Online Supplement for
Fighting Fire with Fire: Commercial Piracy and the Role of File Sharing on Copyright Protection Policy for Digital Goods

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A Mathematical Preliminaries and Proofs of Propositions

Before we start presenting the proofs of the propositions, let us state, without proof, the following lemmas which will be used in the proofs. The proof of both of these lemmas follow directly from equations (3)-(8).

Lemma A.1 *Given p_L and p_C, the consumer market structure is*

(i) $[C]$, *if* $v_{oc} < v_{ol}$, $v_{oc} \leq v_{os}$, *and* $v_{cl} > 1$;

(ii) $[SC]$, *if* $v_{os} < v_{oc} < v_{ol}$, *and* $v_{sc} \leq 1 < v_{cl}$;

(iii) $[S\langle C\rangle]$, *if* $v_{os} < v_{oc} < v_{ol}$, *and* $\min\{v_{cl}, v_{sc}\} > 1$.

(iv) $[CL]$, *if* $v_{oc} < \min\{v_{ol}, v_{os}\}$, *and* $v_{cl} \leq 1$;

(v) $[S\langle C\rangle\langle L\rangle]$, *if* $v_{os} < v_{oc} < v_{ol}$, $v_{cl} \leq 1 < v_{sl}$;

(vi) $[S\langle L\rangle]$, *if* $v_{os} < v_{ol} \leq v_{oc}$, $v_{ol} \in [0, 1]$, *and* $v_{sl} \geq 1$; *and*

(vii) $[L]$, *if* $v_{ol} \leq v_{oc}$, $v_{ol} \in [0, 1]$, *and* $v_{ol} \leq v_{os}$,

where v_{cl}, v_{sc}, v_{ol}, v_{sl}, v_{oc} *and* v_{os} *are as defined in (3)-(8).*

Among the possible set of market structures, three structures will be of particular interest for our analysis, and it is useful to examine the conditions under which these structures will emerge. These three structures are the cases where

(i) Some consumers of type T copy individually, some consumers of both types T and NT purchase copies from the legal publisher, and the commercial pirate is out of the market, i.e., $[SL]$.

(ii) Some consumers of type T copy individually, some consumers of type NT purchase from the commercial pirate but no consumer of type T purchases from the commercial pirate, and the legal publisher sells to some consumers of both types T and NT, i.e., $[S\langle C\rangle L]$.

(iii) Some consumers of type T copy individually, some consumers of both types T and NT purchase copies from the legal publisher, and some consumers of both types will purchase copies from the commercial pirate, i.e., $[SCL]$.

The following lemma summarizes the conditions under which each of these three market structures will emerge in the consumer market.

Lemma A.2 *Given p_L and p_C, the market structure is*

(i) $[SL]$, *if*

$$\frac{f\mu_S}{1-\beta\mu_S} < p_L \le \frac{p_C + f\mu_C}{1-\beta\mu_C}, \quad \text{and} \quad \frac{p_L - f\mu_S}{\beta\mu_S} \le 1; \tag{A.1}$$

(ii) $[S\langle C\rangle L]$, *if*

$$\frac{f\mu_S}{1-\beta\mu_S} \le \frac{p_C + f\mu_C}{1-\beta\mu_C} < p_L, \quad \frac{p_L - p_C - f\mu_C}{\beta\mu_C} < 1, \tag{A.2}$$

$$\frac{p_L - f\mu_S}{\beta\mu_S} \le 1, \quad \text{and} \quad \frac{p_L - p_C - f\mu_C}{\beta\mu_C} \le \frac{p_C - f(\mu_S - \mu_C)}{\beta(\mu_S - \mu_C)}; \tag{A.3}$$

and

(iii) $[SCL]$, *if*

$$\frac{f\mu_S}{1-\beta\mu_S} \le \frac{p_C + f\mu_C}{1-\beta\mu_C} < p_L, \quad \text{and} \quad \frac{p_C - f(\mu_S - \mu_C)}{\beta(\mu_S - \mu_C)} < \frac{p_L - p_C - f\mu_C}{\beta\mu_C} < 1. \tag{A.4}$$

Proof of Proposition 1: Technically, we will prove the following statement:

For any $\alpha \in (0,1)$, there exist $\delta, \bar{\mu}_S$, $\underline{F}(\alpha, \beta\mu_S)$, and $\overline{F}(\alpha, \beta\mu_S)$ such that when $\mu_C < \mu_S < \min\{\bar{\mu}_S, \mu_C + \delta\}$ and $\underline{F}(\alpha, \beta\mu_S) \le F \le \overline{F}(\alpha, \beta\mu_S)$, Π_L increases in α in equilibrium.

