

Maximizing the U.S. Army's Future Contribution to Global Security Using the Capability Portfolio Analysis Tool (CPAT)

Electronic Companion

Scott J. Davis, Shatiel B. Edwards, Gerald E. Teper, David G. Bassett,
Michael J. McCarthy, Scott C. Johnson

United States Army, Warren, Michigan 48397 {scott.j.davis.civ@mail.mil, shatiel.b.edwards.civ@mail.mil,
gerald.e.teper.civ@mail.mil, david.g.bassett.mil@mail.mil, michael.j.mccarthy.civ@mail.mil, scott.c.johnson98.civ@mail.mil}

Craig R. Lawton, Matthew J. Hoffman, Liliana Shelton, Stephen M. Henry,
Darryl J. Melander, Frank M. Muldoon

Sandia National Laboratories, Albuquerque, New Mexico 87185 {crlawto@sandia.gov, mjhoffm@sandia.gov,
lshelto@sandia.gov, smhenry@sandia.gov, djmelan@sandia.gov, fmmuldo@sandia.gov}

Brian D. Alford

Booz Allen Hamilton, Huntsville, Alabama 35806, alford_brian@bah.com

Roy E. Rice

Teledyne Brown Engineering, Huntsville, Alabama 35805, roy.rice@Teledyne.com

THIS electronic companion provides a full enumeration and detailed descriptions of the business rules associated with the Program Executive Office Ground Combat Systems portfolio. Following these business rules, we give a full description of CPAT's mixed-integer linear programming formulation, including all parameters, decision variables, expressions, objective functions, and constraints.

Business Rules

This section is adapted from Melander et al. (2015).

Vehicle Flow

- **Constant Vehicle Population:** Each mission within a group maintains a constant number of vehicles throughout the planning horizon.
- **Group Purity:** At any given time, the vehicles serving a particular mission within a particular group must all be of the same vehicle type.
- **Outflow Availability:** For any year, the number of vehicles of a given type that are upgraded or swapped out may not exceed the number currently exchangeable; that is, the current number present minus the current number in the process of being upgraded. This is true both in storage and in missions in the fleet.
- **Initial Populations:** Each mission has an initial population of vehicles that is already in the fleet and is immediately available to begin modernization. Similarly, there may be some vehicles already in storage; if so, these vehicles immediately available to begin upgrading or swapping into missions.
- **Storage Flow:** Vehicles enter and exit storage via the following means: (1) purchases put new vehicles directly into storage; (2) storage upgrades take one vehicle type already in storage and turn it into another type; and (3) storage exchanges replace one vehicle type in a mission with another from storage, sending the replaced vehicles to storage.

- **No Upgrades Prior to Usage:** Newly purchased vehicles in storage that have not yet been sent to a mission should not be upgraded while in storage.
- **Delivery Implies Fielding:** A vehicle type can only be produced if some of these vehicles are eventually fielded (i.e., taken from storage and placed into the fleet in a specific mission role). An exception is made for vehicle types with zero cost, which are sometimes used to collect different variants of a family of vehicles into one node prior to a secondary upgrade (Figure 1) to reduce the number of upgrade paths when costs do not depend on a vehicle’s prior history.

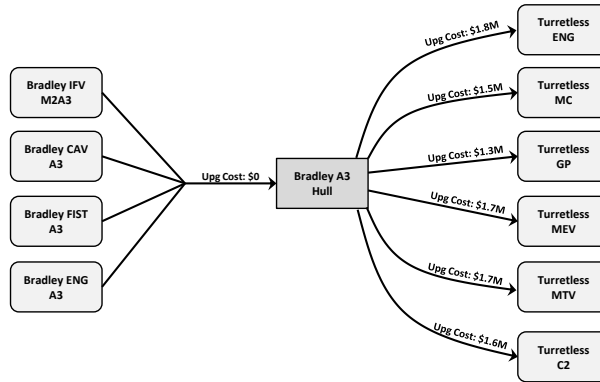


Figure 1: Several vehicle types can be collected into one before upgrading.

- **No Re-Fielding after Retirement:** Vehicles that are retired from a mission to storage cannot be immediately sent back into that same mission during the same year.
- **One-Year Duty Minimum:** Vehicles in a mission must remain for at least one year before they can be swapped out or used for a mission upgrade. This prevents arbitrary fielding and retiring of vehicles in the same year at no cost or performance degradation.

Priority Tiers and Optimization Phases

- **Priority Tiers:** Groups may be allocated to different priority tiers wherein each tier comprises a separate optimization. Subsequent tiers are modernized separately using the remaining budget and production capacity from previous tiers. All other business rules must hold *in toto* across all tiers.
- **Tier Phases:** As we discuss in the main paper, each tier is comprised of schedule, budget, performance, and cost phases.
- **Mission Succession:** One mission can be assigned to succeed another; then nothing can be fielded to this mission until the preceding mission has (1) completely finished fielding, and (2) modernized 100 percent of its original vehicles. This is typically used to enforce programmatic succession for corresponding missions in different tiers.

Modernization Characteristics

- **Delay Partitioning:** When purchasing or upgrading to a new vehicle, there is usually a delay between paying for the new vehicle and its delivery. This delay is comprised of a user-defined administrative delay (i.e., the vehicle has been paid for but is not yet in production) and production delay (i.e., the vehicle is in production but is not yet delivered).
- **Upgrade Administrative Delays:** For any upgrade having nonzero administrative and production delays, the administrative period can begin even if the vehicle to be upgraded is not yet on hand, but the vehicle must be on hand during the production period. That is, upgrade paperwork can be started prior to actual production.
- **Long Lead:** Some vehicle types may have a long lead—a portion of their procurement cost that is incurred one year before any administrative or production delay.

General Scheduling Rules

- **Vehicle Modernization Requirements:** Users may define that some percentage of certain vehicle types in the initial fleet must be modernized by a specific time.
- **Vehicle Mandates:** Users may mandate that a minimum number of a particular vehicle type be in a given mission during the final year.
- **Per-Period Mission Modernization Limit:** For a given mission role, there may a limit to the number of groups that can modernized per period. This reflects programmatic realities, and that not all brigades are concurrently available for modernization.
- **Cumulative Mission Modernization Limit:** Some COAs may require an upper bound on the cumulative number modernized for some vehicle types that exist in the initial fleet.
- **Group Modernization Density:** Missions may require that if a modernization occurs, then it must occur for some number of groups. This reflects that the fleet is partitioned (e.g., Active Army, National Guard) and commonality is important within these partitions (e.g., either upgrade the entire Active Army, or the Active Army and the National Guard). Requirements may be specified for up to three density levels, with NGM representing number of groups modernized, as follows:
 - $\{12, -, -\}$ implies $\text{NGM} = 0$ or $\text{NGM} \geq 12$.
 - $\{12, 16, -\}$ implies $\text{NGM} = \{0, 12\}$ or $\text{NGM} \geq 16$.
 - $\{12, 16, 20\}$ implies $\text{NGM} = \{0, 12, 16\}$ or $\text{NGM} \geq 20$.
- **Group Final Density:** This is analogous to group modernization density, but applies to density in the final year rather than to a cumulative total.
- **Vehicle Obviation:** An obviated vehicle type can only be delivered *prior* to delivery of any obviating vehicle types. This can be used to enforce program succession (e.g., ECP I must be complete before ECP II can start) or XOR behavior (i.e., if A and B obviate each other, they can never be in the same modernization plan).
- **Synchronization Sets:** A collection of missions and vehicles may be required to modernize or divest simultaneously; this collection is called a synchronization set. This is used to respect integrality of families of vehicles for operational and logistics reasons. For example, if mission M_1 uses vehicle S_1 and M_2 uses S_2 , and these missions and vehicles are part of a synchronization set, then the number of groups of S_1 entering or exiting M_1 must equal the number of groups of S_2 entering or exiting M_2 for all years.
- **Storage Consumption Priority:** If multiple vehicles in storage can be upgraded to the same vehicle type, a priority order can be enforced such that available vehicles of one type must all be used before vehicles of another type can be used.
- **Upgrades Trump Purchases:** Some vehicles may be procured both via upgrade of vehicles and via purchase of new vehicle copies. In such cases, users can specify that purchases may only occur only once all vehicles used for upgrades have been exhausted.

