Electronic Companion

Managing the Versions of a Software Product under Variable and Endogenous Demand

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Table EC1: Glossary of Notation

Variable/ Parameter	Description
i	Period, $i \in \{1,2\}$
q_i	Initial features when *i=1* and additional features, i.e., upgrades when *i=2*
y	Design effort
x_i	First-period and second-period demand for *i=1,2*, respectively
b_1	Marginal cost of developing additional product features
b_2	Cost of integrating multiple features
a	Marginal cost of including additional design effort
K_i	First- and second-period fixed software development cost for *i=1,2*, respectively
C_i	First- and second-period total software development cost for *i=1,2*, respectively
P_i	Price of the first- and second-period product for *i=1,2*, respectively
P_u	Price to upgrade to the second-period product for first-period customers
θ	Customer marginal utility for product features
c	Customer set-up and learning cost for a new product
c_u	Customer set-up and learning cost for an upgrade
c_0	Customer set-up and learning cost for an upgrade with no design effort
u_i	Net utility of purchasing the first- and second-period product for *i=1,2*, respectively
$u_{upgrade}$	Net utility of purchasing the upgrade to the first-period customers
δ	Discount factor
λ	Word-of-mouth effect
α	Herding parameter representing the influence of word-of-mouth effect on demand spread.
$\overline{x}_2$	Deterministic portion of the second-period demand
j	Level of the second-period demand, *j* = High (H), Medium (M), Low (L)
ζ	Demand spread parameter showing the deviation of second-period from the moderate level

$\Pr^j$	Probability of event *j*
p	Probability of medium demand
β	Marginal benefit of design effort on fixed cost of software development
γ	Marginal benefit of design effort on customer set-up and learning costs
π_1	First-period profit
π_{2NU}^j	Second-period profit without offering an upgrade for event *j*
$\Delta\pi_2^j$	Incremental profit of offering an upgrade for event *j*
Ω	Software provider's upgrade strategy, $\Omega \equiv$ Never-upgrade (N), Upgrade if x_i^H (H), Upgrade if x_i^H or x_i^M (MH), Upgrade if x_i^H or x_i^M or x_i^L (A)
z^Ω	Indicator variable for software provider's strategy, $z^\Omega \in \{0, 1\}$

Additional Notation

We introduce some additional notation using the parameters that are defined in Table EC1. We do this for making the equations compact. Define $T^j = \theta\left[x_1 + \bar{x}_2 + D^j(\lambda\alpha + \zeta) - b_1/\theta\right]$, $R^1 = (1+\delta)\theta x_1 - b_1$, $R^j = \left[2T^j w\right]/4b_2$, $w = \lambda\theta - 2b_2$, $F^j = \left[(T^j)^2/4b_2\right] - (c_0 x_1 + K_2)$, $S = w^2/4b_2$, $F^1 = cx_1 + K_1$, $S^{NU} = \theta\lambda$, $R^{NU} = \theta\bar{x}_2 - c\lambda$, $F^{NU} = c\bar{x}_2$, $G = c_0\gamma x_1 + K_2\beta$, $X_1 = b_2 - \delta S^{NU} - \delta \Pr^H S - \delta \Pr^M S$
$Y_1 = R^1 + \delta R^{NU} + \delta \Pr^H R^H + \delta \Pr^M R^M$, $X_2 = b_2 - \delta S^{NU} - \delta \Pr^H S$, $Y_2 = R^1 + \delta R^{NU} + \delta \Pr^H R^H$
$X_3 = b_2 - \delta S^{NU}$, $Y_3 = R^1 + \delta R^{NU}$

Expressions of q_2^{j*}, $\Delta\pi_2^{j*}$, $q_1^{\Omega*}$ and $y^{\Omega*}$

Rearranging equation (4), we solve for q_2^{j*} to be:

$$q_2^{j*} = \left[T^j + wq_1\right]/2b_2 \tag{A.1}$$

Substituting (A.1) in equation (3) we get:

$$\Delta\pi_2^{j*} = R^j q_1 + Sq_1^2 + Gy + F^j . \tag{A.2}$$

where T^j, w, R^j, S, G, and F^j are defined in the Additional Notation section above.
From Equation (5), we obtain the following first-order equations for $q_1^{\Omega*}$ and $y^{\Omega*}$:

$$\partial\pi/\partial q_1 = R^1 - 2b_2 q_1 + \delta[R^{NU} + 2S^{NU} q_1] + \delta \Pr^H[R^H + 2Sq_1](z^H + z^{MH} + z^A)$$
$$+ \delta \Pr^M[R^M + 2Sq_1](z^{MH} + z^A) + \delta \Pr^L[R^L + 2Sq_1]z^A = 0$$
$$\partial\pi/\partial y = -2ay + \delta G[\Pr^H(z^H + z^{MH} + z^A) + \Pr^M(z^{MH} + z^A) + \Pr^L z^A] = 0$$

