Online Appendix to: "Effects of Information Revelation Policies under Cost Uncertainty"

A. Variables Used

See Table 6 for the variables used.

	Primary Usage	Vars	Explanation
Exogenous Parameters		n	*Number of bidders in the market*
		c_l	*Cost incurred by a low type*
		c_h	*Cost incurred by a high type*
		θ	*Probability that a bidder is a low cost*
Endogenous Parameters	SPG §4.1	α	Probability that R_o believes each of his opponents is a low-cost
		β	Probability that R_1 believes R_o is a low-cost
	IIS §4.2.1	$F_{1,\text{IIS}}(p)$	CDF of the first period bid distribution
		x	Probability that the first-period winner believes each of his opponents is a low type
	CIS §4.2.2	$F^{\text{sep}}_{1,\text{CIS}}(p)$	CDF of the first period bid distribution under separating equilibrium
		$F^{\text{semi}}_{1,\text{CIS}}(p)$	CDF of the first period bid distribution under semi-pooling equilibrium
		y	Probability that the first-period bidder, who bid $< c_h$, believes each of his opponents is a low type
		$\gamma_{\theta,n}$	Probability that a low cost type fakes
	Comparisons §5	Π_ϕ	Total profits earned by a bidder conditional on being a low type under policy ϕ
		SP_ϕ	Unconditional total profits earned by a bidder under policy ϕ
		EP_ϕ	Expected price paid by a bidder under policy ϕ

Table 6: Variables Used.

B. Deriving the Equilibrium for the Asymmetric Game

Here, we only provide the proof sketch for deriving the equilibrium because the details are identical to those in Maskin and Riley (1985) and Narasimhan (1988). In this section, when we refer to a proposition, it corresponds to the one in Narasimhan (1988). If a pure equilibrium exists, every supplier can be shown to have an incentive to undercut the other suppliers. Beyond a certain price the supplier does not have an incentive to undercut anymore but switch to a high price, leading to another round of undercutting. Thus, only a mixed strategy equilibrium exists (see Proposition

1 in Narasimhan, 1988). As a result, the profit expressions should involve cdfs and, therefore, we obtain $\Pi_{R_1}(p)$ and $\Pi_{R_o}(p)$ in Equations 1 and 2. The strategy sets are convex (Proposition 2 there) and have no mass points in the interior (Proposition 3). The infimum of the strategy space is identical across both types. The strategy sets are identical; if mass points exist, they are only present for one of the two types and that too at the supremum of the strategy space (Proposition 4). By the results established so far, we can conclude that an equilibrium is feasible only if $F_{R_o}(p)$ does not have a mass point at the supremum of the strategy set, i.e., $F_{R_o}(p) = 1$ at the supremum. If we can determine the supremum of R_o's equilibrium strategy, we can exploit the definition of the mixed strategy equilibrium and set the profit from the supremum to be equal to $\Pi_{R_1}(p)$. The supremum strategy is the bid infinitesimally less than c_h (In Maskin and Riley (1985), where a forward auction is considered, its equivalent is the infimum, which is explicitly mentioned to occur at a bid infinitesimally greater than zero). Similar to what Maskin and Riley (1985) have implicitly done, we let the unconditional revenue from the supremum of the strategy to be equal to c_h. The unconditional revenue is so because we assume a continuum of infinitesimally incremental bids in the strategy space. Thus, the profit from the supremum is $(1-\alpha)^n(c_h - c_l)$. By setting up the profits to be equal, we obtain

$$F_{R_o}(p) \quad = \quad 1 - \frac{(1-\alpha)}{\alpha}\left(\left(\frac{c_h - c_l}{p - c_l}\right)^{\frac{1}{n}} - 1\right).$$

The lower support for the bid distribution is obtained by setting $F_{R_o}(p) = 0$, which is $c_l + (1-\alpha)^n(c_h - c_l)$. From the preliminaries (Proposition 4), the lower support of R_1's bid distribution is also the same. If it were anything else, the winner can improve his profits by shifting the lower support. So, $F_{R_1}(c_l + (1-\alpha)^n(c_h - c_l)) = 0$. Substituting for that price, we obtain $\Pi_{R_o}(c_l + (1-\alpha)^n(c_h - c_l))$. This profit should be the same independent of the price bid according to the definition of a mixed strategy equilibrium. By setting them up to be equal, we obtain:

$$F_{R_1}(p) = 1 - \left(\frac{(1-\alpha)}{\beta}\left(\frac{c_h - c_l}{p - c_l}\right)^{\frac{1}{n}} - \frac{1-\beta}{\beta}\right),$$

The cdf is such that a mass point of $\frac{\beta-\alpha}{\beta}$ exists at the supremum.

Substituting these expressions back in the profit expressions, the expected profit for each supplier is equal to $(1-\alpha)^n(c_h - c_l)$.

Suppose one of the low-cost supplier deviates to bid $p = c_h$ while the other low-cost suppliers bid according to the separating equilibrium we identified above. Let it be one of the R_o. Since R_1 is always a low-cost type, such a deviation for R_o would only result in a zero profit, which is less than $(1-\alpha)^n(c_h - c_l)$ it secured before the deviation. Let R_1 deviate and bid $p = c_h$. He secures a profit only every R_o is a low cost type. Even then, the unconditional profit is $(1-\alpha)^n \frac{c_h - c_l}{n+1}$. This profit is less than the profit before deviation. Along similar lines, it can be proved that suppliers have an incentive to deviate from the pooling equilibrium. Thus, only a separating equilibrium exists.

C. IIS Results

C.1 IIS: First Period

Algberaic simplification of Equation 4 yields the right hand side of Equation 5. We next exploit the definition of the mixed strategy equilibrium by computing the payoff for a specific bid in the strategy space and setting it to be the left hand side of Equation 5. The supremum of the strategy space in this game turns out to be the same as that under SPG. Similar to Maskin and Riley (1985) and Section B, we treat the unconditional revenue at the supremum, which is infinitesimally less than c_h, to be equal to c_h (recall that it is a consequence of our implicit assumption regarding the strategy space being the continuum of infinitesimally incremental bids). After substituting the expressions and simplifying the algebra, we obtain the left hand side of Equation 5.

