Submitted to *INFORMS Journal on Applied Analytics*
manuscript 037-06-22-OM.R2

An Optimization Case Study in Analyzing Missouri Redistricting: Supplementary Material

Kiera W. Dobbs, Rahul Swamy, Douglas M. King
Department of Industrial and Enterprise Systems Engineering, University of Illinois Urbana-Champaign, Urbana, IL 61801,
kdobbs2@illinois.edu, rahulswa@illinois.edu, dmking@illinois.edu

Ian G. Ludden, Sheldon H. Jacobson
Department of Computer Science, University of Illinois Urbana-Champaign, Urbana, IL, 61801,
iludden2@illinois.edu, shj@illinois.edu

Key words: Political redistricting; Political fairness; Recombination; Local search

---

This supplementary text is organized as follows. *Additional Optimization Scenarios* describes optimization scenarios that relax/remove Missouri's requirements for state legislative district plans to preserve county lines and satisfy a 15% shifted efficiency gap threshold; subsections *Optimized Senate Plans* and *Optimized House Plans* describe the experimental results from these scenarios. *Optimized Congressional Plans* describes the experimental results from the congressional optimization scenarios. Lastly, *District Plans Originally Provided to the LWV-MO* presents the district plans we originally provided to the LWV-MO, their fairness metric values, and their district vote-shares.

Additional Optimization Scenarios

Recall that the main text presents four optimization scenarios for Missouri's state senate and state house districts: compact plans, shifted efficiency gap plans, partisan asymmetry plans, and competitive plans. These scenarios also follow Missouri's constitutional requirements to preserve county lines and satisfy a 15% shifted efficiency gap threshold. To acknowledge these constraints, we refer to these original scenarios as COMP-C-SEG,

SEG-C-SEG, PA-C-SEG, and CMPTTV-C-SEG, respectively (where "C" indicates county preservation and "SEG" indicates the shifted efficiency gap threshold).

Here we explore additional optimization scenarios that relax the county preservation requirement and/or remove the shifted efficiency gap requirement. By relaxing/removing these requirements, we can observe the extent to which they prevent improvement to political fairness. We refer to these scenarios with the same naming convention:

- Scenarios COMP-C, SEG-C, PA-C, and CMPTTV-C improve compactness, shifted efficiency gap, partisan asymmetry, and competitiveness, respectively. The district plans remain subject to all county preservation constraints, but are no longer required to satisfy the 15% shifted efficiency gap threshold.
- Scenarios COMP-SEG, SEG-SEG, PA-SEG, and CMPTTV-SEG improve compactness, shifted efficiency gap, partisan asymmetry, and competitiveness, respectively. The district plans remain subject to the 15% shifted efficiency gap threshold, but county preservation requirements are relaxed. The state senate plans are constructed entirely from census tracts (rather than a combination of counties and tracts) and counties are no longer required to contain as many whole state senate districts as their population allows. For the state house plans, counties are similarly no longer required to contain as many whole state house districts as their population allows. For both state senate and state house plans, we still apply perimeter penalties as the algorithm optimizes for compactness to encourage districts to follow county lines.
- Scenarios COMP, SEG, PA, and CMPTTV improve compactness, shifted efficiency gap, partisan asymmetry, and competitiveness, respectively. The shifted efficiency gap requirement is removed and the county preservation requirement is relaxed (as described in the previous scenario).

Some optimization scenarios are redundant; SEG-C-SEG and SEG-C represent the same scenario and SEG-SEG and SEG represent the same scenario. The basic optimization algorithm remains the same across all scenarios (see *Methodology* and the appendix in the main text for algorithm details). Note that although the scenarios begin with 30 replications, we winnow the results to satisfy chosen political fairness thresholds; hence, not all scenarios have 30 final plans. We state these thresholds with each set of optimized state senate and state house plans in the following subsections.

Optimized Senate Plans

This subsection describes the metric values for the optimized state senate plans in each optimization scenario. We enforce the following political fairness thresholds:

- Shifted efficiency gap thresholds of 15% and 13.5% for scenarios SEG-C-SEG and SEG, respectively
- Partisan asymmetry thresholds of 0.03, 0.025, 0.02, and 0.015 for scenarios PA-C-SEG, PA-C, PA-SEG, and PA, respectively
- Competitiveness thresholds of 11, 13, 12, and 16 for scenarios CMPTTV-C-SEG, CMPTTV-C, CMPTTV-SEG, and CMPTTV, respectively.

Figure 1 examines how each fairness metric changes with the relaxation/removal of the county preservation and shifted efficiency gap requirements. Figure 1a shows that the compactness values are mostly comparable across COMP-C-SEG, COMP-C, COMP-SEG, and COMP, although there is a large variance in the values each scenario produces. It is possible that enforcing the shifted efficiency gap threshold leads to slightly less compact plans, since COMP-C-SEG and COMP-SEG produce the worst compactness values. Relaxing county preservation allows the algorithm to improve the shifted efficiency gap from 15% to 13.5% (Figure 1b), but this value remains far from the ideal value of zero. Figure 1c shows that both relaxing county preservation and removing the shifted efficiency gap threshold allow the algorithm to find better partisan asymmetry values. For example, the values for PA-C-SEG are around 0.03, while the values for PA are around 0.015. Similarly, the algorithm can substantially improve competitiveness with the relaxation/removal of both requirements. Figure 1d CMPTTV yields 16-18 competitive districts, while CMPTTV-C-SEG only yields 11.

