<u>ONLINE SUPPLEMENT A:</u>
DERIVATION OF THE RESULTS FOR THE BASE MODEL

Proof of Proposition 1

The expected payoffs for the firm in the alarm and no-alarm states and the expected payoff for the user, respectively, are:

$$F_A(\rho_1,\psi) = -\frac{\begin{bmatrix}\psi P_D^I((c\rho_1 + d(1-\rho_1\phi))(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta) \\ +(1-\psi)P_F^I(c\rho_1(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)-\omega(1-\varepsilon+(1-P_F^F)\varepsilon\zeta))\end{bmatrix}}{(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)(\psi P_D^I+(1-\psi)P_F^I)} \tag{A1}$$

$$F_{NA}(\rho_2,\psi) = -\frac{\begin{bmatrix}\psi(1-P_D^I)(c\rho_2 + d(1-\rho_2\phi))(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta) \\ +(1-\psi)(1-P_F^I)(\omega(1-\varepsilon+(1-P_F^F)\varepsilon\zeta)-c\rho_2(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)\end{bmatrix}}{(1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)(\psi P_D^I+(1-\psi)P_F^I-1)} \tag{A2}$$

$$H(\rho_1,\rho_2,\psi) = \psi\mu - \psi\beta(\rho_1 P_D^I + \rho_2(1-P_D^I)) \tag{A3}$$

The first derivatives of payoffs with respect to the decision variables are:

$$\frac{\partial H}{\partial \psi} = \mu - \beta(\rho_1 P_D^I + \rho_2(1-P_D^I)) \tag{A4}$$

$$\frac{\partial F_A}{\partial \rho_1} = (1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)(d\phi P_D^I\psi - c(P_F^I(1-\psi)+P_D^I\psi)) \tag{A5}$$

$$\frac{\partial F_{NA}}{\partial \rho_2} = (1-P_D^F\varepsilon(1-\zeta)-P_F^F\varepsilon\zeta)(d\phi(1-P_D^I)\psi - c(1-P_F^I(1-\psi)-P_D^I\psi)) \tag{A6}$$

We can verify that, for a given ψ, $\frac{\partial F_A}{\partial \rho_1} = 0$ and $\frac{\partial F_{NA}}{\partial \rho_2} = 0$ cannot be satisfied simultaneously. We can also verify that $\partial F_A / \partial \rho_1 \geq \partial F_{NA} / \partial \rho_2$. Consequently, in the equilibrium, $\frac{\partial F_A}{\partial \rho_1} > 0$ and $\frac{\partial F_{NA}}{\partial \rho_2} = 0$, or $\frac{\partial F_A}{\partial \rho_1} = 0$ and $\frac{\partial F_{NA}}{\partial \rho_2} < 0$. Therefore we have two possible equilibrium scenarios: (i) $\rho_1 = 1, 0 < \rho_2 < 1$ and (ii) $0 < \rho_1 \leq 1, \rho_2 = 0$.

(i) $\rho_1 = 1, 0 < \rho_2 < 1$

In this scenario, (A4) and (A6) must be equal to zero, and (A5)>0. Solving (A4) and (A6) for ρ_2 and ψ respectively, we get

$$\rho_2^* = \frac{\mu - P_D^I\beta}{\beta(1-P_D^I)} \tag{A7}$$

$$\psi^* = \frac{c(1-P_F^I)}{c(P_D^I - P_F^I) + (1-P_D^I)d\phi} \tag{A8}$$

Since $0 < \rho_2 < 1$, we get the condition $P_D^I < \frac{\mu}{\beta} < 1$. Substituting (A8) into (A5) shows that (A5) is indeed positive.

(ii) $0 < \rho_1 \leq 1, \rho_2 = 0$

In this scenario, (A4) and (A5) must be equal to zero, and (A6)<0. Solving (A4) and (A5) for ρ_1 and ψ respectively, we get

$$\rho_1^* = \frac{\mu}{P_D^I \beta} \tag{A9}$$

$$\psi^* = \frac{cP_F^I}{d\phi P_D^I - c(P_D^I - P_F^I)} \tag{A10}$$

Since $0 < \rho_1 \leq 1$, we get the condition $0 < \frac{\mu}{\beta} \leq P_D^I$. Substituting (A10) into (A6) shows that (A6) is indeed negative.

Proof of Proposition 2

It follows from the payoff expressions given in Table 3.

Proof of Proposition 3

It follows from the value of IDS expressions given in Table 4.

The Value of Firewall and IDS Combination

From the equilibrium payoffs for the firm in no technology and firewall plus IDS cases given in Table 3, we can calculate the value of firewall and IDS combination. However the expressions for the value of firewall and IDS combination are complex. Instead we compare the value of firewall and IDS combination with the value of firewall only and the value of IDS only in all parameter regions. This comparison gives us the following table. Please note that (Value of IDS) and (Value of F) represent value of individual controls, and can be different in different regions.

Table A1. The Value of IDS and Firewall in Combination

Region	Condition(s)	Value of IDS+F	Comparison
$\frac{\mu}{\beta} > P_D^I$	$\Lambda < \frac{\omega\zeta(1-P_D^I)+c(P_D^I-P_F^I)}{\omega\zeta(1-P_F^I)}$	(Value of IDS) + A	(Value of IDS) < or > (Value of IDS+F) (Value of F) > (Value of IDS+F)
		(Value of F) + B	
	$\frac{\omega\zeta(1-P_D^I)+c(P_D^I-P_F^I)}{\omega\zeta(1-P_F^I)} < \Lambda < 1$	(Value of IDS) + C	
		(Value of F) + B	
	$\Lambda > 1$	(Value of IDS) + C	
		(Value of F) + B	
$\frac{\mu}{\beta} \leq P_D^I$	$\Lambda < 1$	(Value of IDS) + D	(Value of IDS) < or > (Value of IDS+F) (Value of F) < (Value of IDS+F)
		(Value of F) + E	
	$1 < \Lambda < \left(\frac{P_D^I}{P_F^I}\right)$	(Value of IDS) + D	
		(Value of F) + E	
	$\Lambda > \left(\frac{P_D^I}{P_F^I}\right)$	(Value of IDS) + F	
		(Value of F) + E	

$$A = \frac{\varepsilon(cd(1-P_F^I) - c(d\phi - c)(P_D^I - P_F^I))(P_D^F(1-\zeta) + P_F^F\zeta) - (d\phi - c)\omega\varepsilon\zeta(1-P_D^I)P_F^F}{c(P_D^I - P_F^I) + d\phi(1-P_D^I)}$$

$$B = \frac{-c(P_D^I - P_F^I)(d\phi - c)(d(1-\varepsilon(P_D^F(1-\zeta) + P_F^F\zeta))(1-\phi) + \omega(1-\varepsilon(1-\zeta(1-P_F^F))))}{d\phi(c(P_D^I - P_F^I) + d\phi(1-P_D^I))}$$

$$C = \frac{-\varepsilon(cd(1-P_F^I) - c(d\phi - c)(P_D^I - P_F^I))(1 - P_D^F(1-\zeta) - P_F^F\zeta) + (d\phi - c)\omega\varepsilon\zeta(1-P_D^I)(1-P_F^F)}{c(P_D^I - P_F^I) + d\phi(1-P_D^I)}$$

$$D = \frac{cd\varepsilon P_F^I(P_D^F(1-\zeta) + P_F^F\zeta) - (d\phi - c)\omega\varepsilon\zeta P_D^I P_F^F}{(d\phi - c)P_D^I + cP_F^I}$$

$$E = \frac{c(d\phi - c)(P_D^I - P_F^I)(d(1-\varepsilon(P_D^F(1-\zeta) + P_F^F\zeta)) + \omega(1-\varepsilon(1-\zeta(1-P_F^F))))}{d\phi((d\phi - c)P_D^I + cP_F^I)}$$

$$F = \frac{-cd\varepsilon P_F^I(1 - P_D^F(1-\zeta) - P_F^F\zeta) + (d\phi - c)\omega\varepsilon\zeta P_D^I(1-P_F^F)}{(d\phi - c)P_D^I + cP_F^I}$$

Proof of Proposition 4

From Table A1, we know that when $(\mu/\beta) > P_D^I$, IDS and firewall are conflicting. Otherwise (when $(\mu/\beta) \leq P_D^I$) we should investigate each region to determine the interaction effect.

Region 1: $(\Lambda < 1)$

In this region, firm (i) allows all external users in no technology architecture, and (ii) allows all external users in IDS only architecture. We know that Value of (IDS+F) can be less than Value of IDS (see the comparison column in table A1). If this is the case, controls are also conflicting. Otherwise controls can complement or substitute each other. We can write the condition for {Value of (IDS+F)> Value of IDS}from Table A1 as $\Lambda > \frac{P_F^F P_D^I}{(P_D^F(1-\zeta) + P_F^F\zeta)P_F^I}$.

Depending on the value of $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta) + P_F^F\zeta)P_F^I}$, there are two scenarios.

<u>Scenario 1</u>

$$\frac{P_D^F(1-\zeta) + P_F^F\zeta}{P_F^F} > \frac{P_D^I}{P_F^I}$$

<u>Scenario 2</u>

$$\frac{P_D^F(1-\zeta) + P_F^F\zeta}{P_F^F} < \frac{P_D^I}{P_F^I}$$

We can show that in region 1, {Value of (IDS+F) - Value of IDS – Value of F}<0. We also know that Value of F > 0 if $\Lambda > \frac{P_F^F}{P_D^F(1-\zeta) + P_F^F\zeta}$.

In scenario 1, Value of F>0 and Value of (F+IDS)>Value of (IDS) when $\frac{P_F^F}{P_D^F(1-\zeta) + P_F^F\zeta}\frac{P_D^I}{P_F^I} < \Lambda < 1$, and Value of (F+IDS)< Value of (IDS) when $\Lambda < \frac{P_F^F}{P_D^F(1-\zeta) + P_F^F\zeta}\frac{P_D^I}{P_F^I}$. Hence, controls substitute each other when

$\frac{P_F^F}{P_D^F(1-\zeta)+P_F^F\zeta}\frac{P_D^I}{P_F^I} < \Lambda < 1$ since {Value of (IDS+F) - Value of IDS – Value of (F)}<0, and controls conflict with each other when $\Lambda < \frac{P_F^F}{P_D^F(1-\zeta)+P_F^F\zeta}\frac{P_D^I}{P_F^I}$.

In scenario 2, Value of (F+IDS)<Value of IDS when $\Lambda < 1$, and therefore, controls conflict with each other.

Region 2: ($1 < \Lambda < \left(\frac{P_D^I}{P_F^I}\right)$)

In this region, firm (i) does not allow any external user in no technology case, and (ii) allows all external users in IDS only case. From Table A1, the condition for {Value of (IDS+F)> Value of IDS} is $\Lambda > \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$.

Assume that $1<\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$ (i.e., $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} < \frac{P_D^I}{P_F^I}$)

When $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{P_D^I}{P_F^I}$, Value of (IDS+F)> Value of IDS. So we should evaluate {Value of (IDS+F) - Value of IDS – Value of F}) to find the interaction effect. When $1 < \Lambda < \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$, Value of (IDS+F)< Value of IDS. So IDS and firewall conflict with each other.

We also know that Value of F > 0 if $\Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$. Depending on the value of $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$, there are three scenarios in region 2.

<u>Scenario 1:</u>

$$1<\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}<\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$$

<u>Scenario 2:</u>

$$\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}<\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}<\frac{P_D^I}{P_F^I}$$

<u>Scenario 3:</u>

$$\frac{P_D^I}{P_F^I}<\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$$

{Value of (IDS+F) - Value of IDS – Value of F} in region 2 is negative when

$$\Lambda < \frac{P_D^I}{P_F^I} + \frac{c(P_D^I - P_F^I)(1-P_F^I)}{d\phi P_F^I} + \frac{c(P_D^I - P_F^I)(1-(1-\zeta)P_D^F - \zeta P_F^F)}{\omega\zeta\phi P_F^I}$$

Since this condition is always true in region 2, we can conclude that firewall and IDS can only substitute each other when all values are positive.

In scenario 1, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{P_D^I}{P_F^I}$. Therefore controls complement each other when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{P_D^I}{P_F^I}$ since {Value of (IDS+F) - Value of IDS – max(0,Value of F)}>0. Controls conflict with each other when $\omega\zeta < \frac{cd}{d\phi - c} < \omega\zeta \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$.

In scenario 2, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$, and Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\omega\zeta \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \frac{cd}{d\phi - c} < \omega\zeta \frac{P_D^I}{P_F^I}$, and Value of F >0 and Value of (F+IDS)< Value of (IDS) when $\omega\zeta < \frac{cd}{d\phi - c} < \omega\zeta \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$. Therefore, controls substitute each other when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}<0. Controls complement each other when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \Lambda < \frac{P_D^I}{P_F^I}$ since (Value of (IDS+F) - Value of IDS – max{0,Value of F}) >0. Finally controls conflict with each other when $1 < \Lambda < \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$.

In scenario 3, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{P_D^I}{P_F^I}$, and Value of F >0 and Value of (F+IDS)< Value of (IDS) when $1 < \Lambda < \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$. Therefore controls substitute each other when $\frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I} < \Lambda < \frac{P_D^I}{P_F^I}$. Controls conflict with each other when $1 < \Lambda < \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$.

Assume that $1 > \frac{P_F^F P_D^I}{(P_D^F(1-\zeta)+P_F^F\zeta)P_F^I}$ (i.e., $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} > \frac{P_D^I}{P_F^I}$)

Again depending on the value of $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$, there are two additional scenarios in region 2.

<u>Scenario 4:</u>

$$\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \frac{P_D^I}{P_F^I}$$

Scenario 5:

$$\frac{P_D^I}{P_F^I} < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$$

In scenario 4, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \Lambda < \frac{P_D^I}{P_F^I}$, and Value of F >0 and Value of (F+IDS)> Value of (IDS) when $1 < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$. Hence controls complement each other when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \Lambda < \frac{P_D^I}{P_F^I}$ since {Value of (IDS+F) - Value of IDS – max(0,Value of F)}>0. Controls substitute each other when $1 < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}<0**.**

In scenario 5, Value of F >0 and Value of (F+IDS)> Value of (IDS). Therefore controls always substitute each other since {Value of (IDS+F) - Value of IDS – Value of F)}<0**.**

Region 3: ($\Lambda > \left(\frac{P_D^I}{P_F^I}\right)$)

In this region, the firm (i) does not allow any external user in no technology case, and (ii) does not allow any external user in IDS only case. From Table A1, the condition for {Value of (IDS+F)> Value of IDS} is $\Lambda < \frac{(1-P_F^F)P_D^I}{(1-P_D^F(1-\zeta)-P_F^F\zeta)P_F^I}$.

Since $\frac{(1-P_F^F)P_D^I}{(1-P_D^F(1-\zeta)-P_F^F\zeta)P_F^I} > \frac{P_D^I}{P_F^I}$, we can say that when $\frac{P_D^I}{P_F^I} < \Lambda < \frac{(1-P_F^F)P_D^I}{(1-P_D^F(1-\zeta)-P_F^F\zeta)P_F^I}$, Value of (IDS+F)> Value of IDS. So we should evaluate the expression {Value of (IDS+F) - Value of IDS – Value of F}}) to find the interaction effect. When $\Lambda > \frac{(1-P_F^F)P_D^I}{(1-P_D^F(1-\zeta)-P_F^F\zeta)P_F^I}$, Value of (IDS+F)< Value of IDS. So IDS and firewall conflict with each other. We also know that Value of F > 0 if $\Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$. Depending on the value of $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$, there are two scenarios in region 3.

