

Operations Research Techniques for Scheduling Chile's Second Division Soccer League

Guillermo Durán

Departamento de Ingeniería Industrial, Universidad de Chile, Santiago 8370439 Chile

Departamento de Matemática, FCEyN, Universidad de Buenos Aires, Buenos Aires C1428EGA, Argentina

IMAS-CONICET, Argentina, gduran@dii.uchile.cl

Mario Guajardo

Department of Finance and Management Science, NHH Norwegian School of Economics, Bergen 5045

Norway, mario.guajardo@nhh.no

Rodrigo Wolf-Yadlin

Departamento de Ingeniería Industrial, Universidad de Chile, Santiago 8370439 Chile, rwolf@dii.uchile.cl

Appendix A: Match Scheduling Model (MSM)

Sets and parameters

Teams: set of teams.

Rounds: set of rounds.

m : number of times each team plays each other team.

l_{home} : maximum number of home breaks a team can have.

l_{away} : maximum number of away breaks a team can have.

RoundsNoBreak: set of rounds in which no team can have a break.

Clusters: set of geographical clusters.

$TeamsCl(a)$: set of teams in cluster a (defined $\forall a \in Clusters$).

s : a given number of consecutive rounds for which a maximum number of trips by a team to other clusters is defined.

$t_{low}^{a,b}$: minimum number of times a team in cluster b can play away against teams in cluster a in s consecutive rounds (defined $\forall a, b \in Clusters$).

$t_{up}^{a,b}$: maximum number of times a team in cluster b can play away against teams in cluster a in s consecutive rounds (defined $\forall a, b \in Clusters$).

$T_{extreme}$: set of teams for which good trips must be scheduled (teams in far north or south of country).

$T_{trip}(i)$: set of teams suitable as opponents on good trips for team i (teams in zones relatively distant from team i).

$n_{trip}(i)$: number of good trips that must be scheduled for team i .

R_{trip} : set containing all of the rounds k for which a good trip can be scheduled between rounds k and $k + 1$ (i.e., either k or $k + 1$ is a midweek round).

$C(i)$: cluster containing team i .

ComplemenTeams: set of team pairs (i, j) such that i and j must have complementary home-away patterns.

HomeFix: set of pairs (i, k) such that team i must play at home in round k .

AwayFix: set of pairs (i, k) such that team i must play away in round k .

GameFix: set of triples (i, j, k) such that team i must play at home against team j in

round k .

T_{games} : number of matches each team must play during a tournament.

A : set containing the four teams that finish the zonal phase in first or second place in their respective zones (2010 season).

B : set containing the fifth, sixth, and seventh rounds of each half of the final phase double round-robin (2010 season).

Decision variables

The binary decision variables are as follows:

$$x_{i,j,k} = \begin{cases} 1 & \text{if team } i \text{ plays at home against team } j \text{ in round } k, \\ 0 & \text{otherwise.} \end{cases}$$

The variables $x_{i,j,k}$ are defined $\forall i \neq j \in Teams, \forall k \in Rounds$.

To model the breaks and good trips, we also define the following auxiliary variables:

$$w_{i,k} = \begin{cases} 1 & \text{if team } i \text{ has a good trip in rounds } k \text{ and } k + 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{i,k} = \begin{cases} 1 & \text{if team } i \text{ plays at home in rounds } k \text{ and } k + 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{i,k} = \begin{cases} 1 & \text{if team } i \text{ plays away in rounds } k \text{ and } k + 1, \\ 0 & \text{otherwise.} \end{cases}$$

The variables $w_{i,k}$, $y_{i,k}$, and $z_{i,k}$ are defined $\forall i \in Teams, \forall k < |Rounds|$. Although the values of these variables are determined by the values taken by the $x_{i,j,k}$ variables, the

auxiliary variables $y_{i,k}$ and $z_{i,k}$ are useful to formulate easily break constraints (3) and $w_{i,k}$ are useful to obtain good trips as required by geographic constraints (6).