The outline of the proof is as follows: We first show that under the parameters specified by Proposition 1, the legal publisher's profit is maximized at p_{Lc}, where $p_{Lc} < v_{iv}$ and that at $p_L = p_{Lc}$, $\Pi_C[SL] = \Pi_C[S\langle C\rangle L]$. We demonstrate that for $p_L < p_{Lc}$, the market structure is $[SL]$, and for p_L in $[p_{Lc}, v_v]$, the market structure is $[S\langle C\rangle L]$. We then show that $\Pi_L([SL], p_{Lc})$ is greater than the maximum of Π_L over the region $([S\langle C\rangle L])$. Finally, we calculate $\Pi_L([SL], p_{Lc})$, and show that it is increasing in α.

For convenience in notation, first define the following variables. Given $\mu_C \in (0,1)$ and $\mu_S > \mu_C$, define

$$v_i \triangleq \frac{(p_L + f\mu_C)\mu_S - p_L\mu_C}{(1-\beta\mu_C)\mu_S}, \tag{A.5}$$

$$v_{ii} \triangleq 1 - \frac{1-p_L}{1-\beta\mu_C}, \tag{A.6}$$

$$v_{iii} \triangleq 1 - \frac{1-(\beta+f)\mu_S}{1-\beta\mu_C}, \tag{A.7}$$

$$v_{iv} \triangleq 1 - (1-\beta\mu_C)\left(1 - \frac{f\mu_S}{1-\beta\mu_S}\right), \tag{A.8}$$

$$v_v \triangleq (\beta+f)\mu_S\,. \tag{A.9}$$

We start by examining the possible equilibrium market structures as a function of p_L and p_C. By Lemma A.2, Lemma A.1 and the definitions above, there exists $\bar{\mu}_S > 0$ such that when $\mu_S < \bar{\mu}_S$, the market structures that can contain the profit maximizing p_L are

(*i*) $[SCL]$, if $p_C < (1-\beta\mu_C)v_i - f\mu_C$;

(*ii*) $[S\langle C\rangle L]$, if $(1-\beta\mu_C)v_i - f\mu_C \leq p_C < (1-\beta\mu_C)v_{ol} - f\mu_C$;

(*iii*) $[SL]$, if $(1-\beta\mu_C)v_{ol} - f\mu_C \leq p_C \leq 1$,

for $p_L \leq v_{iv}$;

(*i*) $[SC]$, if $p_C < (1-\beta\mu_C)v_{ii} - f\mu_C$;

(*ii*) $[SCL]$, if $(1-\beta\mu_C)v_{ii} - f\mu_C \leq p_C < (1-\beta\mu_C)v_i - f\mu_C$;

(*iii*) $[S\langle C\rangle L]$, if $(1-\beta\mu_C)v_i - f\mu_C \leq p_C < (1-\beta\mu_C)v_{ol} - f\mu_C$;

(*iv*) $[SL]$, if $(1-\beta\mu_C)v_{ol} - f\mu_C \leq p_C \leq 1$,

for $v_{iv} < p_L \leq v_v$; and

(*i*) $[SC]$, if $p_C < (1-\beta\mu_C)v_{iii} - f\mu_C$;

(*ii*) $[S\langle C\rangle]$, if $(1-\beta\mu_C)v_{iii} - f\mu_C \leq p_C < (1-\beta\mu_C)v_{ii} - f\mu_C$;

(*iii*) $[S\langle C\rangle\langle L\rangle]$, if $(1-\beta\mu_C)v_{ii} - f\mu_C \leq p_C < (1-\beta\mu_C)v_{ol} - f\mu_C$;

(*iv*) $[S\langle L\rangle]$, if $(1-\beta\mu_C)v_{ol} - f\mu_C \leq p_C \leq 1$,

for $v_v < p_L \leq 1$, where v_i-v_v are as in (A.5)-(A.9). Now, by (10), $\Pi_C([SCL], p_L, p_C)$ is maximized at

$$p_C([SCL], p_L) = \frac{(p_L - \mu_C(f - \alpha f + \beta p_L))(\mu_S - \mu_C)}{2\mu_S - 2\mu_C(1 - \alpha + \alpha\beta\mu_S)}\,. \tag{A.10}$$

Similarly,

$$\Pi_C([SC], p_L, p_C) = p_C(\alpha(1 - v_{sc}) + (1-\alpha)(1 - v_{oc})) - F\,, \tag{A.11}$$

and hence $\Pi_C([SC], p_L, p_C)$ is maximized at

$$p_C([SC], p_L) = \frac{(\beta + \alpha f - \beta(\beta + f)\mu_C)(\mu_S - \mu_C)}{2(\alpha - \beta\mu_S + (1-\alpha)\beta\mu_S)}. \tag{A.12}$$