Scenario 1:

$$\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \frac{P_D^I}{P_F^I}$$

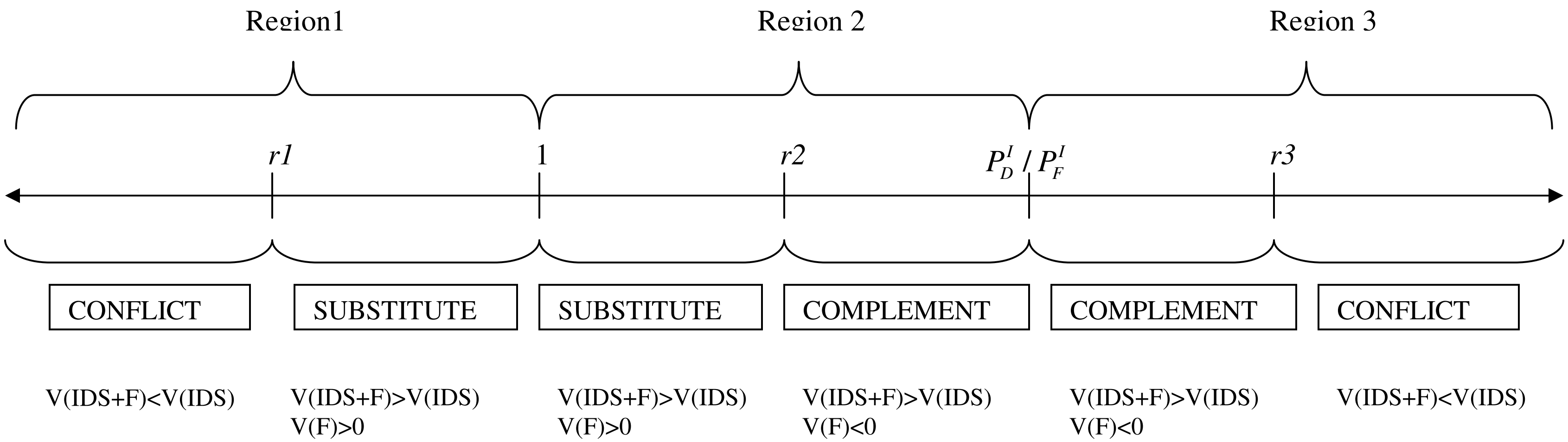
Scenario 2:

$$\frac{P_D^I}{P_F^I} < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I}$$

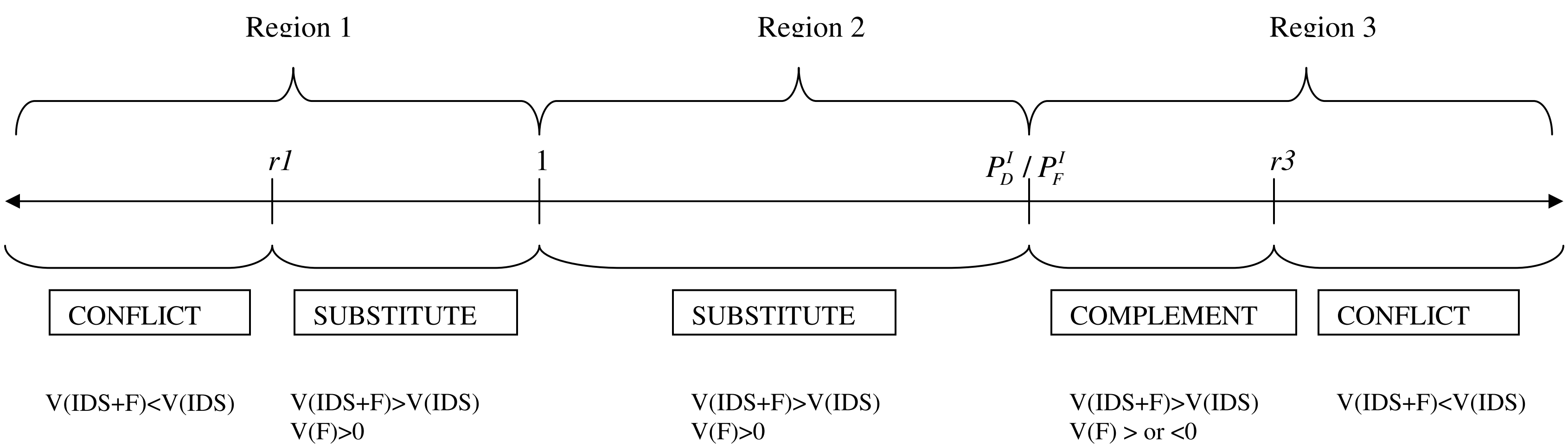
We can show that {Value of (IDS+F) - Value of IDS – Value of F} in region 3 is positive. Therefore firewall and IDS can only complement each other when all costs are positive.
In scenario 1, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_D^I}{P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I}$, and Value of F <0 and Value of (F+IDS)< Value of (IDS) when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I} < \Lambda$. Therefore, controls complement each other when $\frac{P_D^I}{P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I}$ since {Value of (IDS+F) - Value of IDS – Max{0,Value of F}}>0. Controls conflict with each other when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I} < \Lambda$.
In scenario 2, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_D^I}{P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$**,** and Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I}$**,**and Value of F <0 and Value of (F+IDS)< Value of (IDS) when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I} < \Lambda$**.** Therefore, controls complement each other when $\frac{P_D^I}{P_F^I} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}>0. Controls again complement each other when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)} < \Lambda < \frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I}$ since {Value of (IDS+F) - Value of IDS – Max{0,Value of F}}>0. Finally controls conflict with each other when $\frac{(1-P_F^F)}{(1-P_D^F(1-\zeta)-P_F^F\zeta)}\frac{P_D^I}{P_F^I} < \Lambda$**.**

We can summarize the interaction effect after considering all three regions graphically as follows, where Λ is shown on the x-axis.

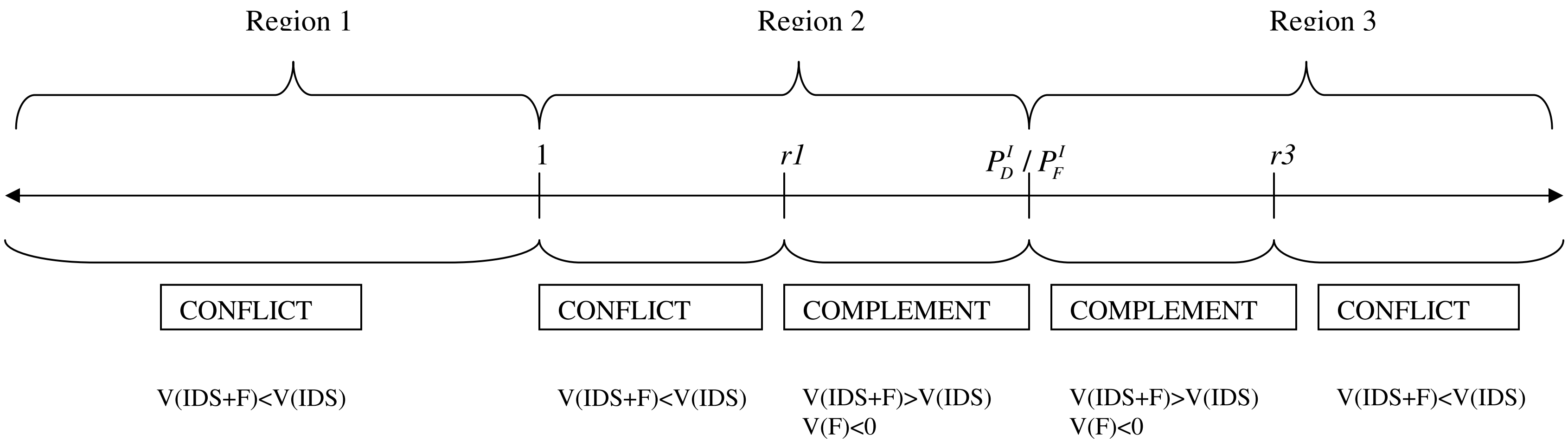
(1) When $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} > \frac{P_D^I}{P_F^I}$ and $\frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta} < \frac{P_D^I}{P_F^I}$



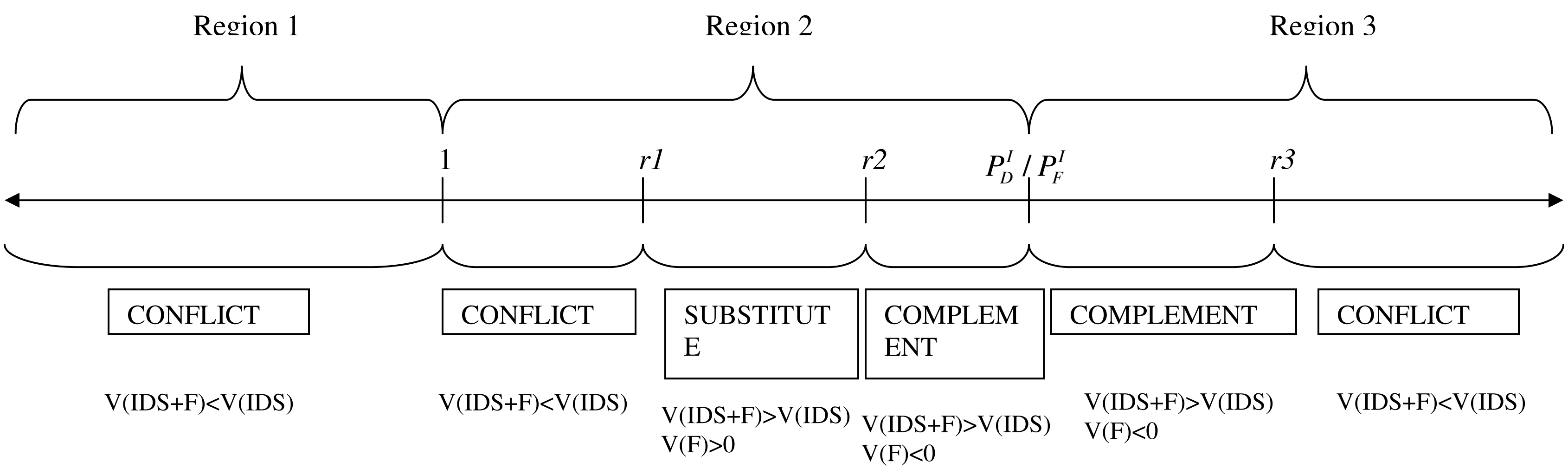
(2) When $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} > \frac{P_D^I}{P_F^I}$ and $\frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta} > \frac{P_D^I}{P_F^I}$



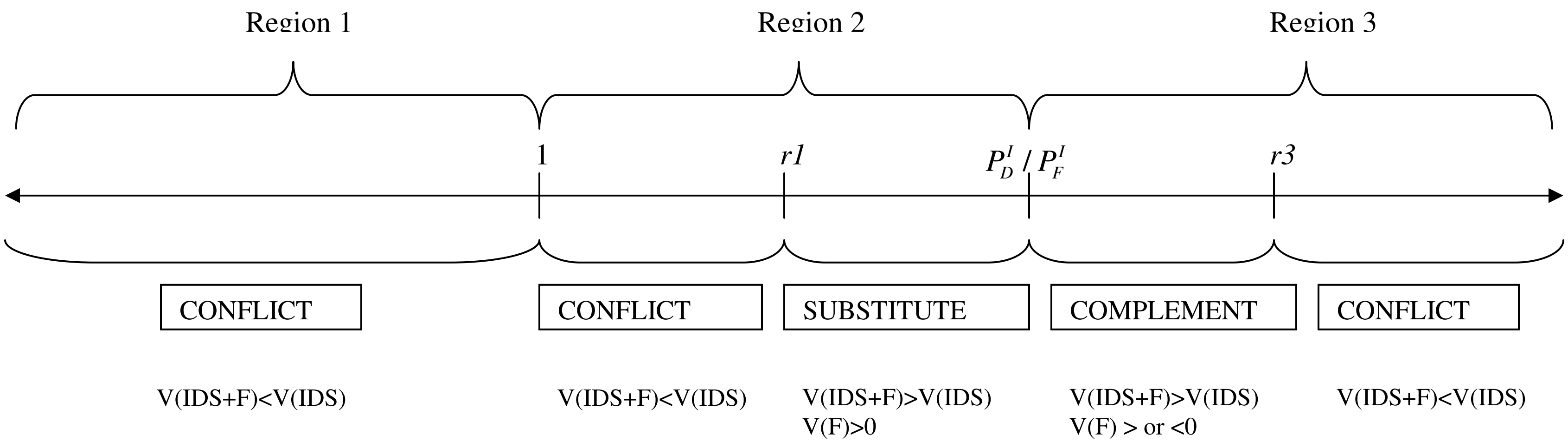
(3) When $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} < \frac{P_D^I}{P_F^I}$ and $\frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta} < \frac{P_F^F}{P_D^F(1-\zeta)+P_F^F\zeta}\frac{P_D^I}{P_F^I}$



(4) When $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} < \frac{P_D^I}{P_F^I}$ and $\frac{P_F^F}{P_D^F(1-\zeta)+P_F^F\zeta}\frac{P_D^I}{P_F^I} < \frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta} < \frac{P_D^I}{P_F^I}$



(5) When $\frac{P_D^F(1-\zeta)+P_F^F\zeta}{P_F^F} < \frac{P_D^I}{P_F^I}$ and $\frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta} > \frac{P_D^I}{P_F^I}$



where $r1 = \frac{P_F^F}{P_D^F(1-\zeta)+P_F^F\zeta}\frac{P_D^I}{P_F^I}$, $r2 = \frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta}$, $r3 = \frac{1-P_F^F}{1-P_D^F(1-\zeta)-P_F^F\zeta}\frac{P_D^I}{P_F^I}$.

Although there are five possibilities depending on the value of firewall and IDS quality parameters, in all of these five possibilities, the transitions of interaction effects is the same. That is, the order of interaction effect from left to right is

CONFLICT->SUBSTITUTE->COMPLEMENT->CONFLICT

After carefully looking at threshold values that separate interaction effects in five possibilities above, we can write the interaction results as in proposition 4.

Proof of Proposition 5

(i) All-external-access scenario

Substituting $\left(P_F^F\right)^{r_F}$ for P_D^F in the value of firewall expression, we get

$\frac{cd\left(P_F^F\right)^{r_F}\varepsilon(1-\zeta)+P_F^F\varepsilon\zeta(c(d+\omega)-d\omega\phi)}{d\phi}$. Taking a partial derivative of this expression with respect to P_F^F and equating it to zero gives $P_F^{F*}=\left(\frac{cdr_F(1-\zeta)}{d\phi\omega\zeta-c(d+\omega)\zeta}\right)^{\frac{1}{1-r_F}}$. Therefore

$P_D^{F*}=\left(\frac{cdr_F(1-\zeta)}{d\phi\omega\zeta-c(d+\omega)\zeta}\right)^{\frac{r_F}{1-r_F}}$. Since both P_F^{F*} and P_D^{F*} must be between zero and one, we have the following conditions.

$\Lambda<\frac{1}{\zeta}$ and $\Lambda<\frac{1}{r_F+(1-r_F)\zeta}$

Both of these conditions are satisfied all the time in the all-external-access scenario. In addition, since $\partial^2(.)/\partial\left(P_F^F\right)^2$ <0, the above solutions constitute the optimal configuration point. The value at the optimal configuration point is always non-negative since the value expression is an increasing concave function.

(ii) No-external-access scenario

Substituting $\left(P_F^F\right)^{r_F}$ for P_D^F in the value of firewall expression, we get

$\varepsilon\left((1-P_F^F)\omega\zeta\left(1-\frac{c}{d\phi}\right)-\left(\frac{c}{\phi}\right)\left(1-\zeta P_F^F-(1-\zeta)\left(P_F^F\right)^{r_F}\right)\right)$. Taking a partial derivative of this expression with respect to P_F^F and equating it to zero gives $P_F^{F*}=\left(\frac{cdr_F(1-\zeta)}{d\phi\omega\zeta-c(d+\omega)\zeta}\right)^{\frac{1}{1-r_F}}$.

Therefore $P_D^{F*}=\left(\frac{cdr_F(1-\zeta)}{d\phi\omega\zeta-c(d+\omega)\zeta}\right)^{\frac{r_F}{1-r_F}}$. Since both P_F^{F*} and P_D^{F*} must be between zero and one, we have the following conditions.

$\Lambda<\frac{1}{\zeta}$ and $\Lambda<\frac{1}{r_F+(1-r_F)\zeta}$.

Both of these conditions may not be satisfied in the no-external-access scenario. The stringent condition is

$\Lambda<\frac{1}{r_F+(1-r_F)\zeta}$

In addition, since $\partial^2(.)/\partial\left(P_F^F\right)^2$ <0, the above solutions constitute the optimal configuration point only when $\Lambda<\frac{1}{r_F+(1-r_F)\zeta}$. The value at the optimal configuration point is always non-negative since the value expression is an increasing concave function.

When $\Lambda > \frac{1}{r_F + (1-r_F)\zeta}$, the point that maximizes the value of firewall is P_F^{F*} >1. Since value expression is an increasing concave function, the value is maximized at P_F^{F*} =1, implying that all external users must be dropped.

Proof of Proposition 6

We know that when $\frac{\mu}{\beta} > P_D^I$**,** the value of IDS is negative, and when $\frac{\mu}{\beta} \le P_D^I$**,** the value of IDS is positive. Therefore firm will always configure its IDS such that the detection rate is higher than or equal to $\frac{\mu}{\beta}$**.** Given $\frac{\mu}{\beta} \le P_D^I$**,** the firm can be in one of the three regions

(i) $\Lambda < 1$ (ii) $1 < \Lambda < \left(\frac{P_D^I}{P_F^I}\right)$ (iii) $\Lambda > \left(\frac{P_D^I}{P_F^I}\right)$

Since the condition for the firm to be in the first region is independent of P_F^I and P_D^I, the firm that is already in region 1 cannot move to another region through configuration. However if the firm is in this region it can play with IDS detection and error rates to find the best point at which the value is maximized. Assuming that $P_D^I = \left(P_F^I\right)^{r_I}$, we can rewrite the value expression in term of P_D^I, and take a partial derivative of the expression w.r.t. P_D^I, we get:

$-\frac{c\left(P_D^I\right)^{1/r_I}(1-r_I)(d+\omega(1-\varepsilon(1-\zeta)))(d\phi - c)}{r_I\left(c\left(\left(P_D^I\right)^{1/r_I} - P_D^I\right) + d\phi P_D^I\right)^2} < 0$. So the firm configures its IDS at the lowest value of P_D^I. Since $\frac{\mu}{\beta} \le P_D^I$**,** the optimal configuration point is $\left(P_D^{I*}, P_F^{I*}\right) = \left(\frac{\mu}{\beta}, \frac{\mu}{\beta}^{1/r_I}\right)$.