Optimized House Plans

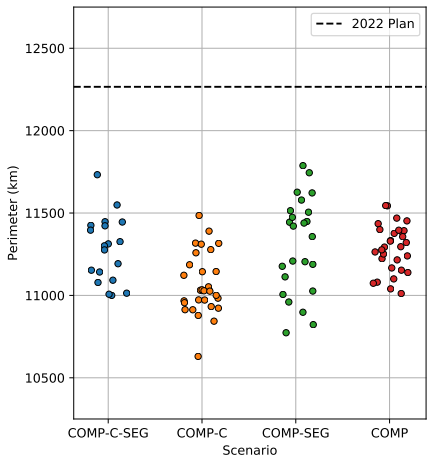
This subsection describes the metric values for the optimized state house plans in each optimization scenario. We enforce the following political fairness thresholds:

- Partisan asymmetry thresholds of 0.035 for scenarios PA-C-SEG, PA-C, and PA-SEG, and 0.03 for PA
- Competitiveness thresholds of 52, 62, 52, and 66 for scenarios CMPTTV-C-SEG, CMPTTV-C, CMPTTV-SEG, and CMPTTV, respectively.

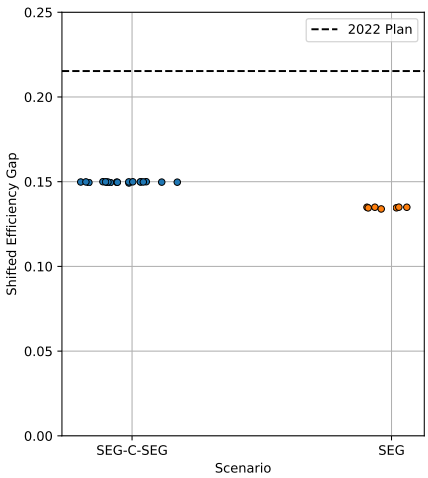
Figure 2 examines how each fairness metric changes with the relaxation/removal of the county preservation and shifted efficiency gap requirements. Note that the algorithm was

Figure 1 **The plots compare optimized fairness metric values for state senate plans across optimization scenarios that relax/remove the county preservation and shifted efficiency gap requirements.**

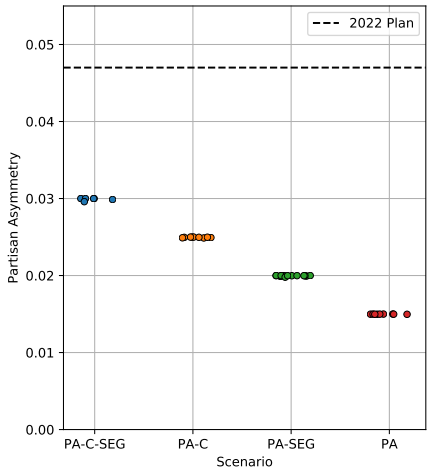
(a) Compactness Comparison



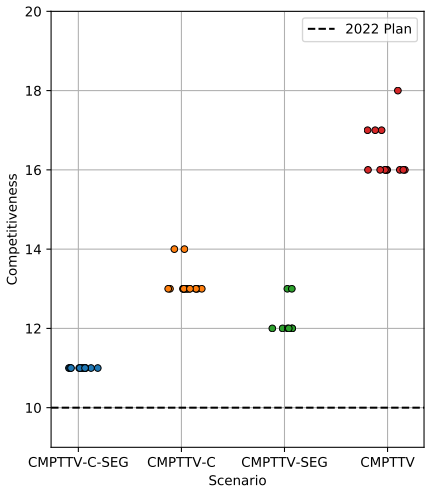
(b) Shifted Efficiency Gap Comparison



(c) Partisan Asymmetry Comparison



(d) Competitiveness Comparison

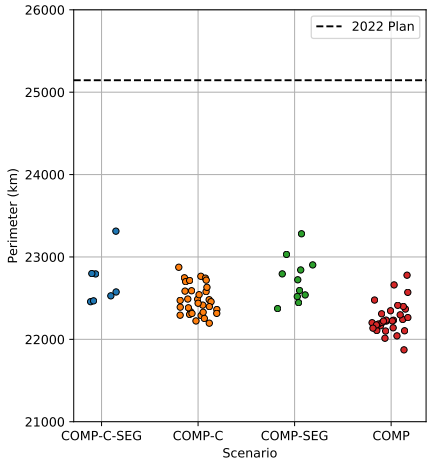


unable to substantially improve the shifted efficiency gap beyond 15%, so Figure 2 does not include a plot analogous to Figure 1b. As with the state senate plans, the compactness values are mostly comparable across all four scenarios, but COMP-C-SEG and COMP-SEG produce the worst compactness values; similarly, there is a large variance in the values each scenario produces. Both relaxing county preservation and removing the shifted efficiency gap threshold allow the algorithm to find slightly better partisan asymmetry values of roughly 0.03 (Figure 2b), but this does not constitute a substantial improvement. Similar to the state senate plans, there is a substantial increase in competitive districts when the

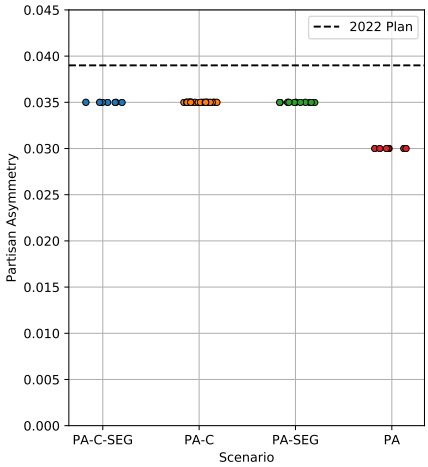
shifted efficiency gap requirement is removed; for example, Figure 2c shows that CMPTTV yields 66-68 competitive districts, while CMPTTV-SEG yields 52-58.

