	1.11e+02
	SDPNAL+	3.69e-06	3.77e-06	1.47e-06	6.2706412e+01	5.81e+01
NH3+.2A2".STO6G m=2964	SDPF+	4.57e-06	9.99e-06	1.62e-06	6.7506589e+01	2.49e+02
	SDPNAL+	5.70e-06	5.03e-06	3.86e-06	6.7506042e+01	1.18e+02
NH4+.1A1.STO6G m=4743	SDPF+	8.86e-06	9.59e-06	1.37e-07	7.2777720e+01	6.19e+02
	SDPNAL+	3.60e-06	9.64e-06	1.62e-05	7.2778616e+01	3.04e+02
Na.2S.STO6G m=4743	SDPF+	9.33e-06	8.13e-06	2.30e-06	1.6107858e+02	5.34e+02
	SDPNAL+	3.48e-06	6.17e-06	3.62e-05	1.6107816e+02	3.11e+02
NaH.1Sigma+.STO6G m=7230	SDPF+	4.64e-06	9.81e-06	2.17e-06	1.6482460e+02	1.18e+03
	SDPNAL+	9.80e-06	9.42e-06	2.49e-05	1.6482336e+02	6.06e+02
Ne.1S.DZ m=7230	SDPF+	6.07e-06	8.94e-06	7.86e-06	1.2864466e+02	1.72e+03
	SDPNAL+	9.15e-07	5.10e-06	5.45e-05	1.2864187e+02	5.86e+02
O.1D.DZ m=7230	SDPF+	9.53e-06	9.19e-06	4.15e-06	7.4791985e+01	1.52e+03
	SDPNAL+	7.11e-06	9.20e-06	3.04e-05	7.4791127e+01	6.53e+02
O.3PDZ m=7230	SDPF+	8.03e-06	9.56e-06	3.24e-06	7.4873477e+01	1.69e+03
	SDPNAL+	6.27e-07	7.84e-06	1.99e-05	7.4872796e+01	7.02e+02
O.3PSZ0.DZ m=7230	SDPF+	3.13e-06	5.00e-06	5.08e-06	7.4873116e+01	1.91e+03
	SDPNAL+	7.50e-06	7.48e-06	2.01e-06	7.4874502e+01	8.63e+02
O2+.2Pig.STO6G m=7230	SDPF+	7.29e-06	9.93e-06	2.59e-06	1.7913508e+02	1.96e+03
	SDPNAL+	5.86e-06	8.66e-06	2.94e-06	1.7913201e+02	6.84e+02

Table EC.7: Test results for SDP problems from Hans Mittelmann benchmarks. Here comp' denotes $\min\{\text{comp}, \text{pdgap}\}$.

problem	algorithm	pfeas	dfeas	comp'	fval	time
Alh_1.r20 nblk=22 m=7230	SDPF+	1.45e-06	4.35e-06	2.04e-06	2.4568341e+02	3.68e+03
	SDPNAL+	2.82e-07	9.69e-07	7.63e-06	2.4568507e+02	2.06e+03
	SDPT3	-	-	-	-	-
BH2.r14 nblk=22 m=1743	SDPF+	6.78e-07	9.74e-07	5.98e-08	3.0430316e+01	7.60e+02
	SDPNAL+	2.31e-07	1.17e-06	2.52e-06	3.0430377e+01	1.87e+02
	SDPT3	1.71e-09	0.00e+00	5.31e-07	3.0430147e+01	3.99e+02
Bex2_1_5 nblk=33 m=3002	SDPF+	5.10e-07	9.66e-07	2.94e-09	2.9779301e+00	2.04e+02
	SDPNAL+	5.18e-07	1.02e-08	1.13e-10	2.9779225e+00	3.25e+01
	SDPT3	6.22e-05	0.00e+00	1.80e-06	2.9778612e+00	4.16e+02

Table EC.7: Test results for SDP problems from Hans Mittelmann benchmarks. Here comp' denotes $\min\{\text{comp}, \text{pdgap}\}$.