Constraints

$$\sum_i (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall j \in Teams, k \in Rounds. \quad (1)$$

$$\sum_k (x_{i,j,k} + x_{j,i,k}) = m \quad \forall i, j \in Teams. \quad (2)$$

$$\sum_{k < |Rounds|} y_{i,k} \leq l_{home} \quad \forall i \in Teams. \quad (3a)$$

$$\sum_{k < |Rounds|} z_{i,k} \leq l_{away} \quad \forall i \in Teams. \quad (3b)$$

$$\sum_j (x_{i,j,k} + x_{i,j,k+1}) - y_{i,k} \leq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3c)$$

$$\sum_j (x_{j,i,k} + x_{j,i,k+1}) - z_{i,k} \leq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3d)$$

$$y_{i,k} - \sum_j x_{i,j,k} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3e)$$

$$y_{i,k} - \sum_j x_{i,j,k+1} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3f)$$

$$z_{i,k} - \sum_j x_{j,i,k} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3g)$$

$$z_{i,k} - \sum_j x_{j,i,k+1} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3h)$$

$$y_{i,k-1} = 0 \quad \forall k \in RoundsNoBreak, \forall i \in Teams. \quad (4a)$$

$$z_{i,k-1} = 0 \quad \forall k \in RoundsNoBreak, \forall i \in Teams. \quad (4b)$$

$$t_{low}^{a,b} \leq \sum_{\substack{k_0+1 \leq k \leq k_0+s, \\ i \in TeamsCl(a)}} x_{i,j,k} \leq t_{up}^{a,b} \quad \forall j \in TeamsCl(b), \forall a, b \in Clusters, \quad (5)$$

$$0 \leq k_0 \leq |Rounds| - s.$$

$$\sum_{k \in R_{trip}} w_{i,k} = n_{trip}(i) \quad \forall i \in T_{extreme}. \quad (6a)$$

$$\sum_{j \in T_{trip}(i)} (x_{j,i,k} + x_{j,i,k+1}) - 2w_{i,k} \geq 0 \quad \forall i \in T_{extreme}, \forall k \in R_{trip}. \quad (6b)$$

$$\sum_i (x_{i,j,k} + x_{i,j,k+1}) - \sum_{i \in C(j)} (x_{i,j,k} + x_{i,j,k+1}) \leq 1 \quad \forall j \in Teams \setminus T_{extreme},$$

$$k < |Rounds| \quad \text{and} \quad \forall j \in T_{extreme}, k < |Rounds|, k \notin R_{trip}. \quad (7)$$

$$\sum_h (x_{i,h,k} + x_{j,h,k}) = 1 \quad \forall (i,j) \in ComplenTeams, k \in Rounds. \quad (8)$$

$$\sum_j x_{i,j,k} = 1 \quad \forall (i,k) \in HomeFix. \quad (9)$$

$$\sum_j x_{j,i,k} = 1 \quad \forall (i,k) \in AwayFix. \quad (10)$$

$$x_{i,j,k} = 1 \quad \forall (i,j,k) \in GameFix. \quad (11)$$

$$\sum_{j,k} x_{i,j,k} = \frac{T_{games}}{2} \quad \forall i \in Teams. \quad (12)$$

$$\sum_j (x_{j,i,k} + x_{j,i,k+1} + x_{j,i,k+2}) \leq 2 \quad \forall i, k < |Rounds| - 1. \quad (13)$$

$$\sum_j (x_{i,j,k} + x_{i,j,k+1} + x_{i,j,k+2}) \leq 2 \quad \forall i, k < |Rounds| - 1. \quad (14)$$

Objective function (2010 season)

$$\max \sum_{\substack{i,j \in A, \\ k \in B}} x_{i,j,k}. \quad (15)$$

We numbered the constraints as in the *Generic Conditions* subsection of the main text.

Constraint (1) ensures that exactly one of the variables $x_{i,j,k}$ or $x_{j,i,k}$ must be equal to 1 (i.e., each team i plays one match in each round).

Constraint (2) states that each team plays each other team m times. The value of m depends on the tournament. In 2007 and 2008, it was 4; in 2009 it was 2 for any pair of teams that belonged to the same group and 1 if they belonged to different groups. For the 2010 tournament, it was 4 for teams in the same zone and 0 for teams in different zones (in addition, teams that classify for the final phase meet twice).