Product Families

- **RDT&E Costs:** For each product family, there may be an RDT&E cost, and vehicles from the family can be delivered if and only if the RDT&E cost profile of the family is incurred. It may be optimal to delay certain RDT&E costs to avoid budgetary bottlenecks. For each year that RDT&E may be delayed, a separate cost profile must be supplied. For family delay $D \geq 0$, at least one vehicle type within that family must have delay $d = D$ and all others must meet $d \geq D$.
- **LRIP Profiles:** LRIP profiles define fixed amounts of vehicles that must be produced in the years before full-rate production (**FRP**) begins, and how they should be produced (purchased or upgraded from a certain vehicle in storage). Per user specifications, some portion of these vehicles may be delivered to storage, while others may disappear (e.g., retained for training purposes or destructive testing).
- **LRIP Timing:** All LRIP profiles incurred by a product family must have final LRIP delivery come exactly one year prior to the first FRP delivery for the family.
- **Active Product Families:** A product family is considered active during a year if any member vehicles are (1) in administrative delay, (2) in production delay, or (3) being delivered (in cases where the production delay is 0). This includes LRIP.
- **Family Start-Up Costs:** Each family may have an associated start-up cost profile, with expenditures over multiple years. This profile is indexed to the year the family first begins work for FRP.
- **Family Per-Period Costs:** Each product family may have an associated per-period cost that must be incurred every year that the family is active.
- **Family Per-Period Capacity:** For each product family and year, there may be an upper limit on the number of member vehicles delivered from FRP.
- **Family Cumulative Capacity:** For each product family, there may be an upper limit on the cumulative number of member vehicles that are ever delivered from FRP.
- **Production Ramp-Up:** A family may have a ramp-up period prior to FRP. During this ramp-up, delivery output must be nondecreasing with time.
- **Minimum Sustaining Rate:** There may be a lower bound on the number of vehicles delivered from a family in any year in which delivery occurs. This bound does not apply to LRIP, ramp-up, or the final production period (to allow ramp-down).
- **Delivery Gaps:** Product families may be restricted so that they cannot start delivering vehicles, stop, and then subsequently restart; all vehicles within such a family must be delivered during a set of contiguous years.
- **Production Smoothing:** For each product family, there may be a limit on the yearly variation in number of vehicles delivered from FRP. This restriction does not apply to LRIP, ramp-up, or the final production period (to allow ramp-down).

Budgets

- **Per-Period Budgets:** Budgets can be defined per year for Procurement, O&S, and (or) RDT&E costs, and for a user-defined combination of the three.
- **Cumulative Budgets:** Cumulative budgets may also be defined in these categories.
- **No Early or Late Modernization Charging:** No modernization may occur so early or late that any associated costs would be incurred outside the time horizon.

Future Programs

Future programs represent potential modernization programs occurring so far in the future that they do not warrant the full sophistication of the production modeling described above. An extended time horizon can be defined, during which lower-detail decisions regarding future programs are made according to the following business rules.

- **Future Program Activation:** Future vehicles are grouped together into future programs. If a future program is activated, at least one (or if user specified, all) of the future vehicles associated with the program must be activated, and predefined start-up, RDT&E, and recurring cost profiles are incurred.
- **Future Vehicle Fielding:** When a future vehicle is activated, systems are fielded to the assigned mission (and LRIP is produced) according to a fixed, user-defined schedule. Performance and per-vehicle costs (purchase, upgrade, and O&S) work the same as their conventional vehicle counterparts.
- **Future Obviates Present:** Once a future vehicle starts fielding to a mission, no other nonfuture vehicles may be fielded to that mission.

Mixed-Integer Linear Programming Formulation

The MILP formulation used by CPAT has the following components. This section has been adapted from Henry et al. 2015.

Indices

i, j = vehicles.

m = missions.

t = years.

p = product families.

d = years delayed for RDT&E.

s = synchronization sets.

\mathcal{F} = future programs.

\mathcal{J} = future vehicles.

$f \in \{\text{Proc, OS, RDTE, Comb}\}$ = budget fund types.

$\pi \in \{\text{Sched, Budg, Perf, Cost}\}$ = optimization phases.

Sets

$\text{TIER}_{\text{Higher}}$ = the set of missions that have already been optimized.

$\text{TIER}_{\text{Current}}$ = the set of missions that are currently being optimized.

$\text{TIER}_{\text{Lower}}$ = the set of missions that will be optimized subsequently.

MS = the set (m^*, m) where mission m occurs in succession after mission m^* .

IR = the set (i, m) where vehicle i serves mission m as an interim (not initial or final) vehicle.

SO = the set (i, j) where vehicle i obviates vehicle j .

SM_s = the set of missions m is sync set s .

SV_s = the set of vehicles i is sync set s .

SPP = the vehicle pairs (i, j) such that i and j can both be upgraded to the same vehicle type, and in performing these upgrades all vehicles of type i must be depleted from storage before j can be used.

UBP = the vehicles i that must be depleted from storage before any purchases (of those types) can be made.

MDL $_m$ = allowable number of brigades for mission m required to modernize, if modernization occurs.

FDL $_m$ = allowable number of brigades in mission m required at the final conventional year, if modernization occurs.

PF $_p$ = the set of vehicles i in product family p .

PPG = the set of product families p that allow delivery gaps.

AD $_p$ = the set of delays d for which a family p has defined RDT&E profiles.

FP $_F$ = the set of future vehicles \mathcal{J} in future program \mathcal{F} .

ALL = the set of future programs \mathcal{F} which must field all associated vehicles when activated.

MF = the set of future vehicles \mathcal{J} that are mandated to be fielded.

Fleet Structure Parameters

$A_{i,m}$ = performance of vehicle i in mission m .

BPM $_m$ = number of brigades in mission m .

VPB $_m$ = number of vehicles in a brigade in mission m .

IB $_{i,m}$ = initial number of brigades of vehicle i in mission m .