Figure 2 **The plots compare optimized fairness metric values for state house plans across optimization scenarios that relax/remove the county preservation and shifted efficiency gap requirements.**

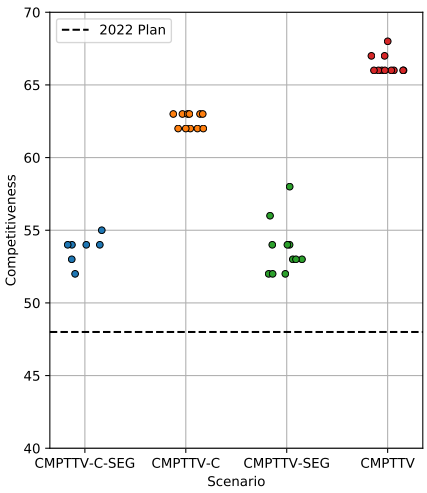
(a) Compactness Comparison



(b) Partisan Asymmetry Comparison



(c) Competitiveness Comparison



Optimized Congressional Plans

This section describes the metric values for the optimized congressional plans in each optimization scenario. In contrast to the state legislative plans, the congressional plans are not subject to the shifted efficiency gap threshold or county preservation requirement; therefore, we do not need to examine scenarios that remove/relax these constraints. Instead, this section discusses how optimizing one fairness metric can affect other fairness metrics.

Figure 3 demonstrates how optimizing for compactness, efficiency gap, partisan asymmetry, and competitiveness affect the other fairness metric values. Optimizing for fairness metrics other than compactness tend to yield worse compactness values (Figure 3a). Figure 3b shows that optimizing compactness, partisan asymmetry, or competitiveness can yield a variety of efficiency gap values; some values are nearly as good as the optimized efficiency gap values, while others are worse than the new 2022 congressional plan's value. Figure 3c shows that optimizing compactness can yield mediocre partisan asymmetry values; since Democratic voters are concentrated in the Kansas City and St. Louis areas, optimizing compactness tends to skew the Democratic vote-shares. Lastly, Figure 3d suggest that optimizing efficiency gap and partisan asymmetry can yield more competitive districts than simply optimizing for compactness.

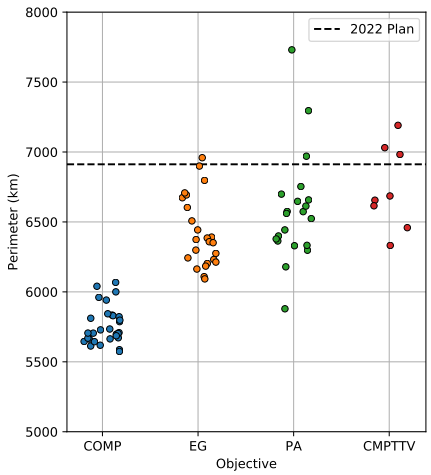
District Plans Originally Provided to the LWV-MO

We originally provided the LWV-MO with a collection of two state senate plans, two state house plans, and eight congressional plans, following Missouri's constitutional requirements and optimized for different fairness metrics. We constructed these plans without the GerryChain Python package and with fewer replications per optimization scenario; see the main text for a more comprehensive analysis of Missouri redistricting. This section presents images of each plan and reports their fairness metric values. Note that all compactness scores (f_{comp}) here are reported in miles and include perimeter penalties. The optimized plans, their fairness metric values, and their district vote-shares are presented as follows:

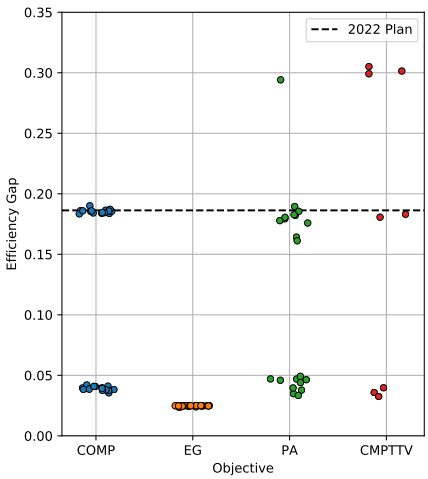
- Tables 1, 2, and 3 report the fairness metric values for the state senate, state house, and congressional plans, respectively.
- Figures 4, 5, and 6 show the estimated fraction of votes won by each party in the state senate, state house, and congressional districts, respectively.
- Figures 7 and 8 display the state senate district plans optimized for compactness: Senate-COMP1 and Senate-COMP2, respectively.
- Figures 9 and 10 display the state house district plans optimized for compactness: House-COMP1 and House-COMP2, respectively.
- Figures 11, 12, 13, 14, 15, 16, 17, and 18 display eight congressional plans optimized for compactness (Congress-COMP1 and Congress-COMP2), the efficiency gap (Congress-EG1 and Congress-EG2), partisan asymmetry (Congress-PA1 and Congress-PA2), and competitiveness (Congress-CMPTTV1 and Congress-CMPTTV2), respectively.

