Online Supplement to
Blog, Blogger, and the Firm: Can Negative Posts by Employees Lead to Positive Outcomes?

APPENDIX A: MEASURING THE WEIGHTED CITATIONS OF A POST

Consider the citations to a post as shown below.

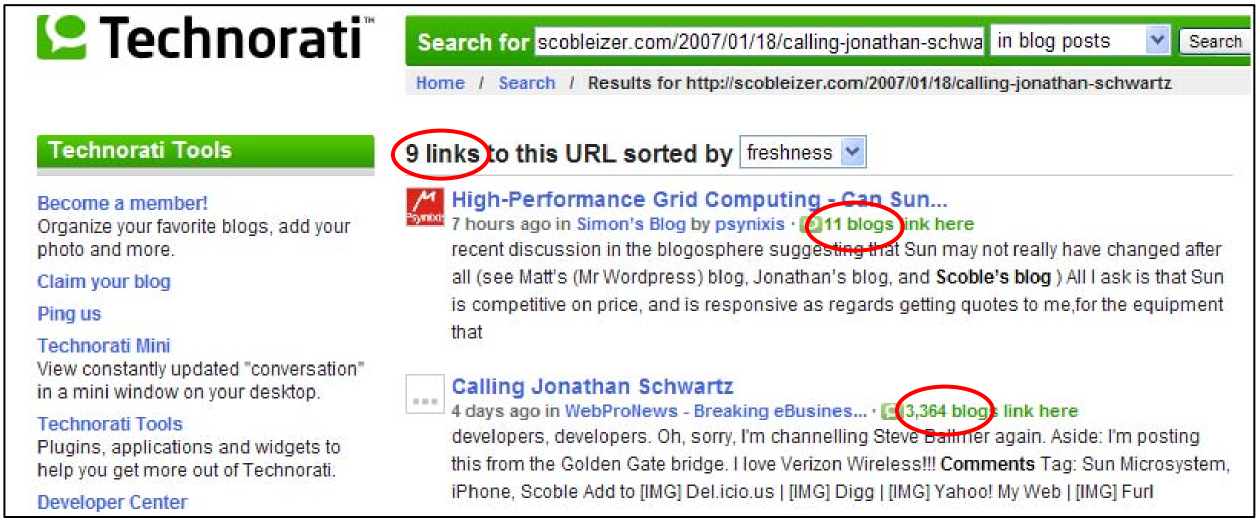

Figure A-1: Snapshot of the blog search engine, Technorati

Influence of a post, $S \quad \# of\ \ citations + \sum \ln(\#\ of\ \ 2^{nd}\ \ level\ \ citations)$

For example, influence of the above post, $S \equiv 9 + \ln(11) + \ln(3364) + \ldots$.

The literature suggests two ways of accounting for second order citations: exponential smooth or logarithmic smoothing (Shirky 2003; Drezner and Farrell 2004). We have taken the log of second-level citations. Our results are robust regarding the choice of method for measuring weighted citations to a post. We have also done the analysis without considering second-level citations. The results are the same, except that the coefficients are larger without considering second-level citations than those with these citations.

APPENDIX B: PROOF OF PROPOSITION

The proposition states that there exists a cutoff value of the average negative influence $\bar{I}^{\,*}$ such that (a) for all values of the average negative influence on a reader $\bar{I}^{-}\varepsilon\left(0,\bar{I}^{-*}\right)$, the decision of allowing negative posts creates more average net positive influence on readers than the decision of no negative posting, whereas (b) for $\bar{I}$ $\quad \bar{I}^{\,*}$, the decision of no negative posting offers a higher average net positive influence on readers than the decision of allowing negative posts. To prove this proposition, we need to prove that $\mathrm{NPI_n} - \mathrm{NPI_p}$ is positive and concave ($\frac{\partial^2(\mathrm{NPI_n} - \mathrm{NPI_p})}{\partial^2(\bar{I}^{-})}$ <0. Note that $\mathrm{NPI_p}$ is always positive. Hence if we prove that the difference between the two NPI is concave it is a sufficient to prove the proposition.

For simplicity we would assume that vn=vm=1.Once we assume this Weighted citations of a blog is exactly equal to its influence. Note that if vn>vm would just reduce the space under which allowing negative posts is optimal. If vm>vn it would increase the space under which allowing negative posts is optimal.

For Proof simplification, we would just work with the reduced model where

$$\bar{P}_n = \frac{1}{1 \quad \beta_1}\left(\beta_9 + \beta_2\bar{W}_n + (\beta_5\bar{W}_n\ )\left(\frac{\bar{W}_n^{-}}{\bar{W}_n + \beta_8\bar{W}_n^{+}}\right)\right)$$

$$\bar{P}_p = \frac{1}{1-\beta_1}\left(\beta_9 + \beta_2\bar{W}_p + (\beta_5\bar{W}_p\ )\left(\frac{\bar{W}_p^{-}}{\bar{W}_p^{-} + \beta_8\bar{W}_p^{+}}\right)\right)$$

Note that this assumption is quite conservative as it reduces the effect of negative post in increasing page views to as much as that by positive or neutral post. This assumption biases $\mathrm{NPI_n}$ towards zero. Under these assumptions:

$$\frac{\partial^2(NPI_n - NPI_p)}{\partial\bar{I}^{-2}} = -\frac{2\left(\beta_2\bar{I}^{-3} + 3\beta_2\beta_4\bar{I}^{-2}\bar{I}^{+} + 3\beta_2\beta_4^2\bar{I}^{-}\bar{I}^{+2} + \beta_2\beta_4^3\bar{I}^{+3} + \beta_3\bar{I}^{-3} + 3\beta_3\beta_4\bar{I}^{-2}\bar{I}^{+} + 3\beta_3\beta_4^2\bar{I}^{-}\bar{I}^{+2} + \beta_3\beta_4^2\bar{I}^{0}\bar{I}^{+2} + \beta_3\beta_4\bar{I}^{+3} + \beta_3\beta_4\bar{I}^{+2}\bar{I}^{0}\right)}{\left(\bar{I}^{-} + \beta_4\bar{I}^{+}\right)^3}$$

(1)

In the above expression, note that all values in the numerator and denominator are positive, and the expression overall has a negative sign.