problem	algorithm	pfeas	dfeas	comp'	fval	time
Bst_jcbpaf2	SDPF+	3.61e-07	6.23e-07	2.87e-09	7.9487455e-02	2.55e+02
nblk=35	SDPNAL+	3.19e-07	3.46e-08	1.51e-08	7.9483190e-02	4.17e+01
m=3002	SDPT3	1.30e-07	0.00e+00	1.18e-08	7.9485765e-02	4.66e+02
CH2_l.r14	SDPF+	7.01e-07	9.10e-07	2.30e-07	4.4853922e+01	6.35e+02
nblk=22	SDPNAL+	9.03e-07	1.23e-06	2.32e-06	4.4853953e+01	1.52e+02
m=1743	SDPT3	8.45e-10	0.00e+00	4.59e-07	4.4853801e+01	3.87e+02
G40_mb	SDPF+	1.00e-08	2.63e-08	7.28e-10	-2.8643232e+03	1.18e+01
nblk= 1	SDPNAL+	3.99e-07	7.52e-11	5.55e-07	-2.8643297e+03	3.60e+03
m=2001	SDPT3	1.96e-12	0.00e+00	6.28e-07	-2.8643196e+03	3.80e+01
G40mc	SDPF+	8.14e-09	1.95e-08	2.90e-10	-5.7295791e+03	7.20e+00
nblk= 1	SDPNAL+	3.19e-07	8.35e-08	3.86e-08	-5.7295795e+03	2.05e+02
m=2000	SDPT3	1.40e-11	0.00e+00	1.93e-07	-5.7295769e+03	3.45e+01
G48_mb	SDPF+	2.22e-09	0.00e+00	5.52e-07	8.2171572e+00	1.07e+01
nblk= 1	SDPNAL+	5.56e-07	1.26e-09	2.94e-05	8.2191558e+00	1.02e+03
m=3001	SDPT3	2.31e-12	0.00e+00	1.26e-07	8.2171648e+00	7.90e+01
G48mc	SDPF+	7.53e-12	0.00e+00	4.82e-08	-1.1999999e+04	6.85e+00
nblk= 1	SDPNAL+	1.55e-07	7.85e-11	1.54e-08	-1.2000000e+04	9.17e+02
m=3000	SDPT3	1.22e-15	0.00e+00	1.73e-08	-1.2000000e+04	4.19e+01
G55mc	SDPF+	1.06e-09	2.27e-08	6.44e-08	-2.2078920e+04	2.26e+01
nblk= 1	SDPNAL+	4.55e-10	1.09e-05	6.25e-06	-2.2078888e+04	3.61e+03
m=5000	SDPT3	3.45e-14	0.00e+00	2.33e-07	-2.2078911e+04	3.09e+02
G59mc	SDPF+	6.55e-09	4.61e-08	1.36e-08	-1.4624653e+04	3.01e+01
nblk= 1	SDPNAL+	6.63e-07	1.53e-06	5.46e-06	-1.4624654e+04	3.66e+03
m=5000	SDPT3	4.13e-12	0.00e+00	2.04e-07	-1.4624647e+04	4.24e+02
G60_mb	SDPF+	4.14e-09	1.09e-08	1.29e-07	1.9280682e+03	5.64e+01
nblk= 1	SDPNAL+	-	-	-	-	-
m=7001	SDPT3	3.05e-13	1.65e-18	2.53e-07	1.9280690e+03	7.71e+02
G60mc	SDPF+	3.16e-09	1.00e-08	4.78e-08	-3.0444536e+04	5.49e+01
nblk= 1	SDPNAL+	-	-	-	-	-
m=7000	SDPT3	1.89e-14	0.00e+00	5.35e-07	-3.0444504e+04	7.85e+02
H3O+_r16	SDPF+	9.11e-07	9.83e-07	7.93e-07	9.0107034e+01	1.97e+03
nblk=22	SDPNAL+	1.27e-07	1.38e-06	1.62e-06	9.0106969e+01	2.55e+02
m=2964	SDPT3	1.03e-11	0.00e+00	6.70e-07	9.0106681e+01	1.87e+03
NH2_-r14	SDPF+	3.49e-07	9.99e-07	4.10e-07	6.2706409e+01	1.02e+03
nblk=22	SDPNAL+	4.14e-07	1.23e-06	8.43e-07	6.2706337e+01	8.62e+01
m=1743	SDPT3	3.11e-08	4.98e-07	6.24e-07	6.2706343e+01	5.46e+02
NH3_-r16	SDPF+	6.41e-07	9.62e-07	6.87e-07	6.7925265e+01	2.62e+03
nblk=22	SDPNAL+	3.42e-07	1.39e-06	1.91e-06	6.7925297e+01	2.49e+02
m=2964	SDPT3	1.13e-11	0.00e+00	4.00e-07	6.7924925e+01	1.87e+03
NH4+_r18	SDPF+	2.81e-05	5.60e-05	1.43e-05	7.2777756e+01	3.63e+03
nblk=22	SDPNAL+	9.72e-07	9.42e-07	2.54e-06	7.2776721e+01	9.33e+02