Constraints (3a) and (3b) model the maximum number of breaks at home and away, respectively. The values of l_{home} and l_{away} are set at 1 for each of the “parts” into which the tournaments are divided: the quarters in 2007 and 2008, the zonal phase and the national phase in 2009, and the zonal phase quarters and the final phase halves in 2010 (see the *Types of Second Division Tournaments* subsection in the main text). Because the two parameters in some future tournament might not have the same value, we prefer to maintain l_{home} and l_{away} as separate parameters. In some cases, such as the zonal phases of the 2009 opening and closing tournaments, an upper bound l_{total} for the total number of breaks was used via the inequality $\sum_{k < |Rounds|} (y_{i,k} + z_{i,k}) \leq l_{total} \quad \forall i \in Teams$. For the 2009 zonal phases, the l_{total} parameter was fixed at 1.

Constraints (3c)–(3h) are logical relationships for calculating variables y and z , as follows: Constraints (3c) and (3d) impose that $y_{i,k}$ will be 1 if both $x_{i,j,k}$ and $x_{i,j,k+1}$ are equal to 1, and z will be 1 if both $x_{j,i,k}$ and $x_{j,i,k+1}$ equal 1; Constraints (3e) and (3f) impose that $y_{i,k}$ is 0 if $x_{i,j,k}$ or $x_{i,j,k+1}$ equals 0; and Constraints (3g) and (3h) impose that $z_{i,k}$ is 0 if $x_{j,i,k}$ or $x_{j,i,k+1}$ equals 0.

Constraints (4a) and (4b) assure that no team has a break in a round belonging to the set *RoundsNoBreak*. This set includes the second and last rounds of each tournament. These restrictions have generally been imposed for reasons of scheduling equity in all the tournaments. The exception was 2008 when one team requested that its first two matches of the season be played away because its home stadium was not yet ready for use, implying that another team played its first two matches at home.

Constraint (5) attempts to evenly distribute the number of trips teams make to relatively distant places by setting a lower bound $t_{low}^{a,b}$ and an upper bound $t_{up}^{a,b}$. Three clusters are typically defined: North, Center, and South. The values of the bounds have varied from year to year. In 2007, for example, it was specified that $t_{low}^{a,b} = 0$, $t_{up}^{a,b} = 1$ and $s = 4$, for $a = \text{North}$ and $b = \text{South}$. Thus, a team from the South cluster could not play more than one match away against teams from the North cluster in four consecutive rounds. Analogous values were used for the remaining combinations of different a and b clusters. In 2008, the values used were $s = 2$ and $t_{low}^{a,b} = 0$ for all a and b clusters, while $t_{up}^{a,b}$ was 1 if a differed from b and 2 if not. For the zonal phase of 2009, this constraint was not applied because the teams were located relatively close to one other. In the 2009 national phase, the constraint was applied for teams in different clusters, with parameter values of $s = 4$, $t_{low}^{a,b} = 0$, and $t_{up}^{a,b} = 1$. In 2010, the constraint was not imposed in the zonal phase because the teams were again relatively close together geographically. In the 2010 final phase, only two clusters, North and South, were defined (teams from the Center were assigned to either the North or the South cluster), and it was imposed that if a team plays two consecutive games away, at most one of them is against a team of a different cluster (which is equivalent to using parameter values of $s = 2$, $t_{low}^{a,b} = 0$, and $t_{up}^{a,b} = 1$ if a differs from b).

Constraints (6a) and (6b) are aimed at scheduling good trips. The way good trips are scheduled varies from season to season. However, together with ANFP officials, we usually define both the set $T_{extreme}$ of teams from the far north and south of the country for which good trips must be scheduled and, for each such team i , the set $T_{trip}(i)$ of teams suitable as opponents on these trips. The number $n_{trip}(i)$ of good trips that must be scheduled for team i will depend mainly on the number of midweek rounds in the tournament. Constraint (6a) then imposes the number of good trips that must be scheduled for team i and Constraint (6b) indicates the logical relationship between the variables x and w . The definition of the parameter sets and values in these constraints varies from year to year depending on the home locations of the teams participating in the tournaments. For example, in 2008 the set $T_{trip}(Arica)$ was defined for the Arica team, based in the far north, by South zone clubs Lota Schwager, Fernández Vial, and Puerto Montt. With rounds 3 and 5 in the set R_{trip} , the value

of $n_{trip}(Arica)$ was fixed at 1.