Note that the prices that are given in (A.10) and (A.12) are unrestricted maximizers of the commercial pirate's profits ignoring the range of validity constraints assuming market structures $[SCL]$ and $[SC]$, and his profits considering the feasibility constraints for these market structures will be at most these amounts. However notice that when $\mu_S - \mu_C$ is sufficiently small, the global maximum for Π_C is negative ignoring its feasibility, and hence imposing feasibility constraints, there exists δ, such that when $\mu_C < \mu_S < \mu_C + \delta$, under market structures $[SCL]$ and $[SC]$, $\Pi_C(p_L, p_C) < 0$. Therefore neither can be the equilibrium market structure at any p_L. Now,

$$\Pi_C([S\langle C\rangle L], p_L, p_C) = p_C(1-\alpha)(v_{cl} - v_{oc}) - F\,, \tag{A.13}$$

and therefore

$$p_C([S\langle C\rangle L], p_L) = (p_L - (f + \beta p_L)\mu_C)/2. \tag{A.14}$$

Plugging (A.14) in (A.13), we have

$$\Pi_C([S\langle C\rangle L], p_L, p_C) = \frac{(1-\alpha)(p_L - (f+\beta p_L)\mu_C)^2}{4\beta\mu_C(1-\beta\mu_C)} - F\,. \tag{A.15}$$

By (A.15), there exists $\bar{\mu}_S > 0$ such that when $\mu_C < \bar{\mu_S}$, then $\Pi_C([S\langle C\rangle L], p_L, p_C) \geq 0$ if and only if

$$p_L \geq p_{Lc} \triangleq \frac{f\mu_C}{1-\beta\mu_C} + \frac{2\sqrt{F(1-\alpha)(1-\beta\mu_C)\beta\mu_C}}{(1-\alpha)(1-\beta\mu_C)}\,. \tag{A.16}$$

Comparing (A.16) with (A.9), we can see that there exists $\lambda \in (0,1)$ and $\bar{\mu}_S > 0$ such that when $\beta = \lambda/\mu_S$ and $\mu_C < \mu_S < \bar{\mu}_S$, $p_{Lc} \leq v_v$ if and only if

$$\alpha \leq 1 - \frac{4F}{\beta\mu_S(1-\beta\mu_S)}\,. \tag{A.17}$$

When $p_{Lc} \leq v_v$, there exists a $p_L \in (p_{Lc}, v_v)$ such that the market structure is $[S\langle C\rangle L]$ and $\Pi_C([S\langle C\rangle L], p_L, p_C) \geq 0$ and hence, $v_{cl} - v_{sc} \geq 0$. Further, by (3), $\partial v_{cl}/\partial p_L > 0$. Therefore, and since $\Pi_C([S\langle L\rangle], p_L, p_C) = 0$, for all p_L, p_C, it follows that for $p_L \geq v_v$, $[S\langle L\rangle]$ cannot be the equilibrium market structure.

Now for a given p_L, for market structures $[S\langle C\rangle L]$ and $[S\langle C\rangle\langle L\rangle]$,

$$p_C([S\langle C\rangle L], p_L) = p_C([S\langle C\rangle\langle L\rangle], p_L) = \frac{p_L(\mu_S - \mu_C)}{2\mu_S}\,. \tag{A.18}$$

Further, $v_{sl} = 1$ at $p_L = v_v$, hence

$$\Pi_L([S\langle C\rangle L], p_L, p_C)|_{p_L=v_v} = \Pi_L([S\langle C\rangle\langle L\rangle], p_L, p_C)|_{p_L=v_v}\,. \tag{A.19}$$

We also know that

$$\Pi_L([S\langle C\rangle\langle L\rangle], p_L, p_C) = p_L((1-\alpha)(1-v_{cl}))\,. \tag{A.20}$$

Plugging (A.18) in (A.20) we can see that $\Pi_L([S\langle C\rangle\langle L\rangle], p_L, p_C)$ is maximized at

$$p_L = \frac{(\beta+f)\mu_C\mu_S}{\mu_C+\mu_S} < v_v\,, \tag{A.21}$$

where the last inequality follows from (A.9). Therefore, Π_L cannot be maximized at $p_L \geq v_v$. Next we have

$$\Pi_L([S\langle C\rangle L], p_L, p_C) = p_L(\alpha(1 - v_{sl}) + (1 - \alpha)(1 - v_{cl}))\,. \tag{A.22}$$

Plugging in (3), (6) and (A.14), and writing the first order condition with respect to p_L, we find that $\Pi_L([S\langle C\rangle L], p_L, p_C)$ is maximized at

$$p_L = \frac{(2\beta + f + \alpha f)\mu_S\mu_C}{2(2\alpha\mu_C + \mu_S + \alpha(1 + \beta\mu_C)\mu_S - \beta\mu_C\mu_S)}\,. \tag{A.23}$$

