Syngenta Uses a Cover Optimizer to Determine Production Volumes for Its European Seed Supply Chain

Supplemental Material – Proofs

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Proof of Proposition 1

A discrete function f(**x**) is said to be concave (i=j) and submodular (i≠j) in the components of **x** if $f(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-f(\mathbf{x}+\mathbf{e}_i)-f(\mathbf{x}+\mathbf{e}_j)+f(\mathbf{x})\leq 0$ for all **x**, i, and j; see Koole (2004). The expected profit in our model is the expectation over several random variables, some of which depend on **u,** thus complicating the proof. We therefore must use the concept of stochastic (sample-path) concavity; see Shanthikumar and Yao (1989).

Note that the expectation of the first two terms in function $\Pi(\mathbf{u})$ in Equation (3), $\mathbf{E}\left(\tilde{p}SV(\boldsymbol{u})+\tilde{r}\min(CO(\boldsymbol{u}),\mathbf{1}\cdot\boldsymbol{FC}^{N+2})\right)$, depend on **u** only through the production volume PR(**u**). We begin by showing that this term is concave in PR. From the definitions of SV and CO, we can write the first two terms in Equation (3) as:

$$\tilde{p}SV(PR)+\tilde{r}\min(CO(PR),\mathbf{1}\cdot\boldsymbol{FC}^{N+2})=\tilde{p}\min\big(D,(1-PI)(CI+PR)\big)+\tilde{r}\min\Big(CI+PR-\min\big(D,(1-PI)(CI+PR)\big),\mathbf{1}\cdot\boldsymbol{FC}^{N+2}\Big)=$$

$$\begin{cases}\tilde{p}(1-PI)(CI+PR)+\tilde{r}\min(PI\cdot(CI+PR),\mathbf{1}\cdot\boldsymbol{FC}^{N+2}), & PR<\frac{D}{1-PI}-CI\\ \tilde{p}D+\tilde{r}\min(CI+PR-\mathrm{D},\mathbf{1}\cdot\boldsymbol{FC}^{N+2}), & PR\geq\frac{D}{1-PI}-CI\end{cases}. \quad (6)$$

Because the minimum of a constant and an increasing linear function is increasing and concave (see Simchi-Levi et al. 2004, p.15), it is obvious that the function in Equation (6) is increasing and concave in PR in each interval, left and right of point D/(1-PI)-CI.

The function is increasing and concave on its whole domain if (1) the left-side and the right-side limit of the function at point D/(1-PI)-CI are identical, and (2) if the left-side limit of the derivative at point D/(1-PI)-CI is greater than or equal to its right-side limit. Condition (1) clearly holds. To check for Condition (2), let us differentiate between two cases: $\mathbf{1}\cdot\boldsymbol{FC}^{N+2} > D\cdot PI/(1-PI)$ and $\mathbf{1}\cdot\boldsymbol{FC}^{N+2} \leq D\cdot PI/(1-PI)$. In the first case, the derivative left of point D/(1-PI)-CI is given by $\tilde{p}(1-PI)+\tilde{r}PI$ and right of this point by $\tilde{r}$. From $\tilde{p} > \tilde{r}$ (see the problem definition), it follows that $\tilde{p}(1-PI)+\tilde{r}PI > \tilde{r}$. In the second case, the derivative left of point D/(1-PI)-CI is given by $\tilde{p}(1-PI) > 0$, while it is zero to the right of this point. In all cases, the derivative left of point D/(1-PI)-CI is greater than to the right of this point, verifying condition (2) and consequently proving that the function in Equation (6) is concave and increasing in PR on its whole domain. The result extends to the expectation over D and CI.

Next, consider the production volume PR(**u**) as a function of the harvested fields, **HF(u)**, instead of as a function of the planted fields, **u**. Let Y_{ij} be a normally distributed random variable with index i, mean $(1+PB_j)FS$, and standard deviation $FS(1+PB_j)PV_j$. Let Z_j be a normal random variable with mean zero and standard deviation $FS(1+PB_j)CPV_j$. The production yield PY_j in country j can then be written as $PY_j = \sum_{i=1}^{HF_j} Y_{i,j} + Z_j HF_j$. With this expression, PY_j is strongly stochastically concave (see Shanthikumar and Yao 1989) in HF_j, which follows from the following argumentation: Let us define four random variables (PR_1~PR(**HF**), PR_2~PR(**HF**+$\mathbf{e}_i$), PR_3~PR(**HF**+$\mathbf{e}_j$), and PR_4~PR(**HF**+$\mathbf{e}_i$+$\mathbf{e}_j$)) and let us couple these random variables such that $PR_2=PR_1+Y_{1,i}+Z_i$, $PR_3=PR_1+Y_{1,j}+Z_j$, and $PR_4=PR_1+Y_{1,i}+Y_{1,j}+Z_i+Z_j$. Clearly, $PR_1+PR_4=PR_2+PR_3$

implies that PR(**HF**) is strongly stochastically linear (and therefore also nonstrictly stochastically concave) in HF_j. Because we have shown above that $\mathbf{E}_{\mathrm{D,CI}}\left(\tilde{p}SV(PR) + \tilde{r}\min(CO(PR), \mathbf{1} \cdot \boldsymbol{FC}^{N+2})\right)$ is an increasing and concave function in PR, it follows from the definition of strong stochastic concavity that $\mathbf{E}_{\mathrm{D,CI}}\left(\tilde{p}SV(PR(\boldsymbol{HF})) + \tilde{r}\min(CO(PR(\boldsymbol{HF})), \mathbf{1} \cdot \boldsymbol{FC}^{N+2})\right)$ is concave and submodular in the components of vector **HF** (Shanthikumar and Yao 1989). The third term in the profit function, as we show in Equation (3), $\left(\boldsymbol{c}^{Field} + FS\boldsymbol{c}^{Prod} \circ (\mathbf{1} + \boldsymbol{PB})\right) \cdot \boldsymbol{HF}$), is linear and separable in the components of **HF**. This proves that the expected profit, $\mathbf{E}_{\mathrm{D,CI,PR}(\mathbf{u})}\Pi(\mathbf{u})$, as defined in Equation (1), is concave and submodular in the number of harvested fields **HF**.