In Constraint (7), the first summation expresses two away matches for team j in two consecutive rounds against any other team, while the second summation adds the requirement that the other team must be in the same cluster as team j . By imposing that the difference between the two summations is no more than 1, we ensure that if team j plays two consecutive away matches, at least one will be played in its cluster. Note that for any team j in $T_{extreme}$, this constraint is not applicable to a given round k in the set R_{trip} when scheduling good trips.

Constraint (8) imposes that team i of a complementary pair will play at home whenever its complement j plays away, and vice versa.

Constraints (9) and (10) model place constraints at home and away, respectively. Clearly, if team i must play at home in round k , for some other team j , the variable $x_{i,j,k}$ must be 1. Analogously, if team i must play away in round k , for some other team j , the variable $x_{j,i,k}$ must be 1.

Constraint (11) specifies that team i plays at home against team j in round k . The ANFP has occasionally requested that teams i and j only meet in round k , regardless of which one is at home. Therefore, we write $x_{i,j,k} + x_{j,i,k} = 1$.

Constraint (12) states the number of games each team must play during the tournament, assuming that T_{games} is even. In 2007, for example, each team played 40 matches of which 20 had to be at home and 20 away. In 2008, each team played 44 matches, 22 at home and 22 away. If T_{games} is odd, which to date has only occurred once (in the 2009 national phase when each club played 13 matches), we write $\frac{T_{games} - 1}{2} \leq \sum_{j,k} x_{i,j,k} \leq \frac{T_{games} + 1}{2}$.

Constraints (13) and (14) impose that in three consecutive rounds, no team can play more than two away or two home matches, respectively. Note that the effect of these constraints, might already have been obtained depending on which set and parameters values were defined for Constraints (3a)–(3b) and (4a)–(4b). This was the case in the 2007, 2009, and 2010 seasons, for which a run of three consecutive home or away matches was avoided by not allowing breaks in the second and last rounds of each part of a tournament, permitting no more than one break at home, and permitting no more than one break away per part.

However, Constraints (4a)–(4b), which ban breaks in the second and last rounds, were not applied in 2008. Therefore, Constraints (13) and (14) had to be incorporated explicitly for sets of three consecutive rounds (including as consecutive the two rounds that end one part and begin the next) to prevent the scheduling of two consecutive breaks with the same home-away status, which is considered undesirable. Alternatively, the two restrictions can be formulated using the auxiliary variables as $y_{i,k} + y_{i,k+1} \leq 1$ for Constraint (13) and $z_{i,k} + z_{i,k+1} \leq 1$ for Constraint (14).

The objective function (15) was used in the 2010 season. Recall that for 2010, in addition to satisfying all the constraints, the ANFP asked that as many attractive matches as possible be concentrated in the last three rounds of the final phase. Because the schedule for the final phase is generated before the start of the zonal phase, it cannot identify the teams that will play each final phase date by name and must therefore specify them in terms of their final positions in the zonal phase standings.

To honor the ANFP’s request, we began by defining an attractive match as one between any two of the four teams that finish the zonal phase in first or second place in their respective zones. Six such matches are in each half of the final phase double round-robin. To the extent possible, these games are to be concentrated between the fifth and seventh rounds (inclusive) of each half of the final phase double round-robin. The objective function captures this condition. The problem is easily solved (note that the dimension of the final phase is relatively low given that only eight teams participate), generating an optimal value of six matches.

Appendix B: Pattern Generating Model (PGM)

A home-away pattern (or just pattern) is a sequence that indicates, for each round in a tournament, whether a team plays at home or away. It can be represented by an array (H, A, \dots, H) generated for each team, with as many components as there are rounds in the tournament. If the component in position k of the array assigned to a given team is the letter H , that team plays at home in round k ; if the letter is A , the team plays away. In the pattern generating model (PGM), the indexes i and j are used to denote teams and their corresponding patterns.