IV $_{i,m}$ = initial number of vehicles of type i in mission m .

IVS $_i$ = initial number of vehicles of type i in storage.

PBS $_i$ = size of a purchase batch for vehicle i .

TVP = the total number of vehicles in all missions.

M = an upper bound for a big-M constraints, customized as appropriate for that constraint.

Scheduling Parameters

T = the last year in the conventional time horizon.

\mathcal{T} = the last year in the extended time horizon, where $\mathcal{T} \geq T$.

FA $_i$ = the year in which vehicle i first becomes available to field to a mission.

PPT $_i$ = number of years vehicle i is in production when purchased.

PT $_i$ = number of years between when vehicle i is purchased and when it is delivered.

UPT $_{i,j}$ = number of years vehicle i is in production when being upgraded to j .

UT $_{i,j}$ = number of years between when vehicle i begins upgrading and when j is delivered.

LPT $_{p,i}$ = number of years LRIP vehicle i is in production for family p .

LT $_{p,i}$ = number of years between when LRIP vehicle i for family p is paid for and when delivered.

MLP = the maximum number of LRIP years for all families.

MP $_{i,m,t}$ = minimum percent of vehicles i in mission m that must be modernized by year t .

FM $_{i,m}$ = minimum mandated number of vehicles i in m at the final conventional year.

BMLY $_m$ = maximum yearly number of brigades that can be modernized in mission m .

BMLC $_m$ = maximum cumulative number of brigades that can be modernized in mission m .

Product Family Parameters

DMAX $_{p,t}$ = maximum delivery capacity of product family p in year t .

DMIN $_p$ = minimum allowable delivery amount of product family p , if active.

DCMAX $_p$ = maximum cumulative delivery capacity of product family p .

DVAR $_p$ = maximum yearly delivery variance allowed for product family p .

RU $_p$ = number of years that the family p take to ramp up to full production.

PLRIP $_{p,i,t}$ = number of vehicles i produced for family p at t years before FRP.

DLRIP $_{p,i,t}$ = number of vehicles i delivered to family p at t years before FRP.

SLRIP $_{p,i}$ = index of the vehicle consumed when family p produces LRIP vehicle i .

Cost and Budget Parameters

CVP_i = cost of purchasing one vehicle of type i .
 $CVU_{i,j}$ = cost of upgrading one vehicle from i to j .
 $CVL_{p,i}$ = cost of getting one LRIP vehicle of type i for family p .
 $CVO_{i,m}$ = cost of operating and sustaining one vehicle of type i in mission m for one year.
 $CPS_{p,t}$ = start-up cost for family p during its t^{th} year of activity.
 CPA_p = cost of keeping product family p active for one year.
 $CPR_{p,d,t}$ = RDT&E cost for product family p in year t when delayed by d years.
 LL_i = long lead fraction of CVP_i charged one year prior to purchasing vehicle i .
 $LL_{i,j}$ = long lead fraction of $CVU_{i,j}$ charged one year prior to upgrading vehicle i to j .
 $LL_{p,i}$ = long lead fraction of $CVL_{p,i}$ charged one year prior to LRIP vehicle i in family p .
 $B_{f,t}$ = budget of fund type f in year t .
 BC_f = cumulative budget of fund type f .
 I_f = a binary flag indicating if fund f is included in the combined budget.
 IP_f = a binary flag indicating if fund f is included in the cost phase minimization.

Future-Program Parameters

$M_{\mathcal{J}}$ = the single mission m to which future vehicle \mathcal{J} can be fielded.
 $FFP_{\mathcal{J},t}$ = number of brigades of \mathcal{J} fielded in year t , if the future vehicle is activated.
 $FA_{\mathcal{J}}$ = the year in which future vehicle \mathcal{J} first fields.
 $CVT_{i,\mathcal{J},m}$ = cost of modernizing one vehicle from i to future vehicle \mathcal{J} in mission m .
 $CVO_{\mathcal{J}}$ = cost of operating and sustaining one future vehicle of type \mathcal{J} for one year.
 $CVL_{\mathcal{J}}$ = cost of getting one LRIP future vehicle of type \mathcal{J} .
 $LL_{i,\mathcal{J},m}$ = long lead fraction of $CVT_{i,\mathcal{J},m}$ charged a year prior to modernizing i to \mathcal{J} in m .
 $LL_{\mathcal{J}}$ = long lead fraction of $CVL_{\mathcal{J}}$ charged a year prior to getting future LRIP \mathcal{J} .
 $CPS_{\mathcal{F},t}$ = start-up cost for future program \mathcal{F} in year t .
 $CPA_{\mathcal{F},t}$ = active cost for future program \mathcal{F} in year t .
 $CPR_{\mathcal{F},t}$ = RDT&E cost for future program \mathcal{F} in year t .
 $FT_{i,\mathcal{J},m}$ = number of years taken to replace i with future vehicle \mathcal{J} in m .
 $FT_{\mathcal{J}}$ = number of years taken to get future LRIP vehicle \mathcal{J} .
 $PLRIP_{\mathcal{F},t}$ = number of LRIP vehicles produced for future program \mathcal{F} in year t .

Integer Variables

$u_{i,j,m,t}$ = number of brigades upgraded from vehicle i to vehicle j in mission m in year t .
 $u_{i,j,t}$ = number of vehicles in storage upgraded from vehicle i to vehicle j in year t .
 $s_{i,j,m,t}$ = number of brigades swapped from vehicle i to vehicle j in mission m in year t .
 $p_{i,t}$ = number of batches purchased of vehicle i in year t .
 $r_{i,\mathcal{J},m,t}$ = number of brigades of i replaced by future vehicle \mathcal{J} in mission m in year t .
 $d_{i,m,t}$ = number of deficit vehicles for the modernization requirement, $MP_{i,m,t}$.
 $d_{i,m}$ = number of deficit brigades for the end-of-conventional-horizon mandate, $FM_{i,m}$.
 $d_{\mathcal{J}}$ = number of deficit brigades for the mandated future vehicle \mathcal{J} .

Binary Indicator Variables

$b_{i,m,\ell}^1$ flags if mission m ever has $\ell \in MDL_m$ brigades of vehicle i .
 $b_{i,m,\ell}^2$ flags if mission m has $\ell \in FDL_m$ brigades of vehicle i at the last conventional year.

$b_{i,t}^3$ flags if at least 1 vehicle of type i is delivered in year t .
 $b_{i,t}^4$ flags if there is at least 1 vehicle of type i exchangeable in storage in year t .
 $b_{p,i,t}^5$ flags if family p first delivers non-LRIP vehicle i in year t .
 $b_{p,t}^6$ flags if product family p is active in year t .
 $b_{p,t}^7$ flags if product family p first becomes active in year t .
 $b_{m,t}^8$ flags if mission m is allowed to field in year t .
 $b_{i,m,t}^9$ flags if intermediate vehicles i in mission m are allowed to field in year t .
 $b_{\mathcal{F}}^{10}$ flags if future program \mathcal{F} is activated.
 $b_{\mathcal{J}}^{11}$ flags if future vehicle \mathcal{J} is activated.
 $b_{\mathcal{J}}^{12}$ flags if mandated future vehicle \mathcal{J} does not field.
 $b_{\mathcal{J},m,\ell}^{13}$ flags if mission m ever has $\ell \in MDL_m$ brigades of future vehicle \mathcal{J} .
 $b_{\mathcal{J},m,\ell}^{14}$ flags if mission m has $\ell \in FDL_m$ brigades of future vehicle \mathcal{J} in the last future year.
 b_i^{15} flags if vehicle i is ever delivered.
 $b_{p,t}^{16}$ flags if any vehicle from family p is delivered in year t .
 $b_{p,t}^{17}$ flags if non-LRIP delivery begins for family p in year t .
 $b_{p,d}^{18}$ flags if family p is delayed by d years.