If $\Lambda > 1$, then through configuration the firm can move between (ii) and (iii).The derivative of the value of IDS in (ii) wrt P_D^I, is

$$-\frac{c\left(P_D^I\right)^{1/r_I}(1-r_I)(d+\omega(1-\varepsilon(1-\zeta)))(d\phi - c)}{r_I\left(c\left(\left(P_D^I\right)^{1/r_I} - P_D^I\right) + d\phi P_D^I\right)^2} < 0$$

The derivative of the value of IDS in (iii) w.r.t. P_D^I, is

$$-\frac{c\left(P_D^I\right)^{1/r_I}(1-r_I)(d+\omega)(1-\varepsilon)(d\phi - c)}{r_I\left(c\left(\left(P_D^I\right)^{1/r_I} - P_D^I\right) + d\phi P_D^I\right)^2} < 0$$

We know that at the boundary between regions (ii) and (iii) (i.e., $\Lambda = \left(\frac{P_D^I}{P_F^I}\right)$), Value of IDS in (ii) = Value of IDS in (iii). Since the value is decreasing function of P_D^I in both regions, and the value expression is a continuous function in regions (ii) and (iii), the firm chooses the minimum value of

P_D^I at optimal configuration. Given that $\frac{\mu}{\beta} \le P_D^I$, the optimal configuration point is $\left(P_D^{I*}, P_F^{I*}\right) = \left(\frac{\mu}{\beta}, \frac{\mu}{\beta}^{1/r_I}\right)$. To sum up, irrespective of the region where the firm lies in, the optimal configuration point for IDS in IDS only case is $\left(P_D^{I*}, P_F^{I*}\right) = \left(\frac{\mu}{\beta}, \frac{\mu}{\beta}^{1/r_I}\right)$.

Proof of Proposition 7

We know that, for a given cost-to-benefit-ratio-for-external-access, (1) the value of firewall is the same for any value of P_D^I and (2) the value of firewall plus IDS is always greater than the value of firewall when $\frac{\mu}{\beta} > P_D^I$. Therefore the firm should configure its IDS such that $\frac{\mu}{\beta} \le P_D^I$ when IDS is deployed together with firewall.

(i) When $\Lambda < 1$ (i.e., when the firm finds it optimal to allow all external users in no technology case)

Value of (IDS+F) =

Value of IDS+ $\frac{cd\varepsilon P_F^I(P_D^F(1-\zeta)+P_F^F\zeta)-(d\phi-c)\omega\varepsilon\zeta P_D^I P_F^F}{(d\phi-c)P_D^I+cP_F^I}$

or

Value of F+ $\frac{c(d\phi-c)(P_D^I-P_F^I)(d(1-\varepsilon(P_D^F(1-\zeta)+P_F^F\zeta))+\omega(1-\varepsilon(1-\zeta(1-P_F^F))))}{d\phi((d\phi-c)P_D^I+cP_F^I)}$

Substituting the value of either firewall or IDS when it is used alone in that region gives

Value of (IDS+F) =

$$\frac{c(d+\omega(1-\varepsilon(1-\zeta)))}{d\phi} - \frac{cP_F^I(\omega(1-\varepsilon+(1-P_F^F)\varepsilon\zeta)+d(1-\varepsilon(P_D^F(1-\zeta)+P_F^F\zeta)))}{(d\phi-c)P_D^I+cP_F^I} - P_F^F\omega\varepsilon\zeta$$

Writing the above value in terms of P_D^F and P_D^I by substituting $\left(P_D^F\right)^{\frac{1}{r_F}}$ for P_F^F and $\left(P_D^I\right)^{\frac{1}{r_I}}$ for P_F^I, we get the value of (IDS+F) ~ $f(P_D^F, P_D^I)$. To find the values of P_D^F and P_D^I that maximize Value of (IDS+F), we take partial derivatives as

$$\frac{\partial \text{Value of(IDS+F)}}{\partial P_D^I} = \frac{c\left(P_D^I\right)^{\frac{1}{r_I}}(1-r_I)\left(d(1-P_D^F\varepsilon)+\omega(1-\varepsilon)+d\varepsilon\zeta(P_D^F-\left(P_D^F\right)^{\frac{1}{r_F}})+\omega\varepsilon\zeta(1-\left(P_D^F\right)^{\frac{1}{r_F}})\right)}{r_I\left((d\phi-c)P_D^I+c\left(P_D^I\right)^{\frac{1}{r_I}}\right)^2}$$

$\frac{\partial \text{Value of(IDS+F)}}{\partial P_D^I} < 0$ for any value of P_D^F. Therefore the firm tries to minimize P_D^I. Since P_D^I cannot be less than $\frac{\mu}{\beta}$, the firm configures the IDS at $(P_D^{I*}, P_F^{I*}) = (\frac{\mu}{\beta}, \left(\frac{\mu}{\beta}\right)^{\frac{1}{r^i}})$.

The partial derivative w.r.t. P_D^F gives

$$\frac{\partial \text{Value of(IDS+F)}}{\partial P_D^F} = \frac{cdP_D^F\left(P_D^I\right)^{\frac{1}{r_I}} r_F\varepsilon(1-\zeta) - \left(P_D^F\right)^{\frac{1}{r_F}}\varepsilon\zeta\left((d\phi-c)\omega P_D^I - cd\left(P_D^I\right)^{\frac{1}{r_I}}\right)}{P_D^F r_F\left((d\phi-c)P_D^I + c\left(P_D^I\right)^{\frac{1}{r_I}}\right)}.$$

Substituting the optimal value of P_D^I to the above expression, we get

$$\frac{cdP_D^F\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}} r_F\varepsilon(1-\zeta) - \left(P_D^F\right)^{\frac{1}{r_F}}\varepsilon\zeta\left((d\phi-c)\omega\frac{\mu}{\beta} - cd\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)}{P_D^F r_F\left((d\phi-c)\frac{\mu}{\beta} + c\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)}.$$

Since $\left((d\phi-c)\omega P_D^I - cd\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)$ is always positive in this region, there is always a P_D^F that will make the partial derivative zero. So we can conclude that

$$(P_D^{F*}, P_F^{F*}) = (\left(\frac{cdr_F(1-\zeta)}{(d\phi-c)\omega\zeta\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}} - cd\zeta}\right)^{\frac{r_F}{1-r_F}}, \left(\frac{cdr_F(1-\zeta)}{(d\phi-c)\omega\zeta\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}} - cd\zeta}\right)^{\frac{1}{1-r_F}}).$$

These probabilities must be between zero and one. The condition for that is

$$\Lambda < \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$$

Since this is always true in this region, there is no additional constraint for this equilibrium.

(ii) When $\Lambda > 1$ (i.e., when the firm finds it optimal to disallow all external users in no technology case)

Value of (IDS+F)

$$= \text{Value of IDS+ } \frac{cd\varepsilon P_F^I(P_D^F(1-\zeta) + P_F^F\zeta) - (d\phi-c)\omega\varepsilon\zeta P_D^I P_F^F}{(d\phi-c)P_D^I + cP_F^I} \quad \text{when } 1 < \Lambda < \left(\frac{P_D^I}{P_F^I}\right),$$

$$= \text{Value of IDS+ } \frac{-cd\varepsilon P_F^I(1 - P_D^F(1-\zeta) - P_F^F\zeta) + (d\phi-c)\omega\varepsilon\zeta P_D^I(1-P_F^F)}{(d\phi-c)P_D^I + cP_F^I} \quad \text{when } \Lambda > \left(\frac{P_D^I}{P_F^I}\right)$$

and

$$= \text{Value of F + } \frac{c(d\phi-c)(P_D^I - P_F^I)(d(1-\varepsilon(P_D^F(1-\zeta) + P_F^F\zeta)) + \omega(1-\varepsilon(1-\zeta(1-P_F^F))))}{d\phi((d\phi-c)P_D^I + cP_F^I)}$$

Substituting the value of either firewall or IDS when it is used alone in that region gives

Value of (IDS+F) =

$$(1-P_F^F)\omega\varepsilon\zeta+\frac{c(1-\varepsilon)(d+\omega)}{d\phi}-\frac{cP_F^I(\omega(1-\varepsilon+(1-P_F^F)\varepsilon\zeta)+d(1-\varepsilon(P_D^F(1-\zeta)+P_F^F\zeta)))}{(d\phi-c)P_D^I+cP_F^I}.$$

Writing the above in terms of P_D^F and P_D^I by substituting $\left(P_D^F\right)^{\frac{1}{r_F}}$ for P_F^F and $\left(P_D^I\right)^{\frac{1}{r_I}}$ for P_F^I, we get (IDS+F) ~ $f(P_D^F, P_D^I)$. To find the values of P_D^F and P_D^I that maximize Value of (IDS+F), we take partial derivatives as,

$$\frac{\partial\text{Value of(IDS+F)}}{\partial P_D^I}=\frac{c\left(P_D^I\right)^{\frac{1}{r_I}}(1-r_I)\left(d(1-P_D^F\varepsilon)+\omega(1-\varepsilon)+d\varepsilon\zeta(P_D^F-\left(P_D^F\right)^{\frac{1}{r_F}})+\omega\varepsilon\zeta(1-\left(P_D^F\right)^{\frac{1}{r_F}})\right)}{r_I\left((d\phi-c)P_D^I+c\left(P_D^I\right)^{\frac{1}{r_I}}\right)^2}$$

which is the same expression as in (i). Therefore, firm configures the IDS at

$$(P_D^{I*},P_F^{I*})=\left(\frac{\mu}{\beta},\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right).$$

The partial derivative w.r.t. P_D^F gives

$$\frac{\partial\text{Value of(IDS+F)}}{\partial P_D^F}=\frac{cdP_D^F\left(P_D^I\right)^{\frac{1}{r_I}}r_F\varepsilon(1-\zeta)-\left(P_D^F\right)^{\frac{1}{r_F}}\varepsilon\zeta\left((d\phi-c)\omega P_D^I-cd\left(P_D^I\right)^{\frac{1}{r_I}}\right)}{P_D^F r_F\left((d\phi-c)P_D^I+c\left(P_D^I\right)^{\frac{1}{r_I}}\right)},$$

which is the same expression as in (i).
Substituting the optimal value of P_D^I to the above expression, we get

$$\frac{cdP_D^F\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}r_F\varepsilon(1-\zeta)-\left(P_D^F\right)^{\frac{1}{r_F}}\varepsilon\zeta\left((d\phi-c)\omega\frac{\mu}{\beta}-cd\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)}{P_D^F r_F\left((d\phi-c)\frac{\mu}{\beta}+c\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)}.$$

$\left((d\phi-c)\omega P_D^I-cd\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)$ is positive when $\Lambda<\frac{1}{\zeta}P_D^I\left(\frac{\mu}{\beta}\right)^{\frac{-1}{r_I}}$. We know that in (ii), $\Lambda>1$.

Therefore $\zeta<P_D^I\left(\frac{\mu}{\beta}\right)^{\frac{-1}{r_I}}$ must be true. We also know that $P_D^I=\frac{\mu}{\beta}$, so we get a condition

$\zeta < \left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$, which is always true. Hence $\left((d\phi - c)\omega P_D^I - cd\left(\frac{\mu}{\beta}\right)^{\frac{1}{r_I}}\right)$ is positive when $1 < \Lambda < \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$ and negative when $\Lambda > \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$. When $\Lambda > \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$, the partial derivative is always positive, implying that (P_D^{F*}, P_F^{F*}) =(1, 1) since the value is an increasing concave function. So firewall configuration is not an issue in that case. The firm should not use a firewall and disallow external access. So we can conclude that

$$(P_D^{F*}, P_F^{F*}) = (\left(\frac{cdr_F(1-\zeta)}{(d\phi - c)\omega\zeta\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}} - cd\zeta}\right)^{\frac{r_F}{1-r_F}}, \left(\frac{cdr_F(1-\zeta)}{(d\phi - c)\omega\zeta\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}} - cd\zeta}\right)^{\frac{1}{1-r_F}})$$

when $1 < \Lambda < \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$. The condition that for these probabilities to be between zero and one is

$$\Lambda < \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$$

This condition may not be satisfied when $1 < \Lambda < \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$ because $\left(\frac{1}{r_F + (1-r_F)\zeta}\right)$ is less than $\frac{1}{\zeta}$. So we should restrict the region to $1 < \Lambda < \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$. When $\left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}} < \Lambda < \frac{1}{\zeta}\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$, the partial derivative is always positive, implying that (P_D^{F*}, P_F^{F*}) =(1, 1) since the value function is increasing. So firewall configuration is not an issue. The firm should not use a firewall and disallow external access.

Proof of Corollary 3

We also know from proposition 7 that both IDS and firewall are deployed at their optimal configuration points when $\Lambda < \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$. Therefore we focus on this region to analyze the interaction effect (i.e., configurable region).

From proposition 4, we know that

- if $\Lambda < r1$, controls conflict with each other
- if $r1 < \Lambda < \min(\frac{P_D^I}{P_F^I}, \max(r1, r2))$, controls substitute each other

- if $\min(\frac{P_D^I}{P_F^I}, \max(r1, r2)) < \Lambda < r3$, controls complement each other
- if $\Lambda > r3$, controls conflict with each other

If we substitute the optimal configuration points into the expression for $r1$, we get

$$r1(\text{at optimal configuration}) = \frac{cdr_F}{(d\phi - c)\omega\zeta - cd\zeta(1-r_F)\left(\frac{\mu}{\beta}\right)^{\frac{1-r_I}{r_I}}}.$$

In the configurable region, $r1(\text{at optimal configuration})$ is *always* less than Λ. Therefore first alternative is not possible. Similarly,

$$r3(\text{at optimal configuration}) = \left(\frac{1-P_F^{F*}}{1-(1-\zeta)P_D^{F*} - \zeta P_F^{F*}}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$$

We want to show that $\Lambda < \left(\frac{1-P_F^{F*}}{1-(1-\zeta)P_D^{F*} - \zeta P_F^{F*}}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$ is true in the configurable region.

When $\Lambda = 0$ (i.e., at the lower limit), the inequality holds. When $\Lambda = \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$ (i.e., at the upper limit), the inequality also holds.

$\lim_{P_D^{F*} -> 1} \left(\frac{1-P_F^{F*}}{1-(1-\zeta)P_D^{F*} - P_F^{F*}\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}} = \Lambda = \left(\frac{1}{r_F + (1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$. As Λ increases, $\left(\frac{1-P_F^{F*}}{1-(1-\zeta)P_D^{F*} - P_F^{F*}\zeta}\right)$ also increases, before reaching the upper limit, Λ is always less than $\left(\frac{1-P_F^{F*}}{1-(1-\zeta)P_D^{F*} - \zeta P_F^{F*}}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$. Therefore the last alternative is not possible either.

So, optimally configured firewall and IDS can only complement or substitute each other.

Since $r1(\text{at optimal configuration}) < 1$ and $r2(\text{at optimal configuration}) > 1$, at optimal configuration, $\max(r1, r2) = r2$. Therefore, $\min(\frac{P_D^I}{P_F^I}, \max(r1, r2))$ is either $r2$ or $\frac{P_D^I}{P_F^I} = \left(\frac{\mu}{\beta}\right)^{\frac{r_I - 1}{r_I}}$.

We know that $\frac{P_D^I}{P_F^I}$ is greater than one and does not change with Λ. We also know that $r2(\text{at optimal configuration})$ is greater than one and increases with Λ. The maximum value of $r2(\text{at optimal configuration})$ is $\left(\frac{1}{r_F + (1-r_F)\zeta}\right)$, which is less than the maximum value of Λ in the configurable region. Therefore both complementary and substitution effects are possible.

Since we cannot compare $\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$ and $r2(\text{at optimal configuration})$, we have:

- Optimally configured controls <u>substitute</u> each other when

  $$\Lambda < \min\left(\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}, \left(\frac{1-P_F^{F*}}{1-(\zeta P_F^{F*}+(1-\zeta)P_D^{F*})}\right)\right)$$

- Optimally configured controls <u>complement</u> each other when

  $$\min\left(\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}, \left(\frac{1-P_F^{F*}}{1-(\zeta P_F^{F*}+(1-\zeta)P_D^{F*})}\right)\right) < \Lambda < \left(\frac{1}{r_F+(1-r_F)\zeta}\right)\left(\frac{\mu}{\beta}\right)^{\frac{r_I-1}{r_I}}$$

ONLINE SUPPLEMENT B:
DERIVATION OF THE RESULTS FOR THE ALTERNATIVE MODEL 1

The following table provides all probability expressions we use in our new analysis.