Solving the above first-order conditions provides the following optimum solutions:

$$q_1^{\Omega*} = \frac{R^1 + \delta R^{NU} + \delta \Pr^H R^H(z^H + z^{MH} + z^A) + \delta \Pr^M R^M(z^{MH} + z^A) + \delta \Pr^L R^L z^A}{2(b_2 - \delta S^{NU} - \delta \Pr^H S(z^H + z^{MH} + z^A) - \delta \Pr^M S(z^{MH} + z^A) - \delta \Pr^L S z^A)} \tag{A.3}$$

$$y^{\Omega*} = \frac{\delta G(\Pr^H(z^H + z^{MH} + z^A) + \Pr^M(z^{MH} + z^A) + \Pr^L z^A)}{2a} \tag{A.4}$$

Proof of Proposition 1

From Equation (A.1), we get $q_2^{j*} = \frac{T^j + (\lambda\theta - 2b_2)q_1}{2b_2}$. Then, $\frac{\partial q_2^{j*}}{\partial q_1} = \frac{(\lambda\theta - 2b_2)}{2b_2}$. Clearly $\frac{\partial q_2^{j*}}{\partial q_1}$ is negative when $\lambda < \frac{2b_2}{\theta}$ and positive otherwise. QED

Proof of Proposition 2

For the second order conditions, we require that $X_i > 0 \ (i = 1,2,3)$ and $X_1 - \delta \Pr^L S > 0$. Then from Equation (A.3), we can write $q_1^{A*} = \frac{Y_1 + \delta \Pr^L R^L}{2(X_1 - \delta \Pr^L S)}$, $q_1^{MH*} = \frac{Y_1}{2X_1}$, $q_1^{H*} = \frac{Y_2}{2X_2}$ and $q_1^{N*} = \frac{Y_3}{2X_3}$.

Since $q_1^{\Omega*}$ is positive, we further have $Y_i > 0 \ (i = 1,2,3)$ and $Y_1 + \delta \Pr^L R^L > 0$.

We present each part of the proposition as follows.

***Part 1**:* When $z^N = 1$, the second order condition is $\frac{\partial^2 \pi}{\partial q_1^2} = 2(\delta S^{NU} - b_2) = 2(\delta\lambda\theta - b_2) > 0$, if $\lambda > \frac{b_2}{\delta\theta}$.

Also, $\pi_1(q_1^{\max}) > 0$ and $\pi_1(q_1 = 0) < 0$, thus, $q_1^{N*} = q_1^{\max}$. Following a similar track, we can show that $q_1^{A*} = q_1^{MH*} = q_1^{H*} = q_1^{\max}$. This establishes Proposition 1.1.

Part 2**:* First we have $q_1^{A*} - q_1^{MH*} = \frac{\delta \Pr^L S}{2(X_1 - \delta \Pr^L S)}\left[\frac{R^L}{S} + \frac{Y_1}{X_1}\right]$ by using the above expressions of q_1^{A*} and q_1^{MH*}. Since $\frac{\delta \Pr^L S}{2(X_1 - \delta \Pr^L S)} > 0$, we need to show that $\frac{R^L}{S} + \frac{Y_1}{X_1} > 0$. Plugging in values of R^L and *S*, the last term is equal to $\frac{R^L}{S} + \frac{Y_1}{X_1} = \frac{2T^L}{w} + \frac{Y_1}{X_1}$. We have $\frac{Y_1}{X_1} > 0$ and $w > 0$ since $\frac{2b_2}{\theta} < \lambda$. If $T^L > 0$, then $\frac{2T^L}{w} + \frac{Y_1}{X_1} > 0$, and thus $q_1^{A*} - q_1^{MH*} > 0$. If $T^L < 0$, $\frac{2T^L}{w} + \frac{Y_1}{X_1} = \frac{2(T^L + wq_1^{MH*})}{w}$ since $q_1^{MH*} = \frac{Y_1}{2X_1}$. For any feasible solution of strategy ***A, $q_2^{L,A*} = \frac{T^L + wq_1^{A*}}{2b_2} > 0$. Thus, $T^L + wq_1^{A*} > 0$.

Suppose $q_1^{A*} < q_1^{MH*}$. This implies $T^L + wq_1^{A*} < T^L + wq_1^{MH*}$, which means $T^L + wq_1^{MH*} > 0$. Then $q_1^{A*} > q_1^{MH*}$. This is a contradiction. Therefore, we should have $q_1^{A*} > q_1^{MH*}$.