C.2 Proof for Lemma 1

Lemma 1 *Consider two settings which may have a different θ and/or n. If the winning bid is the same in both settings, the winner's second belief that none of the suppliers is a low-cost type across the two settings is the same, and independent of both θ and n.*

Proof. Recall that the winner's second period belief about every supplier being a high cost type is x as defined in Equation 3. From this, the belief that none of the suppliers is a low cost type is $x' = x^n$. With that, let us now rearrange Equation 5:

$$1 - (\log x') = \frac{1}{x'} \frac{p - c_l}{c_h - c_l}.$$

x' is a solution to the above equation. It is clear that x' is independent of θ and n, but is a constant for a given p. ■

D. CIS Results

D.1 CIS: Condition for Separating versus Semi-Pooling Equilibrium

We first prove the restriction on the separating equilibrium. The profit from following the separating equilibrium strategy, as we showed before, is: $\Pi^{\text{sep}}_{\text{CIS}} = 2(1-\theta)^n(c_h - c_l)$. While all other suppliers are playing that strategy, let us consider the incentive for one of the suppliers to deviate and bid c_h. The profit from such deviation is:

$$\begin{aligned}\Pi^{\text{dev}} \quad &= \quad \overbrace{(1-\theta)^n}^{\text{All } n \text{ suppliers are high-cost type}} \left(\overbrace{\frac{1}{n+1}(c_h - c_l) +}^{\text{First period profit}} \quad \overbrace{(c_h - c_l)}^{\text{Second period profit: from SPG}} \right) + \\ &\qquad \underbrace{n\theta(1-\theta)^{n-1}}_{\text{Only one other supplier is a low-cost type}} \left(\underbrace{0}_{\text{First period profit}} + \underbrace{c_h - c_l}_{\text{Second period profit}} \right).\end{aligned}$$

The first line in the profit expression corresponds to the scenario when all n suppliers are high-cost types. The supplier under consideration, say i, wins the first period with a probability of $\frac{1}{n+1}$, realizes before the beginning of the second period that none of the other suppliers is a high-cost type, and wins the second period with a bid which is infinitesimally smaller than c_h. The second line corresponds to the case when only one of the other suppliers is a low-cost type, whom we refer to as j. Because j wins the first period, i obtains zero profits in the first period. Since, by assumption, j continues its separating equilibrium strategy in the second period, i can outbid j in the second period with a bid price two infinitesimal increments below c_h and hence, the last term (see the previous sections in the appendix for discussions on infinitesimal bids and the assumptions when considering the unconditional revenue). We obtain θ^{fake}_n by solving for θ when $\Pi^{\text{dev}} > \Pi^{\text{sep}}_{\text{CIS}}$.

We next prove the existence of the semi-pooling equilibrium. The previous part of the proof has already established the incentive for a low-cost supplier to bid $p = c_h$. It is not clear whether or not only a pooling equilibrium exists. Suppose there indeed was a pooling equilibrium where both types of suppliers bid the same price, i.e., $p = c_h$, then the profit from the pooling scenario would be:

$$\Pi^{\text{pool}} = \overbrace{\frac{1}{n+1}(c_h - c_l)}^{\text{First period profit}} + \overbrace{(1-\theta)^n (c_h - c_l)}^{\text{Second period profit from SPG since beliefs do not change}}.$$

While all suppliers are playing the pooling equilibrium, let one of the suppliers deviate to bid in the first period a price infinitesimally smaller than c_h. The expected profit in that case would be:

$$\Pi^{\text{dev}} = \overbrace{(c_h - c_l)}^{\text{First period profit}} + \overbrace{(1-\theta)^n (c_h - c_l)}^{\text{Second period profit: SPG with } R_1 \text{ being the deviating supplier and } R_o \text{ refers to the others}}.$$

Comparing the two profits, it should be clear that the supplier will have an incentive to deviate. Therefore, only a semi-pooling equilibrium exists in our model.

D.2 CIS: Semi-pooling Equilibrium

There are two unknowns, $F^{\text{semi}}_{1,\text{CIS}}(p)$ and $\gamma_{\theta,n}$. To determine them, we need at least two equations. These two equations are obtained by setting three different expressions to be equal. For this, we invoke the definition of mixed strategy equilibrium, in which a supplier's expected profits from all his actions are identical. The first expression is the expected profit for any $p_1 < c_h$, which is Equation 9. The second expression we use is the expected profit faking, which is Equation 10. The third expression is the profit from submitting the maximum feasible bid less than c_h. Consistent with Section B and Maskin and Riley (1985), we consider the point to be a price infinitesimally less than c_h (because of the infinitesimally incremental bid assumption) and its unconditional revenue to be c_h. At that bid, $F^{\text{semi}}_{1,\text{CIS}}(p) = 1 - \gamma_{\theta,n}$. Equation 11 is obtained by setting the first two expressions equal. Equation 12 is obtained by setting the last two expressions equal.

D.3 Proof of Proposition 1

Proposition 1 *For any arbitrary n, the following properties of Equation 12 are observed:*

1. $\gamma_{\theta,n} > 0 \ \forall \ \theta \in (\theta_n^{fake}, 1)$. *There is only one point of inflection, which occurs at* $\theta_n^{\max\gamma} = \frac{1}{1+\frac{n^{1+n}}{n^n+(1+n)^n}}$.
2. $\frac{\partial \theta_n^{fake}}{\partial n} < 0$.
3. *For* $\theta > \theta_1^{\max\gamma}$, $\gamma_{\theta,1} \geq \gamma_{\theta,n}$ *for any* $n > 1$.

