Appendix A – Examples for Propositions 4, 5, 6, 8

A minimal description of $P_{3,3}(G)$ for an instance of the problem (see Figure 1) is provided below by employing PORTA.

$$
\begin{array}{cccccccccccccccc}
-x_1 & & & & & & & & & & & & & & & \leq 0 \\
 & -x_2 & & & & & & & & & & & & & & \leq 0 \\
 & & -x_3 & & & & & & & & & & & & & \leq 0 \\
 & & & -x_4 & & & & & & & & & & & & \leq 0 \\
 & & & +x_4 & & & & & & & & & & & -y_8 & \leq 0 \\
 & & +x_3 & & & & & & & & & & & -y_7 & & \leq 0 \\
 & +x_2 & & & & & & & & & & & -y_6 & & & \leq 0 \\
+x_1 & & & & & & & & & & & -y_5 & & & & \leq 0 \\
 & +x_2 & & +x_4 & & -y_1 & & & & & & & & & & \leq 0 \\
 & +x_2 & & +x_4 & & & & -y_3 & & & & & & & & \leq 0 \\
+x_1 & & +x_3 & & & & -y_2 & & & & & & & & & \leq 0 \\
+x_1 & & +x_3 & & & & & & -y_4 & & & & & & & \leq 0 \\
 & +x_2 & & +x_4 & & & & & +y_4 & & & & & & & \leq 1 \\
 & +x_2 & & +x_4 & & & +y_2 & & & & & & & & & \leq 1 \\
+x_1 & & +x_3 & & & & & +y_3 & & & & & & & & \leq 1 \\
+x_1 & & +x_3 & & & +y_1 & & & & & & & & & & \leq 1 \\
 & +x_2 & +x_3 & +x_4 & & & & & & & +y_5 & & & & & \leq 1 \\
+x_1 & & +x_3 & +x_4 & & & & & & & & +y_6 & & & & \leq 1 \\
+x_1 & +x_2 & & +x_4 & & & & & & & & & +y_7 & & & \leq 1 \\
+x_1 & +x_2 & +x_3 & & & & & & & & & & & +y_8 & & \leq 1 \\
 & & & & & +y_1 & +y_2 & +y_3 & +y_4 & +y_5 & +y_6 & +y_7 & +y_8 & & & \leq 3
\end{array}
$$

As these inequalities provide a minimal description of the anchored 3-core polytope with a budget of 3 and the polytope is full dimensional, the following observations show the tightness of conditions in Propositions 4–6 and Proposition 8.

- For Proposition 4, inequalities $x_1 + y_1 \leq 1$, $x_2 + y_2 \leq 1$, $x_3 + y_3 \leq 1$, or $x_4 + y_4 \leq 1$ are not present.
- For Proposition 5, inequalities $y_5 \leq 1$, $y_6 \leq 1$, $y_7 \leq 1$, or $y_8 \leq 1$ are not present.
- For Proposition 6, inequalities $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, or $y_4 \geq 0$ are not present.
- For Proposition 8, inequalities $x_1 \leq x_2 + y_2$, $x_1 \leq x_3 + y_3$, $x_2 \leq x_1 + y_1$, $x_2 \leq x_4 + y_4$, $x_3 \leq x_4 + y_4$, $x_3 \leq x_1 + y_1$, $x_4 \leq x_3 + y_3$ and $x_4 \leq x_2 + y_2$ are not present.

Thus, we showed that our conditions for Propositions 4–6 and Proposition 8 are minimal.

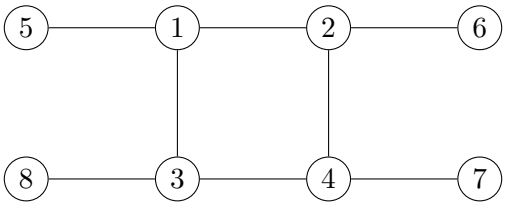

Figure 1 **An instance of the maximum anchor k-core problem with $k = 3$ and $b = 3$**