m=4743	SDPT3	-	-	-	-	-
biggs	SDPF+	-	-	-	-	-
nblk=48	SDPNAL+	-	-	-	-	-
m=1819	SDPT3	1.31e-09	0.00e+00	2.90e-07	-1.4142583e+03	3.21e+02
broyden25	SDPF+	3.14e-07	8.03e-10	2.18e-07	3.5536241e+01	1.46e+03
nblk=19	SDPNAL+	-	-	-	-	-
m=15971	SDPT3	2.10e-10	0.00e+00	8.41e-07	3.5536255e+01	1.10e+03
buck3	SDPF+	4.94e-07	1.42e-12	7.10e-07	-6.0760518e+02	9.35e+02
nblk= 3	SDPNAL+	-	-	-	-	-
m=544	SDPT3	2.41e-06	0.00e+00	3.13e-06	-6.0760227e+02	5.36e+00
buck4	SDPF+	9.89e-07	3.75e-08	7.83e-08	-4.8613148e+02	5.78e+02
nblk= 3	SDPNAL+	-	-	-	-	-
m=1200	SDPT3	2.57e-07	0.00e+00	7.34e-07	-4.8614176e+02	1.96e+01
buck5	SDPF+	9.95e-07	6.10e-08	8.37e-07	-4.3616210e+02	2.22e+03
nblk= 3	SDPNAL+	-	-	-	-	-
m=3280	SDPT3	1.49e-06	0.00e+00	7.63e-05	-4.3619638e+02	1.85e+02
cancer	SDPF+	2.39e-06	3.17e-06	6.18e-08	-2.7623801e+04	3.60e+03
nblk= 1	SDPNAL+	9.93e-15	2.82e-11	1.89e-05	-2.7625376e+04	3.60e+03
m=10469	SDPT3	9.26e-08	0.00e+00	2.89e-04	-2.7607459e+04	8.59e+01
checker	SDPF+	4.66e-09	9.69e-11	2.87e-07	3.3038867e+03	4.54e+01
nblk= 1	SDPNAL+	1.28e-14	2.21e-06	3.33e-04	3.3065285e+03	3.61e+03
m=3970	SDPT3	3.13e-11	0.00e+00	2.28e-07	3.3038861e+03	1.23e+02
cnhil10	SDPF+	8.13e-07	9.37e-07	2.51e-10	0.0000000e+00	1.68e+02
nblk= 1	SDPNAL+	8.94e-08	2.23e-07	3.04e-06	0.0000000e+00	1.95e+01
m=5005	SDPT3	3.45e-08	0.00e+00	2.73e-04	0.0000000e+00	1.70e+01
cnhil8	SDPF+	9.46e-07	7.37e-07	1.60e-09	0.0000000e+00	1.72e+01
nblk= 1	SDPNAL+	2.70e-07	3.13e-07	7.95e-07	0.0000000e+00	9.04e+00
m=1716	SDPT3	1.72e-08	0.00e+00	6.77e-07	0.0000000e+00	1.90e+00
cphil10	SDPF+	1.73e-13	1.29e-10	3.64e-13	0.0000000e+00	9.00e+00
nblk= 1	SDPNAL+	1.59e-16	9.13e-17	1.52e-16	0.0000000e+00	2.80e-01
m=5005	SDPT3	1.93e-14	0.00e+00	2.04e-08	0.0000000e+00	9.87e+00
cphil12	SDPF+	3.09e-13	4.69e-10	6.70e-13	0.0000000e+00	5.70e+01
nblk= 1	SDPNAL+	1.63e-16	1.26e-16	1.59e-16	0.0000000e+00	4.16e-01
m=12376	SDPT3	1.48e-13	0.00e+00	2.01e-07	0.0000000e+00	6.47e+01
dia_patch	SDPF+	2.28e-07	3.30e-07	4.83e-07	5.4174249e-06	1.53e+03
nblk= 1	SDPNAL+	-	-	-	-	-
m=5478	SDPT3	-	-	-	-	-
e*quad*	SDPF+	2.93e-05	1.96e-04	1.43e-04	4.9544525e+03	3.60e+03
nblk= 3	SDPNAL+	1.20e-09	1.72e-06	1.73e-04	4.9270250e+03	3.22e+02
m=5984	SDPT3	1.43e-07	3.87e-08	8.90e-07	4.9273596e+03	9.21e+01
e*stable*	SDPF+	5.59e-07	1.35e-07	9.69e-07	-1.9843376e-01	1.29e+03
nblk=19	SDPNAL+	9.20e-07	4.58e-07	1.38e-05	-1.9846413e-01	1.16e+02
m=5984	SDPT3	1.05e-09	6.08e-08	9.34e-06	-1.9843397e-01	8.21e+01
fap09	SDPF+	9.99e-07	8.68e-07	9.57e-12	1.0780819e+01	2.81e+02
nblk= 2	SDPNAL+	2.74e-07	4.27e-04	2.15e-06	1.0809393e+01	4.74e+00
m=30276	SDPT3	-	-	-	-	-
foot	SDPF+	5.74e-09	0.00e+00	1.36e-08	-5.8529815e+05	6.14e+01

