Online E-Companion
Lock-in Strategy in Software Competition
Open Source Software vs. Proprietary Software

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A1 Proof of Lemma 1: No Customer Switches to the Other Under the Optimal Pricing of Vendor A

Suppose that some customers switch to the other software in the second period, then there are three possible scenarios in equilibrium [7]:

(1) Some customers of A switch to O in period 2. Nobody chooses not to upgrade in period 2.

In this case, there are three types of customers: (a) type AA customers, who use A in period 1 and upgrade it in period 2, (b) type OO customers, who use O in both periods and (c) type AO customers, who use A in period 1 but switch to O in period 2 (See Figure A1).

Denote by U_{AA}, U_{AO} and U_{OO} the net surplus of a type AA customer, a type AO customer and a type OO customer respectively. The marginal customer (r, θ) who uses A in the first period must be indifferent of being an AO customer and being an OO customer (that is, $U_{AO} = U_{OO}$).

$$U_{AO} = [r - \theta - p_{A1}] + [r + v_{up} - (1 - \theta) - s],$$

[7] Since a customer's net surplus for Software A (or Software O) is a monotone function of θ, The following case is impossible: some customers switch from Software A to Software O and some customers switch from Software O to Software A.

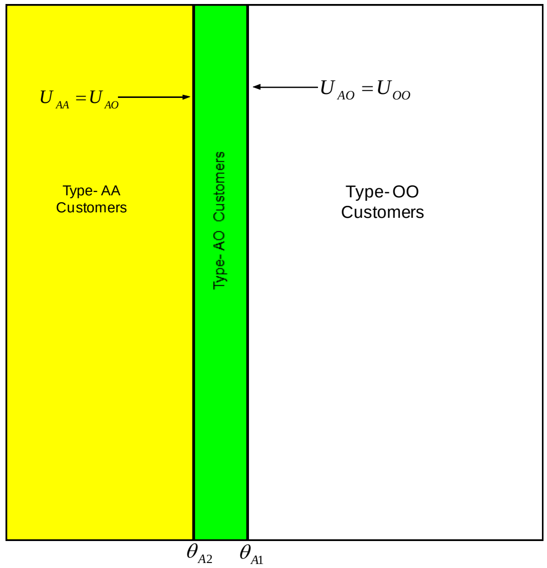


Figure A1: Three types customers: type-AA, type-AO, and type-OO customers

where $[r - \theta - p_{A1}]$ is the net surplus obtained in the first period while $[r + v_{up} - (1 - \theta) - s]$ is the net surplus obtained in the second period, noting that the customer incurs a switching cost of s when it switches from A to O.

$$U_{OO} = [r - (1 - \theta)] + [r + v_{up} - (1 - \theta)],$$

where $[r - (1 - \theta)]$ is the net surplus obtained in the first period while $[r + v_{up} - (1 - \theta)]$ is the net surplus obtained in the second period.

Solving $U_{AO} = U_{OO}$ for the marginal customer, we get $\theta \triangleq \theta_{A1} = \frac{1}{2}(1 - s - p_{A1})$. We have assumed that all customers afford using O, hence $U_{OO} > 0$. It follows that $U_{AO}|_{\theta \leq \theta_{A1}} \geq U_{OO} > 0$. Thus, all customers with $\theta \leq \theta_{A1}$ will use A in period 1. It follows that the demand for A in period 1 is $\theta_{A1} \cdot \frac{R\lambda}{R\lambda} = \theta_{A1}$.

Now, consider the second period. The marginal customer of period 2 must satisfy $U_{AA} = U_{AO}$, where

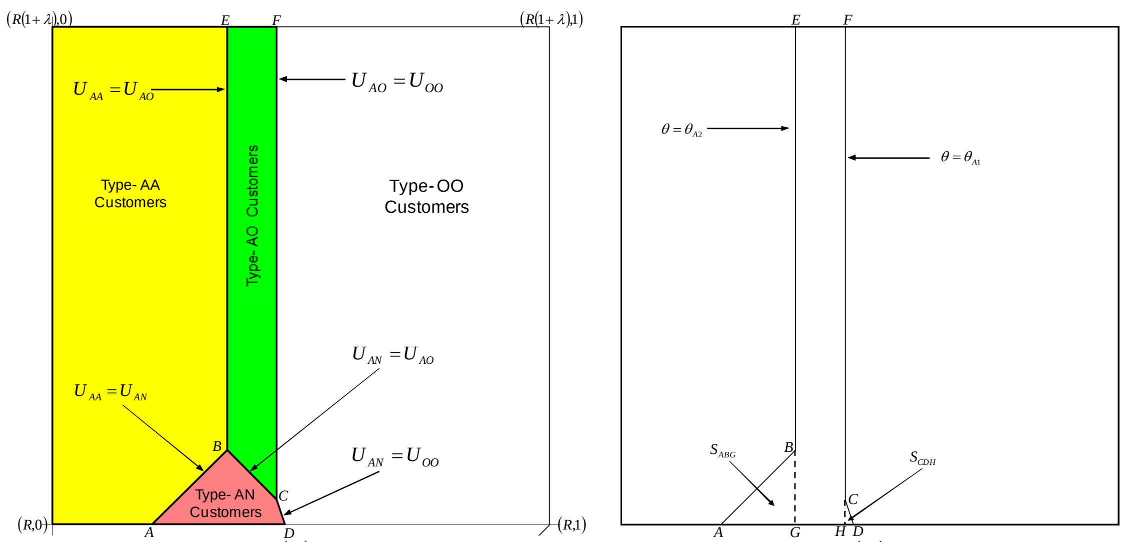
$$U_{AA} = [r - \theta - p_{A1}] + [r + v_{up} - \theta - p_{A2}]$$

Solving $U_{AA} = U_{AO}$ for the marginal customer, we obtain $\theta \triangleq \theta_{A2} = \frac{1}{2}(1 + s - p_{A2})$. Since we have assumed that nobody refuses to upgrade in period 2, the demand for A in period 2 is θ_{A2}. Vendor A's problem in period 2 is $\max_{p_{A2}} p_{A2}\theta_{A2}$. Solving the F.O.C. for p^*_{A2}, we get $p^*_{A2} = \frac{1}{2}(1+s)$. Thus, $\theta^*_{A2} = \frac{1}{4}(1+s)$, $\pi^*_{A2} = \frac{1}{8}(1+s)^2$.

Given the optimal strategy in period 2, vendor A's problem in period 1 is $\max_{p_{A1}} p_{A1}\theta_{A1} + \pi^*_{A2}$, or $\max_{p_{A1}} \frac{1}{2}(1-s-p_{A1})\,p_{A1} + \frac{1}{8}(1+s)^2$. Solving the F.O.C. for p^*_{A1}, we get $p^*_{A1} = \frac{1}{2}(1-s)$ and $\theta^*_{A1} = \frac{1}{4}(1-s)$.

Now, we find a contradiction. Since some customers switch to O, the demand for A in period 1 should be greater than that in period 2. But $\theta^*_{A1} = \frac{1}{4}(1-s) < \frac{1}{4}(1+s) = \theta^*_{A2}$, a contradiction.