Figure 3 **The plots compare the fairness metric values of congressional plans optimized for compactness, efficiency gap, partisan asymmetry, and competitiveness.**

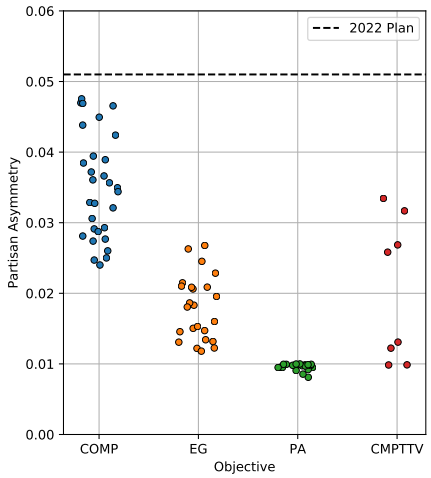
(a) Compactness Comparison



(b) Efficiency Gap Comparison



(c) Partisan Asymmetry Comparison



(d) Competitiveness Comparison

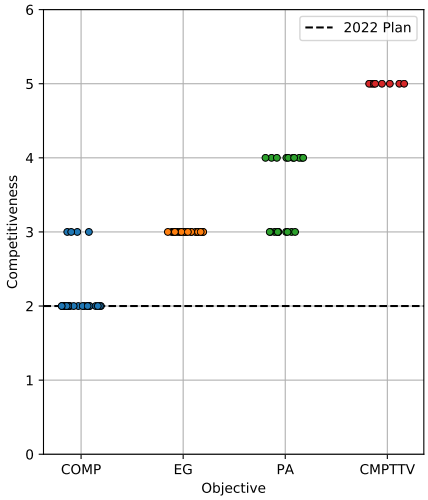


Table 1 The table reports metric values for the state senate plans.

Plan	f_{comp}	f_{eg}	f_{seg}	f_{pa}	f_{cmpttv}	D/R	Whole	MM
Senate-COMP1	15,240	12.89%	14.97%	0.039	10	10/24	107	3
Senate-COMP2	15,574	13.07%	14.99%	0.040	10	10/24	107	3

From left to right, the table lists each plan's compactness (f_{comp}), efficiency gap (f_{eg}), shifted efficiency gap (f_{seg}), partisan asymmetry (f_{pa}), number of seats within a 10% margin of victory (f_{cmpttv}), number of Democratic and Republican seats (D/R), number of whole counties (Whole), and number of majority-minority districts (MM).

Table 2 The table reports metric values for the state house plans.

Plan	f_{comp}	f_{eg}	f_{seg}	f_{pa}	f_{cmpttv}	D/R	Whole	Spanned	MM
House-COMP1	55,759	14.83%	14.97%	0.038	51	46/117	63	293	21
House-COMP2	57,449	14.89%	14.94%	0.038	47	46/117	61	295	19

From left to right, the table lists each plan's compactness (f_{comp}), efficiency gap (f_{eg}), shifted efficiency gap (f_{seg}), partisan asymmetry (f_{pa}), number of seats within a 10% margin of victory (f_{cmpttv}), number of Democratic and Republican seats (D/R), number of whole counties (Whole), number of counties spanned by the districts (Spanned), and number of majority-minority districts (MM).

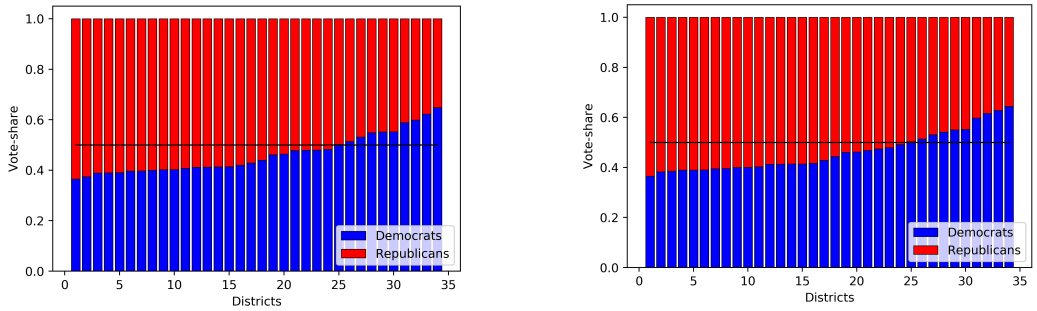
Table 3 The table reports metric values for the congressional plans.

Plan	f_{comp}	f_{eg}	f_{pa}	f_{cmpttv}	D/R	Whole	MM-B	MM-W
Congress-COMP1	4,219	17.83%	0.032	3	2/6	110	44.42%	42.53%
Congress-COMP2	4,398	18.50%	0.038	2	2/6	109	45.97%	40.67%
Congress-EG1	4,506	2.47%	0.015	3	3/5	110	37.27%	49.19%
Congress-EG2	4,991	2.48%	0.013	3	3/5	109	39.35%	47.48%
Congress-PA1	4,478	3.96%	0.010	3	3/5	109	39.73%	47.48%
Congress-PA2	4,740	15.46%	0.009	4	2/6	109	38.33%	48.76%
Congress-CMPTTV1	4,725	30.38%	0.033	4	1/7	110	46.46%	40.39%
Congress-CMPTTV2	5,349	18.49%	0.029	4	2/6	110	39.5%	47.40%

From left to right, the table lists each plan's compactness (f_{comp}), efficiency gap (f_{eg}), partisan asymmetry (f_{pa}), number of seats within a 10% margin of victory (f_{cmpttv}), number of Democratic and Republican seats (D/R), number of whole counties (Whole), the percentage of the majority-minority district that is Black/African American (MM-B), and the percentage of the majority-minority district that is non-Hispanic white (MM-W).