Following a similar track, we show that $q_1^{MH*} > q_1^{H*}$ and. $q_1^{H*} > q_1^{N*}$. This establishes Proposition 1.2.

***Part 3**:* We have $q_1^{A*} - q_1^{MH*} = \frac{\delta \Pr^L S}{2(X_1 - \delta \Pr^L S)}\left[\frac{R^L}{S} + \frac{Y_1}{X_1}\right]$. Since $\frac{\delta \Pr^L S}{2(X_1 - \delta \Pr^L S)} > 0$, we need to show that $\frac{R^L}{S} + \frac{Y_1}{X_1} < 0$. Plugging in values of R^L and *S*, the last term is equal to $\frac{2T^L}{w} + \frac{Y_1}{X_1}$. Note that we have $w < 0$ since $\lambda < \dfrac{2b_2}{\theta}$. We then follow the proof by contradiction track as in Part 2 above. This establishes Proposition 1.3. QED

Proof of Proposition 3

Using Equation (A.4), we have the optimum values as $y^{A*} = \dfrac{\delta G}{2a}$, $y^{MH*} = \dfrac{\delta G(1+p)}{4a}$, $y^{H*} = \dfrac{\delta G(1-p)}{4a}$ and $y^{N*} = 0$. As $\dfrac{\delta G}{2a} > \dfrac{\delta G(1+p)}{4a} > \dfrac{\delta G(1-p)}{4a} > 0$ for all $G, a > 0$ and $p, \delta \in (0,1)$, if follows that $y^{A*} > y^{MH*} > y^{H*} > y^{N*} = 0$. QED

Proof of Lemma 1

Noting that $q_2^{L,A*} = \dfrac{T^L + wq_1^{A*}}{2b_2} = \dfrac{\theta\left[x_1 + \overline{x}_2 - (\lambda \cdot \alpha + \zeta) - b_1/\theta\right] + (\theta\lambda - 2b_2)q_1^{A*}}{2b_2} < 0$ and using assumption A1 the last inequality follows establishing the statement of Lemma 1. QED

Proof of Lemma 2

Noting that $\Delta\pi_2^{H,H*} = R^H q_1^{NU} + S(q_1^{NU})^2 + F^H = \dfrac{(T^H + w \cdot q_1^{NU})^2}{4b_2} - (c_0 x_1 + K_2) > 0$ where the last inequality follows by assumption A2 establishing Lemma 2. QED

Proof of Lemma 3

An interior solution $y*$ requires $1 - \beta y* > 0$ and $1 - \gamma y* > 0$. The optimum values are $y^{MH*} = \dfrac{\delta G(1+p)}{4a}$ and $y^{H*} = \dfrac{\delta G(1-p)}{4a}$ for strategies ***MH*** and ***H***. Substituting for *G* and using assumption A3, the statement of Lemma 3 is established. QED

Proof of Proposition 4

Using Lemmas 1 and 2 the only feasible strategies are ***MH*** and ***H***.
To prove the first part of Proposition 4, i.e. $z^H = 1$, we need to show that $\pi^{H*} > \pi^{MH*}$.
After substituting for y^{MH*} and y^{H*} and rearranging we have,

$$\pi^{MH*} - \pi^{H*} = \delta \Pr{}^M [R^M q_1^{MH*} + Sq_1^{MH*2} + F^M + \frac{\delta G^2}{4a}] - (b_2 - \delta S^{NU} - \delta \Pr{}^H S)(q_1^{MH*} - q_1^{H*})^2 \qquad \text{(A.5)}$$

Under the condition that the word-of-mouth effect is low or moderate, q_1^{MH*} and q_1^{H*} are interior according to Propositions 2.2 or 2.3. Plugging q_1^{MH*} and q_1^{H*} from Equation (A.3), we have

$$\pi^{MH*} - \pi^{H*} = \delta \Pr{}^M \left\{ \frac{(\delta\theta(1-p)S)^2}{16b_2 X_1 X_2} \left[\zeta + f^{\Delta}\right]^2 + \frac{\delta G^2}{4a} - (c_0 x_1 + K_2) \right\}$$

where we define $f^{\Delta} = \lambda\alpha + \left[8b_2(b_2 - \delta S^{NU})T^M + 4b_2 w(R^1 + \delta R^{NU})\right] / (\delta\theta(1-p)w^2)$.