Parameters and sets in the PGM that also appear in the MSM maintain the same definition.

Decision variables

The following binary decision variables are defined:

$$\alpha_{i,k} = \begin{cases} 1 & \text{if pattern } i \text{ indicates a home match in round } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$\beta_{i,k} = \begin{cases} 1 & \text{if pattern } i \text{ indicates a home match in rounds } k \text{ and } k + 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\gamma_{i,k} = \begin{cases} 1 & \text{if pattern } i \text{ indicates an away match in rounds } k \text{ and } k + 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\delta_{i,j,k} = \begin{cases} 1 & \text{if pattern } i \text{ differs from pattern } j \text{ for round } k, \\ 0 & \text{otherwise.} \end{cases}$$

Similar to the MSM definitions, this new model includes a group of main decision variables, denoted $\alpha_{i,k}$, whose values determine the values of the auxiliary variables, $\beta_{i,k}$, $\gamma_{i,k}$, and $\delta_{i,j,k}$. The auxiliary variables are useful to model some of the constraints, as we detail below.

Constraints

$$\sum_i \alpha_{i,k} = \frac{|Teams|}{2} \quad \forall k \in Rounds. \quad (1')$$

$$\sum_k \delta_{i,j,k} \geq m \quad \forall i, j \in Teams. \quad (2a')$$

$$\alpha_{i,k} + \alpha_{j,k} - \delta_{i,j,k} \geq 0 \quad \forall i, j \in Teams, \forall k \in Rounds. \quad (2b')$$

$$\alpha_{i,k} + \alpha_{j,k} + \delta_{i,j,k} \leq 2 \quad \forall i, j \in Teams, \forall k \in Rounds. \quad (2c')$$

$$\alpha_{i,k} - \alpha_{j,k} - \delta_{i,j,k} \leq 0 \quad \forall i, j \in Teams, \forall k \in Rounds. \quad (2d')$$

$$\delta_{i,j,k} - \delta_{j,i,k} = 0 \quad \forall i, j \in Teams, \forall k \in Rounds. \quad (2e')$$

$$\sum_{k < |Rounds|} \beta_{i,k} \leq l_{home} \quad \forall i \in Teams. \quad (3a')$$

$$\sum_{k < |Rounds|} \gamma_{i,k} \leq l_{away} \quad \forall i \in Teams. \quad (3b')$$

$$\alpha_{i,k} + \alpha_{i,k+1} - \beta_{i,k} \leq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3c')$$

$$\beta_{i,k} - \alpha_{i,k} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3d')$$

$$\beta_{i,k} - \alpha_{i,k+1} \leq 0 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3e')$$

$$\alpha_{i,k} + \alpha_{i,k+1} + \gamma_{i,k} \geq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3f')$$

$$\alpha_{i,k} + \gamma_{i,k} \leq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3g')$$

$$\alpha_{i,k+1} + \gamma_{i,k} \leq 1 \quad \forall i \in Teams, \forall k < |Rounds|. \quad (3h')$$

$$\beta_{i,k-1} + \gamma_{i,k-1} = 0 \quad \forall i \in Teams, \forall k \in RoundsNoBreak. \quad (4')$$

$$\sum_{k \in R_{trip}} \gamma_{i,k} \geq n_{trip}(i) \quad \forall i \in T_{extreme}. \quad (6')$$

$$\alpha_{i,k} + \alpha_{j,k} = 1 \quad \forall (i, j) \in ComplenTeams, k \in Rounds. \quad (8')$$

$$\alpha_{i,k} = 1 \quad \forall (i, k) \in HomeFix. \quad (9')$$

$$\alpha_{i,k} = 0 \quad \forall (i, k) \in AwayFix. \quad (10')$$

$$\alpha_{i,k} = 1 \quad \forall (i, k) \text{ such that } (i, j, k) \in GameFix. \quad (11a')$$

$$\alpha_{j,k} = 0 \quad \forall (j, k) \text{ such that } (i, j, k) \in GameFix. \quad (11b')$$

$$\sum_k \alpha_{i,k} = \frac{T_{games}}{2} \quad \forall i \in Teams. \quad (12')$$

$$\alpha_{i,k} + \alpha_{i,k+1} + \alpha_{i,k+2} \geq 1 \quad \forall i \in Teams, \forall k < |Rounds| - 1. \quad (13')$$

$$\alpha_{i,k} + \alpha_{i,k+1} + \alpha_{i,k+2} \leq 2 \quad \forall i \in Teams, \forall k < |Rounds| - 1. \quad (14')$$