(2) Some customers of A switch to O in period 2. And some customers do not upgrade in the second period.



(a) (b)

Figure A2: Four Types of Customers in Other Scenarios

In this case, there are four types of customers: (a) type AA customers, (b) type OO customers, (c) type AO customers, as defined above, and (d) type AN customers who use A in period 1 but does not upgrade it in period 2 (See Figure A2(a)). Denote by U_{AN} the net surplus of a type AN customer.

Since some customers switch to O in period 2 while some customers do not upgrade, there are two types of marginal customers in period 2: (1) those who are indifferent of upgrading A and not upgrading A (that is, $U_{AA} = U_{AN}$) and (2) those who are indifferent of upgrading A and switching to O (that is, $U_{AA} = U_{AO}$).

Consider the first group of marginal customers described above. Type AN customers do not upgrade in period 2, so they obtain a basic utility of v_b. Hence,

$$U_{AN} = [r - \theta - p_{A1}] + v_b,$$

where $[r - \theta - p_{A1}]$ is the net surplus obtained in period 1 while v_b is the net surplus obtained in period 2. Solving $U_{AA} = U_{AN}$, we obtain $\theta = r - p_{A2} + (v_{up} - v_b)$. That is, marginal customers (r, θ) should satisfy $\theta = r - p_{A2} + (v_{up} - v_b)$. This group of customers locates on $\overline{AB}$ of Figure A2(a), where $\overline{AB}$ is defined by $\theta = r - p_{A2} + (v_{up} - v_b)$.

The second group of marginal customers satisfy $U_{AA} = U_{AO}$, or $\theta \triangleq \theta_{A2} = \frac{1}{2}(1 + s - p_{A2})$. This group of customers locates on $\overline{BE}$ of Figure A2(a).

Now, consider period 1. The marginal customers include: (1) those who are indifferent of being type-AO customers and being type-OO customers (that is, $U_{AO} = U_{OO}$). This group of customers locates on $\overline{CF}$ of Figure A2(a), where $\overline{CF}$ is defined by $U_{AO} = U_{OO}$, or $\theta \triangleq \theta_{A1} = \frac{1}{2}(1 - s - p_{A1})$, and (2) those who are indifferent of being type-AN customers and being type-OO customers (that is, $U_{AN} = U_{OO}$). This group of customers locates on $\overline{CD}$ of Figure A2(a), where $\overline{CD}$ is defined by $U_{AN} = U_{OO}$, or $\theta = \frac{1}{3}(2 - r - p_{A1} + v_b - v_{up})$.

Lastly, the marginal customers who are indifferent of being type-AO customers and being type-AN customers locate on $\overline{BC}$ of Figure A2(a), where $\overline{BC}$ is defined by $U_{AO} = U_{AN}$, or $\theta = 1 - r + s + v_b - v_{up}$.

As illustrated by Figure A2(a), type-AO customers locate on the right of the type-AA customers because a customer (r, θ) with a smaller θ is more likely to use A in period 2 than that with a larger θ. Hence, we must have $\theta_{A1} > \theta_{A2}$ in equilibrium. Next, we show a contradiction: $\theta_{A1} < \theta_{A2}$ in equilibrium.

The demand for A in period 2 is $\theta_{A2} - \frac{1}{\lambda R} S_{ABG}$ (see Figure A2(b)), where $\theta_{A2} = \frac{1}{2}(1 + s - p_{A2})$, and $S_{ABG} = \frac{1}{2}[\theta_{A2} - (R - p_{A2} - v_b + v_{up})]^2$ is the size of Triangle ABG. Vendor A's problem in period 2 is $\max_{p_{A2}} p_{A2} \left(\theta_{A2} - \frac{1}{\lambda R} S_{ABG}\right)$. The F.O.C. can be written as $\frac{1}{2}(1 + s) - p_{A2} - \frac{1}{\lambda R} S_{ABG} - p_{A2} \frac{1}{\lambda R}(dS_{ABG}/dp_{A2}) = 0$. The optimal pice in equilibrium, p^*_{A2}, must satisfy $p^*_{A2} = \frac{1}{2}(1 + s) -$

$\frac{1}{\lambda R} S_{ABG} - p_{A2}^* \frac{1}{\lambda R} \left(dS_{ABG}/dp_{A2} \right)$. We claim that $p_{A2}^* < \frac{1}{2}(1+s)$. This is because (1) $S_{ABG} > 0$, and (2) $dS_{ABG}/dp_{A2} = \frac{1}{2}\left[\theta_{A2} - (R - p_{A2} - v_b + v_{up})\right] = \frac{1}{2}\overline{AG} > 0$. Hence, we have $\theta_{A2}^* = \frac{1}{2}\left(1 + s - p_{A2}^*\right) > \frac{1}{4}(1+s)$.

The demand for A in period 1 is $\theta_{A1} + \frac{1}{\lambda R} S_{CHD}$ (see Figure A2(b)), where $\theta_{A1} = \frac{1}{2}(1 - s - p_{A1})$, and $S_{CHD} = \frac{3}{2}\left[\frac{1}{3}(2 - R - p_{A1} + v_b - v_{up}) - \theta_{A1}\right]^2$. Vendor A's problem in period 1 is $\max_{p_{A1}} p_{A1}\left(\theta_{A1} + \frac{1}{\lambda R} S_{CHD}\right) + \pi_{A2}^*\left(p_{A2}^*\right)$. The F.O.C. can be written as $\frac{1}{2}(1-s) - p_{A1} + \frac{1}{\lambda R} S_{CHD} + p_{A1}\frac{1}{\lambda R}\left(dS_{CHD}/dp_{A1}\right) = 0$. Since $S_{CHD} > 0$ and $dS_{CHD}/dp_{A1} = \frac{1}{2}\overline{HD} > 0$, we have $p_{A1}^* > \frac{1}{2}(1-s)$. Hence, $\theta_{A1}^* = \frac{1}{2}\left(1 - s - p_{A1}^*\right) < \frac{1}{4}(1-s)$.

Now, we get a contradiction: $\theta_{A2}^* > \frac{1}{4}(1+s) > \frac{1}{4}(1-s) > \theta_{A1}^*$.

We have used Figure A2(a) to show a contradiction (that is, $\theta_{A2}^* > \theta_{A1}^*$). Using a similar argument as above, we may show $\theta_{A2}^* > \theta_{A1}^*$ in the following scenarios illustrated by Figure A3(a) and Figure A3(b).