Figure 4 **The graphs show the estimated fraction of votes won by Democrats/Republicans in each district for the state senate plans (based on past election results).**

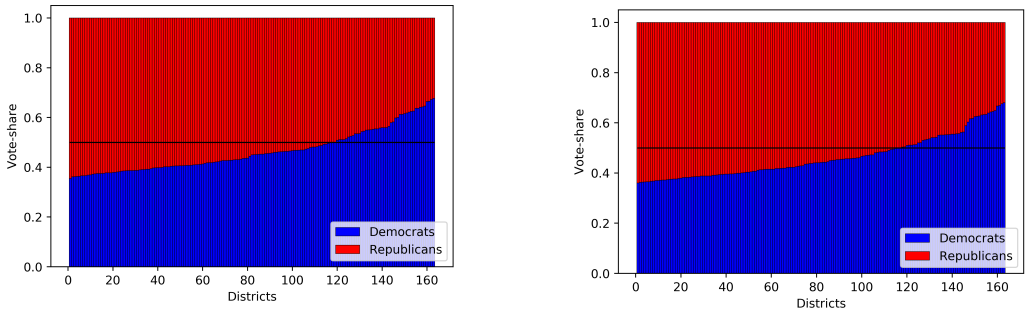
(a) Vote-shares for Senate-COMP1 (b) Vote-shares for Senate-COMP2



Note. Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.

Figure 5 **The graphs show the estimated fraction of votes won by Democrats/Republicans in each district for the state house plans (based on past election results).**

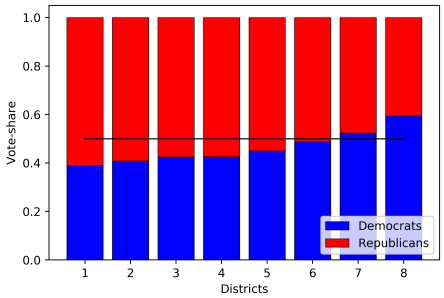
(a) Vote-shares for House-COMP1 (b) Vote-shares for House-COMP2



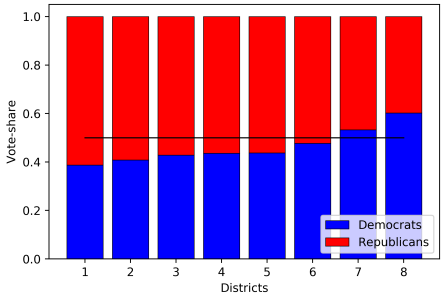
Note. Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.

Figure 6 **The graphs show the estimated fraction of votes won by Democrats/Republicans in each district for the congressional plans (based on past election results). Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.**