The constraints in the PGM aim to ensure that once patterns are assigned to each team, certain conditions in the MSM are satisfied. This means that when the MSM is run, the constraints already satisfied can be omitted, thus simplifying the search for a solution.

We have labeled the constraints such that the number in the label of a PGM constraint corresponds to the number in the label of the related MSM constraint.

Constraint (1') states that in each round, half the teams must play at home (implying, because we assume an even number of teams, that the other half play away).

Constraint (2a') imposes that the patterns of any pair of teams must differ at least m times.

Constraints (2b')–(2e') are logical relationships to relate variables α and δ , as follows: Constraint (2b') imposes that when $\alpha_{i,k}$ and $\alpha_{j,k}$ are both 0, $\delta_{i,j,k}$ must also be 0, given that the patterns of teams i and j do not differ for round k ; Constraint (2c') imposes that when $\alpha_{i,k}$ and $\alpha_{j,k}$ are 1, $\delta_{i,j,k}$ is 0, given that the patterns of teams i and j do not differ for round k ; Constraint (2d') imposes that when $\alpha_{i,k}$ is 1 and $\alpha_{j,k}$ is 0, $\delta_{i,j,k}$ is 1, given that the patterns of teams i and j differ for round k (note that the constraint is valid for all i and j in *Teams*, meaning that the case where $\alpha_{j,k}$ is 1 and $\alpha_{i,k}$ is 0 is also possible in this formulation); Constraint (2e') imposes that if i 's pattern differs from j 's in round k , then j 's pattern also obviously differs from i 's in round k .

Constraints (3a') and (3b') allow the patterns to satisfy the condition setting the maximum number of permitted home and away breaks.

Constraints (3c')–(3h') are logical relationships to relate variables α and β , as follows: Constraint (3c') imposes that if $\alpha_{i,k}$ and $\alpha_{i,k+1}$ are 1 (i.e., i 's pattern indicates home matches in rounds k and $k+1$), then $\beta_{i,k}$ must also be 1; Constraints (3d') and (3e') impose that if at least one of the variables $\alpha_{i,k}$ or $\alpha_{i,k+1}$ is 0, then $\beta_{i,k}$ must also be 0; Constraint (3f') imposes that if $\alpha_{i,k}$ and $\alpha_{i,k+1}$ are 0 (i.e., i 's pattern indicates away matches in rounds k and $k+1$), then $\gamma_{i,k}$ must be 1; Constraints (3g') and (3h') impose that if at least one of the variables $\alpha_{i,k}$ or $\alpha_{i,k+1}$ is 1, then $\gamma_{i,k}$ must be 0.

Constraint (4') ensures the patterns satisfy the condition setting the rounds in which no breaks can be scheduled.

To schedule good trips for a team i , its pattern must have at least $n_{trip}(i)$ sequences of two consecutive away matches for rounds indicated by the set R_{trip} , which is modeled by Constraint (6').

Constraint (8') indicates that pairs of complementary teams must have complementary patterns.

Constraints (9') and (10') ensure the patterns are consistent with the condition of a team i that must play round k at home or away, respectively.

Constraints (11a') and (11b') facilitate the satisfaction of the game constraints by the home match dates of the patterns. In cases in which the ANFP requests that teams i and j meet in round k without specifying which is to play at home, we write: $\alpha_{i,k} + \alpha_{j,k} = 1$.

Constraint (12') imposes the number of games each team must play in the case T_{games} is even. If T_{games} is odd, we impose $\frac{T_{games} - 1}{2} \leq \sum_k \alpha_{i,k} \leq \frac{T_{games} + 1}{2}$.

Constraints (13') and (14') ensure the patterns satisfy the condition that no team play more than two consecutive away or two consecutive home matches, respectively.