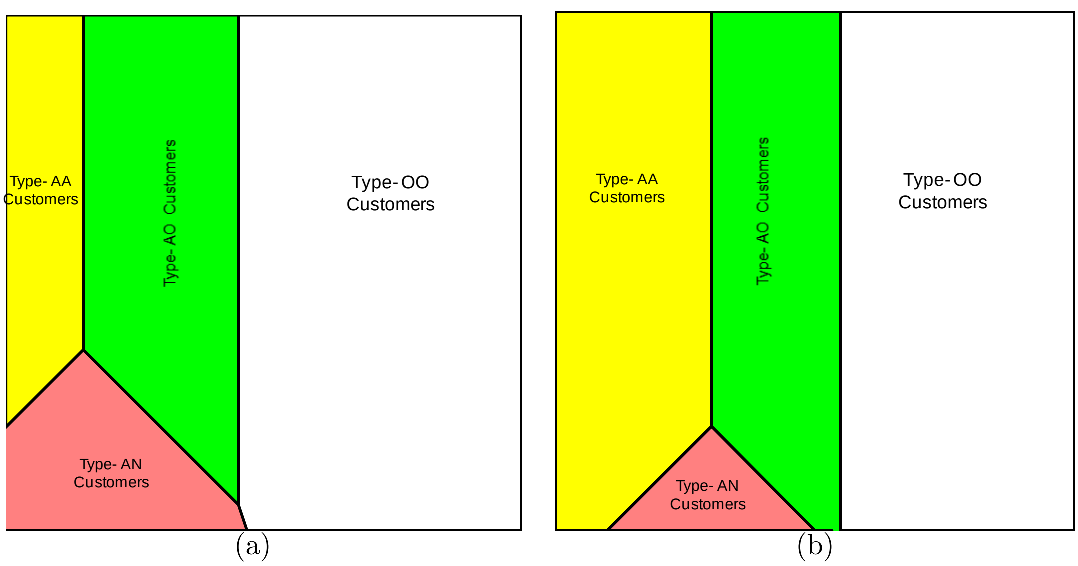


Figure A3: Four Types of Customers in Other Scenarios

(3) Some customers of O switch to A in period 2.

In this case, $\theta_{A2} = \frac{1}{2}(1 - s - p_{A2})$ is obtained by solving $U_{OA} = U_{OO}$, where $U_{OA} = [r - (1-\theta)] + [r + v_{up} - \theta - p_{A2} - s]$; $\theta_{A1} = \frac{1}{2}(1 + s - p_{A1})$ is obtained by solving $U_{AA} = U_{OA}$. Since some customers switch from O to A, we must have $\theta_{A1}^* < \theta_{A2}^*$ in equilibrium. However, using a similar proof as above, we may show that $\theta_{A1}^* > \theta_{A2}^*$ in equilibrium, a contradiction.

A2 Lemma 2: Some Customers Choose not to Upgrade in Period 2

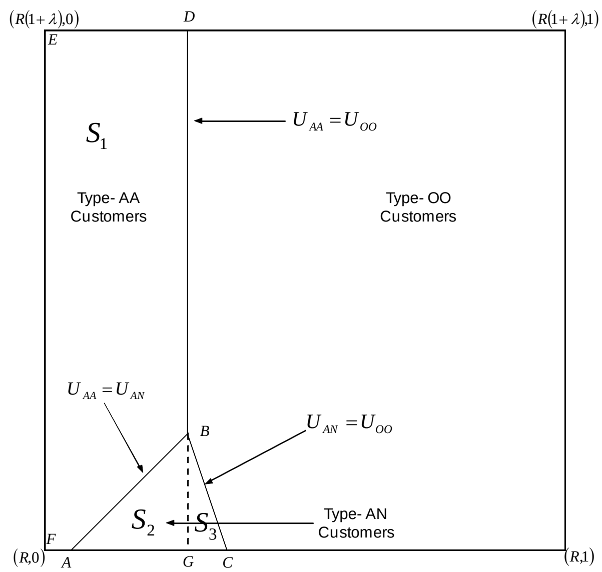


Figure A4: Demand for Software A in Period 1 and Period 2

Let S_1 be the size of Region FABDE (See Figure A4), S_2 be the size of Region ABG, S_3 be the size of Region BGC. Vendor A obtains a profit of $p_{A1} \cdot \frac{1}{\lambda R}(S_1 + S_2 + S_3)$ in period 1 and a profit of $p_{A2} \cdot \frac{1}{\lambda R}(S_1)$ in period 2: $\pi_A^* = p_{A1} \cdot \frac{1}{\lambda R}(S_1 + S_2 + S_3) + p_{A2} \cdot \frac{1}{\lambda R}(S_1)$. Since $S_2 = \frac{1}{2}\left(\overline{BG}\right)^2$, $S_3 = \frac{1}{2}\overline{BG} \cdot \overline{GC} = \frac{1}{2}\overline{BG} \cdot \frac{1}{3}\overline{BG} = \frac{1}{6}\left(\overline{BG}\right)^2$, and $p_{A1} < p_{A2}$, we have: $p_{A1} \cdot S_3 < p_{A2} \cdot S_2$ and thus $\pi_A < \frac{1}{\lambda R}(p_{A1} + p_{A2})(S_1 + S_2) = \frac{1}{4}(p_{A1} + p_{A2})(2 - p_{A1} - p_{A2})$, noting that $U_{AA} = U_{oo}$ gives $\theta_A = \frac{1}{4}(2 - p_{A1} - p_{A2})$. Let $p_A^E = \frac{1}{2}(p_{A1} + p_{A2})$, then $\frac{1}{4}(p_{A1} + p_{A2})(2 - p_{A1} - p_{A2}) = p_A^E\left(1 - p_A^E\right)$. Since $\max_{p_A^E} p_A^E\left(1 - p_A^E\right) = \frac{1}{4}$, we have $\pi_A^* < \frac{1}{4}(p_{A1} + p_{A2})(2 - p_{A1} - p_{A2}) = p_A^E\left(1 - p_A^E\right) \leq \frac{1}{4} = \pi_A^*(s = 0)$.

A3 Proposition 1: Conditions for Two Cases in Equilibrium

Suppose that all of vendor A's customers upgrade in period 2, then the marginal customers satisfy $U_{AA} = U_{OO}$, or $\theta_A = \frac{1}{4}(2 - p_{A1} + p_{A2})$. That is, customers (r, θ) with $\theta \leq \frac{1}{4}(2 - p_{A1} + p_{A2})$ are type AA customers.

If all customers upgrade in equilibrium, then the optimal price in period 2 (p^*_{A2}) must be $\min(1+s-2\theta_A, R-v_b+v_{up}-\theta_A)$. The first price $1+s-2\theta_A$ ensures that nobody switches to O. It is obtained by solving $r+v_{up}-\theta_A-p_{A2}=r+v_{up}-(1-\theta_A)-s$, where the left side of equation is the net surplus of a marginal customer if it upgrades A while the right side of equation is the net surplus if it switches to O. The second price $R-v_b+v_{up}-\theta_A$ ensures that nobody refuses to upgrade. It is obtained by solving $R+v_{up}-\theta_A-p_{A2}=v_b$, where the left side of equation is the net surplus of customer (R,θ_A) who gets the lowest net surplus from upgrading A while the right side of equation is the net surplus of keeping using A without upgrading it.