Continuous Variables

$o_{f,t}$ = the dollar overage of the fund type f budget in year t .
 o_f = the dollar overage of the cumulative budget for fund type f .
 δ_p = the median delivery level for family p .

Variable Expressions

obj_{π} = the objective function for phase π of the optimization run.
 $ex_{f,t}$ = expense of fund type f in year t because of conventional vehicles.
 $fex_{f,t}$ = expense of fund type f in year t because of future vehicles.
 $nbim_{i,m,t}$ = number of brigades of vehicle i in mission m in year t .
 $nvld_{p,i,t}$ = number of vehicles i delivered from family p LRIP in year t .
 $nvpsc_{i,t}$ = number of seed vehicles i consumed by all LRIP profiles in year t .
 $nvla_{i,t}$ = number of vehicles i active because of LRIP in year t .
 $nvis_{i,t}$ = number of vehicles i in storage in year t .
 $nvip_{i,t}$ = number of vehicles i in production in year t .
 $nvia_{i,t}$ = number of non-LRIP vehicles i in administration periods in year t .
 $nvdi_{i,t}$ = number of vehicles i delivered in year t .

Optimization Phases

The notation obj_f^* represents the optimal value of objective function obj_f from a preceding optimization phase.

1) Schedule violations	2) Budget violations
$\begin{aligned} \min \quad & obj_{\text{Sched}} \\ \text{s.t.} \quad & x \in \Omega \end{aligned}$	$\begin{aligned} \min \quad & 0.00001 * obj_{\text{Budg}} \\ \text{s.t.} \quad & obj_{\text{Sched}} \leq obj_{\text{Sched}}^* + 0.001 \\ & x \in \Omega \end{aligned}$
3) Fleet performance	4) Fleet cost
$\begin{aligned} \max \quad & obj_{\text{Perf}} \\ \text{s.t.} \quad & obj_{\text{Budg}} \leq obj_{\text{Budg}}^* + 0.001 \\ & obj_{\text{Sched}} \leq obj_{\text{Sched}}^* + 0.001 \\ & x \in \Omega \end{aligned}$	$\begin{aligned} \min \quad & 0.00001 * obj_{\text{Cost}} \\ \text{s.t.} \quad & obj_{\text{Perf}} \geq obj_{\text{Perf}}^* - 0.001 \\ & obj_{\text{Budg}} \leq obj_{\text{Budg}}^* + 0.001 \\ & obj_{\text{Sched}} \leq obj_{\text{Sched}}^* + 0.001 \\ & x \in \Omega \end{aligned}$

General MILP Constraints $x \in \Omega$

- $obj_{\text{Sched}} =$

$$\sum_{\substack{i,m,t: \\ m \in \text{TIER}_{\text{Current}} \\ t \leq T}} d_{i,m,t} + \sum_{\substack{i,m: \\ m \in \text{TIER}_{\text{Current}}}} d_{i,m} + \sum_{\mathcal{J}} d_{\mathcal{J}} \quad (1)$$

- $obj_{\text{Budg}} =$

$$\sum_t (o_t^p + o_t^o + o_t^r + o_t^c) + o^p + o^o + o^r + o^c \quad (2)$$

- $obj_{\text{Perf}} =$

$$\sum_{i,m,t} A_{i,m} \cdot VPB_m \cdot nbim_{i,m,t} + \sum_{i,\mathcal{J},m,t} A_{\mathcal{J},m} \cdot VPB_m \cdot (\mathcal{T} - t + 1) \cdot r_{i,\mathcal{J},m,t} \quad (3)$$

- $obj_{\text{Cost}} =$

$$\sum_{\substack{f,t: \\ f \neq \text{Comb} \\ t \leq T}} IP_f \cdot ex_{f,t} + \sum_{\substack{f,t: \\ f \neq \text{Comb} \\ t \leq T}} IP_f \cdot fex_{f,t} \quad (4)$$

• $ex_{\text{Proc},t} =$

$$\begin{aligned}
& \sum_i (PBS_i \cdot (LL_i \cdot CVP_i \cdot p_{i,t+PT_i+1} + (1 - LL_i) \cdot CVP_i \cdot p_{i,t+PT_i})) \\
& + \sum_{i,j,m} (VPB_m \cdot (LL_{i,j} \cdot CVU_{i,j} \cdot u_{i,j,m,t+UT_{i,j}+1} + (1 - LL_{i,j}) \cdot CVU_{i,j} \cdot u_{i,j,m,t+UT_{3i,j}})) \\
& + \sum_{i,j} (LL_{i,j} \cdot CVU_{i,j} \cdot u_{i,j,t+UT_{i,j}+1} + (1 - LL_{i,j}) \cdot CVU_{i,j} \cdot u_{i,j,t+UT_{i,j}}) \\
& \quad + \sum_{\substack{p,i,t^*: \\ t+t^*+LT_{p,i}+1 \leq T}} PLRIP_{p,i,t^*} \cdot LL_{p,i} \cdot CVL_{p,i} \cdot b_{p,i,t+t^*+LT_{p,i}+1}^5 \\
& \quad + \sum_{\substack{p,i,t^*: \\ t+t^*+LT_{p,i}+1 \leq T}} PLRIP_{p,i,t^*} \cdot (1 - LL_{p,i}) \cdot CVL_{p,i} \cdot b_{p,i,t+t^*+LT_{p,i}}^5 \\
& \quad + \sum_p b_{p,t}^6 \cdot CPA_p + \sum_{\substack{p,t^*: \\ t^* \leq T}} b_{p,t^*}^7 \cdot CPS_{p,t-t^*} \\
& \qquad \qquad \qquad \forall t \leq T \quad (5)
\end{aligned}$$

• $flex_{\text{Proc},t} =$

$$\begin{aligned}
& \sum_{i,\mathcal{J},m} VPB_m \cdot LL_{i,\mathcal{J},m} \cdot CVT_{i,\mathcal{J},m} \cdot r_{i,\mathcal{J},m,t+FT_{i,\mathcal{J},m}+1} \\
& + \sum_{i,\mathcal{J},m} VPB_m \cdot (1 - LL_{i,\mathcal{J},m}) \cdot CVT_{i,\mathcal{J},m} \cdot r_{i,\mathcal{J},m,t+FT_{i,\mathcal{J},m}} \\
& \quad + \sum_{\mathcal{F}} (CPS_{\mathcal{F},t} + CPA_{\mathcal{F},t}) \cdot b_{\mathcal{F}}^{10} \\
& + \sum_{\mathcal{J}} (PLRIP_{\mathcal{F},t+FT_{\mathcal{J}+1}} \cdot LL_{\mathcal{J}} + PLRIP_{\mathcal{F},t+FT_{\mathcal{J}}} \cdot (1 - LL_{\mathcal{J}})) \cdot CVL_{\mathcal{J}} \cdot b_{\mathcal{J}}^{11} \\
& \qquad \qquad \qquad \forall t \leq \mathcal{T} \quad (6)
\end{aligned}$$