Plugging (A.23) back into (A.22), we have

$$\max_{p_L} \Pi_L([S\langle C\rangle L], p_L, p_C) = \frac{(2\beta + f + \alpha f)^2\mu_S\mu_C}{8\beta(2\alpha\mu_C + (1 - \alpha)(1 + \beta\mu_C)\mu_S)}\,. \tag{A.24}$$

Further by (6),

$$\Pi_L([SL], p_L, p_C) = \frac{p_L(-\alpha p_L + (\beta + \alpha f - (1 - \alpha)\beta p_L))}{\beta\mu_S}\,. \tag{A.25}$$

Plugging in (A.16) and comparing with (A.24), it follows that

$$\Pi_L([SL], p_{Lc}, p_C) \geq \max_{p_{Lc} \leq p_L \leq v_v} \Pi_L([S\langle C\rangle L], p_L, p_C) \tag{A.26}$$

if and only if

$$\frac{\beta\mu_S}{-1 + \alpha - 2\alpha - (1 - \alpha)\beta\mu_S} + F\left(\frac{4\beta\mu_S}{\sqrt{(1 - \alpha)F\beta\mu_S(1 - \beta\mu_S)}} - \frac{8\beta\mu_S}{1 - \beta\mu_S} - \frac{8\alpha}{1 - \alpha - \beta\mu_S - \alpha\beta\mu_S}\right) \geq 0\,. \tag{A.27}$$

Now notice that

$$\frac{d\Pi_L([SL], p_{Lc}, p_C)}{d\alpha} = \frac{\partial\Pi_L([SL], p_{Lc}, p_C)}{\partial\alpha} + \frac{\partial\Pi_L([SL], p_{Lc}, p_C)}{\partial p_{Lc}} \cdot \frac{dp_{Lc}}{d\alpha}\,. \tag{A.28}$$

Further, by (A.16) and (A.25), for any $\varepsilon > 0$, there exists $\bar{\mu}_S$ and δ such that,

$$\left|\frac{\partial\Pi_L([SL], p_{Lc}, p_C)}{\partial\alpha} + \frac{4F(1 - \beta\mu_S)}{(1 - \alpha)(1 - \beta\mu_S)}\right| < \varepsilon\,, \tag{A.29}$$

and

$$\left|\frac{dp_{Lc}}{d\alpha} - \frac{F\beta\mu_S}{(1 - \alpha)\sqrt{(1 - \alpha)F(1 - \beta\mu_S)\beta\mu_S}}\right| < \varepsilon\,. \tag{A.30}$$

By (A.29) and (A.30) it follows that $\partial\Pi_L([SL], p_{Lc}, p_C)/\partial\alpha < 0$ and $dp_{Lc}/d\alpha > 0$. Hence if $d\Pi_L([SL], p_{Lc}, p_C)/d\alpha < 0$, we must have $\partial\Pi_L([SL], p_{Lc}, p_C)/\partial p_{Lc} > 0$. Further, since by (A.25), $\Pi_L([SL], p_L, p_C)$ is concave in p_L, it follows that if $d\Pi_L([SL], p_{Lc}, p_C)/d\alpha < 0$, then p_{Lc} is the maximizer of $\Pi_L([SL], p_L, p_C)$ over $[0, p_{Lc}]$. Now again by (A.16) and (A.25), for any $\varepsilon > 0$, there exists $\bar{\mu}_S$ and δ

small enough such that

$$\left| \frac{d\Pi_L([SL], p_{Lc}, p_C)}{d\alpha} + \frac{4F - \sqrt{(1-\alpha)F(1-\beta\mu_S)\beta\mu_S}}{(1-\alpha)^2(1-\beta\mu_S)} \right| < \varepsilon \,, \tag{A.31}$$

and hence $d\Pi_L([SL], p_{Lc}, pC)/d\alpha > 0$ if

$$\alpha \leq 1 - \frac{16F}{\beta\mu_S(1-\beta\mu_S)} \,. \tag{A.32}$$

To summarize, we conclude that there exists $\bar{\mu}_S, \delta > 0$ such that when $\mu_C < \mu_S < \min\{\bar{\mu}_S, \mu_C + \delta\}$ and (A.17), (A.27) and (A.32) are satisfied, then we have (i) for $0 \leq p_L \leq p_{Lc}$, the market structure is $[SL]$ and for $p_{Lc} \leq p_L \leq v_v$, the market structure is $[S\langle C\rangle L]$; (ii) Π_L is maximized at $p_L = p_{Lc}$; and (iii) $\max_{\{0 \leq p_L \leq 1\}} \Pi_L$ is increasing in α.