Next, we analyze how the expected profit depends on vector **u** of planted fields instead of on the random vector of harvested fields **HF**(**u**). Recall that $\mathrm{HF}_j(\mathrm{u}_j)$ is a binomially distributed random variable with u_j trials and success probability 1-FL_j, and that HF_j depends only on u_j and not on any other components of **u**. Consequently, we can write $\mathrm{HF}_j(\mathrm{u}_j)$ as the sum of u_j independent Bernoulli (binary) random variables, $\mathrm{P}_{j,n}$ (i.e., $\mathrm{HF}_j\left(\mathrm{u}_j + 1\right) = \sum_{n=1}^{u_j+1} P_{j,n} = \mathrm{HF}_j\left(\mathrm{u}_j\right) + P_{j,u_j+1}$) , and the expected profit for a given P_{j,u_j+1} as $\boldsymbol{E}\Pi\left(\mathbf{u} + \mathbf{e}_j \middle| P_{j,u_j+1}\right) = \boldsymbol{E}\Pi\left(\mathbf{HF}(\mathbf{u}) + P_{j,u_j+1}\boldsymbol{e_j}\right)$. Let us define two random Bernoulli random variables P_i and P_j with success probabilities of FL_i and FL_j, respectively. Then, we can write the second difference function (in i and j) of the expected profit, $\Delta_{i,j}\mathbf{E}\Pi(\mathbf{u})$, for given values of P_i and P_j as $\Delta_{i,j}\mathbf{E}\Pi\left(\mathbf{u}\middle|\mathrm{P}_i, \mathrm{P}_j\right) = \mathbf{E}\Pi\left(\mathbf{u} + \boldsymbol{e}_i + \boldsymbol{e}_j\middle|\mathrm{P}_i, \mathrm{P}_j\right) - \mathbf{E}\Pi(\mathbf{u} + \boldsymbol{e}_i|\mathrm{P}_i) - \mathbf{E}\Pi\left(\mathbf{u} + \boldsymbol{e}_j\middle|\mathrm{P}_j\right) + \mathbf{E}\Pi(\mathbf{u}) = \mathbf{E}\Pi\left(\mathbf{HF}(\mathbf{u}) + \mathrm{P}_i\boldsymbol{e}_i + \mathrm{P}_j\boldsymbol{e}_j\right) - \mathbf{E}\Pi(\mathbf{HF}(\mathbf{u}) + \mathrm{P}_i\boldsymbol{e}_i) - \mathbf{E}\Pi\left(\mathbf{HF}(\mathbf{u}) + \mathrm{P}_j\boldsymbol{e}_j\right) + \mathbf{E}\Pi\left(\mathbf{HF}(\mathbf{u})\right)$.

Two cases are possible:

1. If either $P_i = 0$ or $P_j = 0$ (or both), we obtain $\Delta_{i,j}\mathbf{E}\Pi\left(\mathbf{u}\middle|\mathrm{P}_i, \mathrm{P}_j\right) = 0$.

2. If $P_i = P_j = 1$, we obtain $\Delta_{i,j}\mathbf{E}\Pi\left(\mathbf{u}\middle|\mathrm{P}_i, \mathrm{P}_j\right) = \mathbf{E}\Pi\left(\mathbf{HF}(\mathbf{u}) + \boldsymbol{e}_i + \boldsymbol{e}_j\right) - \mathbf{E}\Pi(\mathbf{HF}(\mathbf{u}) + \boldsymbol{e}_i) - \mathbf{E}\Pi\left(\mathbf{HF}(\mathbf{u}) + \boldsymbol{e}_j\right) + \mathbf{E}\Pi(\mathbf{HF}(\mathbf{u})) \leq 0$, which follows from the concavity and submodularity of the expected profit in the components of **HF**, as we showed in the first part of the proof.

Consequently, the expected value of $\Delta_{i,j}\mathbf{E}\Pi(\mathbf{u})$ over P_i and P_j is nonpositive. This completes the proof that $\mathbf{E}_{\mathrm{D,CI,PR}(\mathbf{u})}\Pi(\mathbf{u})$ is concave and submodular in the components of **u**.

Proof of Proposition 2

For some vector **u**(k) to minimize the deviation from the target production share, it must hold that $\left|u_j(k) - kt_j\right| < 1$ for any j=1…J. Otherwise, if $\left|u_j(k) - kt_j\right| \geq 1$, Equation (2) could be further reduced by removing one unit from $u_j(k)$ and adding this unit to some $u_i(k)$ (if $u_j(k) > kt_j$) or by adding one unit to $u_j(k)$ and removing this unit from some $u_i(k)$ (if $u_j(k) < kt_j$). Consequently, it also holds for any j that

$$u_j(k') \geq m_j\left(\boldsymbol{u}(k')\right) > k't_j - 1 > kt_j - 1 \geq m_j\left(\boldsymbol{u}(k)\right) - 1$$

$$\Rightarrow u_j(k') \geq m_j\left(\boldsymbol{u}(k')\right) \geq m_j\left(\boldsymbol{u}(k)\right),$$

thus completing the proof.

References

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