Table B1. The Probability Expressions

Event	**Probability Expression**
a user gains access to the system	$P_{Access} = (1-\varepsilon) + \left(\varepsilon\left[(1-\zeta)(1-P_D^F) + \zeta(1-P_F^F)\right]\right)$
a user that has gained access is an internal user	$P_{I\mid Access} = (1-\varepsilon)/P_{Access}$
a user that has gained access is an external legal user	$P_{E,legal\mid Access} = \varepsilon\zeta(1-P_F^F)/P_{Access}$
a user that has gained access is an external illegal user	$P_{E,illegal\mid Access} = \varepsilon(1-\zeta)(1-P_D^F)/P_{Access}$
a user that has gained access generates an alarm from the IDS	$P_{alarm\mid Access} = (P_D^I\psi_{EU} + P_F^I(1-\psi_{EU}))P_{E,illegal\mid Access} + P_F^I(P_{E,legal\mid Access} + P_{I\mid Access})$
hack by an internal user given that IDS has generated an alarm	$P_{I,hack\mid Alarm} = 0$
normal use by an internal user given that IDS has generated an alarm	$P_{I,no-hack\mid Alarm} = \dfrac{P_F^I P_{I\mid Access}}{P_{alarm\mid Access}}$
hack by an external legal user given that IDS has generated an alarm	$P_{E,legal,hack\mid Alarm} = 0$
normal use by an external user given that IDS has generated an alarm	$P_{E,legal,no-hack\mid Alarm} = \dfrac{P_F^I P_{E,legal\mid Access}}{P_{alarm\mid Access}}$
hack by an external illegal user given that IDS has generated an alarm	$P_{E,illegal,hack\mid Alarm} = \dfrac{P_D^I\psi_{EU}P_{E,illegal\mid Access}}{P_{alarm\mid Access}}$
normal use by an external illegal user given that IDS has generated an alarm	$P_{E,illegal,no-hack\mid Alarm} = \dfrac{P_F^I(1-\psi_{EU})P_{E,illegal\mid Access}}{P_{alarm\mid Access}}$
hack by an internal user given that IDS has not generated an alarm	$P_{I,hack\mid No-alarm} = 0$
normal use by an internal user given that IDS has not generated an alarm	$P_{I,no-hack\mid No-alarm} = \dfrac{(1-P_F^I)P_{I\mid Access}}{\left(1-P_{alarm\mid Access}\right)}$

hack by an external legal user given that IDS has not generated an alarm	$P_{E,legal,hack\|No-alarm} = 0$
normal use by an external legal user given that IDS has not generated an alarm	$P_{E,legal,no-hack\|No-alarm} = \frac{(1-P_F^I)P_{E,legal\|Access}}{\left(1-P_{alarm\|Access}\right)}$
hack by an external illegal user given that IDS has not generated an alarm	$P_{E,illegal,hack\|No-alarm} = \frac{(1-P_D^I)\psi_{EU}P_{E,illegal\|Access}}{\left(1-P_{alarm\|Access}\right)}$
normal use by an external illegal user given that IDS has not generated an alarm	$P_{E,illegal,no-hack\|No-alarm} = \frac{(1-P_F^I)(1-\psi_{EU})P_{E,illegal\|Access}}{\left(1-P_{alarm\|Access}\right)}$
hack given that IDS has generated an alarm	$P_{hack\|Alarm} = \frac{P_D^I\psi_{EU}P_{E,illegal\|Access}}{P_{alarm\|Access}}$
hack given that IDS has not generated an alarm	$P_{hack\|No-alarm} = \frac{(1-P_D^I)\psi_{EU}P_{E,illegal\|Access}}{\left(1-P_{alarm\|Access}\right)}$

The firm's expected payoff per user in the alarm and no-alarm states given that the user gains access are given by the following:

$$F_A(\rho_1,\psi) = \omega\left(P_{I,no-hack|Alarm} + P_{E,legal,no-hack|Alarm}\right) - \rho_1 c - \left(P_{hack|Alarm}\right)(1-\rho_1)d - P_{hack|Alarm}\rho_1(1-\phi)d$$

$$F_{NA}(\rho_2,\psi) = \omega\left(P_{I,no-hack|No-alarm} + P_{E,legal,no-hack|No-alarm}\right) - \rho_2 c - P_{hack|No-alarm}(1-\rho_2)d - P_{hack|No-alarm}\rho_2(1-\phi)d$$

The firm's overall expected payoff per user is:

$$F(\rho_1,\rho_2,\psi) = P_{Access}\left(P_{alarm|Access}F_A(\rho_1,\psi) + P_{no-alarm|Access}F_{NA}(\rho_2,\psi)\right)$$

An illegal external user's payoff from hacking, after gaining access, is given by

$$H(\rho_1,\rho_2,\psi) = \mu\psi - \beta(\rho_1 P_D^I + \rho_2(1-P_D^I))\psi$$

The firm maximizes $F_A(\rho_1,\psi)$ when it gets an alarm from the IDS, and $F_{NA}(\rho_2,\psi)$ when it does not get an alarm from the IDS. A user maximizes his/her payoff.

The following Proposition shows the Nash equilibrium strategies for the firm and an external unauthorized user.

Proposition B1 (similar to Proposition 1) *The equilibrium when the firm implements a firewall and an IDS is given by the following.*

$$\text{if } \frac{\mu}{\beta} \le P_D^I, \text{then } \psi^* = \frac{(1-\varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F)}{\varepsilon(1-\zeta)(1-P_D^F)} \frac{cP_F^I}{d\phi P_D^I - c(P_D^I - P_F^I)}, \quad \rho_1^* = \frac{\mu}{P_D^I \beta}, \rho_2^* = 0$$

$$\text{Otherwise}, \ \psi^* = \frac{(1-\varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F)}{\varepsilon(1-\zeta)(1-P_D^F)} \frac{c(1-P_F^I)}{c(P_D^I - P_F^I) + (1-P_D^I)d\phi}, \rho_1^* = 1, \rho_2^* = \frac{\mu - P_D^I \beta}{(1-P_D^I)\beta}$$

When compared with Proposition 1, we can conclude that the external unauthorized user hacks at a higher rate than before since $\frac{(1-\varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F)}{\varepsilon(1-\zeta)(1-P_D^F)} > 1$. However the firm does not change its monitoring strategy.

Proof of Proposition B1

The expected payoffs for the firm in the alarm and no-alarm states and the expected payoff for the user, respectively, are:

$$F_A(\rho_1, \psi) = \frac{\begin{bmatrix} -\varepsilon(-1+\zeta)(d + c\rho_1 - d\rho_1\phi)\psi(-1+P_D^F)P_D^I + (\omega(1+\varepsilon(-1+\zeta)) \\ +c\rho_1(-1+\varepsilon(\psi-\zeta\psi)) + c\varepsilon(-1+\zeta)\rho_1(-1+\psi)P_D^F + \varepsilon\zeta(-\omega+c\rho_1)P_F^F)P_F^I \end{bmatrix}}{\varepsilon(-1+\zeta)\psi(-1+P_D^F)P_D^I + P_F^I - \varepsilon(\psi-\zeta\psi+(-1+\zeta)(-1+\psi)P_D^F + \zeta P_F^F)P_F^I} \quad \text{(B1)}$$

$$F_{NA}(\rho_2, \psi) = \frac{\begin{bmatrix} \omega(-1+\varepsilon-\varepsilon\zeta) + c\rho_2 + d\varepsilon(-1+\zeta)(-1+\rho_2\phi)\psi + \varepsilon(-1+\zeta)((c\rho_2 + d(1-\rho_2\phi)\psi)P_D^F \\ -(d + c\rho_2 - d\rho_2\phi)\psi(-1+P_D^F)P_D^I) + \varepsilon\zeta(\omega - c\rho_2)P_F^F + (\omega(1+\varepsilon(-1+\zeta)) \\ +c\rho_2(-1+\varepsilon(\psi-\zeta\psi)) + c\varepsilon(-1+\zeta)\rho_2(-1+\psi)P_D^F + \varepsilon\zeta(-\omega+c\rho_2)P_F^F)P_F^I \end{bmatrix}}{\begin{bmatrix} -1-\varepsilon(-1+\zeta)\psi P_D^I + \varepsilon\zeta P_F^F + (1+\varepsilon(-1+\zeta)\psi - \varepsilon\zeta P_F^F)P_F^I \\ +\varepsilon(-1+\zeta)P_D^F(-1+\psi P_D^I + P_F^I - \psi P_F^I) \end{bmatrix}}$$

(B2)

$$H(\rho_1, \rho_2, \psi) = \psi\mu - \psi\beta(\rho_1 P_D^I + \rho_2(1-P_D^I)) \quad \text{(B3)}$$

The first derivatives of payoffs with respect to the decision variables are:

$$\frac{\partial H}{\partial \psi} = \mu - \beta(\rho_1 P_D^I + \rho_2(1-P_D^I)) \quad \text{(B4)}$$

$$\frac{\partial F_A}{\partial \rho_1} = -\varepsilon(-1+\zeta)(c-d\phi)\psi(-1+P_D^F)P_D^I + c(-1+\varepsilon\psi-\varepsilon\zeta\psi+\varepsilon(-1+\zeta)(-1+\psi)P_D^F + \varepsilon\zeta P_F^F)P_F^I$$

(B5)

$$\frac{\partial F_{NA}}{\partial \rho_2} = -c - d\varepsilon(-1+\zeta)\phi\psi + \varepsilon(-1+\zeta)(-(c-d\phi\psi)P_D^F + (c-d\phi)\psi(-1+P_D^F)P_D^I) \quad \text{(B6)}$$
$$+c\varepsilon\zeta P_F^F - c(-1+\varepsilon\psi-\varepsilon\zeta\psi+\varepsilon(-1+\zeta)(-1+\psi)P_D^F + \varepsilon\zeta P_F^F)P_F^I$$

We can verify that, for a given ψ, $\frac{\partial F_A}{\partial \rho_1}$ = 0 and $\frac{\partial F_{NA}}{\partial \rho_2}$=0 cannot be satisfied simultaneously. We can also verify that $\partial F_A / \partial \rho_1 \geq \partial F_{NA} / \partial \rho_2$. Consequently, in the equilibrium, $\frac{\partial F_A}{\partial \rho_1}$>0 and $\frac{\partial F_{NA}}{\partial \rho_2}$=0, or $\frac{\partial F_A}{\partial \rho_1}$=0 and $\frac{\partial F_{NA}}{\partial \rho_2}$<0. Therefore we have two possible equilibrium scenarios: (i) $\rho_1 = 1, 0 < \rho_2 < 1$ and (ii) $0 < \rho_1 \leq 1, \rho_2 = 0$.

(i) $\rho_1 = 1, 0 < \rho_2 < 1$

In this scenario, (B4) and (B6) must be equal to zero, and (B5)>0. Solving (B4) and (B6) for ρ_2 and ψ respectively, we get

$$\rho_2^* = \frac{\mu - P_D^I \beta}{\beta(1 - P_D^I)} \tag{B7}$$

$$\psi^* = \frac{(1 - \varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F)}{\varepsilon(1-\zeta)(1-P_D^F)} \frac{c(1-P_F^I)}{c(P_D^I - P_F^I) + (1-P_D^I)d\phi} \tag{B8}$$

Since $0 < \rho_2 < 1$, we get the condition $P_D^I < \frac{\mu}{\beta} < 1$. Substituting (B8) into (B5) shows that (B5) is indeed positive.

(ii) $0 < \rho_1 \leq 1, \rho_2 = 0$

In this scenario, (B4) and (B5) must be equal to zero, and (B6)<0. Solving (B4) and (B5) for ρ_1 and ψ respectively, we get

$$\rho_1^* = \frac{\mu}{P_D^I \beta} \tag{B9}$$

$$\psi^* = \frac{(1 - \varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F)}{\varepsilon(1-\zeta)(1-P_D^F)} \frac{cP_F^I}{d\phi P_D^I - c(P_D^I - P_F^I)} \tag{B10}$$

Since $0 < \rho_1 \leq 1$, we get the condition $0 < \frac{\mu}{\beta} \leq P_D^I$. Substituting (B10) into (B6) shows that (B6) is indeed negative.

Corollary B1. (Similar to Corollary 1)

(a) The equilibrium for the IDS only case

- *For full-external-access scenario*

$$if\ \frac{\mu}{\beta} \leq P_D^I, then\ \psi^* = \frac{1}{\varepsilon(1-\zeta)} \frac{cP_F^I}{d\phi P_D^I - c(P_D^I - P_F^I)},\quad \rho_1^* = \frac{\mu}{P_D^I \beta}, \rho_2^* = 0$$

$$Otherwise,\ \psi^* = \frac{1}{\varepsilon(1-\zeta)} \frac{c(1-P_F^I)}{d\phi(1-P_D^I) + c(P_D^I - P_F^I) +}, \rho_1^* = 1, \rho_2^* = \frac{\mu - P_D^I \beta}{(1-P_D^I)\beta}$$

- *For no-external-access scenario*

$$\psi^* = 0,\quad \rho_1^* = 0, \rho_2^* = 0$$

(b) The equilibrium for the firewall only case

$$\psi^* = \frac{1-\varepsilon(1-\zeta)P_D^F - \varepsilon\zeta P_F^F}{\varepsilon(1-\zeta)(1-P_D^F)}\frac{c}{d\phi}, \quad \rho^* = \frac{\mu}{\beta}$$

(c) The equilibrium for the no technology case

- *For full-external-access scenario*

$$\psi^* = \frac{1}{\varepsilon(1-\zeta)}, \quad \rho^* = \frac{\mu}{\beta}$$

- *For no-external-access scenario*

$$\psi^* = 0, \quad \rho^* = 0$$

Proof of Corollary B1.

The equilibrium when the firm implements only a firewall, only an IDS, or neither an IDS nor a firewall can be derived from Proposition B1 by making appropriate substitutions to the firewall and IDS quality parameters. By substituting $P_D^I = P_F^I = 0$ in Proposition B1, we get the equilibrium when the firm implements only a firewall. The substitutions imply that no alarm is generated, and, by implication, no false alarm is generated. Notice that in the firewall only case, ρ_1^* is not meaningful because it represents the probability of investigation when there is an alarm. The case when the firm implements only an IDS is more complex because of two possibilities that arise when there is no firewall. In the first possibility, which we refer to as the *no-external-access* (NEA) scenario, the firm does not use a firewall because it does not allow access to any external user and restricts access only to internal users. The second possibility, which we refer to as the *full-external-access* (FEA) scenario, is one in which the firm does not use a firewall because it allows access to every external user, and hence, it does not need a firewall to selectively block or allow access. The former scenario can be analyzed by setting $P_D^F = P_F^F = 1$ in our model, and the latter scenario is equivalent to substituting $P_D^F = P_F^F = 0$. For the case when the firm does not implement either a firewall or an IDS, we substitute $P_D^I = P_F^I = 0$, $\rho_1 = 0$, $\rho_2 = \rho$, and, depending on how we view the absence of a firewall, either $P_D^F = P_F^F = 0$ (FEA) or $P_D^F = P_F^F = 1$ (NEA). Based on these substitutions, we obtain the result given in Corollary B1.

Equilibrium Payoffs (Similar to Table 3)

The firm's expected payoffs under various security architectures are given in Table B2.

Table B2. Firm's Equilibrium Payoff Under Various Security Architectures

Security Architecture		**Firm's Payoff**
No Technology	NEA	$\omega - \omega\varepsilon$
	FEA	$\omega(1+\varepsilon(-1+\zeta)) - \frac{c}{\phi}$
Firewall Only		$\frac{-c+\omega(1+\varepsilon(-1+\zeta))\phi - c\varepsilon(-1+\zeta)P_D^F + \varepsilon\zeta(c-\omega\phi)P_F^F}{\phi}$
IDS Only	NEA	$\omega - \omega\varepsilon$
	FEA	$d+\omega(1+\varepsilon(-1+\zeta)) + \frac{d(c-d\phi)P_D^I}{(c-d\phi)P_D^I - cP_F^I} \quad if\ \frac{\mu}{\beta} \le P_D^I$ $\frac{d(c+\omega(-1+\varepsilon-\varepsilon\zeta)\phi) + (c+\omega(-1+\varepsilon-\varepsilon\zeta))(c-d\phi)P_D^I - c(c+d+\omega(-1+\varepsilon-\varepsilon\zeta)-d\phi)P_F^I}{-d\phi+(-c+d\phi)P_D^I + cP_F^I} \quad if\ \frac{\mu}{\beta} > P_D^I$
IDS and Firewall		$\frac{-\omega(c-d\phi)P_D^I(-1+\varepsilon-\varepsilon\zeta+\varepsilon\zeta P_F^F) + c(d+\omega(-1+\varepsilon-\varepsilon\zeta) + d\varepsilon(-1+\zeta)P_D^F + (-d+\omega)\varepsilon\zeta P_F^F)P_F^I}{(c-d\phi)P_D^I - cP_F^I} \quad if\ \frac{\mu}{\beta} \le P_D^I$ $\frac{\left[\begin{array}{l}(c-d\phi)P_D^I(c+\omega(-1+\varepsilon-\varepsilon\zeta)+(-c+\omega)\varepsilon\zeta P_F^F) + d(c+\omega(-1+\varepsilon-\varepsilon\zeta)\phi + \varepsilon\zeta(-c+\omega\phi)P_F^F \\ +c(-c+\omega(1+\varepsilon(-1+\zeta)) + d(-1+\phi) + \varepsilon\zeta(c+d-\omega-d\phi)P_F^F)P_F^I + c\varepsilon(-1+\zeta)P_D^F(d+(c-d\phi)P_D^I - (c+d-d\phi)P_F^I)\end{array}\right]}{-d\phi+(-c+d\phi)P_D^I + cP_F^I}$ $if\ \frac{\mu}{\beta} > P_D^I$

Table B3. The Value of IDS Expressions

Region	Condition	Value of IDS
$\frac{\mu}{\beta} > P_D^I$	$1 < \Lambda$	0
	$\frac{d\phi(1-P_D^I)+c(P_D^I-P_F^I)}{\phi\left[d(1-P_F^I)-(d\phi-c)(P_D^I-P_F^I)\right]} < \Lambda < 1$	$-\frac{\omega\varepsilon\phi\zeta-c}{\phi}$
	$\Lambda < \frac{d\phi(1-P_D^I)+c(P_D^I-P_F^I)}{\phi\left[d(1-P_F^I)-(d\phi-c)(P_D^I-P_F^I)\right]}$	$-\frac{c(1-\phi)(d\phi-c)(P_D^I-P_F^I)}{\phi\left[d\phi(1-P_D^I)+c(P_D^I-P_F^I)\right]}$
$\frac{\mu}{\beta} \le P_D^I$	$\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I} < \Lambda$	0
	$1 < \Lambda < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$	$\omega\varepsilon\zeta - \frac{cdP_F^I}{(d\phi-c)P_D^I+c\phi P_F^I}$
	$\Lambda < 1$	$\frac{c(d\phi-c)(P_D^I-P_F^I)}{\phi(d\phi-c)P_D^I+c\phi P_F^I}$

Proposition 2B. (Similar to Proposition 2) *Assuming that* $\frac{P_D^F}{P_F^F} > \frac{1-\varepsilon\zeta}{\varepsilon-\varepsilon\zeta}$, *for the default configuration scenario, the value of implementing only a firewall is positive iff* $\frac{P_F^F}{\varepsilon(\zeta P_F^F+(1-\zeta)P_D^F)} < \Lambda < \frac{(1-P_F^F)}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$. *Otherwise the value is not positive.*

Proof of Proposition 2B.