Finally, we need to show that the intersection of the parameter ranges specified by (A.17), (A.27) and (A.32) is nonempty for some $\bar{\mu}_S, \delta > 0$ and when $\mu_C < \mu_S < \min\{\bar{\mu}_S, \mu_C + \delta\}$. First, notice that (A.17) is implied by (A.32). Further (A.32) is equivalent to

$$\sqrt{F} \leq \sqrt{(1-\alpha)\beta\mu_S(1-\beta\mu_S)}/4 \,. \tag{A.33}$$

Rearranging (A.27) and solving for $\sqrt{F}$, we can also see that (A.27) is a quadratic with a strictly positive lead coefficient and has two strictly positive roots given by

$$r_{1,2} = \frac{\beta\mu_S(1-(\alpha+\beta\mu_S(1-\alpha))^2 \pm (1-\alpha)(1-\beta\mu_S)\sqrt{(1-\alpha)(1-\beta\mu_S)(1+\beta\mu_S-\alpha(1-\beta\mu_S)))}}{4(1+\beta\mu_S+\alpha(1-\beta\mu_S))(\alpha(1-\beta\mu_S)+\beta\mu_S)\sqrt{(1-\beta\mu_S)(1-\alpha)\beta\mu_S}} \tag{A.34}$$

In addition, $\sqrt{F}$ has to lie between the two roots given in (A.34). Plugging in the right hand side of (A.33) in (A.27), we conclude that

$$\underline{F}(\alpha, \beta\mu_S) \triangleq r_1^2 < \overline{F}(\alpha, \beta\mu_S) \triangleq \frac{(1-\alpha)\beta\mu_S(1-\beta\mu_S)}{16} \,, \tag{A.35}$$

where r_1 is the smaller of the two roots given in (A.34), and that Π_L will be increasing in α when $\underline{F}(\alpha, \beta\mu_S) < F < \overline{F}(\alpha, \beta\mu_S)$. This completes the proof. ■

Proof of Proposition 2: We will technically prove the following statement:

There exist $\bar{\mu}_S, \underline{\alpha} \in (0,1)$, such that when $0 < \mu_C < \mu_S < \bar{\mu}_S$, $\alpha > \underline{\alpha}$, and

$$\frac{\beta(\mu_S-\mu_C)(1-\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}{8\frac{\mu_C}{\mu_S}(1+\frac{\mu_C}{\mu_S})} \leq F \leq \frac{\beta\mu_C(\mu_S-\mu_C)}{16\mu_S} \,, \tag{A.36}$$

then Π_L is decreasing in μ_S.

The steps of the proof is as follows: First, we show that under the parameters specified by Proposition

2, the legal publisher's profit is maximized at p_{Lc}, where $p_{Lc} < v_{iv}$, and that at $p_L = p_{Lc}$, $\Pi_C[SL] = \Pi_C[SCL]$ holds. We then show that for $p_L < p_{Lc}$, the market structure is $[SL]$ and for p_L in $[p_{Lc}, x]$ where $x \in [v_{iv}, v_v]$, the market structure is $[SCL]$. Finally, we show that $\Pi_L([SL], p_{Lc}) > max\Pi_L([SCL])$, and that $\Pi_L([SL], p_{Lc})$ is decreasing in μ_S to complete the proof.

First, we have

$$\Pi_C([S\langle C\rangle], p_L, p_C) = p_C(1-\alpha)(1-v_{oc}) - F\,. \tag{A.37}$$

By (A.13) and (A.37), there exists $\underline{\alpha} \in (0,1)$ such that when $\alpha > \underline{\alpha}$, $\Pi_C([S\langle C\rangle], p_L, p_C), \Pi_C([S\langle C\rangle L], p_L, p_C) < 0$ and hence $[S\langle C\rangle L]$ or $[S\langle C\rangle]$ can not be the equilibrium market structure for any $p_L \in [0,1]$. Next, by plugging (A.10) into (10), we have

$$\Pi_C([SCL], p_L, p_C) = \frac{(p_L - ((1-\alpha)f + \beta p_L)\mu_C)^2(\mu_S - \mu_C)}{4\beta\mu_C(1-\beta\mu_C)(\mu_S - \mu_C(1-\alpha+\alpha\beta\mu_S))} - F\,. \tag{A.38}$$

Therefore, there exists $\bar{\mu}_S > 0$ such that when $\mu_S < \bar{\mu}_S$, $\Pi_C([SCL], p_L, p_C) \geq 0$ if and only if

$$p_L \geq p_{Lc} \triangleq \frac{1}{1-\beta\mu_C}\left((1-\alpha)f\mu_C + 2\sqrt{\frac{F\beta\mu_C(1-\beta\mu_C)(\mu_S-\mu_C(1-\alpha+\alpha\beta\mu_S))}{\mu_S-\mu_C}}\right)\,. \tag{A.39}$$

