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Redesigning Zoning Systems for Equitable and Efficient Last-Mile Delivery at Ninja Van

John Gunnar Carlsson,[a] Stanley Lim,[b] Sheng Liu,[c] Han Yu,[a] Witsanu Arntong,[d] Ee Hsin Tan[d]

[a]University of Southern California, jcarlsso@usc.edu, hyu376@usc.edu; [b]Michigan State University slim@msu.edu; [c]University of Toronto sheng.liu@rotman.utoronto.ca; [d]Ninja Logistics Pte. Ltd. eehsin@ninjavan.co, witsanuarntong@gmail.com

Appendix A. Optimization Model Formulation and Solution

We present first the general zoning optimization model with deterministic demand $\mathbf{d}$. Specifically, let $\mathbf{d} \in \mathbb{Z}^M$ be a demand vector for M possible demand locations, where d_m is the order quantity from location m. The decision variables are denoted by $\{R_1, \ldots, R_K\}$, where R_k represents zone k and is a subset of the service region. Furthermore, let $\text{Workspan}(\mathbf{d}, p_k, R_k)$ denote the work span function for station k, which depends on the depot location p_k and zone R_k. Then **Problem 1** can be formulated as

$$\min_{R_1,\ldots,R_K} \max_{1\le k\le K} \text{Workspan}(\mathbf{d}, p_k, R_k) \tag{A.1}$$

$$s.t. \bigcup_{k=1}^{K} R_k = \text{service region}, \tag{A.2}$$

$$R_k \cap R_{k'} = 0,\ \forall k \neq k' \quad k, k' = 1, \ldots, K, \tag{A.3}$$

where constraint (A.2) ensures the coverage of the service region and constraints (A.3) ensure the zones are non-overlapping. In practice, the observed order demand varies from day to day, and so does the work span of every station. Following the sample average approximation (SAA) scheme, we can replace $\text{Workspan}(\mathbf{d}, p_k, R_k)$ by the sample average $\sum_{n=1}^{N} \text{Workspan}(\mathbf{d}_n, p_k, R_k)/N$ over N demand samples $\{\mathbf{d}_1, \ldots, \mathbf{d}_N\}$. Accordingly, the sample average work span will be used for gradient computation in Equation (2). Similarly, we can use the sample worst-case work span in the case of robust optimization.

We then present more details about the dual problem for finding the optimal AWVD, which is also discussed in Carlsson et al. (2016). Recall that the dual problem is

$$\min_{\mathbf{w}} \frac{1}{K} \sum_{k=1}^{K} w_i + \iint_{\text{service region}} f(\mathbf{x}) \max_k \left[-dist(\mathbf{x}, k) - w_k\right] d\mathbf{x},$$

and we can verify that a subgradient of the above objective function $G(w_1, \ldots, w_K) = (G_1, \ldots, G_K)$ can be computed by

$$G_k = \frac{1}{K} - \iint_{R_k(\mathbf{w})} f(\mathbf{x}) d\mathbf{x},\ k = 1, \ldots, K.$$

This is because (let $G^\Delta = (G_1^\Delta, \ldots, G_K^\Delta)$ with $G_k^\Delta = -\iint_{R_k(\mathbf{w})} f(\mathbf{x}) d\mathbf{x}$) for any $\mathbf{w}$ and $\mathbf{w}'$, the following inequality holds:

$$\begin{aligned}\iint_{\text{service region}} f(\mathbf{x}) \max_k \left[-dist(\mathbf{x}, k) - w'_k\right] d\mathbf{x} &\geq \sum_{k=1}^{K} \left\{ \iint_{R_k(\mathbf{w})} f(\mathbf{x})(-dist(\mathbf{x}, k) - w_k) d\mathbf{x} + G_k^\Delta (w'_k - w_k) \right\} \\ &= \iint_{\text{service region}} f(\mathbf{x}) \max_k \left[-dist(\mathbf{x}, k) - w_k\right] d\mathbf{x} + G^\Delta (\mathbf{w}' - \mathbf{w}).\end{aligned}$$

With historical data, we can evaluate $\iint_{R_k(\mathbf{w})} f(\mathbf{x}) d\mathbf{x}$ as the normalized work span of zone k.

References

Carlsson, John Gunnar, Erik Carlsson, Raghuveer Devulapalli. 2016. Shadow prices in territory division. *Networks and Spatial Economics* **16** 893–931.