Table EC.7: Test results for SDP problems from Hans Mittelmann benchmarks. Here comp' denotes $\min\{\text{comp}, \text{pdgap}\}$.

problem	algorithm	pfeas	dfeas	comp'	fval	time
nblk= 1 m=2209	SDPNAL+ SDPT3	3.09e-07 3.97e-06	1.89e-10 0.00e+00	1.79e-05 8.20e-06	-5.8534088e+05 -5.8528857e+05	3.60e+03 4.73e+01
hand nblk= 1 m=1297	SDPF+ SDPNAL+ SDPT3	1.84e-08 7.72e-07 1.55e-08	2.56e-08 6.20e-10 0.00e+00	3.90e-09 1.24e-07 5.53e-07	-2.4747779e+04 -2.4747790e+04 -2.4747752e+04	6.84e+00 1.82e+03 1.26e+01
ice_2.0 nblk= 1 m=8113	SDPF+ SDPNAL+ SDPT3	2.43e-09 - 3.87e-12	1.63e-10 - 0.00e+00	4.08e-07 - 3.80e-07	6.8083911e+03 - 6.8083914e+03	2.14e+02 - 7.79e+02
inc_1200 nblk= 2 m=5175	SDPF+ SDPNAL+ SDPT3	2.32e-06 3.79e-12 4.81e-08	4.75e-07 8.09e-06 0.00e+00	8.83e-10 1.90e-03 1.23e-05	-1.1575928e+00 8.3700086e-02 -1.1566568e+00	3.60e+03 3.60e+03 6.57e+01
inc_600 nblk= 2 m=2515	SDPF+ SDPNAL+ SDPT3	9.84e-07 3.65e-14 2.74e-08	2.56e-07 6.75e-06 2.57e-18	4.09e-09 1.81e-03 1.94e-06	-6.6855340e-01 -2.3678686e-02 -6.6843406e-01	2.46e+03 3.60e+03 1.60e+01
neosfbr25 nblk= 1 m=14376	SDPF+ SDPNAL+ SDPT3	2.66e-05 1.41e-09 2.58e-11	1.13e-05 1.81e-07 0.00e+00	4.74e-07 2.78e-06 8.31e-08	3.4537603e+03 3.4535482e+03 3.4536139e+03	3.60e+03 2.50e+03 7.76e+02
neosfbr30e8 nblk= 1 m=25201	SDPF+ SDPNAL+ SDPT3	1.26e-04 3.00e-10 1.98e-12	1.05e-04 1.29e-06 0.00e+00	4.51e-04 1.85e-05 7.35e-08	5.7047831e+03 5.6961603e+03 5.6954664e+03	3.63e+03 3.60e+03 2.82e+03
neu1 nblk= 2 m=3003	SDPF+ SDPNAL+ SDPT3	3.46e-07 1.05e-13 8.32e-09	6.62e-07 2.11e-09 9.23e-09	7.37e-08 1.88e-04 9.12e-05	-5.0331671e-06 -4.6682975e-04 -1.0776363e-05	4.13e+02 5.09e+02 1.27e+02
neu1g nblk= 1 m=3002	SDPF+ SDPNAL+ SDPT3	6.20e-07 1.62e-13 4.19e-10	4.45e-07 1.79e-09 0.00e+00	9.61e-09 1.80e-06 1.99e-07	1.2500000e+02 1.2499954e+02 1.2500000e+02	1.12e+02 4.70e+02 1.04e+02
neu2 nblk= 2 m=3003	SDPF+ SDPNAL+ SDPT3	- - 5.05e-08	- - 1.24e-07	- - 2.12e-04	- - -6.0403737e+02	- - 1.26e+02
neu2c nblk=13 m=3002	SDPF+ SDPNAL+ SDPT3	- - 3.98e-05	- - 0.00e+00	- - 9.13e-03	- - 3.4472963e+04	- - 3.39e+02
neu2g nblk= 1 m=3002	SDPF+ SDPNAL+ SDPT3	1.25e-03 3.06e-08 3.06e-08	2.85e-08 0.00e+00 0.00e+00	1.46e-03 6.80e-07 3.4100002e+04	3.3819730e+04 - - 3.4100002e+04	3.60e+03 - - 1.02e+02
neu3g nblk= 1 m=8007	SDPF+ SDPNAL+ SDPT3	6.94e-07 9.28e-07 6.53e-15	2.89e-07 7.95e-08 0.00e+00	9.76e-07 5.03e-07 8.32e-05	3.1254591e-07 6.4084985e-07 1.3150173e-05	6.88e+02 6.52e+01 1.96e+02
p_auss2_3.0 nblk=1 m=9115	SDPF+ SDPNAL+ SDPT3	2.44e-09 - 1.44e-12	8.92e-11 - 0.00e+00	8.07e-07 - 8.54e-07	8.6181059e+03 - 8.6181093e+03	2.67e+02 - 9.47e+02
rabmo nblk= 2 m=5004	SDPF+ SDPNAL+ SDPT3	1.84e-05 6.04e-13 1.56e-06	7.47e-06 5.61e-07 3.66e-06	3.27e-06 2.44e-07 2.71e-04	-3.7182372e+00 -3.7274501e+00 -3.7272538e+00	3.60e+03 1.60e+02 1.04e+02