NPI$_n$=NPI$_p$ under two conditions: (1) when $\bar{I}^- = 0$, because when there are no negative posts, NPI^{-+} and NPI^+ are identical, and (2) $\bar{I}^- = \bar{I}^{-*}$, where $\bar{I}^{-*}$ solves NPI$_n$=NPI$_p$. This expression is quadratic in $\bar{I}^-$, so solving for $\bar{I}^{-*}$ and ignoring the negative root yields:

$$\frac{\sqrt{{}_2^2\bar{I}^{0^2} \quad 2 \;{}_2\;{}_3\bar{I}^{0^2} \quad 2\;{}_2^2\;{}_4\bar{I}^0\bar{I} \quad {}_3^2\bar{I}^{0^2} \quad 2\;{}_2\;{}_3\;{}_4\bar{I}^0 \quad {}_2^2\;{}_4^2\bar{I}^{\;\;2} \quad 4\;{}_2\;{}_3\bar{I}^{\;\;2} \quad 4\;{}_2\;{}_3\bar{I}^0\bar{I} \quad 4\;{}_3^2\bar{I}^{\;\;2} \quad 4\;{}_3^2\bar{I}^0\bar{I}} \quad {}_2\bar{I}^0 \quad {}_3\bar{I}^0 \quad {}_2\;{}_4\bar{I}}{2\;{}_2\;{}_3}$$

We find that ${}_3 \qquad {}_2\;{}_4$ is a sufficient condition for $\bar{I}^{\;\;*}$ to exist and be positive. From our empirical analysis, the data satisfies the condition ${}_3 \qquad {}_2\;{}_4$. Let

$$a = \sqrt{\beta_2^2\bar{I}^{0^2} + 2\beta_2\beta_3\bar{I}^{0^2} - 2\beta_2^2\beta_4\bar{I}^0\bar{I}^+ + \beta_3^2\bar{I}^{0^2} - 2\beta_2\beta_3\beta_4\bar{I}^0 + \beta_2^2\beta_4^2\bar{I}^{+^2} + 4\beta_2\beta_3\bar{I}^{+^2} + 4\beta_2\beta_3\bar{I}^0\bar{I}^+ + 4\beta_3^2\bar{I}^{+^2} + 4\beta_3^2\bar{I}^0\bar{I}^+}$$

and $b = \beta_1\bar{I}^0 + \beta_2\bar{I}^0 + \beta_1\beta_3\bar{I}^+$.

When ${}_3 \qquad {}_2\;{}_4$, *it can be seen that:*

$$\begin{aligned} a &= \sqrt{\beta_2^2\bar{I}^{0^2} + 2\beta_2\beta_3\bar{I}^{0^2} - 2\beta_2^2\beta_4\bar{I}^0\bar{I}^+ + \beta_3^2\bar{I}^{0^2} - 2\beta_2\beta_3\beta_4\bar{I}^0 + \beta_2^2\beta_4^2\bar{I}^{+^2} + 4\beta_2\beta_3\bar{I}^{+^2} + 4\beta_2\beta_3\bar{I}^0\bar{I}^+ + 4\beta_3^2\bar{I}^{+^2} + 4\beta_3^2\bar{I}^0\bar{I}^+} \\ &= \sqrt{\left(4\beta_2\beta_3 + 4\beta_3^2 + \beta_2^2\beta_4^2\right)\bar{I}^{+^2} + \left(-2\beta_2^2\beta_4\bar{I}^0 + 4\beta_2\beta_3\bar{I}^0 - 2\beta_2\beta_3\beta_4\bar{I}^0 + 4\beta_3^2\bar{I}^0\right)\bar{I}^+ + \beta_2^2\bar{I}^{0^2} + 2\beta_2\beta_3\bar{I}^{0^2} + \beta_3^2\bar{I}^{0^2}} \\ &> \sqrt{\left(4\beta_2\beta_3 + 4\beta_3^2 + \beta_2^2\beta_4^2\right)\bar{I}^{+^2} + \left(2\beta_2^2\beta_4\bar{I}^0 + 2\beta_2\beta_3\beta_4\bar{I}^0\right)\bar{I}^+ + \beta_2^2\bar{I}^{0^2} + 2\beta_2\beta_3\bar{I}^{0^2} + \beta_3^2\bar{I}^{0^2}} > 0 \end{aligned}$$

This proves that a exists and is positive. Moreover, b is also positive. Now, we show that a^2 –b^2 is positive:

$$a^2 - b^2 = 4\left(\beta_2 + \beta_3\right)\bar{I}^+\left(\beta_3\bar{I}^+ + \left(\beta_3 - \beta_2\beta_4\right)\bar{I}^0\right) > 0$$

Algebraically, if a^2 –b^2>0 and a + b >0, then a – b > 0. Hence, $\bar{I}^{-*} > 0$ (Proved).