It follows from the payoff expressions in Table B2.

Proposition 3B. *For the default configuration scenario, the value of implementing only an IDS is positive iff* $(\mu/\beta) \le P_D^I$.

Proof of Proposition 3B

It follows from the value of IDS expressions given in Table B3.

The Value of Firewall and IDs Combination

From the equilibrium payoffs for the firm in no technology and firewall plus IDS cases given in Table B2, we can calculate the value of firewall and IDS combination. However the expressions for the value of firewall and IDS combination are complex. Instead we compare the value of firewall and IDS combination with the value of firewall only and the value of IDS only in all parameter regions. This comparison gives us the following table. Please note that (Value of IDS) and (Value of F) represent value of individual controls, and can be different in different regions.

Table B4. The Value of IDS and Firewall in Combination

Region	Condition(s)	Value of IDS+F	Comparison
$\frac{\mu}{\beta} > P_D^I$	$1 < \Lambda$	(Value of IDS) + F	(Value of IDS) < or > (Value of IDS+F) (Value of F) > (Value of IDS+F)
		(Value of F) + B	
	$\frac{d\phi(1-P_D^I)+c(P_D^I-P_F^I)}{\phi\left[d(1-P_F^I)-(d\phi-c)(P_D^I-P_F^I)\right]} < \Lambda < 1$	(Value of IDS) + F	
		(Value of F) + B	
	$\Lambda < \frac{d\phi(1-P_D^I)+c(P_D^I-P_F^I)}{\phi\left[d(1-P_F^I)-(d\phi-c)(P_D^I-P_F^I)\right]}$	(Value of IDS) + E	
		(Value of F) + B	
$\frac{\mu}{\beta} \le P_D^I$	$\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I} < \Lambda$	(Value of IDS) + D	(Value of IDS) < or > (Value of IDS+F) (Value of F) < (Value of IDS+F)
		(Value of F) + A	
	$1 < \Lambda < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$	(Value of IDS) + C	
		(Value of F) + A	
	$\Lambda < 1$	(Value of IDS) + C	
		(Value of F) + A	

where

$$A = \frac{c(d\phi-c)(1+\varepsilon(-1+\zeta)P_D^F-\varepsilon\zeta P_F^F)(P_D^I-P_F^I)}{\phi((d\phi-c)P_D^I+cP_F^I)} > 0$$

$$B = \frac{-c(1-\phi)(d\phi-c)(1+\varepsilon(-1+\zeta)P_D^F-\varepsilon\zeta P_F^F)(P_D^I-P_F^I)}{\phi(d\phi+(c-d\phi)P_D^I-cP_F^I)} < 0$$

$$C = \frac{\varepsilon(\omega\zeta(d\phi-c)P_D^I P_F^F+c(d(-1+\zeta)P_D^F+(\omega-d)\zeta P_F^F)P_F^I)}{(c-d\phi)P_D^I-cP_F^I}$$

$$D = \frac{-\omega\varepsilon\zeta(d\phi-c)P_D^I(1-P_F^F)+c(d-\omega\varepsilon\zeta+d\varepsilon(-1+\zeta)P_D^F+(\omega-d)\varepsilon\zeta P_F^F)P_F^I}{(c-d\phi)P_D^I-cP_F^I}$$

$$E = \frac{\begin{bmatrix}\varepsilon(\zeta P_F^F(d(c-\omega\phi)+(c-\omega)(c-d\phi)P_D^I+c(-c+\omega+d(-1+\phi))P_F^I) \\ +c(-1+\zeta)P_D^F(-d+(-c+d\phi)P_D^I+(c+d-d\phi)P_F^I))\end{bmatrix}}{d\phi+(c-d\phi)P_D^I-cP_F^I}$$

$$F = -\omega+\omega\varepsilon+\frac{\begin{bmatrix}(c-d\phi)P_D^I(c+\omega(-1+\varepsilon-\varepsilon\zeta)+(\omega-c)\varepsilon\zeta P_F^F)+d(c+\omega(-1+\varepsilon-\varepsilon\zeta)\phi+\varepsilon\zeta(-c+\omega\phi)P_F^F \\ +c(-c+\omega(1+\varepsilon(-1+\zeta))+d(-1+\phi)+\varepsilon\zeta(c+d-\omega-d\phi)P_F^F)P_F^I \\ +c\varepsilon(-1+\zeta)P_D^F(d+(c-d\phi)P_D^I-(c+d-d\phi)P_F^I)\end{bmatrix}}{-d\phi+(d\phi-c)P_D^I+cP_F^I}$$

Proposition 4B. (Similar to Proposition 4) (1) *When* $\frac{\mu}{\beta} \le P_D^I$

- *If*

$$\frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I}<\Lambda<Min\left\{\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}, Max\left\{\frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I}, \frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}\right\}\right\}$$

, then IDS and firewall substitute each other.

- *If*

$$Min\left\{\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}, Max\left\{\frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I}, \frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}\right\}\right\}<\Lambda<\frac{(1-P_F^F)\left[(d\phi-c)P_D^I+cP_F^I\right]}{\left[1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)\right]d\phi P_F^I}$$

, then IDS and firewall complement each other.

- *Otherwise, IDS and firewall conflict with each other.*

(2) When $\frac{\mu}{\beta} > P_D^I$**:** IDS and firewall conflict with each other.

Proof of Proposition 4B.

From Table B4, we know that when $(\mu/\beta) > P_D^I$, IDS and firewall are conflicting. Otherwise (when $(\mu/\beta) \le P_D^I$) we should investigate each region to determine the interaction effect.

Region 1: ($\Lambda < 1$)

In this region, firm (i) allows all external users in no technology architecture, and (ii) allows all external users in IDS only architecture. We know that Value of (IDS+F) can be less than Value of IDS (see the comparison column in table B4). If this is the case, controls are also conflicting. Otherwise controls can complement or substitute each other. We can write the condition for {Value of (IDS+F)> Value of IDS}from Table B4 as $\Lambda > \frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$.

Depending on the value of $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$, there are two scenarios.

Scenario 1

$$\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I} > 1$$

Scenario 2

$$\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I} < 1$$

We can show that in region 1, {Value of (IDS+F) - Value of IDS – Value of F}<0. We also know that Value of F > 0 if $\Lambda > \frac{P_F^F}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)}$.

In scenario 2, Value of F>0 and Value of (F+IDS)>Value of (IDS) when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I} < \Lambda < 1$, and Value of (F+IDS)< Value of (IDS) when

$\Lambda < \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$. Hence, controls substitute each other when $\frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I} < \Lambda < 1$ since {Value of (IDS+F) - Value of IDS – Value of (F)}<0, and controls conflict with each other when $\Lambda < \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$.

In scenario 1, Value of (F+IDS)<Value of IDS when $\Lambda < 1$, and therefore, controls conflict with each other.

Region 2: ($1 < \Lambda < \left(\frac{(d\phi - c)P_D^I + cP_F^I}{d\phi P_F^I}\right)$)

In this region, firm (i) does not allow any external user in no technology case, and (ii) allows all external users in IDS only case. From Table B4, the condition for {Value of (IDS+F)> Value of IDS} is $\Lambda > \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$.

First, assume that $1 < \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$ (i.e., $\frac{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)}{P_F^F} < \frac{(d\phi - c)P_D^I + cP_F^I}{d\phi P_F^I}$)

When $\frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I} < \Lambda < \frac{(d\phi - c)P_D^I + cP_F^I}{d\phi P_F^I}$, Value of (IDS+F)> Value of IDS. So we should evaluate {Value of (IDS+F) - Value of IDS – Value of F}) to find the interaction effect. When $1 < \Lambda < \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$, Value of (IDS+F)< Value of IDS. So IDS and firewall conflict with each other.

We also know that Value of F > 0 if $\Lambda < \frac{1 - P_F^F}{1 - \varepsilon P_D^F + \varepsilon\zeta(P_D^F - P_F^F)}$. Depending on the value of $\frac{1 - P_F^F}{1 - \varepsilon P_D^F + \varepsilon\zeta(P_D^F - P_F^F)}$, there are three scenarios in region 2.

<u>Scenario 1:</u>

$$1 < \frac{1 - P_F^F}{1 - \varepsilon P_D^F + \varepsilon\zeta(P_D^F - P_F^F)} < \frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I}$$

<u>Scenario 2:</u>

$$\frac{P_F^F((d\phi - c)P_D^I + cP_F^I)}{\varepsilon((1-\zeta)P_D^F + \zeta P_F^F)d\phi P_F^I} < \frac{1 - P_F^F}{1 - \varepsilon P_D^F + \varepsilon\zeta(P_D^F - P_F^F)} < \frac{(d\phi - c)P_D^I + cP_F^I}{d\phi P_F^I}$$

<u>Scenario 3:</u>

$$\frac{(d\phi - c)P_D^I + cP_F^I}{d\phi P_F^I} < \frac{1 - P_F^F}{1 - \varepsilon P_D^F + \varepsilon\zeta(P_D^F - P_F^F)}$$

{Value of (IDS+F) - Value of IDS – Value of F} in region 2 is always negative. Therefore, we can conclude that firewall and IDS can only substitute each other when all values are positive.

In scenario 1, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$. Therefore controls complement each other when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ since {Value of (IDS+F) - Value of IDS – max(0,Value of F)}>0. Controls conflict with each other when $1<\Lambda<\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$.

In scenario 2, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$, and Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, and Value of F >0 and Value of (F+IDS)< Value of (IDS) when $1<\Lambda<\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$. Therefore, controls substitute each other when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}<0. Controls complement each other when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ since (Value of (IDS+F) - Value of IDS – max{0,Value of F}) >0. Finally controls conflict with each other when $1<\Lambda<\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$.

In scenario 3, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, and Value of F >0 and Value of (F+IDS)< Value of (IDS) when $1<\Lambda<\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$. Therefore controls substitute each other when $\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}<\Lambda<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$. Controls conflict with each other when $1<\Lambda<\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$.

Next, assume that $1>\frac{P_F^F((d\phi-c)P_D^I+cP_F^I)}{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)d\phi P_F^I}$ (i.e., $\frac{\varepsilon((1-\zeta)P_D^F+\zeta P_F^F)}{P_F^F}>\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$)

Again depending on the value of $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$, there are two additional scenarios in region 2.

Scenario 4:

$$\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$$

Scenario 5:

$$\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I} < \frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$$

In scenario 4, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \Lambda < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, and Value of F >0 and Value of (F+IDS)> Value of (IDS) when $1<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$. Hence controls complement each other when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \Lambda < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ since {Value of (IDS+F) - Value of IDS – max(0,Value of F)}>0. Controls substitute each other when $1<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}<0**.**

In scenario 5, Value of F >0 and Value of (F+IDS)> Value of (IDS). Therefore controls always substitute each other since {Value of (IDS+F) - Value of IDS – Value of F)}<0**.**

Region 3: ($\Lambda > \left(\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}\right)$)

In this region, the firm (i) does not allow any external user in no technology case, and (ii) does not allow any external user in IDS only case. From Table B4, the condition for {Value of (IDS+F)> Value of IDS} is $\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$.

Since $\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}$>1, we can say that when $\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, Value of (IDS+F)> Value of IDS. So we should evaluate the expression {Value of (IDS+F) - Value of IDS – Value of F}}) to find the interaction effect. When $\Lambda>\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, Value of (IDS+F)< Value of IDS. So IDS and firewall conflict with each other. We also know that Value of $F>0$ if $\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$. Depending on the value of $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$, there are two scenarios in region 3.

Scenario 1:

$$\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$$

Scenario 2:

$$\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$$

We can show that {Value of (IDS+F) - Value of IDS – Value of F} in region 3 is positive. Therefore firewall and IDS can only complement each other when all costs are positive. In scenario 1, Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$, and Value of F <0 and Value of (F+IDS)< Value of (IDS) when $\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda$.

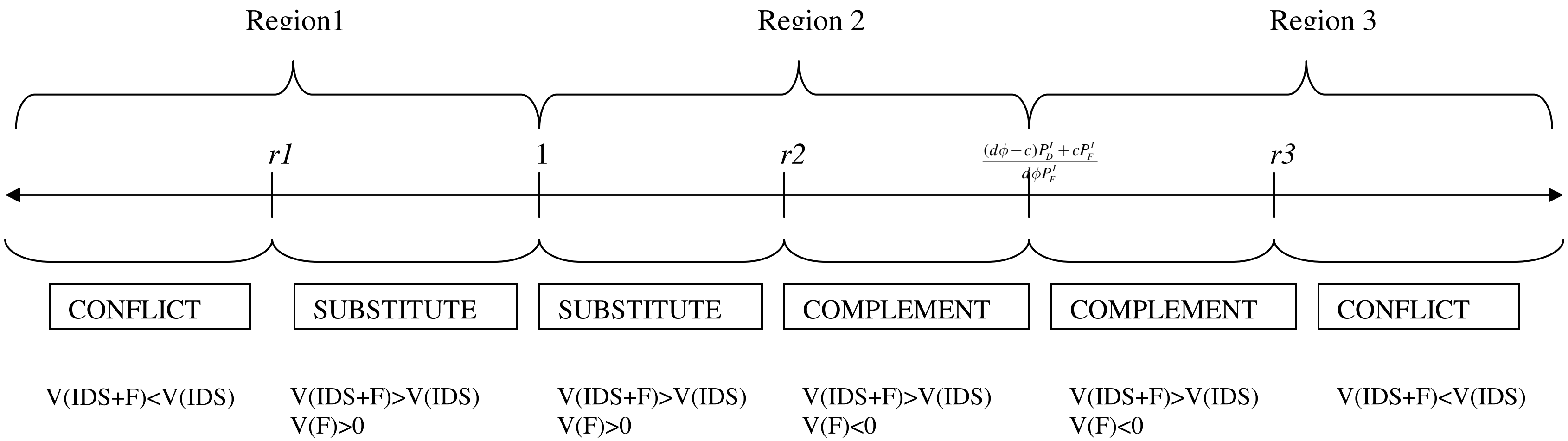
Therefore, controls complement each other when $\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ since {Value of (IDS+F) - Value of IDS – Max{0,Value of F}}>0. Controls conflict with each other when $\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda$.

In scenario 2, Value of F >0 and Value of (F+IDS)> Value of (IDS) when $\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$, and Value of F <0 and Value of (F+IDS)> Value of (IDS) when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$,and Value of F <0 and Value of (F+IDS)< Value of (IDS) when $\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda$.

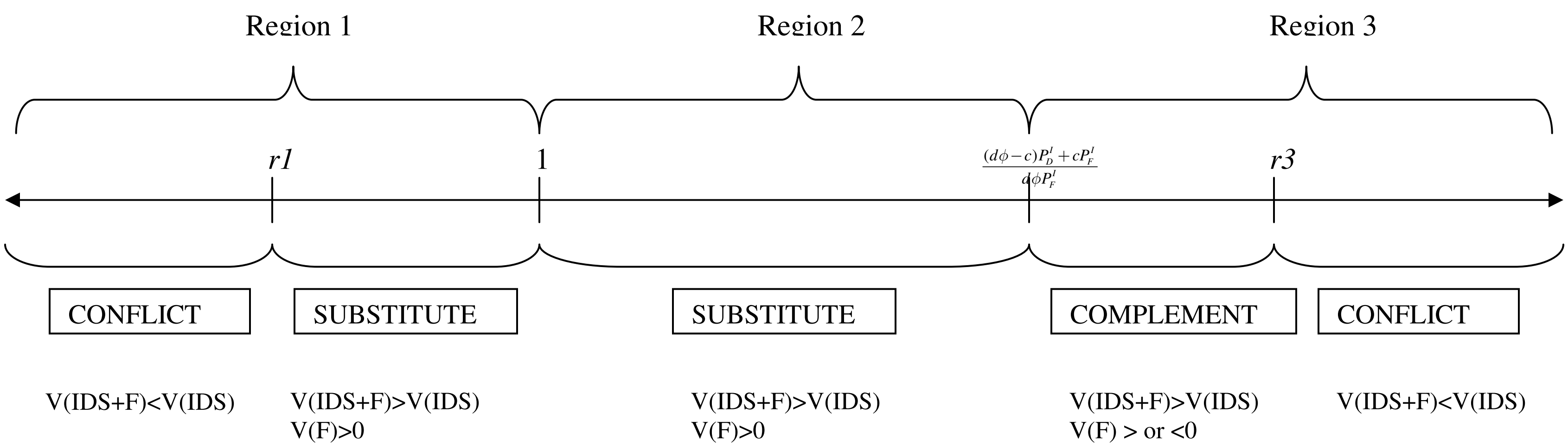
Therefore, controls complement each other when $\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda<\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$ since {Value of (IDS+F) - Value of IDS – Value of F)}>0. Controls again complement each other when $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}<\Lambda<\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ since {Value of (IDS+F) - Value of IDS – Max{0,Value of F}}>0. Finally controls conflict with each other when $\frac{(1-P_F^F)}{(1-\varepsilon((1-\zeta)P_D^F+\zeta P_F^F))}\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}<\Lambda$.