reimer5 nblk= 2 m=6187	SDPF+ SDPNAL+ SDPT3	3.77e-03 - 8.11e-06	1.64e-04 - 1.03e-07	4.33e-04 - 1.40e-05	-1.5163447e+01 - -1.5183781e+01	3.60e+03 - 2.71e+03
rl_2000 nblk= 1 m=2001	SDPF+ SDPNAL+ SDPT3	3.49e-09 1.21e-04 1.04e-08	1.40e-07 9.09e-12 0.00e+00	2.34e-05 2.71e-07 3.93e-07	-4.8997334e+05 -4.9029307e+05 -4.9029215e+05	3.60e+03 3.60e+03 3.52e+01
rose13 nblk= 1 m=2379	SDPF+ SDPNAL+ SDPT3	8.89e-07 1.84e-15 8.38e-11	6.16e-07 8.86e-09 0.00e+00	5.60e-07 6.13e-05 3.72e-07	1.1999921e+01 1.1998465e+01 1.2000000e+01	8.05e+01 1.97e+02 7.83e+00
rose15 nblk= 2 m=3860	SDPF+ SDPNAL+ SDPT3	8.40e-07 4.59e-16 3.14e-09	1.52e-07 6.76e-09 4.16e-08	9.37e-07 8.53e-04 1.44e-03	-8.1462754e-05 -1.9993087e-03 -6.3462959e-06	2.84e+03 3.56e+02 3.46e+01
sensor_1000b nblk= 2 m=5549	SDPF+ SDPNAL+ SDPT3	9.68e-07 4.08e-07 5.60e-10	8.62e-07 1.35e-07 0.00e+00	2.60e-09 1.87e-05 6.15e-08	2.3355746e+00 2.3347799e+00 2.3355818e+00	2.42e+02 1.38e+03 6.28e+01
sensor_500b nblk= 2 m=3540	SDPF+ SDPNAL+ SDPT3	8.63e-07 9.10e-07 7.40e-11	1.57e-07 1.32e-08 0.00e+00	2.32e-08 2.30e-09 1.12e-07	1.7389448e+01 1.7389458e+01 1.7389454e+01	6.67e+01 9.99e+01 2.26e+01
shmup3 nblk= 3 m=420	SDPF+ SDPNAL+ SDPT3	4.07e-07 - 2.06e-09	6.66e-08 - 0.00e+00	9.88e-07 - 7.23e-07	-2.0988377e+03 - -2.0988351e+03	5.27e+02 - 3.53e+01
shmup4 nblk= 3 m=800	SDPF+ SDPNAL+ SDPT3	6.37e-07 - 6.91e-09	5.68e-08 - 1.89e-14	9.88e-07 - 3.25e-07	-7.9925517e+03 - -7.9925465e+03	1.83e+03 - 1.80e+02
shmup5 nblk= 3 m=1800	SDPF+ SDPNAL+ SDPT3	2.30e-05 - 4.58e-08	3.23e-07 - 0.00e+00	8.69e-05 - 5.11e-07	-2.3859149e+04 - -2.3858904e+04	3.60e+03 - 1.14e+03
spar060 nblk= 2 m=7261	SDPF+ SDPNAL+ SDPT3	4.19e-07 6.41e-07 4.04e-12	0.00e+00 1.76e-12 0.00e+00	1.89e-09 6.14e-08 6.40e-08	-1.2780000e+03 -1.2779990e+03 -1.2779999e+03	4.65e+00 1.99e+01 7.36e+01
swissroll nblk= 1 m=3380	SDPF+ SDPNAL+ SDPT3	7.46e-04 - - - -	8.92e-10 - - - -	3.25e-03 - - - -	-5.5490868e+05 - - - -	3.60e+03 - - - -
taha1a nblk=14 m=3002	SDPF+ SDPNAL+ SDPT3	1.38e-07 4.89e-07 1.26e-05	8.89e-07 9.25e-08 8.84e-18	9.70e-07 2.65e-08 8.69e-07	-9.9999539e-01 -9.9999872e-01 -1.0000070e+00	1.83e+02 5.85e+01 2.32e+02
taha1b nblk=22 m=8007	SDPF+ SDPNAL+ SDPT3	5.00e-07 1.00e-13 4.64e-12	2.00e-07 3.85e-08 0.00e+00	1.66e-06 4.08e-05 7.16e-06	-7.7329713e-01 -7.7352821e-01 -7.7328713e-01	3.60e+03 3.60e+03 2.77e+02
taha1c nblk=14 m=6187	SDPF+ SDPNAL+ SDPT3	6.25e-07 2.73e-12 9.51e-03	8.71e-07 1.79e-07 1.06e-17	2.39e-07 1.58e-06 2.02e-03	-9.9999795e-01 -1.0000046e+00 -1.0000122e+00	7.87e+02 3.41e+02 2.44e+03
theta12 nblk= 1	SDPF+ SDPNAL+	5.89e-07 9.24e-07	4.07e-08 7.80e-09	6.92e-10 2.59e-10	-9.2801485e+01 -9.2801694e+01	2.30e+01 1.74e+01