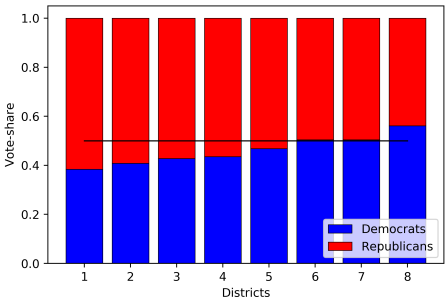
(a) Vote-shares for Congress-COMP1



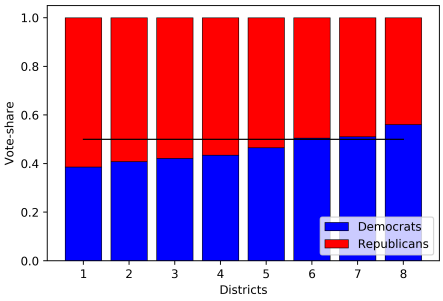
(b) Vote-shares for Congress-COMP2



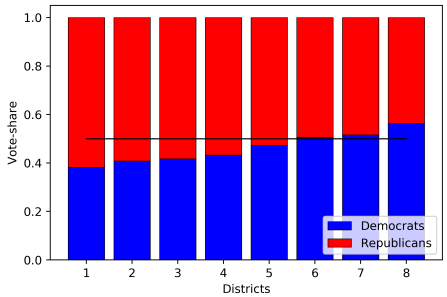
(c) Vote-shares for Congress-EG1



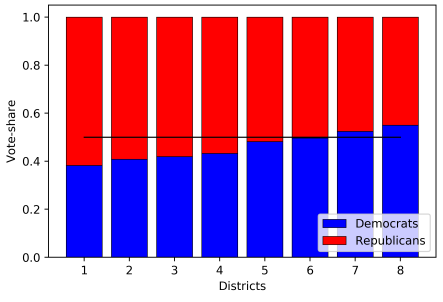
(d) Vote-shares for Congress-EG2



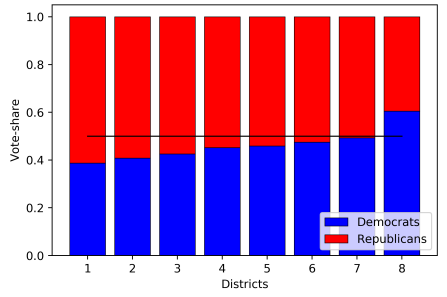
(e) Vote-shares for Congress-PA1



(f) Vote-shares for Congress-PA2



(g) Vote-shares for Congress-CMPTTV1



(h) Vote-shares for Congress-CMPTTV2

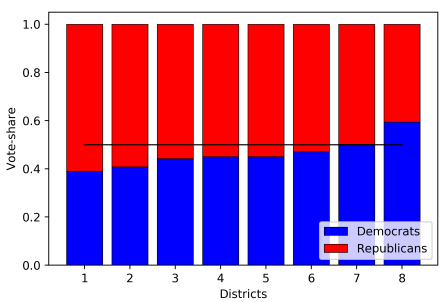
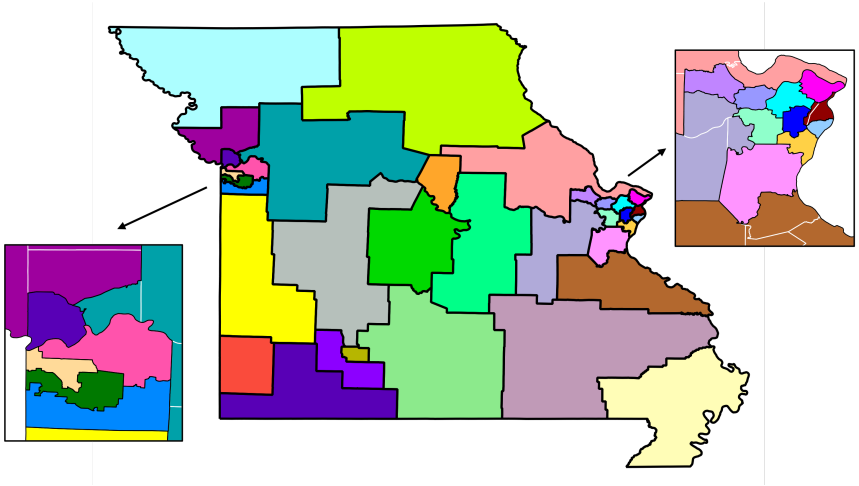
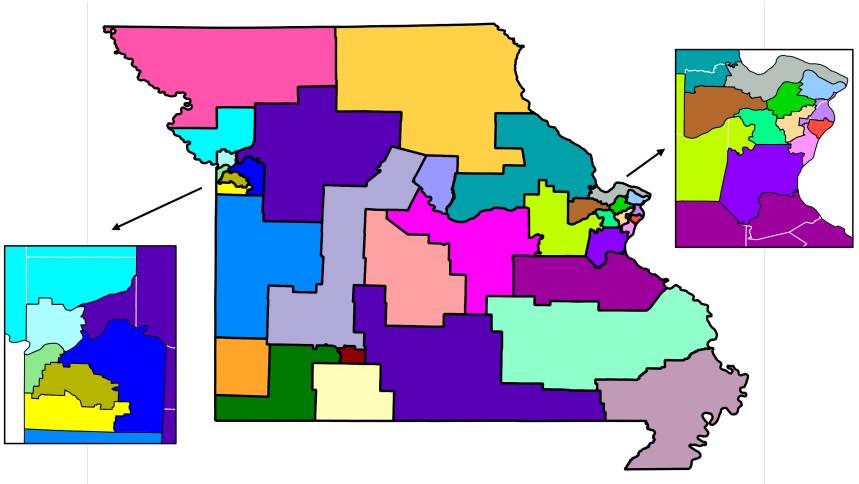


Figure 7 **This map shows Senate-COMP1, a state senate plan optimized for compactness, with a closer view of populated areas.**



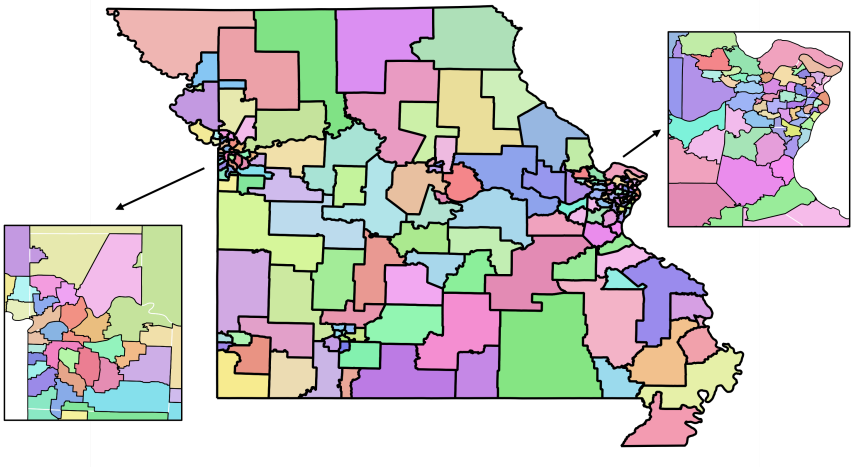
Note. District boundaries are shown in black. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 8 **This map shows Senate-COMP2, a state senate plan optimized for compactness, with a closer view of populated areas.**



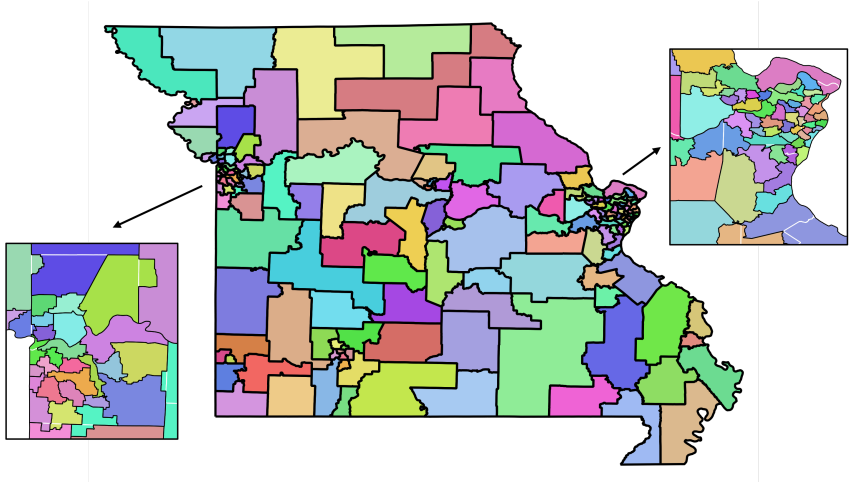
Note. District boundaries are shown in black. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 9 **This map shows House-COMP1, a state house plan optimized for compactness, with a closer view of populated areas.**