Proof. The proof for Property 1 is as follows. Note from Equation 12 that $\gamma_{\theta,n} = 0$ both at $\theta = 1$ and again at θ_n^{fake}. Next, we obtain the first order derivative from Equation 12:

$$\frac{\partial \gamma_{\theta,n}}{\partial \theta} = \frac{(1-\gamma_{\theta,n})(1-\theta-n\theta\gamma_{\theta,n})}{(1-\theta)\theta(1+n\theta(1-\gamma_{\theta,n}))}.$$

When the derivative is computed at θ_n^{fake}, the slope is found to be positive. We will use the same first order derivative expression to show that only one point of inflection occurs for every $\theta \in (\theta_n^{\text{fake}}, 1)$. Since both the numerator and denominator are polynomial expressions, to demonstrate that $\gamma_{\theta,n}$ is continuous in the range of θ of interest to us, consider when the denominator becomes zero. The denominator is zero for $\gamma_{\theta,n} = 1 + \frac{1}{n\theta} > 1$ or for $\theta = 0$ or for $\theta = 1$. The probability of faking can never be greater than 1. Faking occurs only for $\theta \geq \theta_n^{\text{fake}} > 0$. At $\theta = 1$, the numerator also becomes zero (because of the last term in the numerator). One can apply L'Hospital's rule to determine the slope. We will not concern about the nature of the function at $\theta = 1$ for two reasons. First, we already know that $\gamma_{\theta,n} = 0$ at $\theta = 1$. Second, comparing the policies for $\theta = 1$ is uninteresting. So, we next consider the zeros of the numerator. The value $\gamma_{\theta,n} = 1$ is ruled out by the definition of a semi-pooling equilibrium. The zero in the numerator occurs when $\gamma_{\theta,n}^{\max} = \frac{1-\theta}{n\theta}$. Substituting back in Equation 12, we solve for θ and obtain the expression for $\theta_n^{\max\gamma}$; the corresponding second order derivative is negative.

The proof for Property 2 is as follows. $\frac{\partial \theta_n^{\text{fake}}}{\partial n} = -\frac{1}{(2+n)^2}$.

Now, the proof for Property 3. The solution, $\gamma_{\theta,n}$, is the value of x that makes $(1-\theta+\theta x)^{n+1}$, referred to as LHS, equal to $(1-\theta)^n\theta(n+1)(1-x)$, referred to as RHS. Notice that LHS is an increasing function of x while RHS is a decreasing function of x. The point of intersection of LHS and RHS yields us the solution $x^* = \gamma_{\theta,n}$. First, note that $LHS < RHS$, at $x = 0$ for $\theta > \theta_1^{\max\gamma}$. It is also obvious that, at $x = 1$, $LHS > RHS$. These two statements mean that a solution exists in the $x \in (0, 1)$ range for the case we consider. If we set $x = y$, for some y, and observe $LHS > RHS$, then it must be that $y > x^*$. We use this logic to prove the property. Specifically, we set $x = \gamma_{\theta,1}$ and show that $LHS > RHS$ for $\theta > \theta_1^{\max\gamma}$ and $n > 1$. Of course, when $x^* = \gamma_{\theta,1}$ and $n = 1$, $LHS = RHS$.

We first prove the property for $n \geq 3$ and then for $n = 2$. When $x = \gamma_{\theta,1}$, $LHS - RHS$ is:

$$\begin{aligned}(1-\theta+\theta\gamma_{\theta,1})^{n+1} - (1-\theta)^n\theta(n+1)(1-\gamma_{\theta,1}) &= (n+1)(1-\theta)^n\left(-2+\theta+\sqrt{(3-4\theta+\theta^2)}\right) \\ &\quad + \left(-1+\theta+\sqrt{(3-4\theta+\theta^2)}\right)^{n+1}.\end{aligned}$$

Because $\left(-1+\theta+\sqrt{(3-4\theta+\theta^2)}\right) > 0$, $LHS > RHS$ so long as the following is true:

$$\begin{aligned}-(1+n)(1-\theta)^n + (-1+\theta+\sqrt{(3-4\theta+\theta^2)})^{n+1} &\overset{?}{>} 0. \\ L' = \left(\frac{(\sqrt{(3-\theta)}-\sqrt{1-\theta})^{n+1}}{n+1}\right)^{\frac{2}{n-1}} &\overset{?}{>} (1-\theta).\end{aligned}$$

Independent of n, it is observed that $\frac{\partial L'}{\partial \theta} > 0$. Since $L' > 0$, if we can establish that $L' = 1-\theta$ for some specific θ' and n'; and L' at θ' is always increasing in n; then we have established that $LHS > RHS$ for all $n \geq n'$ and $\theta \geq \theta'$. Compute L' at $\theta = \theta_1^{\max\gamma}$.

$$L'|_{\theta_1^{\max\gamma}} = \left(\frac{1}{n+1}\right)^{\frac{2}{n-1}}.$$

From this, it can be shown that $\left(\frac{1}{n+1}\right)^{\frac{2}{n-1}} = 1 - \theta_1^{\max\gamma}$ when $n = 3$. It can also be seen that $\frac{\partial}{\partial n}\left(\frac{1}{n+1}\right)^{\frac{2}{n-1}} > 0$. Thus, we have proved that $LHS > RHS$ for $n \geq 3$ and $\theta \geq \theta_1^{\max\gamma}$. For $n = 2$ and $\theta \geq \theta_1^{\max\gamma}$, LHS can be shown to be greater than RHS through direct substitution. ■

E. Proofs for the Comparisons

E.1 Proof of Theorem 1

Theorem 1 $\Pi_{IIS} > \Pi_{CIS}^{sep}$ *and* $EP_{IIS} > EP_{CIS}^{sep}$ *for* $\theta \in [0, \theta_n^{fake})$.