Table EC.7: Test results for SDP problems from Hans Mittelmann benchmarks. Here comp' denotes $\min\{\text{comp}, \text{pdgap}\}$.

problem	algorithm	pfeas	dfeas	comp'	fval	time
m=17979	SDPT3	1.28e-12	0.00e+00	2.72e-07	-9.2801537e+01	1.86e+02
theta123 nblk= 1 m=90020	SDPF+ SDPNAL+ SDPT3	3.72e-07 5.74e-07 -	1.56e-07 2.76e-08 -	1.60e-12 7.42e-11 -	-2.4668675e+01 -2.4668640e+01 -	3.83e+02 1.61e+01 -
t_texture nblk= 1 m=1802	SDPF+ SDPNAL+ SDPT3	1.31e-04 1.23e-07 4.67e-08	7.23e-10 3.52e-09 0.00e+00	6.85e-04 8.79e-04 3.53e-04	3.4445867e+02 3.4410041e+02 3.4460806e+02	3.60e+03 3.60e+03 4.44e+01
trto3 nblk= 2 m=544	SDPF+ SDPNAL+ SDPT3	8.21e-07 - 2.32e-05	3.86e-12 - 0.00e+00	5.01e-07 - 9.32e-07	-1.2799974e+04 - -1.2799982e+04	1.24e+02 - 2.11e+00
trto4 nblk= 2 m=1200	SDPF+ SDPNAL+ SDPT3	4.51e-07 - 7.52e-06	6.26e-17 - 0.00e+00	4.49e-07 - 1.08e-06	-1.2765821e+04 - -1.2765805e+04	1.11e+03 - 9.69e+00
trto5 nblk= 2 m=3280	SDPF+ SDPNAL+ SDPT3	5.01e-04 - 8.39e-05	1.57e-10 - 0.00e+00	2.13e-03 - 1.41e-05	-1.2808388e+04 - -1.2799641e+04	3.60e+03 - 7.08e+01
vibra3 nblk= 3 m=544	SDPF+ SDPNAL+ SDPT3	3.70e-07 - 3.11e-06	6.34e-12 - 0.00e+00	9.90e-07 - 1.19e-06	-1.7261308e+02 - -1.7261270e+02	6.98e+02 - 4.59e+00
vibra4 nblk= 3 m=1200	SDPF+ SDPNAL+ SDPT3	1.16e-03 - 9.61e-07	1.59e-09 - 0.00e+00	5.54e-03 - 9.72e-07	-1.6547537e+02 - -1.6561301e+02	3.60e+03 - 2.66e+01
vibra5 nblk= 3 m=3280	SDPF+ SDPNAL+ SDPT3	5.10e-04 - 9.14e-06	4.78e-07 - 0.00e+00	1.49e-03 - 2.58e-05	-1.2819633e+02 - -1.6589585e+02	3.60e+03 - 2.91e+02
yalsdp nblk= 3 m=5051	SDPF+ SDPNAL+ SDPT3	4.88e-07 4.91e-07 2.86e-10	6.88e-11 1.19e-09 0.00e+00	8.80e-08 6.86e-10 4.55e-08	-1.7921268e+00 -1.7921268e+00 -1.7921263e+00	8.04e+01 6.17e+01 8.78e+01