• $ex_{\text{OS},t} =$

$$\sum_{i,m} CVO_{i,m} \cdot VPB_m \cdot nbim_{i,m,t} \quad \forall t \leq T \quad (7)$$

• $flex_{\text{OS},t} =$

$$\begin{aligned}
& \sum_{\substack{i,\mathcal{J},m,t^*: \\ t^* \leq t}} VPB_m \cdot CVO_{\mathcal{J}} \cdot r_{i,\mathcal{J},m,t^*} \\
& + \begin{cases} 0 & t \leq T \\ ex_{\text{OS},T} - \sum_{\substack{i,\mathcal{J},m,t^*: \\ T < t^* \leq t}} VPB_m \cdot CVO_{i,m} \cdot r_{i,\mathcal{J},m,t^*} & T < t \leq T \end{cases} \\
& \qquad \qquad \qquad \forall t \leq \mathcal{T} \quad (8)
\end{aligned}$$

- $ex_{RDTE,t} =$

$$\sum_{p,d} CPR_{p,d,t} \cdot b_{p,d}^{18} \quad \forall t \leq T \quad (9)$$

- $flex_{RDTE,t} =$

$$\sum_{\mathcal{F}} CPR_{\mathcal{F},t} \cdot b_{\mathcal{F}}^{10} \quad \forall t \leq \mathcal{T} \quad (10)$$

- $ex_{Comb,t} =$

$$\sum_{\substack{f: \\ f \neq \text{Comb}}} I_f \cdot ex_{f,t} \quad \forall t \leq T \quad (11)$$

- $flex_{Comb,t} =$

$$\sum_{\substack{f: \\ f \neq \text{Comb}}} I_f \cdot flex_{f,t} \quad \forall t \leq \mathcal{T} \quad (12)$$

- $nbim_{i,m,t} =$

$$IB_{i,m} + \sum_{\substack{j,t^*: \\ t^* \leq t}} (s_{j,i,m,t^*} + u_{j,i,m,t^*} - s_{i,j,m,t^*} - u_{i,j,m,t^*}) - \sum_{\substack{\mathcal{J},t^*: \\ t^* \leq t}} r_{i,\mathcal{J},m,t^*} \quad \forall i, m, t \quad (13)$$

- $nvld_{p,i,t} =$

$$\sum_{t^* \leq \min(MLY, T-t)} DLRIP_{p,i,t^*} \cdot b_{p,i,t+t^*}^5 \quad \forall p, i, t \leq T \quad (14)$$

- $nvpsc_{i,t} =$

$$\sum_{\substack{p,j,t^*: \\ SLRIP_{p,j}=i \\ t^* \leq \min(MLY, T-t-LPT_{p,j})}} PLRIP_{p,j,t^*} \cdot b_{p,j,t+t^*+LPT_{p,j}}^5 \quad \forall i, t \leq T \quad (15)$$

- $nvla_{i,t} =$

$$\sum_{\substack{p,t^*,t^{**}: \\ t^* \leq MLY \\ t+t^*-\beta_{p,i} < t^{**} \leq t+t^*+LT_{p,i}}} PLRIP_{p,i,t^*} \cdot b_{p,i,t^{**}}^5 \quad \forall i, t \leq T \quad (16)$$

- $nvls_{i,t} =$

$$\begin{aligned}
& IVS_i + \sum_{t^* \leq t} (PBS_i \cdot p_{i,t^*} - nvlsc_{i,t^*}) + \sum_{\substack{j,t^*: \\ t^* \leq t}} (u_{j,i,t^*} - u_{i,j,t^*}) \\
& + \sum_{\substack{p,t^*: \\ t^* \leq t}} nvl d_{p,i,t^*} + \sum_{\substack{j,m,t^*: \\ t^* \leq t}} VPB_m \cdot (s_{i,j,m,t^*} - s_{j,i,m,t^*}) + \sum_{\substack{\mathcal{J},m,t^*: \\ t^* \leq t}} VPB_m \cdot r_{i,\mathcal{J},m,t^*} \\
& \qquad \qquad \qquad \forall i, t \leq T \quad (17)
\end{aligned}$$

- $nvip_{i,t} =$

$$\begin{aligned}
& \left\{ \begin{array}{ll} PBS_i \cdot p_{i,t} & \text{if } PPT_i = 0 \\ \sum_{t < t^* \leq t + PPT_i} PBS_i \cdot p_{i,t^*} & \text{if } PPT_i > 0 \end{array} \right. + \left\{ \begin{array}{ll} \sum_{j,m} VPB_m \cdot u_{j,i,m,t} & \text{if } UPT_{j,i} = 0 \\ \sum_{\substack{j,m,t^*: \\ t < t^* \leq t + UPT_{j,i}}} VPB_m \cdot u_{j,i,m,t^*} & \text{if } UPT_{j,i} > 0 \end{array} \right. + \left\{ \begin{array}{ll} \sum_{j,t} u_{j,i,t} & \text{if } UPT_{j,i} = 0 \\ \sum_{\substack{j,t^*: \\ t < t^* \leq t + UPT_{j,i}}} u_{j,i,t^*} & \text{if } UPT_{j,i} > 0 \end{array} \right. \\
& \qquad \qquad \qquad \forall i, t \leq T \quad (18)
\end{aligned}$$

- $nvia_{i,t} =$

$$\begin{aligned}
& \sum_{t + PPT_i < t^* \leq t + PT_i} PBS_i \cdot p_{i,t^*} + \sum_{\substack{j,t^*: \\ t + UPT_{j,i} < t^* \leq t + UT_{j,i}}} u_{j,i,t^*} + \sum_{\substack{j,m,t^*: \\ t + UPT_{j,i} < t^* \leq t + UT_{j,i}}} VPB_m \cdot u_{j,i,m,t^*} \\
& \qquad \qquad \qquad \forall i, t \leq T \quad (19)
\end{aligned}$$

- $nvd_{i,t} =$

$$PBS_i \cdot p_{i,t} + \sum_j u_{j,i,t} + \sum_{j,m} VPB_m \cdot u_{j,i,m,t} \quad \forall i, t \leq T \quad (20)$$