By (A.8) and (A.39), there exists $\bar{\mu}_S > 0$ such that when $\mu_S < \bar{\mu}_S$ and $\alpha > \underline{\alpha}$, $p_{Lc} < v_{iv}$ if and only if

$$F \leq \frac{\beta\mu_C(\mu_S - \mu_C)}{4\mu_S}\,. \tag{A.40}$$

Now by (A.8) and (A.9), when $\mu_S > \mu_C$, $v_{iv} < v_v$. Then, when $p_{Lc} \leq v_{iv}$, there exists a $p_L \in (p_{Lc}, v_v)$ such that the market structure is $[SCL]$ and $\Pi_C([SCL], p_L, p_C) \geq 0$ and hence $v_{cl} - v_{sc} \geq 0$ or $v_{cl} - v_{oc} \geq 0$. In addition, by (3) and (7), $\partial v_{cl}/\partial p_L > 0$ and $\partial v_{oc}/\partial p_L > 0$. As a result since $\Pi_C([S\langle L\rangle], p_L, p_C) = 0$, for all p_L, p_C, it follows that $[S\langle L\rangle]$ cannot be the equilibrium market structure for $p_L \geq v_v$. Plugging (A.10) into (9) and writing the first order condition, we find that Π_L is maximized at

$$p_L = \frac{\mu_C(-(2\beta+f+\alpha f)\mu_S + \mu_C((1-\alpha)(2\beta+f) + 2\alpha\beta(\beta+f)\mu_S))}{-2\mu_S + 2\mu_C(\beta\mu_C + (1-2\alpha)(1-\beta\mu_S))}\,. \tag{A.41}$$

Plugging (A.41) into (9), we then have

$$\begin{aligned}\Pi_L([SL], p_L, p_C) - \max_{p_{Lc}\leq p_L \leq v_v} \Pi_L([SCL], p_L, p_C) \\ = \frac{4(1+\frac{\mu_C}{\mu_S})\sqrt{F\beta\frac{\mu_C}{\mu_S}(\mu_S-\mu_C)} - \beta\frac{\mu_C}{\mu_S}(\mu_S-\mu_C) - 8F(1+\frac{\mu_C}{\mu_S})\frac{\mu_C}{\mu_S}}{2(1-(\frac{\mu_C}{\mu_S})^2)\frac{\mu_C}{\mu_S}}\,.\end{aligned} \tag{A.42}$$

(A.42) is nonnegative if and only if

$$\frac{\beta(\mu_S-\mu_C)(1-\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}{8\frac{\mu_C}{\mu_S}(1+\frac{\mu_C}{\mu_S})} \leq F \leq \frac{\beta(\mu_S-\mu_C)(1+\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}{8\frac{\mu_C}{\mu_S}(1+\frac{\mu_C}{\mu_S})}\,. \tag{A.43}$$

In addition, we have

$$\frac{d\Pi_L([SL],p_{Lc},p_C)}{d\mu_S}=\frac{\partial\Pi_L([SL],p_{Lc},p_C)}{\partial\mu_S}+\frac{\partial\Pi_L([SL],p_{Lc},p_C)}{\partial p_{Lc}}\cdot\frac{dp_{Lc}}{d\mu_S}\,. \tag{A.44}$$

Now by (A.25),

$$\frac{\partial\Pi_L([SL],p_{Lc},p_C)}{\partial\mu_S}=\frac{\alpha p_{Lc}^2}{\beta\mu_S^2}>0\,, \tag{A.45}$$

and by (A.39),

$$\frac{dp_{Lc}}{d\mu_S}=-\frac{\alpha\beta F\mu_C^2(1-\beta\mu_C)}{(\mu_S-\mu_C)\sqrt{\beta F\mu_C(1-\beta\mu_C)(\mu_S-\mu_C)(\mu_S-\mu_C(1-\alpha(1+\beta\mu_S)))}}<0\,. \tag{A.46}$$

Further, by (A.25) and (A.39) for small enough $\bar{\mu}_S>0$, when $\mu_S<\bar{\mu}_S$, $d\Pi_L([SL],p_{Lc},p_C)/d\mu_S\le 0$ if

$$F\le\frac{\beta\mu_C(\mu_S-\mu_C)}{16\mu_S}\,. \tag{A.47}$$

Therefore, when (A.47) is satisfied, by (A.44), (A.45) and (A.46), we have $\partial\Pi_L([SL],p_{Lc},p_C)/\partial p_{Lc}>0$. Moreover, by (A.25), $\Pi_L([SL],p_L,p_C)$ is concave in p_L and hence p_{Lc} is the maximizer of $\Pi_L([SL],p_L,p_C)$ over $[0,p_{Lc}]$. In summary, we have shown that there exist $\underline{\alpha},\bar{\mu}_S>0$, such that when $\alpha>\underline{\alpha}$, $\mu_S<\bar{\mu}_S$ and under (A.40), (A.43) and (A.47), Π_L is maximized at p_{Lc} and $\max_{\{0\le p_L\le 1\}}\Pi_L$ is decreasing in μ_S.