Proof. $\Pi_{\text{IIS}} > \Pi_{\text{CIS}}^{\text{sep}}$ can be established through the Taylor series expansion of the logarithmic expression, the result can be easily observed. Given this result, the comparison involving the expected prices are obtained directly by considering the expressions. ■

E.2 Proof of Proposition 2

Proposition 2 *For* $n = 1$, *there exists a* $\theta_1^{equal} < 0.87$ *such that* $\Pi_{CIS}^{semi} > \Pi_{IIS}$ *and* $EP_{CIS}^{semi} > EP_{IIS}$ *for* $\theta > \theta_1^{equal}$ *and;* $\Pi_{CIS}^{semi} < \Pi_{IIS}$ *for* $\theta_1^{fake} < \theta < \theta_1^{equal}$.

Proof. So long as $(\Pi_{\text{CIS}}^{\text{semi}} - \Pi_{\text{IIS}})/(1-\theta) > 0$, it implies $(\Pi_{\text{CIS}}^{\text{semi}} - \Pi_{\text{IIS}}) > 0$. The function $(\Pi_{\text{CIS}}^{\text{semi}} -$

$\Pi_{\text{IIS}})/(1-\theta)$ evaluated at $n = 1$ has only one point of inflection. This implies that there are only two θ values when the function is zero. The first one occurs at some $\theta < 0$ and the other is at $\theta \approx 0.8688$. The directionality of the difference can be verified by substituting the value of θ. From this, it should be evident that $\theta_1^{\text{equal}} < 0.87$. Thus, when $n = 1$, we have proved that, for $\theta > \theta_1^{\text{equal}}$, $\Pi_{\text{CIS}}^{\text{semi}} > \Pi_{\text{IIS}}$; and that, for $\theta_1^{\text{fake}} < \theta < \theta_1^{\text{equal}}$, $\Pi_{\text{CIS}}^{\text{semi}} < \Pi_{\text{IIS}}$. The comparisons involving $EP_{\text{CIS}}^{\text{semi}}$ and EP_{IIS} can be obtained from how the expected payments vary with Π_ϕ. ■

E.3 Proof of Theorem 2

Theorem 2 *Suppose $n > 0$. There always exists a $\theta_n^{equal} < 0.87$ such that $\Pi_{IIS} < \Pi_{CIS}^{semi}$ and $EP_{IIS} < EP_{CIS}^{semi}$ for any $\theta \in [\max\{\theta_1^{\max\gamma}, \theta_n^{equal}\}, 1)$.*

Proof. There are two parts to prove the supplier profit comparison. We begin with the first part. For any n:

$$\begin{aligned}\Pi_{\text{CIS}}^{\text{semi}} &= \left(((n+1)(1-\gamma_{\theta,n})(1-\theta)^n\theta)^{\frac{n}{n+1}} + (1-\theta)^n\right)(c_h - c_l) \\ &\underbrace{\geq}_{\text{Property 3 in Proposition 1}} \left(((n+1)(1-\gamma_{\theta,1})(1-\theta)^n\theta)^{\frac{n}{n+1}} + (1-\theta)^n\right)(c_h - c_l) = \kappa.\end{aligned}$$

Of course, Property 3 can only be invoked for $\theta > \theta_1^{\max\gamma}$. Thus, κ is an envelope to the expected profit expression for the semi-pooling equilibrium under CIS and that too for a specific range of θ.

For the second part of the proof, we compare κ and Π_{IIS}. Specifically, consider the following difference function:

$$\begin{aligned}\frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n} &= \left(\frac{(n+1)(2-\theta-\sqrt{3-4\theta-\theta^2})}{1-\theta}\right)^{\frac{n}{n+1}} - 1 + n\log(1-\theta) \\ (1+n)^2\left(\frac{\partial}{\partial n}\frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n}\right) &= (1+n)^2\log[1-\theta] + A^{\frac{n}{1+n}}(n + \log A)\end{aligned}$$

where $A = \frac{(1+n)\left(-2+\theta+\sqrt{3-4\theta+\theta^2}\right)}{-1+\theta}$. Rearranging the terms, we can find $(1+n)^2\left(\frac{\partial}{\partial n}\frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n}\right) > 0$ for any arbitrary n and $\theta > \theta_1^{\max\gamma}$. This states that, suppose the difference between κ and IIS is positive for a given $n = n'$, and $\theta > \theta_1^{\max\gamma}$; it will remain positive for all $n > n'$. Now, we combine the results from Proposition 2 with that from the two parts proved here:

$$\Pi_{\text{CIS}}^{\text{semi}} \overbrace{>}^{\text{By Part 1}} \kappa \overbrace{>}^{\text{By Part 2}} \Pi_{\text{IIS}} \text{ for } \theta > 0.87.$$

As in the previous theorems, the expected payment comparisons simply follow the expected supplier profit comparisons under the specific cases. ■

E.4 Proof of Theorem 3

Theorem 3 *For $\theta > \theta_n^{equal}$, the first period bid distribution under* CIS *first order stochastically dominates that under* IIS*.*

Proof. The condition $\theta > \theta_n^{\text{equal}}$, implies that $\Pi_{\text{CIS}}^{\text{semi}} \geq \Pi_{\text{IIS}}$, which is the same as

$$\begin{aligned}\left[1-\theta+\theta(1-F_{1,\text{CIS}}^{\text{semi}}(p))\right]^n (p-c_l) - \left[1-\theta+\theta(1-F_{1,\text{IIS}}(p))\right]^n (p-c_l) + \\ (1-\theta)^n n \log\left(1-\theta+\theta(1-F_{1,\text{IIS}}(p))\right)(c_h-c_l) \quad &\geq \quad 0. \qquad (19)\end{aligned}$$