We claim that if $1+s-2\theta_A > R-v_b+v_{up}-\theta_A$, then some customers will be dropped by vendor A in period 2. That is, *not* all of vendor A's customers upgrade. The reason is as follows. Given θ_A, then $p_{A2}=R-v_b+v_{up}-\theta_A$ is the max price ensuring that nobody switches to O or is dropped by vendor A. Vendor A may increase p_{A2} a little bit to $p_{A2}+\varepsilon$ ($\varepsilon>0, \varepsilon\to 0$) such that nobody switches to O (i.e. $p_{A2}+\varepsilon<1+s-2\theta_A$ still holds) while some customers choose not to upgrade (see Figure A5, the demand for A in period 2 shrinks from Region EFGD to Region EFABD when vendor A increases the price from p_{A2} to $p_{A2}+\varepsilon$). Then $\pi_{A2}(p_{A2}+\varepsilon) = (p_{A2}+\varepsilon)\left\{\theta_A-\frac{1}{2\lambda R}\left[\theta_A-(R-p_{A2}-\varepsilon-v_b+v_{up})\right]^2\right\}$. Inserting $p_{A2}=R-v_b+v_{up}-\theta_A$ in $\pi_{A2}(p_{A2}+\varepsilon)$, we obtain $\pi_{A2}(p_{A2}+\varepsilon)=(R+\varepsilon-v_b+v_{up}-\theta_A)\left(\theta_A-\frac{\varepsilon^2}{2R\lambda}\right)$. It can be shown that $\lim_{\varepsilon\to 0}\left[\pi_{A2}(p_{A2}+\varepsilon)-\pi_{A2}(p_{A2})\right]/\varepsilon=\theta_A>0$. This means that vendor A has incentives to charge a price p_{A2} ($p_{A2}>R-v_b+v_{up}-\theta_A$) such that some customers are dropped by vendor A in period 2. This violates the assumption that all of vendor A's customers upgrade in period 2.

Therefore, we must have $\min(1+s-2\theta_A, R-v_b+v_{up}-\theta_A)=1+s-2\theta_A$. We claim that $p^*_{A2}=1+s-2\theta_A$. This is because (1) if $p^*_{A2}>1+s-2\theta_A$, then some customers switch to O, violating the assumption that all of vendor A's customers upgrade in period 2. And we have shown that customers do not switch in equilibrium (see Lemma 1), and (2) if $p^*_{A2}<1+s-2\theta_A$, the demand for A do not change[8] while vendor A obtains a lower profit than otherwise it charges $p^*_{A2}=1+s-2\theta_A$.

Foresighted customers know vendor A's optimal pricing strategy in period 2. Thus, $\theta_A =$

[8] Again, we have shown in Lemma 1 that customers do not switch in equilibrium. So vendor A would not charge a low price such that some customers switch to A from O in period 2.

$\frac{1}{4}(2 - p_{A1} + p^*_{A2})$ holds in period 1. Inserting $p^*_{A2} = 1 + s - 2\theta_A$ in $\theta_A = \frac{1}{4}(2 - p_{A1} + p^*_{A2})$, we obtain $\theta_A = \frac{1}{2}(1 - s - p_{A1})$. Thus, $\pi_A = p_{A1}\theta_A + p_{A2}\theta_A = (p_{A1} + 1 + s - 2(1 - s - p_{A1})) \cdot \frac{1}{2}(1 - s - p_{A1})$. Solving the F.O.C. $d\pi_A/dp_{A1} = 0$, we get $p^*_{A1} = \frac{1}{2} - s$. It follows that $p^*_{A2} = \frac{1}{2} + s$, $\theta^*_A = \frac{1}{4}$, $\pi^*_A = \frac{1}{4}$.

Now, we examine the necessary condition: $\min(1 + s - 2\theta_A, R - v_b + v_{up} - \theta_A) = 1 + s - 2\theta_A$, which implies that $1 + s - 2\theta_A \leq R - v_b + v_{up} - \theta_A$. Inserting $\theta^*_A = \frac{1}{4}$ in this inequality, we get $s \leq R - v_b + v_{up} - \frac{3}{4}$.

We claim that $s \leq R - v_b + v_{up} - \frac{3}{4}$ is also the sufficient condition for the optimal strategy described above: $p^*_{A1} = \frac{1}{2} - s$, $p^*_{A2} = \frac{1}{2} + s$. If it is not an optimal strategy, then the optimal strategy should be characterized by some A's customers not upgrading A in period 2. The reason is that $\left\{p^*_{A1} = \frac{1}{2} - s, p^*_{A2} = \frac{1}{2} + s\right\}$ has already been the optimal strategy under the assumption that all vendor A's customers upgrade. However, as shown in Lemma 2, if some customers choose not to upgrade, then vendor A obtains a profit lower than $\frac{1}{4}$. Thus, $\left\{p^*_{A1} = \frac{1}{2} - s, p^*_{A2} = \frac{1}{2} + s\right\}$ must be the global optimal strategy for vendor A.

When $s > R - v_b + v_{up} - \frac{3}{4}$, the necessary condition for "all A's customers upgrade" is violated. Thus, the equilibrium must be characterized by some A's customers choose not to upgrade. According to Lemma 2, we have $\pi^*_A < \frac{1}{4}$.

A4 Proposition 2: Vendor A's Profit in $s > s_L$

When $s > R - v_b + v_{up} - \frac{3}{4}$, some of vendor A's customers choose not to upgrade. Since we have shown in Lemma 1 that nobody switches to O in equilibrium, there are only two possible cases in period 2: (1) vendor A charges a price such that high-reservation-utility customers are indifferent of upgrading A and switching to O, (2) vendor A charges a monopolistic price when switching cost is so high that high-reservation-utility customers are locked in by A in period 1. Denote by ST_1 the optimal pricing strategy in the first case and ST_2 for the second case.

That is, vendor A's problem in period 2 is $\max_{p_{A2}} \pi_{A2}(p_{A2})$, $s.t. p_{A2} \leq 1 + s - 2\theta_A$, where $\theta_A = \frac{1}{4}(2 - p_{A1} + p_{A2})$ is the solution of $U_{AA} = U_{OO}$. $p_{A2} \leq 1 + s - 2\theta_A$ ensures that high-reservation-utility marginal customers do not switch to O. If $p_{A2} \leq 1 + s - 2\theta_A$ is binding, then $p^*_{A2} = 1 + s - 2\theta_A$. This is the case ST_1. If $p_{A2} \leq 1 + s - 2\theta_A$ is non-binding, then the switching cost is so high that vendor A charges a monopolistic price in period 2 (i.e. p^*_{A2} is an interior solution of $\max_{p_{A2}} \pi_{A2}(p_{A2})$). This is the case ST_2.