- **Priority Tiers constraints:**

$$u_{i,j,m,t} = 0 \quad \forall i, j, m \in \text{TIER}_{\text{Lower}}, t \leq T \quad (21)$$

$$s_{i,j,m,t} = 0 \quad \forall i, j, m \in \text{TIER}_{\text{Lower}}, t \leq T \quad (22)$$

$$d_{i,m,t} = 0 \quad \forall i, m \in \text{TIER}_{\text{Lower}}, t \quad (23)$$

$$d_{i,m} = 0 \quad \forall i, m \in \text{TIER}_{\text{Lower}} \quad (24)$$

$$u_{i,j,m,t} = u_{i,j,m,t}^* \quad \forall i, j, m \in \text{TIER}_{\text{Higher}}, t \leq T \quad (25)$$

$$s_{i,j,m,t} = s_{i,j,m,t}^* \quad \forall i, j, m \in \text{TIER}_{\text{Higher}}, t \leq T \quad (26)$$

$$d_{i,m,t} = d_{i,m,t}^* \quad \forall i, m \in \text{TIER}_{\text{Higher}}, t \quad (27)$$

$$d_{i,m} = d_{i,m}^* \quad \forall i, m \in \text{TIER}_{\text{Higher}} \quad (28)$$

- **Mission Succession** constraints:

$$\sum_{i,j} (s_{i,j,m,t} + u_{i,j,m,t}) \leq \max_m (BMLY_m) \cdot b_{m,t}^8 \quad \forall m, t \leq T \quad (29)$$

$$1 - b_{m,t}^8 \leq \sum_{\substack{i,m^*: \\ (m^*,m) \in \text{MS} \\ IB_{i,m^*} > 0}} nbim_{i,m^*,t} + \sum_{\substack{i,j,m^*,t^*: \\ (m^*,m) \in \text{MS} \\ t \leq t^* \leq T}} (s_{i,j,m^*,t^*} + u_{i,j,m^*,t^*}) \leq M \cdot (1 - b_{m,t}^8) \quad \forall m, t \leq T \quad (30)$$

- **Outflow Availability** constraints:

$$nvis_{i,t} - \sum_{\substack{j,t^*: \\ t < t^* \leq t + UPT_{i,j}}} u_{i,j,t^*} \geq 0 \quad \forall i, t \leq T \quad (31)$$

$$VPB_m \cdot \left(nbim_{i,m,t} - \sum_j (u_{j,i,m,t} + s_{j,i,m,t}) - \sum_{\substack{j,t^*: \\ t < t^* \leq t + UPT_{i,j}}} u_{i,j,m,t^*} \right) \geq 0 \quad \forall i, m, t \leq T \quad (32)$$

$$\sum_{\substack{\mathcal{J},t: \\ t \leq T}} r_{i,\mathcal{J},m,t} \leq nbim_{i,m,T} \quad \forall i, m \quad (33)$$

$$\sum_{\mathcal{J}} r_{i,\mathcal{J},m,t} \leq nbim_{i,m,t-1} \quad \forall i, m, 1 < t \leq T \quad (34)$$

- **No Upgrades Prior to Usage** constraints:

$$\sum_{\substack{j,t^*: \\ t^* \leq t + UPT_{i,j}}} u_{i,j,t^*} \leq IVS_i + \sum_{\substack{j,m,t^*: \\ t^* \leq t}} VPB_m \cdot s_{i,j,m,t^*} + \sum_{\substack{j,t^*: \\ t^* \leq t}} u_{j,i,t^*} + \sum_{\substack{\mathcal{J},m,t^*: \\ t^* \leq t}} VPB_m \cdot r_{i,\mathcal{J},m,t^*} \quad \forall i, t \leq T \quad (35)$$

- **No Re-Fielding after Retirement** constraints:

$$BMLY_m \cdot b_{i,m,t}^9 \geq \sum_j s_{j,i,m,t} \quad \forall (i, m) \in \text{IR}, t \leq T \quad (36)$$

$$BMLY_m \cdot (1 - b_{i,m,t}^9) \geq \sum_j s_{i,j,m,t} \quad \forall (i, m) \in \text{IR}, t \leq T \quad (37)$$

- **No Early or Late Modernization Charging** constraints:

$$p_{i,t} = 0 \quad \forall i, t \leq \begin{cases} PT_i + 1 & \text{if } LL_i > 0 \\ PT_i & \text{if } LL_i = 0 \end{cases} \quad (38)$$

$$u_{i,j,m,t} = 0 \quad \forall i, j, m, t \leq \begin{cases} UT_{i,j} + 1 & \text{if } LL_{i,j} > 0 \\ UT_{i,j} & \text{if } LL_{i,j} = 0 \end{cases} \quad (39)$$

$$u_{i,j,t} = 0 \quad \forall i, j, t \leq \begin{cases} UT_{i,j} + 1 & \text{if } LL_{i,j} > 0 \\ UT_{i,j} & \text{if } LL_{i,j} = 0 \end{cases} \quad (40)$$

$$\begin{cases} b_{p,i,t}^5 = 0 & \text{if } LL_{p,i} = 0 \\ b_{p,i,t}^5 + b_{p,i,t+1}^5 = 0 & \text{if } LL_{p,i} > 0 \end{cases} \quad \forall p, i, t \mid PLRIP_{p,i,t} > 0 \quad (41)$$

$$b_{p,t}^7 = 0 \quad \forall p, t \leq T \mid \sum_{t \leq t^* < T} CPS_{p,-t^*} > 0 \text{ or } \sum_{T-t < t^* < T} CPS_{p,t^*} > 0 \quad (42)$$

- **Vehicle Modernization Requirements** constraints:

$$VPB_m \cdot \left(\sum_{\substack{j,t^*: \\ t^* \leq t}} (u_{i,j,m,t^*} + s_{i,j,m,t^*}) + \sum_{\substack{\mathcal{J},t^*: \\ t^* \leq t}} r_{i,\mathcal{J},m,t^*} \right) + d_{i,m,t} \geq MP_{i,m,t} \cdot IV_{i,m} \quad \forall i, m \in \text{TIER}_{\text{Current}}, t \quad (43)$$

- **Per-Period and Cumulative Mission Modernization Limit** constraints:

$$\sum_{i,j} (u_{i,j,m,t} + s_{i,j,m,t}) \leq BMLY_m \quad \forall m \in \text{TIER}_{\text{Current}}, t \leq T \quad (44)$$

$$\sum_{\substack{i,j,t: \\ IV_{i,m} > 0 \\ t \leq T}} (u_{i,j,m,t} + s_{i,j,m,t}) + \sum_{\substack{i,\mathcal{J},t: \\ t \leq T}} r_{i,\mathcal{J},m,t} \leq BMLC_m \quad \forall m \in \text{TIER}_{\text{Current}} \quad (45)$$

- **Vehicle Mandates** constraints:

$$VPB_m \cdot (nbim_{i,m,\mathcal{T}} + d_{i,m}) \geq FM_{i,m} \quad \forall i, m \in \text{TIER}_{\text{Current}} \quad (46)$$

- **Vehicle Obviation** constraints:

$$b_{j,t}^3 \leq 1 - b_{i,t^*}^3 \quad \forall (i, j) \in \text{SO}, t, t^* \leq t \leq T \quad (47)$$