We can summarize the interaction effect after considering all three regions graphically as follows, where Λ is shown on the x-axis.

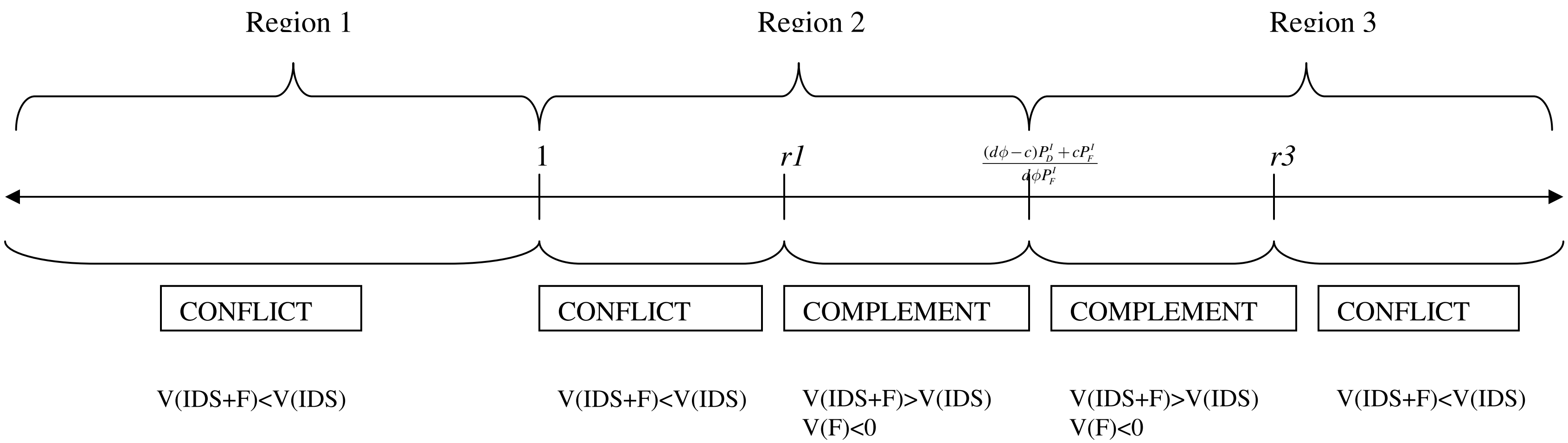
(1) When $\frac{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]}{P_F^F} > \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ and $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$



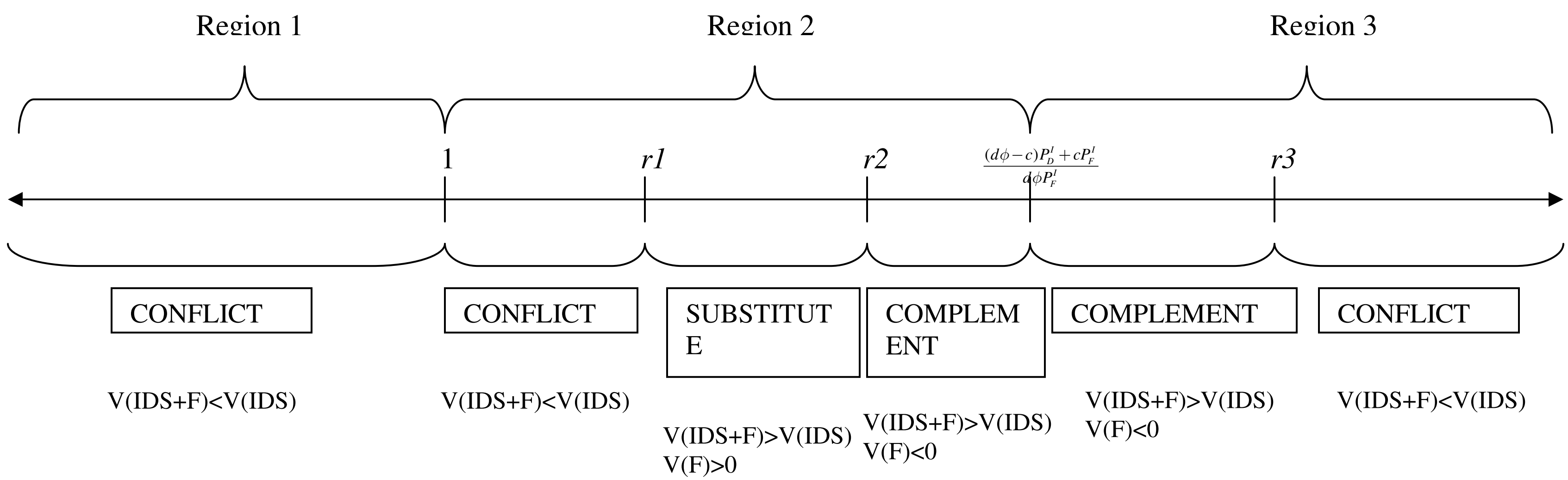
(2) When $\frac{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]}{P_F^F} > \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ and $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} > \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$



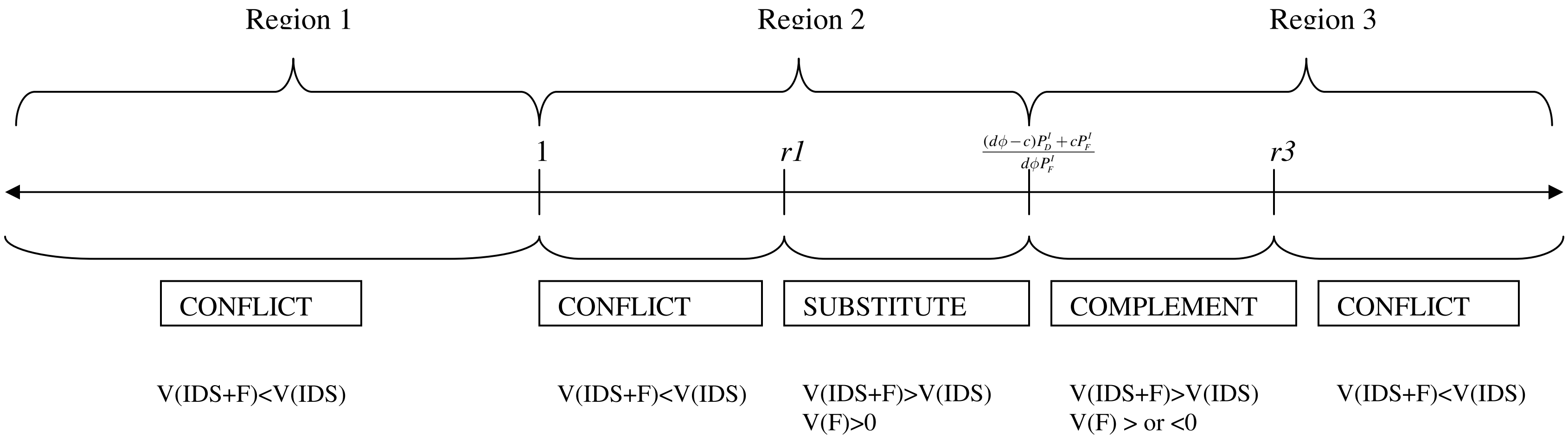
(3) When $\frac{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]}{P_F^F} < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ and $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I}$



(4) When $\frac{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]}{P_F^F} < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ and $\frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I} < \frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)} < \frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$



(5) When $\frac{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]}{P_F^F}<\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$ and $\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}>\frac{(d\phi-c)P_D^I+cP_F^I}{d\phi P_F^I}$



where $r1=\frac{P_F^F\left[(d\phi-c)P_D^I+cP_F^I\right]}{\varepsilon\left[(1-\zeta)P_D^F+\zeta P_F^F\right]d\phi P_F^I}$, $r2=\frac{1-P_F^F}{1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)}$, $r3=\frac{(1-P_F^F)\left[(d\phi-c)P_D^I+cP_F^I\right]}{\left[1-\varepsilon P_D^F+\varepsilon\zeta(P_D^F-P_F^F)\right]d\phi P_F^I}$.

Although there are five possibilities depending on the value of firewall and IDS quality parameters, in all of these five possibilities, the transitions of interaction effects is the same. That is, the order of interaction effect from left to right is

CONFLICT->SUBSTITUTE->COMPLEMENT->CONFLICT

After carefully looking at threshold values that separate interaction effects in five possibilities above, we can write the interaction results as in Proposition 4B.

ONLINE SUPPLEMENT C:
DERIVATION OF THE RESULTS FOR THE ALTERNATIVE MODEL 2

Model Description

We develop a game theoretical model, similar to the one in the paper. We assume that ε fraction of traffic to the system being protected comes from external users, and $(1-\varepsilon)$ fraction originates from internal users. ζ fraction of external traffic comes from unauthorized users. These are the users that should be stopped at the firewall layer. We model the effectiveness of a firewall through two parameters: P_D^F and P_F^F. P_D^F is the probability that the firewall stops an unauthorized external user. P_F^F is the probability that an authorized external user is stopped at the firewall. The firm incurs a cost of σ if an authorized external user is stopped at the firewall.

All internal users are assumed to be authorized users of the system. However they as well as authorized external users can misuse the system by improperly accessing data or programs they are not authorized to use. A fraction λ of the authorized users is assumed to be dishonest. IDS is aimed at detecting unauthorized use by internal as well as external users. We model the effectiveness of an IDS through two parameters P_D^I and P_F^I. The parameter P_D^I denotes the probability that the IDS gives a signal whenever an intrusion occurs. Sometimes the IDS gives false signals. P_F^I is the probability of generating a false signal.

It is assumed that the firm incurs a cost of c each time it monitors the audit trail of a user for a possible intrusion. We assume that manual monitoring detects intrusions with certainty. If monitoring detects the intrusion, the firm recovers all of the damage incurred.[1] We assume that when a hacker breaks into the system he gets a utility of μ. If the intrusion is discovered and the hacker is identified, the hacker incurs a penalty of β. External hackers are hard to be identified and punished compared to internal hackers. $\Delta \leq 1$ captures the probability that an external hacker is punished after an intrusion originated from him(her) is detected. We assume that internal hackers are punished with probability one if their intrusions are detected.

Below we present the main results of the analysis along with their proofs.

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[1] We analyzed a model with an additional parameter ϕ that models the fraction of the damage recovered when an intrusion is detected. The results are qualitatively identical to those discussed here.

Proposition 1C. (Similar to Proposition 1) *The equilibria for the stage 2 of the game when the firm implements a firewall and an IDS are given in the following.*

<table>
<tr><td>Conditions</td><td>$\frac{c}{d}<k_1$</td><td>$k_1<\frac{c}{d}<a_1$</td><td>$a_1<\frac{c}{d}<k_2$</td><td>$k_2<\frac{c}{d}<a_2$</td><td>$\frac{c}{d}>a_2$</td></tr>
<tr><td>$\frac{\mu}{\beta}>1$</td><td colspan="3">$\rho_1=1,\rho_2=1$
$\psi_1=1,\psi_2=1$</td><td rowspan="2">$\rho_1=1,\rho_2=0$
$\psi_1=1,\psi_2=1$</td><td rowspan="5">$\rho_1=0,\rho_2=0$
$\psi_1=1,\psi_2=1$</td></tr>
<tr><td>$P_D^I<\frac{\mu}{\beta}<1$</td><td rowspan="2">$\rho_1=1,\rho_2=1$
$\psi_1=1,\psi_2=0$</td><td colspan="2">$\rho_1=1,\rho_2=K$
$\psi_1=1,\psi_2=T$</td></tr>
<tr><td>$\Delta<\frac{\mu}{\beta}<P_D^I$</td><td rowspan="2">$\rho_1=1,\rho_2=0$
$\psi_1=1,\psi_2=0$</td><td colspan="2" rowspan="3">$\rho_1=H,\rho_2=0$
$\psi_1=1,\psi_2=S$</td></tr>
<tr><td>$P_D^I\Delta<\frac{\mu}{\beta}<\Delta$</td><td>$\rho_1=1,\rho_2=F$
$\psi_1=R,\psi_2=0$</td></tr>
<tr><td>$\frac{\mu}{\beta}<P_D^I\Delta$</td><td colspan="2">$\rho_1=D,\rho_2=0$
$\psi_1=P,\psi_2=0$</td></tr>
</table>

where, $k_1=\dfrac{(1-P_D^I)(1-P_D^F)\varepsilon\zeta}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta+(1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$

$$k_2=\frac{(1-P_D^I)(\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta))}{(1-P_D^I)(\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta))+(1-P_F^I)(1-\lambda)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$$

$$a_1=\frac{P_D^I(1-P_D^F)\varepsilon\zeta}{P_D^I(1-P_D^F)\varepsilon\zeta+P_F^I(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$$

$$a_2=\frac{P_D^I(\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta))}{P_D^I(\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta))+P_F^I(1-\lambda)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$$

$$P=\frac{cP_F^I(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{P_D^I(1-P_D^F)\varepsilon\zeta(d-c)},\ R=\frac{c(1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta(d-c)},$$

$$S=\frac{c(P_F^I(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+P_D^I(1-P_D^F)\varepsilon\zeta)-d(1-P_D^F)P_D^I\varepsilon\zeta}{(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda(dP_D^I-c(P_D^I-P_F^I))},$$

$$T=\frac{c(1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)-(1-P_D^I)(1-P_D^F)(d-c)\varepsilon\zeta}{(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda(d(1-P_D^I)+c(P_D^I-P_F^I))}$$

Proof of Proposition 1C.
The firm's payoff is given by

$$F(\rho_1,\rho_2,\psi_1,\psi_2) = -c\big[(\rho_1 + (1-P_F^I)(\rho_2-\rho_1))(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^I)\varepsilon\zeta)+(1-P_D^F)(\rho_2 - P_D^I(\rho_2-\rho_1))\varepsilon\zeta\psi_1$$
$$+(P_D^I - P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)(\rho_2-\rho_1)\lambda\psi_2\big] - P_F^F(1-\zeta)\varepsilon\sigma$$
$$-d\big[1-\rho_2+P_D^I(\rho_2-\rho_1)\big]\ \big[(1-P_D^F)\varepsilon\zeta\,\psi_1+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\psi_2\big]$$

The payoff functions for the external and the internal hackers are respectively

$$H_E(\rho_1,\rho_2,\psi_1) = (1-P_D^F)\mu\psi_1 - (1-P_D^F)\beta\Delta(P_D^I\rho_1+(1-P_D^I)\rho_2)\psi_1$$
$$H_I(\rho_1,\rho_2,\psi_2) = \mu\psi_2 - \beta(P_D^I\rho_1 + (1-P_D^I)\rho_2)\psi_2$$

There are 16 possible pure strategy equilibriums. These are (monitor, monitor, hack, hack), (monitor, monitor, hack, no hack), **(monitor, monitor, no hack, hack**), **(monitor, monitor, no hack, no hack)**, (monitor, no monitor, hack, hack), **(monitor, no monitor, no hack, hack)**, (monitor, no monitor, hack, no hack), **(monitor, no monitor, no hack, no hack)**, **(no monitor, monitor, hack, hack), (no monitor, monitor, no hack, hack)**, **(no monitor, monitor, hack, no hack), (no monitor, monitor, no hack, no hack)**, (no monitor, no monitor, hack, hack), **(no monitor, no monitor, no hack, hack)**, **(no monitor, no monitor, hack, no hack)**, **(no monitor, no monitor, no hack, no hack)**.