Table EC.8: Detailed test results for non-parametric density estimation.

problem	algorithm	pfeas	dfeas	comp	fval	time
n=500, rank=10 $\sigma = 0.5/\sqrt{2\pi}$	SDPF+ FISTA	6.43e-09 9.91e-07	1.32e-07 2.36e-08	5.79e-09 3.68e-08	1.0215680e+01 1.0215698e+01	2.03e+01 3.36e+01
n=500, rank=5 $\sigma = 1/\sqrt{2\pi}$	SDPF+ FISTA	1.83e-09 2.36e-07	9.62e-07 7.46e-07	1.31e-08 3.38e-07	3.1095986e+00 3.1095982e+00	1.07e+01 4.65e+01
n=500, rank=39 $\sigma = 2/\sqrt{2\pi}$	SDPF+ FISTA	6.49e-09 1.54e-07	1.75e-07 5.33e-07	2.26e-07 3.52e-07	3.2469466e+00 3.2469462e+00	1.06e+01 3.60e+03
n=1000, rank=11 $\sigma = 0.5/\sqrt{2\pi}$	SDPF+ FISTA	2.17e-09 9.59e-07	6.68e-07 4.84e-08	3.33e-08 5.77e-08	9.1833437e+00 9.1833572e+00	3.19e+01 1.46e+02
n=1000, rank=8 $\sigma = 1/\sqrt{2\pi}$	SDPF+ FISTA	3.10e-10 1.07e-07	2.01e-07 0.00e+00	3.48e-08 2.48e-07	3.1707847e+00 3.1707849e+00	3.77e+01 3.47e+02
n=1000, rank=38 $\sigma = 2/\sqrt{2\pi}$	SDPF+ FISTA	5.50e-14 -	4.44e-07 -	2.08e-07 -	3.2405354e+00 -	3.75e+01 -
n=2000, rank=8 $\sigma = 0.5/\sqrt{2\pi}$	SDPF+ FISTA	1.60e-09 9.61e-07	3.37e-07 6.17e-08	2.95e-09 8.08e-08	8.4964317e+00 8.4964428e+00	1.28e+02 7.36e+02
n=2000, rank=7 $\sigma = 1/\sqrt{2\pi}$	SDPF+ FISTA	4.92e-09 8.58e-08	6.69e-07 0.00e+00	9.01e-10 2.51e-07	3.2101141e+00 3.2101142e+00	1.83e+02 2.72e+03
n=2000, rank=30 $\sigma = 2/\sqrt{2\pi}$	SDPF+ FISTA	4.44e-16 -	7.77e-07 -	4.58e-08 -	3.2109010e+00 -	3.87e+02 -