It is infeasible to obtain a simple analytical solution for ST_1 and ST_2. We briefly explain how to get A's optimal pricing strategy for ST_1 and ST_2. For ST_1, let $p^*_{A2} = 1 + s - 2\theta_A$. Inserting it in $\theta_A = \frac{1}{4}(2 - p_{A1} + p^*_{A2})$, we get

$$\theta_A = \frac{1}{2}(1 - s - p_{A1}) \tag{A1}$$

and

$$p^*_{A2} = p_{A1} + 2s. \tag{A2}$$

The demand for A in period 2 (d_{A2}) is Region EFABD in Figure A4. d_{A2} can be written as a function of p_{A2} and θ_A. Inserting $\theta_A = \frac{1}{2}(1 - s - p_{A1})$ and $p^*_{A2} = p_{A1} + 2s$ in d_{A2}, we get $d_{A2}(p_{A1})$. The demand for A in period 1 (d_{A1}) is Region Region EFCBD in Figure A4. Again, we can write d_{A1} as a function of p_{A1}. Thus, $\pi_A = p_{A1}d_{A1} + p_{A2}d_{A2}$ can be written as a function of p_{A1}. Solving the F.O.C. $d\pi_A/dp_{A1} = 0$, we may get p^*_{A1}. Inserting p^*_{A1} in $p^*_{A2}(p_{A1})$, $\theta_A(p_{A1})$, and $\pi_A(p_{A1})$, we may get p^*_{A2}, θ^*_A and π^*_A.

Although we cannot get a simple analytical solution for ST_1, we can show that π^*_A derived from ST_1 is a decreasing function of s. Given s, let $p^*_{A1}(s)$ be the optimal price in period 1. Let s decrease to $s - \varepsilon$ ($\varepsilon > 0, \varepsilon \to 0$). We want to show that $\pi^*_A(s-\varepsilon) > \pi^*_A(s)$. We will show that $\pi^*_A(s-\varepsilon)|_{p_{A1}(s-\varepsilon)=p^*_{A1}(s)+\varepsilon} > \pi^*_A(s)$, vendor A obtains a higher profit by charging $p^*_{A1}(s) + \varepsilon$ in period 1. However, $p^*_{A1}(s) + \varepsilon$ is not necessarily the optimal price for the case of lower switching cost ($s - \varepsilon$), so we must have $\pi^*_A(s-\varepsilon)|_{p^*_{A1}(s-\varepsilon)} \geq \pi^*_A(s-\varepsilon)|_{p_{A1}(s-\varepsilon)=p^*_{A1}(s)+\varepsilon} > \pi^*_A(s)$.

According to Eq.(A1) $\theta^*_A(s) = \frac{1}{2}(1 - s - p^*_{A1}(s))$. When s drops to $s - \varepsilon$ and vendor A chargers $p^*_{A1}(s) + \varepsilon$, we obtain $\theta^*_A(s-\varepsilon) = \frac{1}{2}[1 - (s-\varepsilon) - (p^*_{A1}(s) + \varepsilon)] = \theta^*_A(s)$. The marginal high-reservation-utility customers do not change. But, the marginal low-reservation-utility customers who are indifferent between being type AN customers and type AA customers change. Since $p^*_{A1}(s-\varepsilon) = p^*_{A1}(s) + \varepsilon$, we have $p^*_{A2}(s-\varepsilon) = p^*_{A1}(s-\varepsilon) + 2(s-\varepsilon) = p^*_{A1}(s) + \varepsilon + 2(s-\varepsilon) = p^*_{A1}(s) + 2s - \varepsilon$ (see Eq.(A2)). Figure A6 illustrates the change of demand for A when s changes to $s - \varepsilon$ and p_{A1} increases to $p^*_{A1}(s) + \varepsilon$. The location of marginal type AN customers shifts down

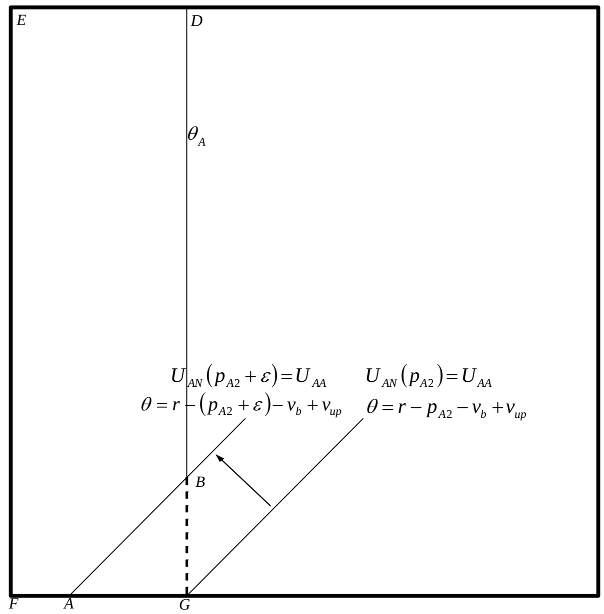


Figure A5: Software A provider increases p_{A2} by ε

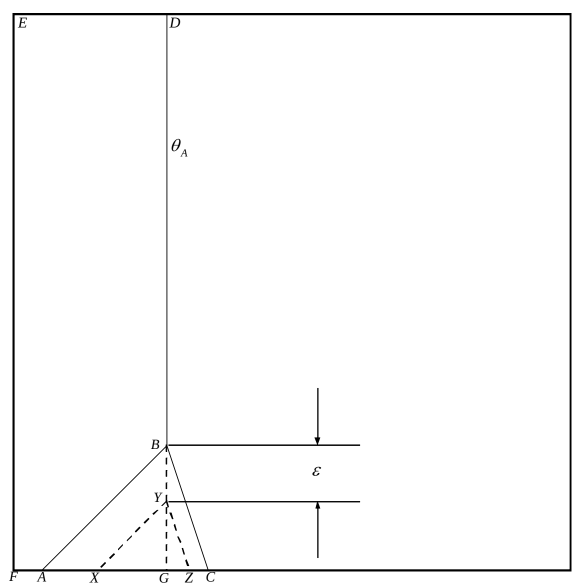


Figure A6: Demand for Software A Changes when s Reduces to $s-\varepsilon$

from $\overline{AB}$ to $\overline{XY}$ and from $\overline{BC}$ to $\overline{YZ}$. The length of $\overline{BY}$ is ε. Note that the slope of $\overline{AB}$ and $\overline{XY}$ is 1 while the slope of $\overline{BC}$ and $\overline{YZ}$ is -3 (see Figure A4), we have:

$$\pi_A^*(s) = p_{A1}^*(s)\left[\theta_A + \frac{1}{6\lambda R}\overline{BG}^2\right] + \left[p_{A1}^*(s) + 2s\right]\left[\theta_A - \frac{1}{2\lambda R}\overline{BG}^2\right]$$
$$\pi_A(s-\varepsilon) = \left[p_{A1}^*(s) + \varepsilon\right]\left[\theta_A + \frac{1}{6\lambda R}\left(\overline{BG}-\varepsilon\right)^2\right] + \left[p_{A1}^*(s) + 2s - \varepsilon\right]\left[\theta_A - \frac{1}{2\lambda R}\left(\overline{BG}-\varepsilon\right)^2\right]$$

$\lim_{\varepsilon\to 0}\left[\pi_A(s-\varepsilon) - \pi_A^*(s)\right]/\varepsilon = 2\overline{BG}\left(\overline{BG} + 3s + p_{A1}^*(s)\right)/(3R\lambda) > 0$. The last inequality holds because $p_{A2}^*(s) = 2s + p_{A1}^*(s) > 0$, otherwise $\pi_A^*(s) < 0$. Therefore, $\pi_A^*(s) < \pi_A(s-\varepsilon)\,|_{p_{A1}(s-\varepsilon)=p_{A1}^*(s)+\varepsilon} \leq \pi_A^*(s-\varepsilon)$. $\pi_A^*(s)$ derived from ST_1 is a decreasing function of s.