To assure the optimality of strategy ***H***, we find the root of ζ^* values where $\pi^{MH*} - \pi^{H*} = 0$. There is only one positive root $\zeta^* = \sqrt{[16b_2X_1X_2((c_0x_1 + K_2) - \delta G^2/(4a))]/(\delta\theta(1-p)S)^2} - f^{\Delta}$ when the following inequality holds: $\frac{(\delta\theta(1-p)S)^2}{16b_2X_1X_2}\left[f^{\Delta}\right]^2 + \frac{\delta G^2}{4a} - (c_0x_1 + K_2) < 0$; It is equivalent to say that a is sufficiently high: $a > a^* = \frac{\delta G^2}{4}\left[(c_0x_1 + K_2) - \frac{(\delta\theta(1-p)S)^2}{16b_2X_1X_2}(f^{\Delta})^2\right]^{-1}$. Then, $\pi^{H*} > \pi^{MH*}$ when $\zeta > \zeta^*$ and $a > a^*$.

To prove the second part of the proposition, i.e. $z^{MH} = 1$, we need to show $\Delta\pi_2^{H,MH*} > \Delta\pi_2^{M,MH*} > 0$ and $\pi^{MH*} > \pi^{H*}$. First, we have $\pi^{MH*} > \pi^{H*}$ when $\zeta < \zeta^*$. We next check the feasibility constraint: $\Delta\pi_2^{H,MH*} > \Delta\pi_2^{M,MH*} > 0$. Using the condition $\pi^{MH*} > \pi^{H*}$, we have

$$\delta \Pr^M [R^M q_1^{MH*} + Sq_1^{MH*2} + F^M + \frac{\delta G^2}{4a}] - (b_2 - \delta S^{NU} - \delta \Pr^H S)(q_1^{MH*} - q_1^{H*})^2 > 0.$$

Define the cutoff point Δ^* as $\Delta^* = R^M q_1^{MH*} + S(q_1^{MH*})^2 + F^M + \frac{\delta G^2}{4a}$. Since $X_2 = b_2 - \delta S^{NU} - \delta \Pr^H S > 0$ for the interior solution of strategy ***H***, then $\Delta^* > 0$. Plugging the value of y^{MH*}, we can have $\Delta\pi_2^{M,MH*} = R^M q_1^{MH*} + S(q_1^{MH*})^2 + F^M + \frac{\delta G^2(1+p)}{4a}$. Then $\Delta\pi_2^{M,MH*} > \Delta^* > 0$. Also, we have $\Delta\pi_2^{H,MH*} > \Delta\pi_2^{M,MH*}$ by definition. Thus the feasibility is established; therefore, strategy ***MH*** is the optimum. QED

Proof of Proposition 5

When the word-of-mouth effect is high, Proposition 2.1 holds, i.e., $q_1^{H*} = q_1^{MH*} = q_1^{max}$. Therefore

$$\pi^{MH*} - \pi^{H*} = \delta p(R^M q_1^{\max} + Sq_1^{\max 2} + F^M + \frac{\delta G^2}{4a})$$

using Equation A.5 given in proposition 4. It is easy to verify that $\pi^{MH*} - \pi^{H*}$ is independent of ζ. QED

Proof of Corollary 1

The proof is embedded in the proof of proposition 5. It is easy to verify that the difference $\pi^{MH*} - \pi^{H*}$ is strictly increasing in the word-of-mouth effect. Once the difference is positive, it does not become negative as λ increases. QED

Proof of Proposition 6

When $z^H = 1$, we get $y^{H*} = \frac{\delta(c_0\gamma x_1 + K_2\beta)(1-p)}{4a}$ and $\frac{\partial y^{H*}}{\partial p} = -\frac{\delta(c_0\gamma x_1 + K_2\beta)}{4a} < 0$. Also, when $z^{MH} = 1$, we have $y^{MH*} = \frac{\delta(c_0\gamma x_1 + K_2\beta)(1+p)}{4a}$ and $\frac{\partial y^{MH*}}{\partial p} = \frac{\delta(c_0\gamma x_1 + K_2\beta)}{4a} > 0$. QED

Results of the Model with the Word-of-Mouth Decreasing Demand Variability

In this section, we report a very brief sketch of the results of an alternative model in which high levels of word-of-mouth can possibly reduce demand variability. The impact of word-of-mouth on demand variance is modeled as $\alpha\lambda(\bar{\lambda}-\lambda)$ instead of $\alpha\lambda$. In this new model, the expressions for the optimal values for the initial and upgrade features change. This change is solely due to the change in the expression of T^j. The new expression is:

$$T^j = \theta\left[x_1 + \bar{x}_2 + D^j(\lambda(\bar{\lambda}-\lambda)\alpha + \zeta) - b_1/\theta\right].$$

Thus, the rest of the expressions for the optimum values of decision variables and cutoff figures change accordingly. For example, the expression of ζ^* remains the same although f^{Δ} is now given by

$$f^{\Delta} = \lambda(\bar{\lambda}-\lambda)\alpha + \left[8b_2(b_2 - \delta S^{NU})T^M + 4b_2 w(R^1 + \delta R^{NU})\right]/(\delta\theta(1-p)w^2).$$