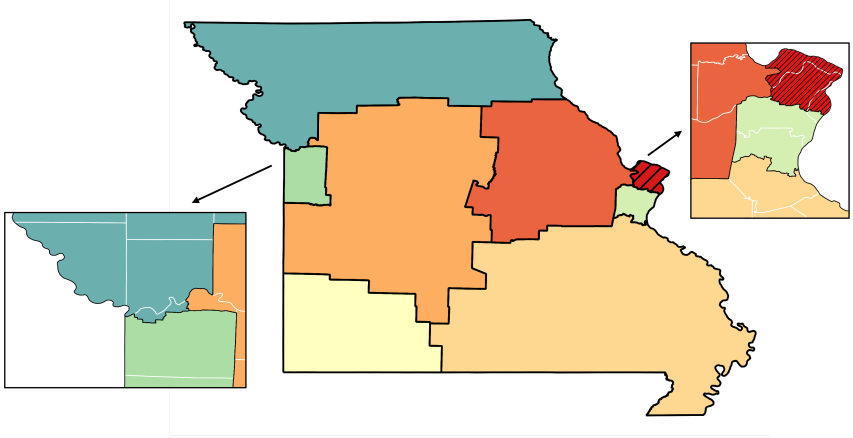
Note. District boundaries are shown in black. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 10 **This map shows House-COMP2, a state house plan optimized for compactness, with a closer view of populated areas.**



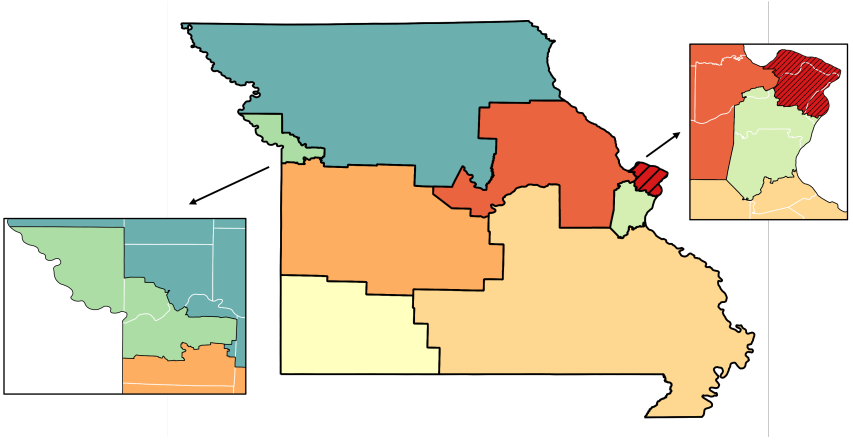
Note. District boundaries are shown in black. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 11 **This map shows Congress-COMP1, a congressional plan optimized for compactness, with a closer view of populated areas.**



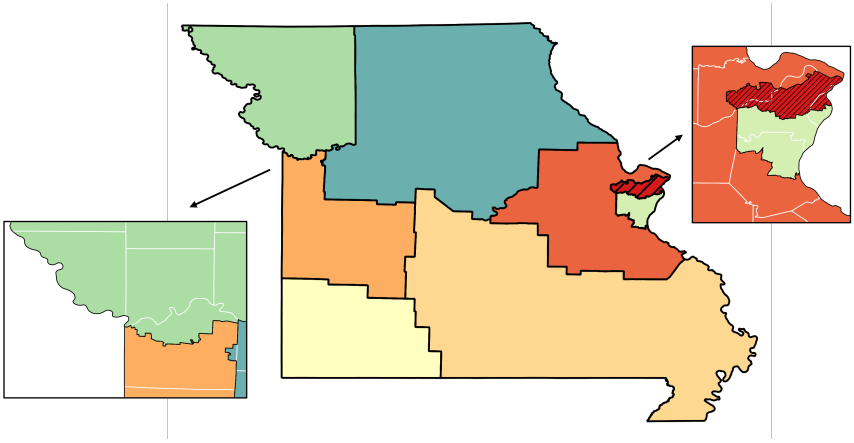
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 12 **This map shows Congress-COMP2, a congressional plan optimized for compactness, with a closer view of populated areas.**



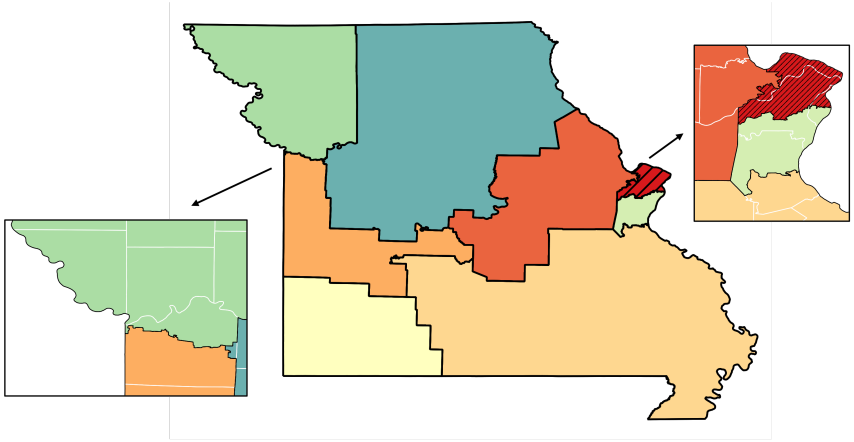
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 13 **This map shows Congress-EG1, a congressional plan optimized for the Efficiency Gap, with a closer view of populated areas.**