In order to complete the proof, we have to show that the parameter region defined by (A.40), (A.43) and (A.47) is nonempty. First, notice that (A.47) implies (A.40). Moreover, for $\mu_C<\mu_S$,

$$\frac{\beta\mu_C(\mu_S-\mu_C)}{16\mu_S}\le\frac{\beta(\mu_S-\mu_C)(1+\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}{8\frac{\mu_C}{\mu_S}(1+\frac{\mu_C}{\mu_S})}\,. \tag{A.48}$$

Therefore the intersection of (A.43) and (A.47) is nonempty if

$$\frac{\beta(\mu_S-\mu_C)(1-\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}{8\frac{\mu_C}{\mu_S}(1+\frac{\mu_C}{\mu_S})}\le F\le\frac{\beta\mu_C(\mu_S-\mu_C)}{16\mu_S}\,. \tag{A.49}$$

(A.49) is nonempty when

$$\frac{1}{(1+\frac{\mu_C}{\mu_S})(1+\sqrt{1-(\frac{\mu_C}{\mu_S})^2})}\le\frac{1}{2}\,, \tag{A.50}$$

which is always satisfied for $\mu_C<\mu_S$. This completes the proof. ■

Proof of Proposition 3: Technically we will sprove the following statement:

There exist $\bar{\mu}_S,\delta\in(0,1)$ such that when $\mu_S<\bar{\mu}_S$,

$$\frac{\mu_S}{2-\alpha-\beta\mu_S(1-\alpha)}-\delta<\mu_C<\frac{\mu_S}{2-\alpha-\beta\mu_S(1-\alpha)}, \tag{A.51}$$

and

$$0 < \alpha \leq \frac{5 - \sqrt{9 - 8\beta\mu_S} - 4\beta\mu_S}{4(1 - \beta\mu_S)}, \tag{A.52}$$

then equilibrium Π_L is decreasing in μ_S.

We will first show that under the specified parameter conditions, the legal publisher's profit is maximized at p_{Lc}, where $p_{Lc} < v_v$ and that at $p_L = p_{Lc}$, $\Pi_C[SCL] = \Pi_C[S\langle C\rangle L]$. We will then demonstrate that for $p_L < p_{Lc}$, the market structure is $[SCL]$, and for p_L in $[p_{Lc}, v_v]$, the market structure is $[S < C > L]$. Finally, we will show that $\Pi_L([S\langle C\rangle L], p_{Lc}) > max\Pi_L([SCL])$, and that $\Pi([S\langle C\rangle L, p_{Lc})$ is decreasing in μ_S.

As shown in the proof of Proposition 1, there exists $\bar{\mu}_S$ such that when $0 < \mu_C < \mu_S < \bar{\mu}_S$, the possible equilibrium market structures are for $p_L \leq v_{iv}$, $[SCL]$, $[S\langle C\rangle L]$ or $[SL]$; for $v_{iv} < p_L \leq v_v$, $[SC]$, $[SCL]$, $[S\langle C\rangle L]$ or $[SL]$; and for $p_L > v_v$, $[SC]$, $[S\langle C\rangle]$, $[S\langle C\rangle\langle L\rangle]$ or $[S\langle L\rangle]$. First, for any given μ_S, μ_C and ε, there exists $K > 0$ such that when $F < K$, by (A.16) and (A.39), $\max_{0 \leq p_L \leq 1} \Pi_L([SL], p_L, p_C) < \varepsilon$ and $\max_{0 \leq p_L \leq 1} \Pi_L([S\langle L\rangle], p_L, p_C) < \varepsilon$ and hence $[SL]$ cannot be an equilibrium market structure. Now, by (A.37),

$$p_C([S\langle C\rangle], p_L) = (1 - (\beta + f)\mu_C)/2\,. \tag{A.53}$$

Also, there exists $0 < \bar{\mu}_S < 1$ such that

$$(1 - \beta\mu_C)\left(1 - \frac{1 - (\beta + f)\mu_S}{1 - \beta\mu_C}\right) - f\mu_C \leq \frac{1 - (\beta + f)\mu_C}{2}\,. \tag{A.54}$$