Among those equilibriums, the ones highlighted in bold are not possible because of the following reasons. First, (monitor, monitor, no hack, hack), (monitor, no monitor, no hack, hack), (no monitor, monitor, no hack, hack), (no monitor, no monitor, no hack, hack) cannot be an equilibrium since ψ_1 always greater than or equal to ψ_2 since $\frac{\partial H_E}{\partial \psi_1} \geq \frac{\partial H_I}{\partial \psi_2}$. Second, (no monitor, monitor, hack, hack), (no monitor, monitor, hack, no hack) are not possible because monitoring probability when there is a signal is always greater than or equal to monitoring probability when there is no signal because $\frac{\partial F}{\partial \rho_1} \geq \frac{\partial F}{\partial \rho_2}$. Third, there is no pure strategy equilibrium in (monitor, monitor, no hack, no hack), (monitor, no monitor, no hack, no hack), (no monitor, monitor, no hack, no hack) since if both types of hackers do not hack, then the firm's dominant strategy is no monitor both in signal and no signal states. Fourth, (no monitor, no monitor, hack, no hack) and (no monitor, no monitor, no hack, no hack) are dominated by (no monitor, no monitor, hack, hack).
First order conditions for the firm are

$$\frac{\partial F}{\partial \rho_1} = -c(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)P_F^I(1-\lambda\psi_2) + (d-c)P_D^I(\lambda\psi_2(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+\varepsilon\zeta\psi_1(1-P_D^F)) \tag{C.1}$$

$$\frac{\partial F}{\partial \rho_2} = -c(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)(1-P_F^I)(1-\lambda\psi_2) + (d-c)(1-P_D^I)(\lambda\psi_2(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+\varepsilon\zeta\psi_1(1-P_D^F)) \tag{C.2}$$

First order conditions for the external and internal hacker are respectively

$$\frac{\partial H_E}{\partial \psi_1} = (1 - P_D^I)(\mu - \beta\Delta(P_D^I(\rho_1 - \rho_2) + \rho_2)) \tag{C.3}$$

$$\frac{\partial H_I}{\partial \psi_2} = \mu - \beta(P_D^I(\rho_1 - \rho_2) + \rho_2) \tag{C.4}$$

(monitor, monitor, hack, hack) (i.e. ρ_1=1, ρ_2=1, ψ_1=1, ψ_2=1) is an equilibrium iff $\frac{\partial F}{\partial \rho_i}\Big|_{\psi_1 \to 1, \psi_2 \to 1} > 0$ for i=1,2 , $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho_1 \to 1, \rho_2 \to 1} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho_1 \to 1, \rho_2 \to 1} > 0$. These conditions are satisfied when (i) $\frac{c}{d} < a_2$ (ii) $\frac{c}{d} < k_2$ (iii) $\frac{\mu}{\beta} > \Delta$ (iv) $\frac{\mu}{\beta} > 1$. Since k_2 is less than a_2 and Δ is less than 1, (ii) and (iv) are sufficient conditions.

(monitor, monitor, hack, no hack) (i.e. ρ_1=1, ρ_2=1, ψ_1=1, ψ_2=0) is an equilibrium iff $\frac{\partial F}{\partial \rho_i}\Big|_{\psi_1 \to 1, \psi_2 \to 0} > 0$ for i=1,2 , $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho_1 \to 1, \rho_2 \to 1} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho_1 \to 1, \rho_2 \to 1} < 0$. These conditions are satisfied when (i) $\frac{c}{d} < a_1$ (ii) $\frac{c}{d} < k_1$ (iii) $\frac{\mu}{\beta} > \Delta$ (iv) $\frac{\mu}{\beta} < 1$. Since k_1 is less than a_1, (ii) (iii) and (iv) are sufficient conditions.

(monitor, no monitor, hack, hack) (i.e. ρ_1=1, ρ_2=0, ψ_1=1, ψ_2=1) is an equilibrium iff $\frac{\partial F}{\partial \rho_1}\Big|_{\psi_1 \to 1, \psi_2 \to 1} > 0$, $\frac{\partial F}{\partial \rho_2}\Big|_{\psi_1 \to 1, \psi_2 \to 1} < 0$, $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho_1 \to 1, \rho_2 \to 0} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho_1 \to 1, \rho_2 \to 0} > 0$. These conditions are satisfied when (i) $\frac{c}{d} < a_2$ (ii) $\frac{c}{d} > k_2$ (iii) $\frac{\mu}{\beta} > P_D^I\Delta$ (iv) $\frac{\mu}{\beta} > P_D^I$. Since $P_D^I\Delta$ is less than Δ, (i) (ii) and (iv) are sufficient conditions.

(monitor, no monitor, hack, no hack) (i.e. ρ_1=1, ρ_2=0, ψ_1=1, ψ_2=0) is an equilibrium iff $\frac{\partial F}{\partial \rho_1}\Big|_{\psi_1 \to 1, \psi_2 \to 0} > 0$, $\frac{\partial F}{\partial \rho_2}\Big|_{\psi_1 \to 1, \psi_2 \to 0} < 0$, $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho_1 \to 1, \rho_2 \to 0} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho_1 \to 1, \rho_2 \to 0} < 0$. These conditions are satisfied when (i) $\frac{c}{d} < a_1$ (ii) $\frac{c}{d} > k_1$ (iii) $\frac{\mu}{\beta} > P_D^I\Delta$ (iv) $\frac{\mu}{\beta} < P_D^I$. Therefore (i) (ii) (iii) and (iv) are all sufficient conditions.

(no monitor, no monitor, hack, hack) (i.e. ρ_1=0, ρ_2=0, ψ_1=1, ψ_2=1) is an equilibrium iff $\frac{\partial F}{\partial \rho_i}\Big|_{\psi_1 \to 1, \psi_2 \to 1} < 0$ for i=1,2 , $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho_1 \to 0, \rho_2 \to 0} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho_1 \to 0, \rho_2 \to 0} > 0$. These conditions are satisfied when (i) $\frac{c}{d} > a_2$ (ii) $\frac{c}{d} > k_2$. Therefore, (i) is the sufficient condition.

Since ρ_1 and ρ_2 or ψ_1 and ψ_2 cannot be in mixed strategies together in any mixed strategy equilibrium and $\rho_1 \geq \rho_2$, $\psi_1 \geq \psi_2$, the mixed strategy equilibriums can be of following forms: (ρ_1=mixed, $\rho_2 = 0$, ψ_1=mixed, $\psi_2 = 0$), (ρ_1=1, ρ_2 = mixed, ψ_1=mixed, $\psi_2 = 0$), (ρ_1=1, ρ_2 = mixed, ψ_1=1, ψ_2 = mixed), and (ρ_1=mixed, $\rho_2 = 0$, ψ_1=1, ψ_2 = mixed).

If (ρ_1=mixed, $\rho_2 = 0$, ψ_1=mixed, $\psi_2 = 0$) is an equilibrium, first order condition for the firm with respect to ρ_1 and first order condition for the external hacker must be satisfied at zero. Equating C.1 to zero gives the relationship between ψ_1 and ψ_2. Plugging the equilibrium value of $\psi_2 = 0$ into that expression and solving for ψ_1 gives

$$\psi_1 = \frac{cP_F^I(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{P_D^I(1-P_D^F)\varepsilon\zeta(d-c)} \tag{C.5}$$

Equating C.3 to zero gives the relationship between ρ_1 and ρ_2. Plugging the equilibrium value of $\rho_2 = 0$ into that expression and solving for ρ_1 gives

$$\rho_1 = \frac{\mu}{P_D^I\Delta\beta} \tag{C.6}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.5) and (C.6) must be between zero and one. Second equilibrium values must make derivative of the payoff function for the firm with respect to ρ_2 and derivative of payoff function for internal hacker given in C.2 and C.4 negative (since ρ_2 =0 and ψ_2 =0). The necessary conditions yield the following sufficient conditions.

$$\frac{\mu}{\beta} < P_D^I\Delta \text{ and } \frac{c}{d} < a_1$$

If (ρ_1=1, ρ_2 = mixed, ψ_1=mixed, $\psi_2 = 0$) is an equilibrium, first order condition for the firm with respect to ρ_2 and first order condition for the external hacker must be satisfied at zero. Equating C.2 to zero gives the relationship between ψ_1 and ψ_2. Plugging the equilibrium value of $\psi_2 = 0$ into that expression and solving for ψ_1 gives

$$\psi_1 = \frac{c(1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta(d-c)} \tag{C.7}$$

Equating C.3 to zero gives the relationship between ρ_1 and ρ_2. Plugging the equilibrium value of $\rho_1 = 1$ into that expression and solving for ρ_2 gives

$$\rho_2 = \frac{\mu - P_D^I\Delta\beta}{(1-P_D^I)\Delta\beta} \tag{C.8}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.7) and (C.8) must be between zero and one. Second equilibrium values must make derivative of the payoff function for the firm with respect to ρ_1 given in C.1 positive and derivative of payoff function for internal hacker given in C.4 negative (since ρ_1 =1 and ψ_2 =0). The necessary conditions yield the following sufficient conditions.

$P_D^I \Delta < \frac{\mu}{\beta} < \Delta$ and $\frac{c}{d} < k_1$

If (ρ_1=1, ρ_2 = mixed, ψ_1=1, ψ_2 = mixed) is an equilibrium, first order condition for the firm with respect to ρ_2 and first order condition for the internal hacker must be satisfied at zero. Equating C.2 to zero gives the relationship between ψ_1 and ψ_2. Plugging the equilibrium value of ψ_1 = 1 into that expression and solving for ψ_2 gives

$$\psi_2 = \frac{c(1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)-(1-P_D^I)(1-P_D^F)(d-c)\varepsilon\zeta}{(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda(d(1-P_D^I)+c(P_D^I-P_F^I))} \quad \text{(C.9)}$$

Equating C.4 to zero gives the relationship between ρ_1 and ρ_2. Plugging the equilibrium value of ρ_1 = 1 into that expression and solving for ρ_2 gives

$$\rho_2 = \frac{\mu - P_D^I \beta}{(1-P_D^I)\beta} \quad \text{(C.10)}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.9) and (C.10) must be between zero and one. Second equilibrium values must make derivative of the payoff function for the firm with respect to ρ_1 and derivative of payoff function for external hacker given in C.1 and C.3 positive (since ρ_1=1 and ψ_1=1). The necessary conditions yield the following sufficient conditions.

$P_D^I < \frac{\mu}{\beta} < 1$ and $k_1 < \frac{c}{d} < k_2$

If (ρ_1=mixed, ρ_2 = 0, ψ_1= 1, ψ_2 = mixed) is an equilibrium, first order condition for the firm with respect to ρ_1 and first order condition for the internal hacker must be satisfied at zero. Equating C.1 to zero gives the relationship between ψ_1 and ψ_2. Plugging the equilibrium value of ψ_1 = 1 into that expression and solving for ψ_2 gives

$$\psi_2 = \frac{c(P_F^I(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+P_D^I(1-P_D^F)\varepsilon\zeta)-d(1-P_D^F)P_D^I\varepsilon\zeta}{(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda(dP_D^I-c(P_D^I-P_F^I))} \quad \text{(C.11)}$$

Equating C.4 to zero gives the relationship between ρ_1 and ρ_2. Plugging the equilibrium value of ρ_2 = 0 into that expression and solving for ρ_1 gives

$$\rho_1 = \frac{\mu}{P_D^I \beta} \quad \text{(C.12)}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.11) and (C.12) must be between zero and one. Second equilibrium values must make derivative of the payoff function for the firm with respect to ρ_2 given in C.2 negative and derivative of payoff function for external hacker given in C.3 positive (since ρ_2 =0 and ψ_1=1). The necessary conditions yield the following sufficient conditions.

$\frac{\mu}{\beta} < P_D^I$ and $a_1 < \frac{c}{d} < a_2$

Corollary 1aC. (Similar to Corollary 1a) *The equilibria for the stage 2 of the game when the firm implements only an IDS are given in the following.*

<table>
<tr><td>Conditions</td><td>$\frac{c}{d}<k_1^*$</td><td>$k_1^*<\frac{c}{d}<a_1^*$</td><td>$a_1^*<\frac{c}{d}<k_2^*$</td><td>$k_2^*<\frac{c}{d}<a_2^*$</td><td>$\frac{c}{d}>a_2^*$</td></tr>
<tr><td>$\frac{\mu}{\beta}>1$</td><td colspan="3">$\rho_1=1,\rho_2=1$
$\psi_1=1,\psi_2=1$</td><td rowspan="2">$\rho_1=1,\rho_2=0$
$\psi_1=1,\psi_2=1$</td><td rowspan="5">$\rho_1=0,\rho_2=0$
$\psi_1=1,\psi_2=1$</td></tr>
<tr><td>$P_D^I<\frac{\mu}{\beta}<1$</td><td rowspan="2">$\rho_1=1,\rho_2=1$
$\psi_1=1,\psi_2=0$</td><td colspan="2">$\rho_1=1,\rho_2=K$
$\psi_1=1,\psi_2=L$</td></tr>
<tr><td>$\Delta<\frac{\mu}{\beta}<P_D^I$</td><td rowspan="2">$\rho_1=1,\rho_2=0$
$\psi_1=1,\psi_2=0$</td><td colspan="2" rowspan="3">$\rho_1=H,\rho_2=0$
$\psi_1=1,\psi_2=J$</td></tr>
<tr><td>$P_D^I\Delta<\frac{\mu}{\beta}<\Delta$</td><td>$\rho_1=1,\rho_2=F$
$\psi_1=G,\psi_2=0$</td></tr>
<tr><td>$\frac{\mu}{\beta}<P_D^I\Delta$</td><td colspan="2">$\rho_1=D,\rho_2=0$
$\psi_1=E,\psi_2=0$</td></tr>
</table>

where, $k_1^*=\dfrac{(1-P_D^I)\varepsilon\zeta}{(1-P_D^I)\varepsilon\zeta+(1-P_F^I)(1-\varepsilon\zeta)}$, $k_2^*=\dfrac{(1-P_D^I)(\varepsilon\zeta+\lambda(1-\varepsilon\zeta))}{(1-P_D^I)(\varepsilon\zeta+\lambda(1-\varepsilon\zeta))+(1-P_F^I)(1-\lambda)(1-\varepsilon\zeta)}$

$$a_1^*=\frac{P_D^I\varepsilon\zeta}{P_D^I\varepsilon\zeta+P_F^I(1-\varepsilon\zeta)}\ ,\ a_2^*=\frac{P_D^I(\varepsilon\zeta+\lambda(1-\varepsilon\zeta))}{P_D^I(\varepsilon\zeta+\lambda(1-\varepsilon\zeta))+P_F^I(1-\lambda)(1-\varepsilon\zeta)}$$

and $D=\dfrac{\mu}{P_D^I\Delta\beta}$, $E=\dfrac{cP_F^I(1-\varepsilon\zeta)}{P_D^I\varepsilon\zeta(d-c)}$, $F=\dfrac{\mu-P_D^I\Delta\beta}{(1-P_D^I)\Delta\beta}$, $G=\dfrac{c(1-P_F^I)(1-\varepsilon\zeta)}{(1-P_D^I)\varepsilon\zeta(d-c)}$, $H=\dfrac{\mu}{P_D^I\beta}$,

$$J=\frac{c(P_F^I(1-\varepsilon\zeta)+P_D^I\varepsilon\zeta)-dP_D^I\varepsilon\zeta}{(1-\varepsilon\zeta)\lambda(dP_D^I-c(P_D^I-P_F^I))},\ K=\frac{\mu-P_D^I\beta}{(1-P_D^I)\beta},\ L=\frac{c(1-P_F^I)(1-\varepsilon\zeta)-(1-P_D^I)(d-c)}{(1-\varepsilon\zeta)\lambda(d(1-P_D^I)+c(P_D^I-P_F^I))}$$

Proof of Corollary 1aC.
Security architecture with IDS is a special case of security architecture with IDS plus firewall. We obtain the results for IDS only case by substituting P_D^F =0 and P_F^F =0 into the results of firewall plus IDS case. All the equilibriums given in firewall plus IDS case converge to corresponding equilibriums in IDS only case after the substitution. Also separating points on the x-axis, k_1, a_1, k_2, a_2, are replaced by $k_1^*, a_1^*, k_2^*, a_2^*$.

Corollary 1bC. (Similar to Corollary 1b) *The equilibria for the stage 2 of the game when the firm implements only a firewall are given in the following.*

<table>
<tr><td>Conditions</td><td>$\frac{c}{d} < m_1^*$</td><td>$m_1^* < \frac{c}{d} < m_2^*$</td><td>$\frac{c}{d} > m_2^*$</td></tr>
<tr><td>$\frac{\mu}{\beta} > 1$</td><td colspan="2">$\rho = 1, \psi_1 = 1, \psi_2 = 1$</td><td rowspan="3">$\rho = 0, \psi_1 = 1, \psi_2 = 1$</td></tr>
<tr><td>$\Delta < \frac{\mu}{\beta} < 1$</td><td>$\rho = 1, \psi_1 = 1, \psi_2 = 0$</td><td rowspan="2">$\rho = \frac{\mu}{\beta}$
$\psi_1 = 1$
$\psi_2 = \mathrm{A}$</td></tr>
<tr><td>$\frac{\mu}{\beta} < \Delta$</td><td>$\rho = \frac{\mu}{\Delta\beta}$
$\psi_1 = \frac{c(1-\varepsilon(P_F^F(1-\zeta)))}{(1-P_D^F)(d-c)\varepsilon\zeta}$
$\psi_2 = 0$</td></tr>
</table>

where $m_1^* = \frac{\varepsilon\zeta(1-P_D^F)}{\varepsilon\zeta(1-P_D^F)+(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$,

$m_2^* = \frac{\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{\varepsilon\zeta(1-P_D^F)+(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$ and

$\mathrm{A} = \frac{c(1-\varepsilon+(1-P_F^F)\varepsilon-(P_D^F-P_F^F)\varepsilon\zeta)-d(1-P_D^F)\varepsilon\zeta}{d(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda}$

Proof of Corollary 1bC.