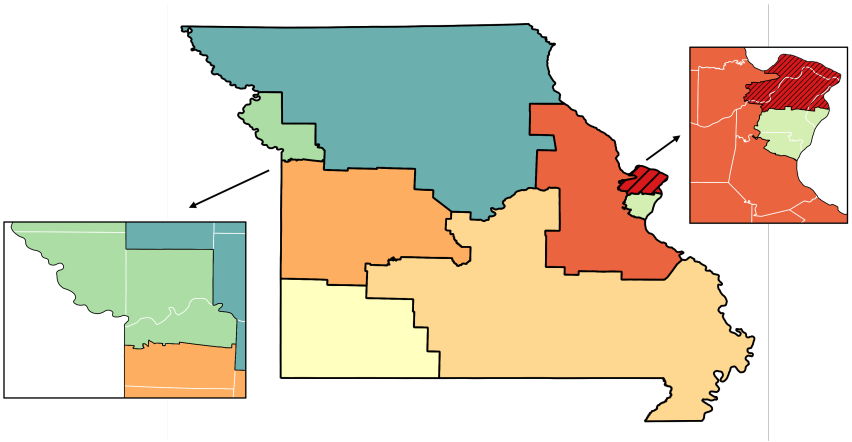
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 14 **This map shows Congress-EG2, a congressional plan optimized for the Efficiency Gap, with a closer view of populated areas.**



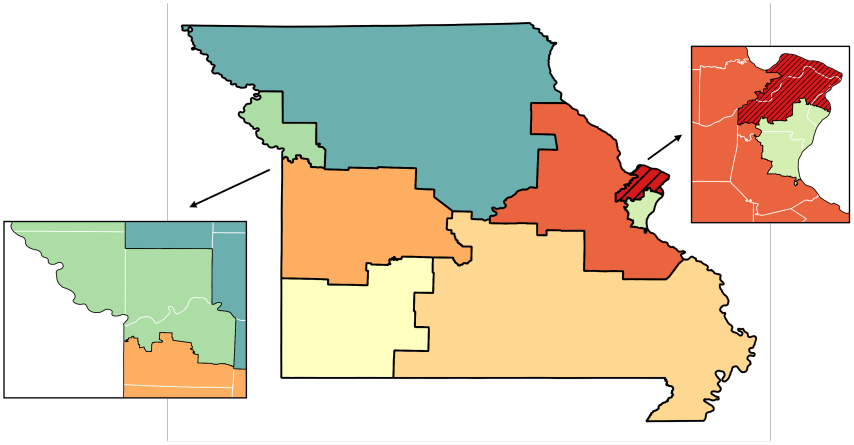
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 15 **This map shows Congress-PA1, a congressional plan optimized for Partisan Asymmetry, with a closer view of populated areas.**



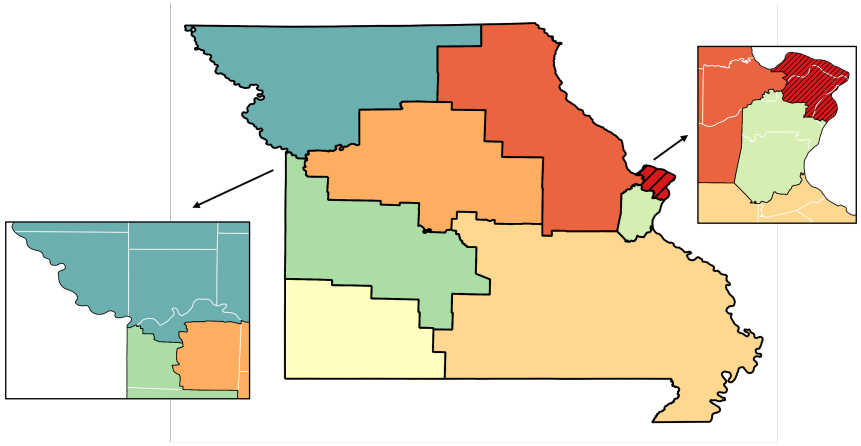
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 16 **This map shows Congress-PA2, a congressional plan optimized for Partisan Asymmetry, with a closer view of populated areas.**



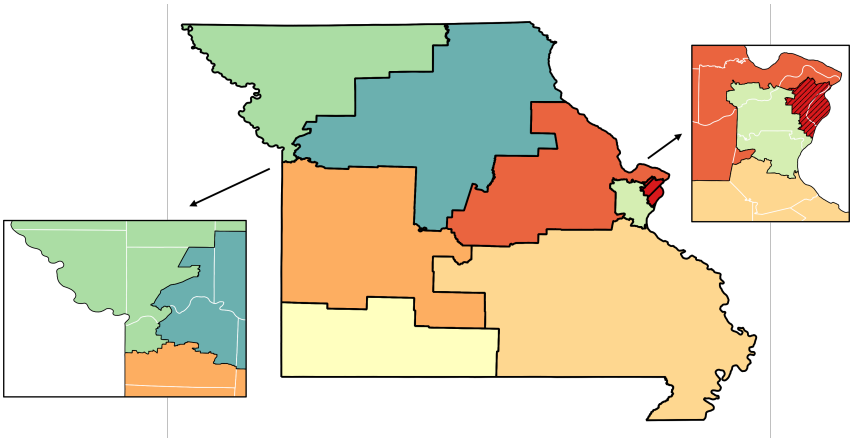
Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 17 **This map shows Congress-CMPTTV1, a congressional plan optimized for competitiveness, with a closer view of populated areas.**



Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.

Figure 18 **This map shows Congress-CMPTTV2, a congressional plan optimized for competitiveness, with a closer view of populated areas.**



Note. District boundaries are shown in black and the majority-minority district is hatched with black stripes. The insets show the Kansas City area to the west and the St. Louis area to the east. The insets also show county boundaries that do not coincide with a district boundary in white.