This condition holds even without the last term on the left hand side equation, since that term is a negative one. Then, for any $p > c_l$,

$$\begin{aligned}\left[1-\theta F_{1,\text{CIS}}^{\text{semi}}(p)\right]^n - \left[1-\theta F_{1,\text{IIS}}(p)\right]^n &\geq 0\\ \theta(F_{1,\text{IIS}}(p) - F_{1,\text{CIS}}^{\text{semi}}(p))(\text{Positive terms.}) &\geq 0\end{aligned}$$

This implies that $F_{1,\text{IIS}}(p) \geq F_{1,\text{CIS}}^{\text{semi}}(p)$ for any given $p > c_l$. ■

E.5 Proof of Theorem 4

Theorem 4 *For $\theta < \theta_n^{fake}$, the first period bid distributions under* IIS *first order stochastically dominates that under* CIS*.*

Proof. The condition $\theta < \theta_n^{\text{fake}}$ implies that $\Pi_{\text{CIS}}^{\text{sep}} \leq \Pi_{\text{IIS}}$. From the expected profit expressions, we compute the actual difference. Therefore,

$$\begin{aligned}\left[1-\theta+\theta(1-F_{1,\text{IIS}}(p))\right]^n (p-c_l) - \left[1-\theta+\theta(1-F_{1,\text{CIS}}^{\text{sep}}(p))\right]^n (p-c_l) = \\ -(1-\theta)^n n \log\frac{1-\theta}{(1-\theta+\theta(1-F_{1,\text{IIS}}(p)))}(c_h-c_l).\end{aligned}$$

The right hand side expression is positive. So,

$$\begin{aligned}\left[1-\theta+\theta(1-F_{1,\text{IIS}}(p))\right]^n (p-c_l) - \left[1-\theta+\theta(1-F_{1,\text{CIS}}^{\text{sep}}(p))\right]^n (p-c_l) &\geq 0\\ \theta(F_{1,\text{CIS}}^{\text{sep}}(p) - F_{1,\text{IIS}}(p))(\text{Positive terms.}) &\geq 0\end{aligned}$$

This implies that $F_{1,\text{IIS}}(p) \leq F^{\text{sep}}_{1,\text{CIS}}(p)$ for any given $p > c_l$. ■

E.6 Proof of Theorem 5

Theorem 5 $SW_{IIS} = SW^{sep}_{CIS}$ *for* $\theta \in [0, \theta^{fake}_n]$. $SW_{IIS} > SW^{semi}_{CIS}$ *for* $\theta \in (\theta^{fake}_n, 1)$.

Proof. The social welfare comparison is the inverse of the probability of faking or bidding $p = c_h$. The only scenario when faking occurs is in CIS under the semi-pooling equilibrium. ■

E.7 Proof of Theorem 6

Theorem 6 *For every* $\theta \in [0, \theta^{fake}_1]$, *there exist* n_1 *and* n_2, *such that* $\frac{\partial SW_{CIS}}{\partial n} < 0$ *for* $n_1 \leq n < n_2$.

Proof. The higher the probability of faking, the lower is the social welfare. By Property 2 in Proposition 1, we know that probability of faking can increase with n for a certain range. Corresponding to that range, the social welfare decreases. ■

F. Three Cost Type Model

Consider the model specification in Section 6.1.

Second Period Game

There are four scenarios in the second period game which are relevant to our analysis. In all the four scenarios, whenever there is uncertainty about the opponent's type, let l' and m' be the beliefs that a supplier has about the opponent being of type c_l and c_m, respectively. An interesting property observed in the four scenarios is as follows. Suppose bidder A of a particular type mixes his bids over the strategy space $[p_1, p_2]$ such that bidder B of type α is indifferent among prices. If $\alpha = c_l$, the expected profit for B of type c_m increases with the bid price in the same range. Instead, if $\alpha = c_m$, the profit for B of type c_l decreases in the range. We refer to this property as profit variation property, or simply *PVP*.

Scenario 1: The suppliers know each other's cost type. The equilibrium in this case is straightforward and the scenario resembles a Bertrand game.

Scenario 2: The uncertainty that each supplier has about his opponent is identical. The expected profit for supplier of type α – independent of whether it is c_l or c_m – from bidding a price of p is: $(1 - l'F^I_{c_l}(p) - m'F^I_{c_m}(p))(p - \alpha)$, where $F^I_\beta(p)$ is the cumulative density function of bid distribution from the opponent when his type is β. In this case, the equilibrium is such that $F^I_{c_m}(p) = \frac{1}{m'}\left(1 - l' - \frac{(1-l'-m')(1-c)}{p-c}\right)$ for $p > (c + \frac{(1-l'-m')}{1-l'}(1-c))$ and $F^I_{c_l}(p) =$

$\frac{1}{l'}\left(1-(c+\frac{1-l'-m'}{1-l'}(1-c))\frac{1-l'}{p}\right)$ defined for $c(1-l')+(1-l'-m')(1-c) < p < c+\frac{(1-l'-m')}{1-l'}(1-c)$. Suppliers of type c_l and c_m respectively generate profits of $\Pi_{c_l}^{\text{Sc:2}} = c(1-l') + (1-l'-m')(1-c)$ and $\Pi_{c_m}^{\text{Sc:2}} = (1-l'-m')(1-c)$. Deviations can be ruled out because of PVP and also because profit for either type from faking as c_h is less than the profit already generated.