Note that a given set of patterns generated by the PGM and assigned to the teams will not necessarily result in the production of a feasible solution that satisfies all of the MSM conditions. Although Constraints (3a')–(3h'), (4'), (8'), (9'), (10'), (12'), (13'), and (14') of the PGM ensure that Constraints (3a)–(3h), (4a)–(4b), (8), (9), (10), (12), (13), and (14), respectively, of the MSM are satisfied, Constraints (1'), (2a')–(2e'), (6'), and (11a')–(11b') of the PGM only impose necessary, not sufficient, conditions for the satisfaction of Constraints (1), (2), (6a)–(6b), and (11) when the MSM is run. Constraints (5) and (7) of the MSM are not captured in the PGM.

In our experiments solving the 2009 tournament of the Second Division, we found it useful to define $\forall i \neq j \in Teams, \forall k \in Rounds$ as an additional family of binary variables as follows:

$$\eta_{i,j,k} = \begin{cases} 1 & \text{if pattern } i \text{ is associated with pattern } j \text{ for round } k, \\ 0 & \text{otherwise.} \end{cases}$$

A pattern i is associated with pattern j for round k when team i plays team j in round k in a schedule (note that the home-away conditions are assigned by the variable α_{ik}).

With these variables, a preliminary schedule can be generated when solving the PGM that satisfies all the original conditions, except for the geographic ones, Constraints (5)–(7). Although this will not necessarily be the definitive schedule determined by the MSM, which will incorporate these remaining conditions, its construction is useful to ensure that

the patterns generated lead to a schedule that complies with all the other conditions.

The constraints on these variables are formulated below.

$$\eta_{i,j,k} - \delta_{j,i,k} \leq 0 \quad \forall i, j \in Teams, k \in Rounds. \quad (15a')$$

$$\sum_j \eta_{i,j,k} = 1 \quad \forall i \in Teams, k \in Rounds. \quad (15b')$$

$$\sum_k \eta_{i,j,k} = m \quad \forall i, j \in Teams. \quad (15c')$$

$$\eta_{i,j,k} - \eta_{j,i,k} = 0 \quad \forall i, j \in Teams, k \in Rounds. \quad (15d')$$

Constraint (15a') is a logical relationship between $\delta_{i,j,k}$ and $\eta_{i,j,k}$, indicating that for a team i to be able to play team j in round k of the preliminary schedule, their home-away patterns must differ for that round. In other words, only when $\delta_{j,i,k}$ is 1 can the variable $\eta_{i,j,k}$ also be 1.

Constraint (15b') imposes that in each round, a pattern i must be associated with a pattern j .

Constraint (15c') imposes that each pair of patterns must be associated m times.

Constraint (15d') imposes reciprocity of variable $\eta_{i,j,k}$ (i.e., if team i is associated with team j in round k , then team j is also associated with team i in round k).

Variables $\eta_{i,j,k}$ enable the game Constraints (11) to be explicitly incorporated, as follows:

$$\eta_{i,j,k} = 1 \quad \forall (i, j, k) \in GameFix \quad (16')$$

(When team i must play team j in round k at home, then $\eta_{i,j,k} = 1$ and $\alpha_{j,k} = 0$.)

Finally, observe that the schedule constructed by introducing the variable $\eta_{i,j,k}$ and Constraints (15a')–(15d') and (16') satisfies all MSM constraints, except the geographic ones, Constraints (5)–(7), which we found to be the most difficult restrictions to manage. In our experience, after assigning to the teams the set of patterns obtained by running the PGM in this fashion (with the incorporation of the η variables), executing the MSM tends to find a feasible solution quickly enough that the combined solution time for the two models is reasonable. An additional procedure that has proved useful with the more difficult-to-solve instances is to assign patterns only to certain teams using the PGM, and then, in the MSM, leave the

teams with fewer constraints without any set pattern. This increases the space for finding a feasible solution with the MSM, although generally at the cost of greater solution times. We found that an effective strategy consists of fixing patterns for the heavily constrained teams and leaving only about six or seven teams without any fixed pattern after the PGM. Then, the MSM still leads to a feasible solution in reasonable time.