- **Synchronization Sets** constraints:

$$\sum_{i \in \text{SV}_s} nbim_{i,m,t} = \sum_{i \in \text{SV}_s} nbim_{i,m^*,t} \quad \forall s, m \in \text{SM}_s, m^* \in \text{SM}_s, t \leq T \mid m \neq m^* \quad (48)$$

- **Storage Consumption Priority** constraints:

$$b_{i,t}^4 \leq nvis_{i,t} - \sum_{\substack{j,t^*: \\ t < t^* \leq t + UPT_{i,j}}} u_{i,j,t^*} \leq TVP \cdot b_{i,t}^4 \quad \forall i, t \leq T \mid \exists(i, j) \in \text{SPP} \quad (49)$$

$$\sum_{\substack{j,j^*: \\ (i,j) \in \text{SPP} \\ t + UPT_{j,j^*} \leq T}} u_{j,j^*,t+UPT_{j,j^*}} \leq TVP^2 \cdot (1 - b_{i,t}^4) \quad \forall i, t \leq T \mid \exists(i, j) \in \text{SPP} \quad (50)$$

- **Upgrades Trump Purchases** constraints:

$$b_{i,t}^4 \leq nvis_{i,t} - \sum_{\substack{j,t^*: \\ t < t^* \leq t + UPT_{i,j}}} u_{i,j,t^*} \leq TVP \cdot b_{i,t}^4 \quad \forall i \in \text{UBP}, t \leq T \quad (51)$$

$$\sum_{\substack{j: \\ t + PPT_j \leq T}} p_{j,t+PPT_j} \leq TVP \cdot (1 - b_{i,t}^4) \quad \forall i \in \text{UBP}, t \leq T \quad (52)$$

- **Per-Period and Cumulative Budgets** constraints:

$$ex_{f,t} + fe_{f,t} \leq B_{f,t} + o_{f,t} \quad \forall f, t \leq T \quad (53)$$

$$fe_{f,t} \leq B_{f,t} + o_{f,t} \quad \forall f, t \in \{T + 1, \dots, \mathcal{T}\} \quad (54)$$

$$\sum_{t \leq T} ex_{f,t} + \sum_{t \leq \mathcal{T}} fe_{f,t} \leq B_f + o_f \quad \forall f \quad (55)$$

- **Group Modernization Density** constraints:

$$\sum_{\ell \in \text{MDL}_m} \ell \cdot b_{i,m,\ell}^1 \leq \sum_{\substack{j,t: \\ t \leq T}} (u_{j,i,m,t} + s_{j,i,m,t}) \leq \sum_{\substack{\ell \in \text{MDL}_m: \\ \ell < \max(\text{MDL}_m)}} \ell \cdot b_{i,m,\ell}^1 + BPM_m \cdot b_{i,m,\max(\text{MDL}_m)}^1 \quad \forall i, m \quad (56)$$

$$\sum_{\ell \in \text{MDL}_m} b_{i,m,\ell}^1 \leq 1 \quad \forall i, m \quad (57)$$

- **Group Final Density** constraints:

$$nbim_{i,m,\mathcal{T}} \geq (b_{i,m,\ell}^2 + \sum_{\ell^* \in \text{MDL}_m} \ell^* \cdot b_{i,m,\ell^*}^1 - 1) \cdot \ell - \begin{cases} \left(\sum_{j,t} (u_{i,j,m,t} + s_{i,j,m,t}) + \sum_{\mathcal{J},t} r_{i,\mathcal{J},m,t} \right) \cdot \ell & \text{if } \ell = \max(\text{FDL}_m) \\ 0 & \text{otherwise} \end{cases} \quad \forall i, m, \ell \in \text{FDL}_m \quad (58)$$

$$nbim_{i,m,\mathcal{T}} \leq \sum_{\substack{\ell \in \text{FDL}_m: \\ \ell < \max(\text{FDL}_m)}} \ell \cdot b_{i,m,\ell}^2 + BPM_m \cdot b_{i,m,\max(\text{FDL}_m)}^2 \quad \forall i, m \quad (59)$$

$$\sum_{\ell \in \text{FDL}_m} b_{i,m,\ell}^2 \leq 1 \quad \forall i, m \quad (60)$$

- **Delivery Implies Fielding** constraints:

$$b_{i,t}^3 \leq nvd_{i,t} \leq TVP \cdot b_{i,t}^3 \quad \forall i, t \leq T \quad (61)$$

$$\sum_{\substack{j,m,t^*: \\ t^* \leq T}} (u_{j,i,m,t^*} + s_{j,i,m,t^*}) \geq b_{i,t}^3 \quad \forall i, t \leq T \mid CVU_{j,i} > 0 \text{ for some } j \quad (62)$$

- **Active Product Families and Family Per-Period Costs** constraints:

$$b_{p,t}^6 \leq \sum_{i \in \mathbb{PF}_p} (nvip_{i,t} + nvia_{i,t} + nvla_{i,t}) \leq M \cdot TVP \cdot b_{p,t}^6 \quad \forall p, t \leq T \quad (63)$$

- **Family Start-Up Costs** constraints:

$$b_{p,t}^7 \leq \sum_{i \in \mathbb{PF}_p} (nvip_{i,t} + nvia_{i,t}) \leq TVP \cdot \sum_{t^* \leq t} b_{p,t^*}^7 \quad \forall p, t \leq T \quad (64)$$

$$\sum_{t \leq T} b_{p,t}^7 \leq 1 \quad \forall p \quad (65)$$

- **LRIP Timing** constraints:

$$b_i^{15} \geq b_{i,t}^3 \quad \forall i, t \leq T \quad (66)$$

$$b_i^{15} \leq \sum_{t \leq T} b_{i,t}^3 \quad \forall i \quad (67)$$

$$b_{p,t}^{17} \leq \sum_{i \in \mathbb{PF}_p} b_{i,t}^3 \quad \forall p, t \leq T \quad (68)$$

$$\sum_{t^* \leq t} b_{p,t^*}^{17} \geq b_{i,t}^3 \quad \forall p, i \in \mathbb{PF}_p, t \leq T \quad (69)$$

$$\sum_{t \leq T} b_{p,t}^{17} \leq 1 \quad \forall p \quad (70)$$

$$b_i^{15} + b_{p,t}^{17} - 1 \leq b_{p,i,t}^5 \leq \frac{b_i^{15}}{b_{p,t}^{17}} \quad \forall p, i, t \leq T \quad (71)$$

- **Delivery Gaps** constraints:

$$b_{p,t}^{16} - b_{p,t-1}^{16} + b_{p,t^*}^{16} \leq 1 \quad \forall p \in \mathbb{PFG}, t, t^* \mid t^* + 1 < t \leq T \quad (72)$$

- **Family Per-Period Capacity and Family Cumulative Capacity** constraints:

$$\sum_{i \in \mathbb{PF}_p} nvd_{i,t} \leq DMAX_{p,t} \quad \forall p, t \leq T \quad (73)$$

$$\sum_{\substack{i,t: \\ i \in \mathbb{PF}_p \\ t \leq T}} nvd_{i,t} \leq DCMAX_p \quad \forall p \quad (74)$$