Table EC.9: Detailed test results for H-weighted nearest correlation matrix problem with square loss function.

problem	algorithm	pfeas	dfeas	comp	fval	time
n = 100 rank=56	SDPF+ SSNAL	9.94e-09 8.68e-10	8.63e-07 2.00e-08	2.33e-08 6.16e-08	7.6275514e+01 7.6275517e+01	8.63e+00 1.27e+01
n = 200 rank= 100	SDPF+ SSNAL	9.26e-09 6.58e-10	4.01e-07 1.05e-08	1.73e-08 2.58e-08	4.4762000e+02 4.4762000e+02	1.73e+01 8.73e+00
n = 500 rank=222	SDPF+ SSNAL	4.44e-09 1.39e-10	8.36e-07 1.96e-09	2.71e-09 9.51e-09	3.7958633e+03 3.7958627e+03	3.40e+01 2.40e+01
n = 1000 rank=396	SDPF+ SSNAL	8.32e-09 2.51e-10	3.81e-07 4.88e-10	4.60e-10 9.51e-09	1.8637052e+04 1.8637051e+04	7.73e+01 6.92e+01
n = 2000 rank=697	SDPF+ SSNAL	9.13e-09 2.44e-11	3.32e-07 1.95e-10	8.04e-12 9.36e-09	9.0980052e+04 9.0980052e+04	3.06e+02 3.31e+02
n = 5000 rank=1441	SDPF+ SSNAL	1.47e-11 1.00e-10	6.96e-07 5.50e-10	2.41e-08 2.51e-08	7.1678529e+05 7.1678527e+05	2.36e+03 3.26e+03

Table EC.10: Detailed test results for H-weighted nearest correlation matrix problems with the Huber-loss function.

problem	algorithm	pfeas	dfeas	comp	fval	time
n=100 rank=56	SDPF+	8.16e-09	7.22e-07	1.74e-07	5.0989963e+01	2.21e+01
	APG	0.00e+00	9.79e-07	2.68e-16	5.0989960e+01	6.44e+01
n=200 rank=99	SDPF+	9.42e-09	7.42e-07	8.07e-08	2.6947628e+02	2.65e+01
	APG	0.00e+00	9.90e-07	3.50e-16	2.6947628e+02	1.85e+02
n=500 rank=216	SDPF+	1.19e-11	7.82e-07	3.47e-08	2.1158843e+03	7.69e+01
	APG	0.00e+00	9.61e-07	2.97e-16	2.1158842e+03	1.35e+03
n=1000 rank=384	SDPF+	4.44e-09	9.46e-07	8.16e-08	9.8461211e+03	1.84e+02
	APG	0.00e+00	2.32e-04	1.39e-15	9.8461306e+03	3.60e+03
n=2000 rank=654	SDPF+	5.29e-10	6.54e-07	7.68e-08	4.5499748e+04	8.53e+02
	APG	0.00e+00	3.75e-03	6.59e-17	4.5510046e+04	3.60e+03
n=5000 rank=1465	SDPF+	3.47e-09	8.27e-04	9.82e-03	3.3638798e+05	3.61e+03
	APG	-	-	-	-	-

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