Scenario 3: Only one supplier has his type revealed to the other and let the revealed type be c_m. The revealed supplier faces uncertainty about the other supplier and his profit from bidding a price p is $(1-l'F_{c_l}^{H;R=c_m}(p) - m'F_{c_m}^{H;R=c_m}(p))(p-c)$, where $F_{\alpha}^{H;R=c_m}(p)$ is the cdf of the bid distribution for the supplier hiding. The profit for the hiding supplier of type α is $(1-F_{c_m}^{R}(p))(p-\alpha)$ where $F_{c_m}^{R}(p)$ is the bid distribution of the revealed supplier. The equilibrium is as follows: the revealed type bids according to the distribution $F_{c_m}^{R}(p) = \left(1-\frac{(1-l'-m')(1-c)}{(p-c)(1-l')}\right)$ with a mass point of $1-\frac{m'}{1-l'}$ at the supremum (infinitesimally close to 1); the hiding supplier of type c_m bids $F_{c_m}^{H;R=c_m}(p) = \frac{1}{m'}\left(1-l'-\frac{(1-l'-m')(1-c)}{p-c}\right)$ for $p > c+\frac{1-l'-m'}{1-l'}(1-c)$; and the hiding supplier of type c_l submits a pure strategy bid of $c+\frac{1-l'-m'}{1-l'}(1-c)$. The profits for the respective cases are: $\Pi_{c_m}^{\text{Sc:3; R}} = (1-l'-m')(1-c)$, $\Pi_{c_m}^{\text{Sc:3; H}} = \frac{(1-l'-m')}{1-l'}(1-c)$ and $\Pi_{c_l}^{\text{Sc:3; H}} = c+\frac{(1-l'-m')}{1-l'}(1-c)$. Deviations can be ruled out because of PVP. Also, a mixed strategy for c_l type is unsustainable.

Scenario 3a: A variant of Scenario 3 is useful later for considering deviations. Let a hiding supplier, B, be of type c_m or c_h and a revealed supplier, A, be uncertain about it. Suppose the beliefs A has about B are $l' = 0$ and m', and they be common knowledge. Also, suppose B incorrectly believes that A is of type c_m when the type actually is c_l. In this case, $F_{c_m}^{H;R=c_m}(p)$, as before, is the bid distribution for B when he is type of c_m. Given the strategy by B, A's strategy (because of PVP) is to bid $c+\frac{1-l'-m'}{1-l'}(1-c)$, generating the same as the profit $\Pi_{c_l}^{\text{Sc:3-a; H}}$. This is the only profit of interest to us.

Scenario 4: Only one supplier has his type revealed to the other and let the type revealed be c_l. The revealed supplier generates a profit of $(1-l'F_{c_l}^{H;R=c_l}(p) - m'F_{c_m}^{H;R=c_l}(p))p$ from bidding a price p, where $F_{\alpha}^{H;R=c_l}(p)$ is the cdf of the bid distribution for the supplier hiding who is of type α when the revealed type is c_l. The profit for the hiding supplier is $(1-F_{c_l}^{R}(p))(p-\alpha)$ where α is the cost of the opponent (=0 or c depending on the supplier's type) and $F_{c_l}^{R}(p)$ is the bid distribution of the revealed supplier. Two different equilibria are feasible depending on whether or not $(1-l'-m') > (1-l')c$:

- If the condition is valid, the equilibrium bids are as follows: for the revealed supplier, the cdf is $F^R_{c_l}(p) = \left(1 - (\frac{(1-l'-m')}{1-l'} - c)\frac{1}{p-c}\right)$ for $p > \frac{1-l'-m'}{1-l'}$ and with a masspoint of $1 - \frac{m'}{(1-c)(1-l')}$ at the supremum (infinitesimally close to 1); the cdf for the hiding supplier, whose type is c_m, is $F^{H;R=c_l}_{c_m}(p) = \frac{1}{m'}\left(1 - l' - \frac{(1-l'-m')}{p}\right)$; and the hiding supplier, whose type is c_l, bids $\frac{(1-l'-m')}{1-l'}$. The profits for the respective cases are $\Pi^{\text{Sc:4-(1); R}}_{c_l} = (1 - l' - m')$, $\Pi^{\text{Sc:4-(1); H}}_{c_m} = \frac{(1-l'-m')}{1-l'} - c$, and $\Pi^{\text{Sc:4-(1); H}}_{c_l} = \frac{(1-l'-m')}{1-l'}$.

- If the condition is invalid, i.e., $(1 - l' - m') < (1 - l')c$, the following are the equilibrium strategies: the revealed type bids according to the distribution $F^R_{c_l}(p) = \frac{1}{l'}\left(1 - \frac{(1-l')c}{p}\right)$, whose supremum is c; the hiding supplier of type c_m bids his cost; and the hiding supplier of type c_l bids in the same manner as revealed supplier. The profits in this case are: $\Pi^{\text{Sc:4-(2); R}}_{c_l} = \Pi^{\text{Sc:4-(2); H}}_{c_l} = (1 - l')c$, and $\Pi^{\text{Sc:4-(1); H}}_{c_m} = 0$.

First Period Game: IIS

It turns out that only a separating equilibrium exists in the first period here. We first characterize the equilibrium and later argue that deviations from that equilibrium do not exist.

$$
\begin{aligned}
\pi_{\text{IIS},c_m}(p) &= \overbrace{(1 - l - m)(p - c + \Pi^{\text{Sc:3; R}}_{c_m})}^{\text{Opponent is type } c_h} + \overbrace{m(1 - F_m(p))(p - c + \Pi^{\text{Sc:3; R}}_{c_m})}^{\text{Opponent is type } c_m \text{ but he lost first period}} \\
&\quad + \underbrace{mF_m(p)(0 + \Pi^{\text{Sc:3; H}}_{c_m})}_{\text{Opponent is type } c_m \text{ but he won first period}} + \underbrace{l\Pi^{\text{Sc:4; H}}_{c_m}}_{\text{Opponent is type } c_l} \\
&= (1 - l - mF_m(p))(p - c + \Pi^{\text{Sc:3; R}}_{c_m}) + mF_m(p)\Pi^{\text{Sc:3; H}}_{c_m} + l\Pi^{\text{Sc:4; H}}_{c_m} \\
\pi_{\text{IIS},c_l}(p) &= \overbrace{((1 - l)(p + \Pi^{\text{Sc:4; R}}_{c_l})}^{\text{Opponent is not type } c_l} + \overbrace{l(1 - F_l(p))(p + \Pi^{\text{Sc:4; R}}_{c_l})}^{\text{Opponent is type } c_l \text{ but he lost first period}} + \overbrace{lF_l(p)(0 + \Pi^{\text{Sc:4; H}}_{c_l})}^{\text{Opponent is type } c_l \text{ but he won}} \qquad (20) \\
&= (1 - lF_l(p))(p + \Pi^{\text{Sc:4; R}}_{c_l}) + lF_l(p)\Pi^{\text{Sc:4; H}}_{c_l}
\end{aligned}
$$