Now consider ST_2, where p_{A2}^* is an interior solution for $\max_{p_{A2}}\pi_{A2}(p_{A2})$. Given θ_A, d_{A2} can be written as a function of p_{A2} and θ_A. Thus, $\pi_{A2} = p_{A2}\cdot d_{A2}(p_{A2},\theta_A) = \pi_{A2}(p_{A2},\theta_A)$. Solving the F.O.C. $d\pi_{A2}/dp_{A2} = 0$, we may get p_{A2}^*, which is a function of θ_A. And π_{A2}^* can be written as a function of θ_A. Inserting $p_{A2}^*(\theta_A)$ in $\theta_A = \frac{1}{4}(2 - p_{A1} + p_{A2})$, we may get $\theta_A = \theta_A(p_{A1})$. d_{A1} can be written as a function of θ_A and p_{A1}. Inserting $\theta_A(p_{A1})$ in $\pi_A = p_{A1}d_{A1} + p_{A2}d_{A2}$, we get $\pi_A = p_{A1}\cdot d_{A1}(p_{A1},\theta_A(p_{A1})) + \pi_{A2}^*(\theta_A(p_{A1}))$, which is a function of p_{A1}. Solving the F.O.C. $d\pi_A/dp_{A1} = 0$, we may get p_{A1}^*. Inserting p_{A1}^* in $p_{A2}^*(p_{A1})$, $\theta_A(p_{A1})$, and $\pi_A(p_{A1})$, we may get p_{A2}^*, θ_A^* and π_A^*. The necessary condition for this solution is $p_{A2}^* \leq 1 + s - 2\theta_A^*$. Let $\hat{s} = p_{A2}^* + 2\theta_A^* - 1$, then when $s \geq \hat{s}$, ST_2 is a candidate optimal strategy for vendor A. Since marginal customers do not depend on s in ST_2, π_A^* derived from ST_2 does not depend on s - $d\pi_A^*/ds = 0$. Let π_A^* for ST_2 be $\pi_A^{LOCK-IN}$.

The global optimal strategy for vendor A is comparing π_A^* derived from ST_1 and $\pi_A^{LOCK-IN}$ and then taking the pricing strategy that results in a higher profit. We claim that when $s = \hat{s}$, π_A^* derived from ST_1 is no less than $\pi_A^{LOCK-IN}$. The reason is as follows. When $s = \hat{s}$, $p_{A2}^*(\hat{s})$ in ST_2 satisfies $p_{A2} = 1 + s - 2\theta_A$. But, the optimal strategy in ST_1 is: given $p_{A2} = 1 + s - 2\theta_A$, search for p_{A1}^* to maximize π_A. Thus, $\{p_{A2}^*(\hat{s}), p_{A1}^*(\hat{s})\}$ derived from ST_2 is only a candidate strategy in ST_1, not necessarily the global optimal strategy for ST_1. Thus, we prove the claim.

Since $\pi_A^*(s)$ is a decreasing function of s in ST_1, $\pi_A^*(s) \to \frac{1}{4}$ when $s \to R - v_b + v_{up} - \frac{3}{4}$, and $\pi_A^{LOCK-IN} < \frac{1}{4}$ (see Lemma 2 and Proposition 1), there must exist an s_H such that π_A^* derived from ST_1 equals to $\pi_A^{LOCK-IN}$. We have shown that π_A^* derived from ST_1 is no less than $\pi_A^{LOCK-IN}$ at

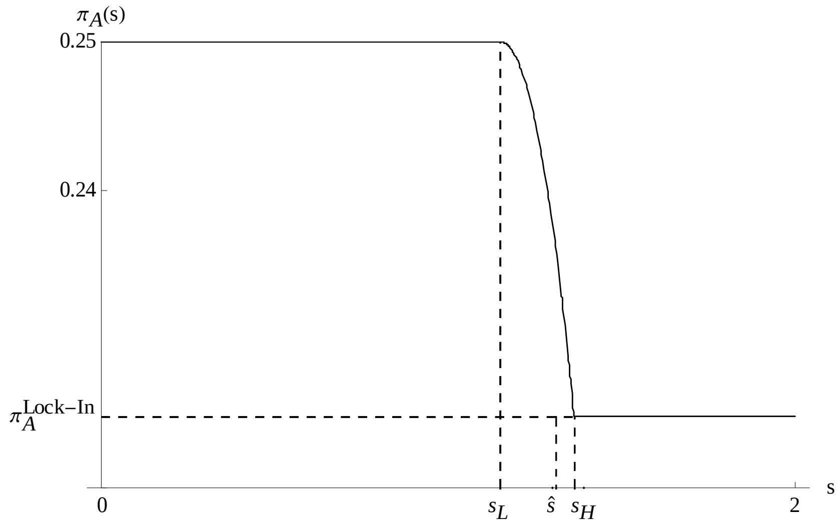


Figure A7: $\pi_A^*(s)$: a non-increasing function of switching cost (s)

$s = \hat{s}$. Hence, we must have $s_H \geq \hat{s}$. The necessary condition for ST_2 holds when $s \geq s_H$.

Figure A7 illustrates the above results. When $s\tilde{}\,(s_L, \hat{s})$, only ST_1 gives the optimal pricing strategy. When $s\tilde{}[\hat{s}, s_H)$, both pricing strategies obtained from ST_1 and ST_2 are feasible, but π_A^* in ST_1 is greater than that in ST_2. When $s\tilde{}[s_H, +\infty)$, both pricing strategies obtained from ST_1 and ST_2 are feasible, but π_A^* in ST_2 is greater than that in ST_1.

A5 Proposition 3: Myopic Customers

When customers are myopic, they only consider the first period payoff. Thus, the marginal customers (r, θ_{A1}) who are indifferent from adopting A and adopting O in period 1 satisfy:

$$r - p_{A1} - \theta_{A1} = r - (1 - \theta_{A1}), \text{ or } \theta_{A1} = (1 - p_{A1})/2.$$

We assume that $R - p_{A1} - \theta_{A1} \geq 0$ (A is affordable to the customer with the lowest reservation utility). Later we will see that this assumption holds in equilibrium. Then the demand for A is θ_{A1} in period 1. Since $\theta_{A1} \in [0, 1]$, it follows that $p_{A1} \in [-1, 1]$. Further,

$$\pi_{A1} = p_{A1}\theta_{A1} = p_{A1}(1 - p_{A1})/2,\ d\pi_{A1}/dp_{A1} = 1/2 - p_{A1}, \text{ and } d\theta_{A1}/dp_{A1} = -1/2.$$

In period 2, customers (r,θ) with $\{\theta \le \theta_{A2} = \theta_{A1}, r \ge p_{A2} + \theta_{A2}\}$ upgrade while other locked-in customers choose not to upgrade.