- **Minimum Sustaining Rate** constraints:

$$b_{p,t}^{16} \leq \sum_{i \in \mathbb{PF}_p} b_{i,t}^3 \quad \forall p, t \leq T \quad (75)$$

$$b_{p,t}^{16} \geq b_{i,t}^3 \quad \forall p, i \in \mathbb{PF}_p, t \leq T \quad (76)$$

$$\sum_{i \in \mathbb{PF}_p} nvd_{i,t} \geq DMIN_p \cdot b_{p,t}^{16} - TVP \cdot (1 - b_{p,t+1}^{16}) \quad \forall p, t < T \quad (77)$$

- **RDT&E Costs** constraints:

$$b_{p,d}^{18} \leq \sum_{\substack{i \in \mathbb{PF}_p: \\ FA_i + d \leq T}} b_{i,FA_i+d}^3 \quad \forall p, d \quad (78)$$

$$\sum_{d^* \leq d} b_{p,d^*}^{18} \geq b_{i,FA_i+d}^3 \quad \forall p, i \in \mathbb{PF}_p \mid FA_i + d \leq T \quad (79)$$

$$\sum_d b_{p,d}^{18} \leq 1 \quad \forall p \quad (80)$$

$$\sum_{d \notin \mathbb{AD}_p} b_{p,d}^{18} = 0 \quad \forall p \quad (81)$$

- **Production Smoothing** constraints:

$$\sum_{i \in \mathbb{PF}_p} nvd_{i,t} \leq \left(1 + \frac{1}{2} \cdot DVAR_p\right) \cdot \delta_p \quad \forall p, t \mid DVAR_p \geq 0 \text{ and } RU_p < t \leq T \quad (82)$$

$$\sum_{i \in \mathbb{PF}_p} nvd_{i,t} \geq \left(1 - \frac{1}{2} \cdot DVAR_p\right) \cdot \delta_p - TVP \cdot \left(\sum_{0 \leq t^* \leq RU_p} (1 - b_{p,t-t^*}^{16}) - (1 - b_{p,t+1}^{16}) \right) \quad \forall p, t \mid DVAR_p \geq 0 \text{ and } RU_p < t < T - 1 \quad (83)$$

- **Production Ramp-up** constraints:

$$\sum_{i \in \mathbb{PF}_p} (nvd_{i,t} - nvd_{i,t-1}) \geq -TVP \cdot \left(1 - b_{p,t}^{16} + \begin{cases} 0 & \text{if } t \leq RU_p + 1 \\ b_{p,t-RU_p-1}^{16} & \text{if } t > RU_p + 1 \end{cases} \right) \quad \forall p, t \mid DVAR_p \geq 0 \text{ and } RU_p > 0 \text{ and } 1 < t \leq T \quad (84)$$

- **Future Program Activation** constraints:

$$b_{\mathcal{F}}^{10} \leq \sum_{\mathcal{J} \in \mathbb{FP}_{\mathcal{F}}} b_{\mathcal{J}}^{11} \leq M \cdot b_{\mathcal{F}}^{10} \quad \forall \mathcal{F} \quad (85)$$

$$b_{\mathcal{F}}^{10} \leq b_{\mathcal{J}}^{11} \quad \forall \mathcal{F} \in \text{ALL}, \mathcal{J} \in \mathbb{FP}_{\mathcal{F}} \quad (86)$$

- **Future Vehicle Fielding** constraints:

$$b_{\mathcal{J}}^{11} \leq \sum_{i,m,t} r_{i,\mathcal{J},m,t} \leq M \cdot b_{\mathcal{J}}^{11} \quad \forall \mathcal{J} \quad (87)$$

$$\sum_{i,m} r_{i,\mathcal{J},m,t} = b_{\mathcal{J}}^{11} \cdot FFP_{\mathcal{J},t} \quad \forall \mathcal{J}, t \quad (88)$$

$$d_{\mathcal{J}} = \sum_t FFP_{\mathcal{J},t} \cdot b_{\mathcal{J}}^{12} \quad \forall \mathcal{J} \quad (89)$$

$$b_{\mathcal{J}}^{11} + b_{\mathcal{J}}^{12} = 1 \quad \forall \mathcal{J} \in \text{MF} \quad (90)$$

- **Future Obviates Present** constraints:

$$\sum_{\substack{i,j,t: \\ FA_{\mathcal{J}} \leq t}} s_{i,j,m,t} \leq M \cdot (1 - b_{\mathcal{J}}^{11}) \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (91)$$

$$\sum_{\substack{i,j,t: \\ FA_{\mathcal{J}} \leq t}} u_{i,j,m,t} \leq M \cdot (1 - b_{\mathcal{J}}^{11}) \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (92)$$

- **Future Group Modernization Density** constraints:

$$\sum_{i,t} r_{i,\mathcal{J},m,t} \geq \ell \cdot (b_{\mathcal{J}}^{11} + b_{\mathcal{J},m,\ell}^{13} - 1) \quad \forall \mathcal{J}, m = M_{\mathcal{J}}, \ell \in \text{MDL}_m \quad (93)$$

$$\sum_{i,t} r_{i,\mathcal{J},m,t} \leq \sum_{\substack{\ell \in \text{MDL}_m: \\ \ell < \max(\text{MDL}_m)}} \ell \cdot b_{\mathcal{J},m,\ell}^{13} + BPM_m \cdot b_{\mathcal{J},m,\max(\text{MDL}_m)}^{13} \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (94)$$

$$\sum_{\ell \in \text{MDL}_m} b_{\mathcal{J},m,\ell}^{13} \leq 1 \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (95)$$

- **Future Group Final Density** constraints:

$$\sum_{i,t} r_{i,\mathcal{J},m,t} \geq \ell \cdot (b_{\mathcal{J}}^{11} + b_{\mathcal{J},m,\ell}^{14} - 1) \quad \forall \mathcal{J}, m = M_{\mathcal{J}}, \ell \in \text{FDL}_m \quad (96)$$

$$\sum_{i,t} r_{i,\mathcal{J},m,t} \leq \sum_{\substack{\ell \in \text{FDL}_m: \\ \ell < \max(\text{FDL}_m)}} \ell \cdot b_{\mathcal{J},m,\ell}^{14} + BPM_M \cdot b_{\mathcal{J},m,\max(\text{FDL}_m)}^{14} \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (97)$$

$$\sum_{\ell \in \text{FDL}_m} b_{\mathcal{J},m,\ell}^{14} \leq 1 \quad \forall \mathcal{J}, m = M_{\mathcal{J}} \quad (98)$$

References

Henry SM, Muldoon FM, Hoffman MJ, Kao GK, Lawton CR, Melander DJ, Rice RE, Shelton L (2015) The capability portfolio analysis tool (CPAT): A mixed integer linear programming formulation for fleet modernization analysis. Report SAND2015-3487, Sandia National Laboratories, Albuquerque, NM.

Melander DJ, Henry SM, Hoffman MJ, Kao GK, Lawton CR, Muldoon FM, Rice RE, Shelton L (2015) The CPAT domain model - How CPAT “thinks” from an analyst perspective. Report SAND2015-4009, Sandia National Laboratories, Albuquerque, NM.