We will continue to assume Bayesian update of beliefs. Consider the first equation first. When Scenario 3 occurs in the second period of IIS, $l' = 0$ and $m' = \frac{m(1-F_m(q))}{1-l-mF_m(q)}$, where q is the winning bid. This implies that $\Pi^{\text{Sc:3; R}}_{c_m} = \frac{1-l-m}{1-l-mF_m(p)}$ since $q = p$ in this case. When computing $\Pi^{\text{Sc:4; H}}_{c_l}$ for supplier A, l' and m' correspond to the beliefs held by the winning opponent B, whose bid now is q. So, if $p^{\text{IIS}}_{l,m}$ is such that $F_m(p^{\text{IIS}}_{l,m}) = 0$, then $\Pi^{\text{Sc:3; H}}_{c_m}$ as a function of a losing bid p submitted by A is: $\frac{1}{F_m(p)}\int_{p^{\text{IIS}}_{l,m}}^{p}(\frac{1-l-m}{1-l-mF_m(q)})f_m(q)(1 - c)\,dq = -(1 - c)\frac{(1-l-m)}{yF_m(p)}\log(\frac{1-l-mF_m(p)}{1-l})$. Substituting for these expressions:

$$\begin{aligned}\pi_{\text{IIS},c_m}(p) &= (1-l-mF_m(p))(p-c)-(1-l-m)(1-c)\log(\frac{1-l-mF_m(p)}{1-l}) \\ &\quad +l\Pi_{c_m}^{\text{Sc:4; H}}+(1-l-m)(1-c) \qquad (21)\end{aligned}$$

Given the separating nature of the equilibrium, $\Pi_{c_m}^{\text{Sc:4; H}}$ is independent of p. The last two terms in Equation 21, which are independent of p, can be ignored while computing the equilibrium. One can easily compute the equilibrium bid distribution by noting that the expected payoff from any price p in the strategy set is the same and that p infinitesimally close to 1 is the supremum of the strategy set. The equilibrium for $F_m(p)$ is thus the solution to the following:

$$(1-l-m)(1-\log(\frac{1-l-m}{1-l})) = (1-l-mF_m(p))\frac{(p-c)}{(1-c)}-(1-l-m)\log(\frac{1-l-mF_m(p)}{1-l}).$$

The infimum of the strategy set is

$$p_{l,m}^{\text{IIS}} = c+(1-c)\frac{(1-l-m)}{1-l}(1-\log(\frac{1-l-m}{1-l})). \qquad (22)$$

The total profit for type c_m is: $(1-l-m)(1-c)(2-\log(\frac{1-l-m}{1-l}))+l\Pi_{c_m}^{\text{Sc:4; H}}$. We discuss about the computation of $\Pi_{c_m}^{\text{Sc:4; H}}$ next.

Computing the profit expressions for Scenario 4 occurring in the second period of IIS has to consider different conditions. Note that, here, $m' = m$ and $l' = \frac{l(1-F_l(q))}{1-lF_l(q)}$. Let $p_{l,l}^{\text{IIS}}$ be such that $F_l(p_{l,l}^{\text{IIS}}) = 0$.

1. If $(1-l-m) > (1-l)c$, then $(1-l'-m') > (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)}-m$, $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{1}{F_l(p)}\int_{p_{l,l}^{\text{IIS}}}^{p}(\frac{1-l-m(1-lF_l(q))}{1-l})f_l(q)\,dq$ and $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)}\int_{p_{l,l}^{\text{IIS}}}^{p}(\frac{1-l-m(1-lF_l(q))}{1-l}-c)f_l(q)\,dq$.
2. If $(1-c) < y$, then $(1-l'-m') < (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)}\int_{p_{l,l}^{\text{IIS}}}^{p}(\frac{1-l}{(1-lF_l(q))})f_l(q)\,dq$.
3. Otherwise, two sub-cases arise. Let p_e be such that $F_l(p_e) = \frac{1}{l}(1-\frac{(1-l)(1-c)}{y})$
   - If $p \le p_e$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)}\int_{p_{l,l}^{\text{IIS}}}^{p}(\frac{1-l}{(1-lF_l(q))})f_l(q)\,dq$.
   - For $p > p_e$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)}-m$, $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)}\int_{p_e}^{p}(\frac{1-l-m(1-lF_l(q))}{1-l}-c)f_l(q)\,dq$, and

$$\Pi_{c_l}^{\text{Sc:4; H}} = \tfrac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^{p_e} (\tfrac{1-l}{(1-lF_l(q))}) f_l(q)\, dq + \tfrac{1}{F_l(p)} \int_{p_e}^{p} (\tfrac{1-l-m(1-lF_l(q))}{1-l}) f_l(q)\, dq.$$

The equilibrium is different in each of the cases. Now, to compute the bid distribution, we invoke the property that the strategies in the mixed equilibrium yield the same profit and that $p_{l,m}^{\text{IIS}}$ is the supremum of the strategy space for type c_l. The explicit expressions for the different cases have not been shown for the sake of simplicity. We use the expression to compare against the bid distribution of a single period game. The single period game has the same bid distribution as Scenario 2 except for $l' = l$ and $m' = m$. One can show the stochastic dominance of IIS bid distribution in each case, validating the presence of the extraction effect.