The basic idea of the next step is showing that $\pi_A|_{p_{A1}=-1} \ge \pi_A^N$, then it must follow that $\pi_A \ge \pi_A|_{p_{A1}=-1} \ge \pi_A^N$.

$\pi_A|_{p_{A1}=-1} = -1+\pi_{A2}|_{\theta_{A2=1}} = -1+(2R-2+w)\left[6\lambda R-(R-1)^2+(R-1)w\right]/(27\lambda R)$, where $w = \sqrt{(2R-1)^2+6\lambda R}$. It straightforward to show that $d\left(\pi_A|_{p_{A1}=-1}\right)/d\lambda = \frac{(1-R+w)^2(2R-2+w)w}{54\lambda^2 Rw} > 0$, and $\pi_A|_{p_{A1}=-1,\lambda\to 0} = R-2$. Apparently $\pi_A \ge \pi_A|_{p_{A1}=-1} \ge \pi_A^N$ holds when $R \ge 2.25$. Numerical results for $R \in [1, 2.25]$ show that $\pi_A \ge \pi_A^N$.

A6 Lemma 3: Equilibrium Prices and Profits for $R >> 1$

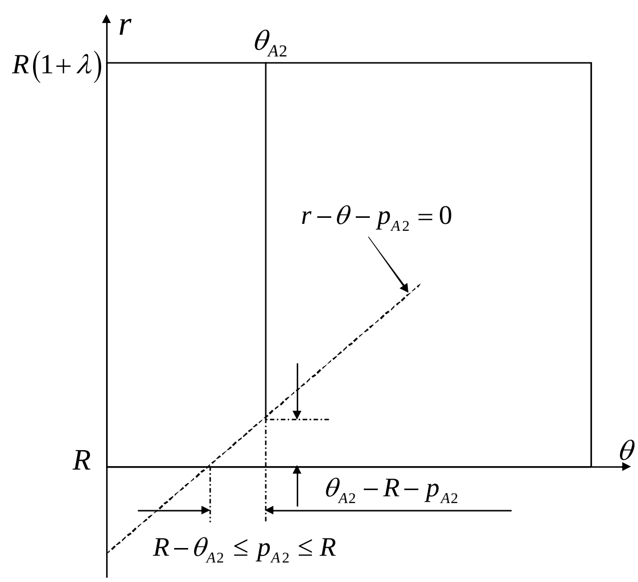


Figure A8: Optimal p_{A2} given θ_{A2}

Given θ_{A2}, it can be shown that the optimal price in period 2 is

$$p_{A2}^* = \frac{1}{3}\left(2R - 2\theta_{A2} + \sqrt{(R-\theta_{A2})^2 + 6\lambda R\theta_{A2}}\right), \tag{A3}$$

which maximizes $\pi_{A2}(p_{A2}) = p_{A2}\left[\theta_{A2} - \frac{1}{2R\lambda}\left(\theta_{A2} - (R-p_{A2})\right)^2\right]$ (see Figure A8). And it can be verified that $0 < R - p_{A2}^{*} < \theta_{A2}$. Solving

$$\begin{cases} p_{A2} = \frac{1}{3}\left(2R - 2\theta_{A2} + \sqrt{\left(R - \theta_{A2}\right)^2 + 6\lambda R\theta_{A2}}\right) \\ \theta_{A2} = \frac{1}{4}\left(2 - p_{A1} - p_{A2}\right) \end{cases}$$

yields

$$\begin{aligned} p_{A2} &= \tfrac{1}{33}\left[7\left(p_{A1} - 2\right) + 4R\left(7 - \lambda\right)\right] \\ &+ \tfrac{4}{33}\sqrt{4 + 8R\left(5\lambda - 2\right) + R^2\left(\lambda^2 - 14\lambda + 16\right) - 4\left(5R\lambda - 2R + 1\right)p_{A1} + p_{A1}^2} \end{aligned} \tag{A4}$$

Thus, A's second period price and profit can be expressed as $p_{A2}\left(p_{A1}\right)$ and $\pi_{A2}\left(p_{A1}\right)$.

The first period profit can be written as

$$\pi_{A1} = \frac{1}{\lambda R}\left[\frac{1}{2}\left(R' - R\right)\left(\theta_{A1} - \theta_{A2}\right) + R\lambda\theta_{A2}\right]p_{A1},$$

where $\theta_{A1} = \frac{1}{3}\left(2 - R - p_{A1}\right)$, $\theta_{A2} = \frac{1}{4}\left(2 - p_{A1} - p_{A2}\right)$, and $R' = \frac{1}{4}\left(2 - p_{A1} + 3p_{A2}\right)$, which solves $\frac{1}{3}\left(2 - R' - p_{A1}\right) = \frac{1}{4}\left(2 - p_{A1} - p_{A2}\right)$. After substituting eq.(A4) into π_{A1}, it can be expressed as a function of p_{A1}: $\pi_{A1}\left(p_{A1}\right)$.

It can be shown that $R - \theta_{A2} \leq p_{A2} \leq R$ when $R\lambda \geq \theta_{A2}$. Noting that R is sufficiently large, we have $R - 1 \leq p_{A2} \leq R$. Using $\theta_{A2} = \frac{1}{4}\left(2 - p_{A1} - p_{A2}\right)$, we have $-2 - R \leq p_{A1} \leq 3 - R$. Let $\pi_A = \pi_{A1}\left(p_{A1}\right) + \pi_{A2}\left(p_{A1}\right)$ and $p_{A1} = t - R$. Solving $d\pi_A/dp_{A1} = 0$ and noting that

$$\sqrt{4 + 16R\left(R - 1\right) + 40R\lambda - 14R^2\lambda + R^2\lambda^2 - \left(R - t\right)\left(7R - 4 - 20R\lambda + t\right)}|_{R>>1} \approx R\left(3 + \lambda\right) - a,$$

where $a = \left(t - 2\right)\left(10\lambda - 3\right)/\left(\lambda + 3\right)$, we have

$$d\pi_A/dp_{A1}|_{R>>1} \approx 2\left[15 + \lambda - 2\left(6 + \lambda\right)t\right]/3\left(3 + \lambda\right)^2 = 0.$$

Thus,

$$p_{A1} = \left(15 + \lambda\right)/\left[2\left(6 + \lambda\right)\right] - R. \tag{A5}$$

It is easy to verify that $\left(d^2\pi_{A2}/dp_{A1}^2|_{R>>1}\right) = -4\left(6 + \lambda\right)/\left[3\left(3 + \lambda\right)^2\right] < 0$. Substituting eq.(A5) into eq.(A4) obtains p_{A2}. Substituting p_{A1} and p_{A2} into $\theta = \theta_{A2} = \frac{1}{4}(2 - p_{A1} - p_{A2})$ obtains θ_{A2}. Using p_{A1}, p_{A2} and θ_{A2}, we can get π_A.