Security architecture with firewall is a special case of security architecture with IDS plus firewall. We obtain the payoff functions for the firm and hackers in firewall only case by substituting P_D^I =1, P_F^I =0, $\rho_1 = \rho$, $\rho_2 = \rho$ into those in firewall plus IDS case. This gives

$$F(\rho,\psi_1,\psi_2) = -c\rho[1-\varepsilon(P_F^F(1-\zeta)-(1-P_D^F)\zeta\psi_1)] - d(1-\rho)\{(1-P_D^F)\varepsilon\zeta\psi_1+\lambda[1-\varepsilon(P_F^F(1-\zeta))]\psi_2\} - P_F^F(1-\zeta)\varepsilon\sigma \quad \text{(C.13)}$$

$$H_E(\rho,\psi_1) = (1-P_D^F)\mu\psi_1 - (1-P_D^F)\beta\Delta\rho\psi_1 \quad \text{(C.14)}$$

$$H_I(\rho,\psi_2) = \mu\psi_2 - \beta\rho\psi_2 \quad \text{(C.15)}$$

Possible pure strategy equilibriums are (monitor, hack, hack), (monitor, hack, no hack) and (no monitor, hack, hack).

First order condition for the firm is

$$\frac{\partial F}{\partial \rho} = -c\left[(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+(1-P_D^F)\varepsilon\zeta\,\psi_1\right]+ \qquad \text{(C.16)}$$
$$d\left[\lambda\psi_2(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+(1-P_D^F)\varepsilon\zeta\,\psi_1\right]$$

First order conditions for the external and internal hacker are respectively

$$\frac{\partial H_E}{\partial \psi_1} = (1-P_D^F)(\mu-\beta\Delta\rho) \qquad \text{(C.17)}$$

$$\frac{\partial H_I}{\partial \psi_2} = \mu-\beta\rho \qquad \text{(C.18)}$$

(monitor, hack, hack) (i.e. ρ=1, ψ_1=1, ψ_2=1) is an equilibrium iff $\frac{\partial F}{\partial \rho}\Big|_{\psi_1\to 1,\psi_2\to 1} > 0$, $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho\to 1} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho\to 1} > 0$. These conditions are satisfied when (i) $\frac{c}{d} < \frac{\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{\varepsilon\zeta(1-P_D^F)+(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$ (ii) $\frac{\mu}{\beta} > \Delta$ (iii) $\frac{\mu}{\beta} > 1$. Hence, (i) and (iii) are sufficient conditions.

(monitor, hack, no hack) (i.e. ρ=1, ψ_1=1, ψ_2=0) is an equilibrium iff $\frac{\partial F}{\partial \rho}\Big|_{\psi_1\to 1,\psi_2\to 0} > 0$, $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho\to 1} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho\to 1} < 0$. These conditions are satisfied when (i) $\frac{c}{d} < \frac{\varepsilon\zeta(1-P_D^F)}{\varepsilon\zeta(1-(1-P_F^F))+(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$ (ii) $\frac{\mu}{\beta} > \Delta$ (iii) $\frac{\mu}{\beta} < 1$. Hence, (i), (ii) and (iii) are all sufficient conditions.

(no monitor, hack, hack) (i.e. ρ=0, ψ_1=1, ψ_2=1) is an equilibrium iff $\frac{\partial F}{\partial \rho}\Big|_{\psi_1\to 1,\psi_2\to 1} < 0$, $\frac{\partial H_E}{\partial \psi_1}\Big|_{\rho\to 0} > 0$ and $\frac{\partial H_I}{\partial \psi_2}\Big|_{\rho\to 0} > 0$. These conditions are satisfied when (i) $\frac{c}{d} > \frac{\varepsilon\zeta(1-P_D^F)+\lambda(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{\varepsilon\zeta(1-P_D^F)+(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$. Hence, (i) is the sufficient condition.

Since ψ_1 and ψ_2 cannot be in mixed strategies together in any mixed strategy equilibrium and $\psi_1 \geq \psi_2$, the mixed strategy equilibriums can be of following forms: (ρ=mixed, ψ_1=mixed, ψ_2 = 0) and (ρ = mixed, ψ_1=1, ψ_2 = mixed).
If (ρ=mixed, ψ_1=mixed, ψ_2 = 0) is an equilibrium, first order condition for the firm and first order condition for the external hacker must be satisfied at zero. Equating C.16 to zero gives the

relationship between ψ_1 and ψ_2. Plugging the equilibrium value of $\psi_2 = 0$ into that expression and solving for ψ_1 gives

$$\psi_1 = \frac{c(1-\varepsilon(P_F^F(1-\zeta)))}{(1-P_D^F)(d-c)\varepsilon\zeta} \qquad \text{(C.19)}$$

Equating C.17 to zero and solving for ρ gives

$$\rho = \frac{\mu}{\Delta\beta} \qquad \text{(C.20)}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.19) and (C.20) must be between zero and one. Second equilibrium values must make derivative of payoff function for internal hacker given in C.18 negative (since ψ_2 =0). The necessary conditions yield the following sufficient conditions.

$$\frac{\mu}{\beta} < \Delta \text{ and } \frac{c}{d} < m_1^*$$

If (ρ=mixed, ψ_1=1, ψ_2 = mixed) is an equilibrium, first order condition for the firm and first order condition for the internal hacker must be satisfied at zero. Equating C.16 to zero gives the relationship between ψ_1 and ψ_2. Plugging the equilibrium value of ψ_1= 1 into that expression and solving for ψ_2 gives

$$\psi_2 = \frac{c(1-\varepsilon+(1-P_F^F)\varepsilon-(P_D^F-P_F^F)\varepsilon\zeta)-d(1-P_D^F)\varepsilon\zeta}{d(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)\lambda} \qquad \text{(C.21)}$$

Equating C.17 to zero and solving for ρ gives

$$\rho = \frac{\mu}{\beta} \qquad \text{(C.22)}$$

There are two necessary conditions for this equilibrium. First equilibrium values given in (C.21) and (C.22) must be between zero and one. Second equilibrium values must make derivative of payoff function for the external hacker given in C.17 positive (since ψ_1=1). The necessary conditions yield the following sufficient conditions.

$$\Delta < \frac{\mu}{\beta} < 1 \text{ and } m_1^* < \frac{c}{d} < m_2^*$$

Corollary 1cC. (Similar to Corollary 1b) *The equilibria for the stage 2 of the game when the firm implements no technology are given in the following.*

<table>
<tr><td>Conditions</td><td>$\frac{c}{d} < m_1 = \varepsilon\zeta$</td><td>$m_1 < \frac{c}{d} < m_2 = \varepsilon\zeta + \lambda(1-\varepsilon\zeta)$</td><td>$\frac{c}{d} > m_2$</td></tr>
<tr><td>$\frac{\mu}{\beta} > 1$</td><td colspan="2">$\rho = 1, \psi_1 = 1, \psi_2 = 1$</td><td rowspan="3">$\rho = 0, \psi_1 = 1, \psi_2 = 1$</td></tr>
<tr><td>$\Delta < \frac{\mu}{\beta} < 1$</td><td>$\rho = 1, \psi_1 = 1, \psi_2 = 0$</td><td rowspan="2">$\rho = \frac{\mu}{\beta}$
$\psi_1 = 1$
$\psi_2 = \frac{c - d\varepsilon\zeta}{d(1-\varepsilon\zeta)\lambda}$</td></tr>
<tr><td>$\frac{\mu}{\beta} < \Delta$</td><td>$\rho = \frac{\mu}{\Delta\beta}$
$\psi_1 = \frac{c(1-\varepsilon\zeta)}{(d-c)\varepsilon\zeta}$,
$\psi_2 = 0$</td></tr>
</table>

Proof of Corollary 1cC. Security architecture with neither a firewall nor an IDS is a special case of security architecture with firewall. We obtain the results for no technology case by substituting P_D^F =0 and P_F^F =0 into the results of firewall only case. All the equilibriums given in firewall only case converge to corresponding equilibriums in no technology case after the substitutions. Also separating points on the x-axis, m_1^*, m_2^*, are replaced by m_1, m_2.

Proposition 2C. (Similar to Proposition 2) *For the default configuration scenario, given any set of parameters, there exists a σ^* such that if the cost of dropping an authorized external user is less than σ^* then the value of firewall to the firm is positive; otherwise, the value is negative.*

Proof of Proposition 2C.
We calculate the value of firewall to the firm as (expected cost in no technology case) – (expected cost in firewall only case). Since different equilibriums exit in different regions of parameter space, we first overlap equilibriums obtained in both cases. It can be easily shown that $m_1^* < m_1$. As a result we get three distinct regions in parameter space. Table C1 summarizes the loss in each region. Table C2 shows value of firewall.

To prove that there is a threshold value for σ below which the firm realizes a positive value from the firewall, all we need to show is the value expressions given in Table C2 is positive as σ goes to zero and those are decreasing in σ. Since the coefficient in front of σ is negative in each expression, we only need to show value at σ=0 is positive. It is obvious from Table C2 that when σ is zero, the value is positive. The last column of Table C2 shows the threshold value of the cost of dropping authorized external users in different regions of parameter space.

Table C1. Equilibriums under No Technology and Firewall Only Cases

Condition	Overlapping Equilibriums			
	No Technology		Firewall Only	
	Equilibrium	Exp. Loss	Equilibrium	Exp. Loss
$\frac{\mu}{\beta} < \Delta$	ρ, ψ_1 mixed $\psi_2 = 0$	$\frac{cd(1-\varepsilon\zeta)}{d-c}$	ρ, ψ_1 mixed $\psi_2 = 0$	$\frac{cd(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}{d-c} + P_F^F(1-\zeta)\varepsilon\sigma$
$1 > \frac{\mu}{\beta} > \Delta$	$\rho = 1, \psi_1 = 1$ $\psi_2 = 0$	c	$\rho = 1, \psi_1 = 1$ $\psi_2 = 0$	$c((1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+(1-P_D^F)\zeta)+P_F^F(1-\zeta)\varepsilon\sigma$
$\frac{\mu}{\beta} > 1$	$\rho = 1, \psi_1 = 1$ $\psi_2 = 1$	c	$\rho = 1, \psi_1 = 1$ $\psi_2 = 1$	$c((1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)+(1-P_D^F)\zeta)+P_F^F(1-\zeta)\varepsilon\sigma$

Table C2. The Value of Firewall to the Firm and the Threshold Value for the Dropping Cost

Condition	The Value of Firewall	The Threshold Value
$\frac{\mu}{\beta} < \Delta$	$\frac{P_F^F(1-\zeta)\varepsilon(dc-(d-c)\sigma)}{d-c}$	$\frac{dc}{d-c}$
$1 > \frac{\mu}{\beta} > \Delta$	$\varepsilon(c(P_F^F+(P_D^F-P_F^F)\zeta)-P_F^F(1-\zeta)\sigma)$	$\frac{c(P_F^F+(P_D^F-P_F^F)\zeta)}{P_F^F(1-\zeta)}$
$\frac{\mu}{\beta} > 1$	$\varepsilon(c(P_F^F+(P_D^F-P_F^F)\zeta)-P_F^F(1-\zeta)\sigma)$	$\frac{c(P_F^F+(P_D^F-P_F^F)\zeta)}{P_F^F(1-\zeta)}$

Proposition 3C. (Similar to Proposition 3) *For the default configuration scenario, the value of implementing only an IDS is positive iff* $(\mu/\beta) < P_D^I \Delta$.

Proof of Proposition 3C.

First we calculate the value of IDS to the firm as (expected cost in no technology case) – (expected cost in IDS only case). After overlapping equilibriums in these cases, we obtain four regions in parameter space. Table C3 shows these regions.

Table C3. Equilibriums under No Technology and IDS only Cases

Condition	Overlapping Equilibriums			
	No Technology		IDS only	
	Equilibrium	Exp. Loss	Equilibrium	Exp. Loss
$\frac{\mu}{\beta} < P_D^I \Delta$	ρ, ψ_1 mixed $\psi_2 = 0$	$\frac{cd(1-\varepsilon\zeta)}{d-c}$	ρ_1, ψ_1 mixed $\rho_2 = \psi_2 = 0$	$\frac{dcP_F^I(1-\varepsilon\zeta)}{(d-c)P_D^I}$
$\Delta > \frac{\mu}{\beta} > P_D^I \Delta$	ρ, ψ_1 mixed $\psi_2 = 0$	$\frac{cd(1-\varepsilon\zeta)}{d-c}$	ρ_2, ψ_1 mixed $\rho_1 = 1, \psi_2 = 0$	$\frac{c(d(1-P_D^I)+c(P_D^I-P_F^I))(1-\varepsilon\zeta)}{(d-c)(1-P_D^I)}$
$1 > \frac{\mu}{\beta} > \Delta$	$\rho = 1, \psi_1 = 1$ $\psi_2 = 0$	c	$\rho_1 = 1, \rho_2 = 1$ $\psi_1 = 1, \psi_2 = 0$	c
$\frac{\mu}{\beta} > 1$	$\rho = 1, \psi_1 = 1$ $\psi_2 = 1$	c	$\rho_1 = 1, \rho_2 = 1$ $\psi_1 = 1, \psi_2 = 1$	c

Table C.4 shows the value of IDS in each parameter region. When $\frac{\mu}{\beta} < P_D^I \Delta$, the value of IDS is positive. On the other hand, the value is negative when $\Delta > \frac{\mu}{\beta} > P_D^I \Delta$. In other two regions the value of IDS is zero since expected costs in both cases are the same.

Table C4. The Value of IDS to the Firm

Condition	The Value of IDS
$\frac{\mu}{\beta} < P_D^I \Delta$	$\frac{dc(P_D^I - P_F^I)(1-\varepsilon\zeta)}{(d-c)P_D^I}$
$\Delta > \frac{\mu}{\beta} > P_D^I \Delta$	$\frac{-c^2(P_D^I - P_F^I)(1-\varepsilon\zeta)}{(d-c)(1-P_D^I)}$
$1 > \frac{\mu}{\beta} > \Delta$	0
$\frac{\mu}{\beta} > 1$	0

Proposition 4C. (Similar to Proposition 4)

(1) *When* $\Delta < \frac{\mu}{\beta} \le P_D^I$

- *If* $\overline{\Lambda} < \frac{(1-P_D^I)(1-P_D^F)\varepsilon\zeta}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta + (1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$ *then IDS and firewall <u>substitute</u> each other.*
- *If*

$$\frac{(1-P_D^I)(1-P_D^F)\varepsilon\zeta}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta + (1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)} < \overline{\Lambda} < \frac{(1-P_D^F)\varepsilon\zeta}{(1-P_D^F)\varepsilon\zeta + (1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$$

then IDS and firewall <u>complement</u> each other.

(2) *When* $\Delta > \frac{\mu}{\beta} > P_D^I\Delta$ & $\overline{\Lambda} < \frac{(1-P_D^I)(1-P_D^F)\varepsilon\zeta}{(1-P_D^I)(1-P_D^F)\varepsilon\zeta + (1-P_F^I)(1-\varepsilon+(1-P_F^F)\varepsilon-(1-P_F^F)\varepsilon\zeta)}$ *: IDS and firewall <u>conflict</u> with each other.*

where $\overline{\Lambda} = c/d$

Proof of Proposition 4C.

To show the interaction effect, we first need to overlap equilibriums in all four cases in parameter space. Although we assume that $m_1^* < k_1^*$ (since no ordering is possible between them) equilibrium regions do not change even if wee assume $m_1^* > k_1^*$. Then we calculate $V_{F+IDS} - (V_F + Max(0, V_{IDS}))$ to assess the extent of interaction between security technologies. Table C5 presents these values in each region.

From Table C5, it is clear that when $P_D^I\Delta < \frac{\mu}{\beta} < \Delta$ and $\frac{c}{d} < k_1$, firewall and IDS conflict with each other because $V_{F+IDS} - (V_F + Max(0, V_{IDS}))$ is negative and V_{IDS} is negative. When $\Delta < \frac{\mu}{\beta} < P_D^I$ and $k_1 < \frac{c}{d} < m_1^*$, firewall and IDS complement each other since $V_{F+IDS} - (V_F + Max(0, V_{IDS}))$ is positive. When $\Delta < \frac{\mu}{\beta} < P_D^I$ and $\frac{c}{d} < k_1$, firewall and IDS substitute each other since $V_{F+IDS} - (V_F + Max(0, V_{IDS}))$ is zero.

Table C5. Interaction Effect between Firewall and IDS

μ/β	c/d	$V_{F+IDS} - (V_F + Max(0, V_{IDS}))$	*Interaction Effect*
($P_D^I\Delta$, Δ)	$(0, k_1)$	$\frac{-c^2(q_1^I + q_2^I - 1)(1-\varepsilon+q_2^F\varepsilon - q_2^F\varepsilon\zeta)}{(d-c)(1-q_1^I)}$	Conflict
(Δ, P_D^I)	$(0, k_1)$	0	Substitute
(Δ, P_D^I)	(k_1, m_1^*)	$\frac{(d-c)(1-q_1^F)(1-\varepsilon\zeta)(\beta-\mu)}{\beta}$	Complement