We next consider deviations from the equilibrium. The deviations for type c_m are straightforward to discuss since they most closely mirror the discussion in the original model. Type c_m faking as type c_h does not yield any benefit since it does not bias the opponent's belief and only the winner's bid is revealed. When type c_m fakes as type c_l, the opponent bids aggressively, resulting in lower profits for the faking bidder. A similar argument holds for type c_l except that Scenario 3a arises.

First Peiod Game: CIS Separating Equilibrium

Recall that all bids are revealed under CIS. If the separating equilibrium is valid in the first period, then the second period is a Bertrand game. The expected payoffs from bidding p are:

$$\begin{aligned}
\pi_{\text{CIS},c_m}^{\text{sep}}(p) &= (1-l-m)(p-c+1-c) + m(1-F_m(p))(p-c) \\
&= (1-l-mF_m(p))(p-c) + (1-l-m)(1-c) \\
\pi_{\text{CIS},c_l}^{\text{sep}}(p) &= (1-l-m)(p+1) + m(p+c) + l(1-F_l(p))p \\
&= (1-lF_l(p))p + ((1-l-m) + mc).
\end{aligned}$$

Note that, in each equation, the last term is independent of p and will not affect the equilibrium calculations. Ignoring that term, the expected profit expressions are similar to the expected profit in Scenario 2 of the second period game. Therefore, the expressions for the equilibrium bid distributions are identical to that scenario. The infimum of type c_m's strategy set is $p_{l,m}^{\text{sep}} = c + \frac{(1-l-m)}{1-l}(1-c)$. The expected profits are $\pi_{\text{CIS},c_l}^{\text{sep}} = 2(1-l-m+mc)$ and $\pi_{\text{CIS},c_m}^{\text{sep}} = 2(1-l-m)(1-c)$. Similar to the original model, the separating equilibrium does not always exist. Given the opponent's action as fixed, a bidder of c_m type generates a profit of $(1-l-m)(\frac{1-c}{2}+(1-c))+m(1-c)+l(1-c)$, which is greater than $2(1-l-m)(1-c)$ if $3l+3m>1$. Along similar lines, fixing the opponent's action, c_l type bidder fakes as c_h if $3l+(3+4c)m>1$. It is important to note that these conditions are feasible in the space of $0<l<1$ and $0<m<1-l$ and, hence, the proof demonstrates the existence of the faking effect. The exact conditions when a separating equilibrium exists can be obtained from the semipooling equilibrium we characterize next. We can easily rule out a pooling equilibrium.

First Period Game: CIS **Semipooling Equilibrium**

Suppose γ_l and γ_m are the respective probabilities that each of type c_l and c_m bid like a c_h type. Type c_l faking as c_m is not sustained as an equilibrium because of a PVP-lie property. Let $F_{c_l}^{\text{semi}}(p)$ and $F_{c_m}^{\text{semi}}(p)$ be the cdfs of the bid distributions for c_l and c_m types, respectively. It can be shown that the lower support for type c_l is lower than that for type c_m. Assume for now that p_m^{CIS} is the lower end of the bid distribution for type c_m. In this case, the expected profits for the different types and different bids are:

$$\begin{aligned}
\Pi_{c_m}^{\text{semi}}(p|p<1) &= (1-l-m+l\gamma_l+m\gamma_m)(p-c+\Pi_{c_m}^{\text{Sc:3; R}})+m(1-F_{c_m}^{\text{semi}}(p)-\gamma_m)(p-c) && (23)\\
\Pi_{c_m}^{\text{semi}}(p=1) &= (1-l-m+l\gamma_l+m\gamma_m)(\frac{1-c}{2}+\Pi_{c_m}^{\text{Sc:2}})+m(1-\gamma_m)\Pi_{c_m}^{\text{Sc:3; H}}+l(1-\gamma_l)\Pi_{c_m}^{\text{Sc:4; H}} && (24)\\
\Pi_{c_l}^{\text{semi}}(p|p<p_m^{\text{CIS}}) &= (1-l-m+l\gamma_l+m\gamma_m)(p+\Pi_{c_l}^{\text{Sc:4; R}})+m(1-F_{c_m}^{\text{semi}}(p)-\gamma_m)(p+c) && (25)\\
&\quad +l(1-F_{c_l}^{\text{semi}}(p)-\gamma_l)p\\
\Pi_{c_l}^{\text{semi}}(p=1) &= (1-l-m+l\gamma_l+m\gamma_m)(\frac{1}{2}+\Pi_{c_l}^{\text{Sc:2}})+m(1-\gamma_m)\Pi_{c_l}^{\text{Sc:3; H}}+l(1-\gamma_l)\Pi_{c_l}^{\text{Sc:4; H}} && (26)
\end{aligned}$$

Equations 24 and 26 are the profit expressions when faking. We note that $F_{c_l}^{\text{semi}}(\approx 1)=1-\gamma_l$, $F_{c_m}^{\text{semi}}(\approx 1)=1-\gamma_m$, $F_{c_l}^{\text{semi}}(p_m^{\text{CIS}})=1-\gamma_l$, and $F_{c_m}^{\text{semi}}(p_m^{\text{CIS}})=0$; and that the expected profits are the same across all prices in the strategy space, to compute the expected profits. In this case, the beliefs l' and m' are the beliefs held by the opponent of a non-revealing bidder (the opponent can be non-revealing also). Unlike in the previous policy, the beliefs are the same independent of the cost type holding it and it is so because all bids are revealed. Using Bayesian update, $(1-l'-m')=\frac{(1-l-m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$ and $(1-l')=\frac{1-l-m(1-\gamma_m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$. One has to solve the four equations subject to the constraints that $1\geq\gamma_l\geq 0$ and $1\geq\gamma_m\geq 0$ and the updated beliefs should be considered for the appropriate Scenario 4. If a solution does not exist, then the equilibrium is a separating one.