A7 Proposition 4: The Lock-in Effect on Individual Customers

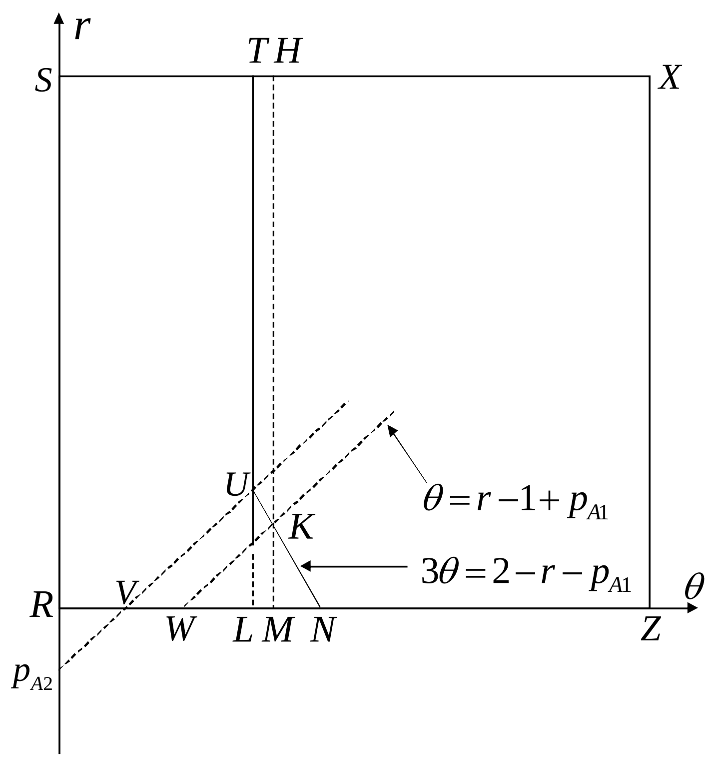


Figure A9: Individual Customer Analysis

Consider the case A vs. O (See Figure A9), we use S^N_{AO} to represent the scenario where there are no switching costs and S^L_{AO} to represent the identical market with lock-in. there are five types of customers: type-1, those who adopt A in S^N_{AO} and S^L_{AO}, and they can afford p_{A2} in S^L_{AO} (area STUVRS); type-2, those who adopt A in S^N_{AO} and S^L_{AO}, but can not afford p_{A2} in S^L_{AO} (area UKMVU); type-3, those who adopt A in S^N_{AO} but adopt O in S^L_{AO} (area THKUT); type-4, those who adopt O in S^N_{AO} but adopt A in S^L_{AO} (area KMNK); and type-5, those who adopt O in S^N_{AO} and S^L_{AO} (the remaining area HKNZXH).

The lock-in effect on type 1 customers is

$$\Delta CS(r,\theta) = (2r - 2\theta - p_{A1} - p_{A2}) - \left(2r - 2\theta - p_{A1}^N - p_{A2}^N\right) < 0,$$

Here p_{A1}, p_{A2} can be obtained from Lemma 3, and $p_{A1}^N = p_{A2}^N = 1/2$. Hence type 1 customers are worse off. The lock-in effect on type 2 customers is

$$\Delta CS(r,\theta) = (r - \theta - p_{A1}) - \left(2r - 2\theta - p_{A1}^N - p_{A2}^N\right) = 1 - p_{A1} + \theta - r.$$

If $\theta > r - 1 + p_{A1}$, then $\Delta CS(r,\theta) > 0$. That is, customers in area WKMW is better off with lock-in. Using the similar argument on other types of customers (type 3 ~type 5 customers), we can prove that customers in area KMNK are also better off with lock-in. Thus, customers in area KNWK are better off with lock-in in the competition A vs. O.

To summarize, for low-reservation-utility customers, vendor lock-in benefits those who locate in area KNWK but hurts those who locate in area VWKUV. For high-reservation-utility customers, vendor lock-in always hurts them.

A8 Proposition 5: Welfare Analysis

Consider the lock-in case. In period 1, the consumer surplus is a sum of A's customers' surplus and O's customers' surplus: $CS_1 = CS_{A1} + CS_{O1}$,

$$CS_{A1} = \int_R^{r_c} \int_0^{(2-r-p_{A1})/3} (r - \theta - p_{A1}) \cdot f(\theta, r)\, d\theta dr + \int_{r_c}^{R(1+\lambda)} \int_0^{\theta_{A2}} (r - \theta - p_{A1}) \cdot f(\theta, r)\, d\theta dr,$$

$$CS_{O1} = \int_R^{r_c} \int_{(2-r-p_{A1})/3}^1 (r - 1 + \theta) \cdot f(\theta, r)\, d\theta dr + \int_{r_c}^{R(1+\lambda)} \int_{\theta_{A2}}^1 (r - 1 + \theta) \cdot f(\theta, r)\, d\theta dr,$$

where $(r_c, \theta_{A2}) = ((2 - p_{A1} + 3p_{A2})/4, (2 - p_{A1} - p_{A2})/4)$, $f(\theta, r) = 1/(\lambda R)$ is the probability density function of customers, and p_{A1}, p_{A2} are given by Lemma 3. Similarly, in period 2, $CS_2 =$

$CS_{A2} + CS_{O2}$,

$$CS_{A2} = \int_{R}^{p_{A2}+\theta_{A2}} \int_{0}^{r-p_{A2}} (r - \theta - p_{A2}) \cdot f(\theta, r)\, d\theta dr + \int_{p_{A2}+\theta_{A2}}^{R(1+\lambda)} \int_{0}^{\theta_{A2}} (r - \theta - p_{A2}) \cdot f(\theta, r)\, d\theta dr,$$

$$CS_{O2} = CS_{O1}.$$

The total consumer surplus $CS = CS_1 + CS_2$.

Now, consider the identical market without any switching costs.

$$CS^N = 2\int_{R}^{R(1+\lambda)} \int_{0}^{\theta_A} (r - \theta - p_A)\, f(\theta, r)\, d\theta dr + 2\int_{R}^{R(1+\lambda)} \int_{\theta_A}^{1} (r - 1 + \theta)\, f(\theta, r)\, d\theta dr,$$

where $\theta_A = 1/4$, $p_A = 1/2$. Lastly we have

$$CS^N - CS|_{R>>1} = [\lambda (12 + \lambda)] / \left[8 (6 + \lambda)^2\right] > 0$$

The open source community always gets a zero profit $\pi_O = 0$. From Lemma 3, A's profit is $\pi_A = 3/[2(6 + \lambda)]$ in the market with lock-in or $\pi_A^N = 1/4$ in the market without any switching costs. Thus, $PS^N - PS = \lambda/[4(6 + \lambda)] > 0$.

Apparently, the social welfare of the market without any switching costs is greater than that with lock-in.