By (7) and (A.37), $\Pi_C([S\langle C\rangle], p_L, p_C)$ is concave in p_C. Further, for $p_L \geq v_v$, the boundary of the market structures $[S\langle C\rangle]$ and $[S\langle C\rangle\langle L\rangle]$ is at $(1 - \beta\mu_C)v_{ii} - f\mu_C$ as shown in the proof of Proposition 1. Then by (A.6), (A.9) and (A.54) it follows that at $p_L = v_v$, $[S\langle C\rangle]$ cannot be the equilibrium market structure. With a similar argument, if

$$(1 - \beta\mu_C)v_{iii} - f\mu_C \leq p_C([SC], p_L)\,, \tag{A.55}$$

then $[SC]$ cannot be the equilibrium market structure. By (A.7), (A.12) and since $\alpha \leq 1$, (A.55) is equivalent to

$$\alpha \leq \frac{5 - \sqrt{9 - 8\beta\mu_S} - 4\beta\mu_S}{4(1 - \beta\mu_S)}\,. \tag{A.56}$$

Moreover, for $p_L < v_v$, by (A.6) and (A.7), we have $v_{ii} < v_{iii}$. Since, when (A.56) is satisfied at $p_L = v_v$, (A.55) holds, it follows that for $p_L < v_v$, $(1 - \beta\mu_C)v_{ii} - f\mu_C \leq p_C([SC], p_L)$. Therefore when (A.56) is satisfied, for $p_L < v_v$, $\max \Pi_C([SC], p_L, p_C) < \max \Pi_C([SCL], p_L, p_C)$ and hence $[SC]$ cannot be the equilibrium market structure for $p_L < v_v$. Now by (A.15) and (A.38), $\Pi_C([SCL], p_L, p_C) \leq \Pi_C([S\langle C\rangle L], p_L, p_C)$ if and only if

$$p_L \geq p_{Lc} \triangleq \frac{f\mu_S}{1 - \beta\mu_C - \sqrt{\frac{(\mu_S - \mu_C)(\mu_S - \mu_C(1 - \alpha + \alpha\beta\mu_S))}{(1-\alpha)\mu_C^2}}}\,. \tag{A.57}$$

Further, by (A.9), $p_L < \beta\mu_S$ ensures that $p_L < v_v$. Plugging (A.57) into (A.22) and (A.41) into (9), we see that there exists $\delta > 0$ such that when

$$\frac{\mu_S}{2-\alpha-\beta\mu_S(1-\alpha)} - \delta < \mu_C < \frac{\mu_S}{2-\alpha-\beta\mu_S(1-\alpha)}, \tag{A.58}$$

$\Pi_L([S\langle C\rangle L], p_{Lc}, p_C) \geq \max_{\{0 \leq p_L \leq p_{Lc}\}} \Pi_L([SCL], p_L, p_C)$ if and only if

$$\frac{\lambda}{2\beta\mu_S} p_{Lc}^2 - p_{Lc} + \frac{\beta\mu_S}{2(4-\lambda)} \leq 0\,, \tag{A.59}$$

where $\lambda = 2 - \alpha(1-\alpha)(1-\beta\mu_S)$. Further, plugging in (A.57), (A.59) is satisfied if and only if

$$\frac{4-\lambda-\sqrt{2}\sqrt{8-6\lambda+\lambda^2}}{4\lambda-\lambda^2}\beta\mu_S \leq p_{Lc} \leq \frac{4-\lambda+\sqrt{2}\sqrt{8-6\lambda+\lambda^2}}{4\lambda-\lambda^2}\beta\mu_S\,. \tag{A.60}$$

When (A.58) is satisfied, and since, by (3), v_{cl} is monotonically increasing in p_L, and since $[S\langle C\rangle\langle L\rangle]$ is the market structure at $p_{Lc} < v_v$, $[S\langle L\rangle]$ cannot be the equilibrium market structure for $p_L \geq v_v$. Hence, the only possible equilibrium market structure for $p_L \geq v_v$ is $[S\langle C\rangle\langle L\rangle]$. Now notice that

$$\frac{d\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)}{d\mu_S} = \frac{\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)}{\partial\mu_S} + \frac{\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)}{\partial p_{Lc}} \cdot \frac{dp_{Lc}}{d\mu_S}\,. \tag{A.61}$$

Further, by (A.22) and (A.57), for any $\varepsilon > 0$, there exists $\bar{\mu}_S, \delta \in (0,1)$ such that when (A.58) is satisfied and $\mu_S < \bar{\mu}_S$,

$$\left|\frac{\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)}{\partial\mu_S} - \frac{\alpha p_{LC}^2}{\beta\mu_S^2}\right| < \varepsilon\,, \tag{A.62}$$

and hence $\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/\partial\mu_S > 0$. We also know that $dp_{Lc}/d\mu_S > 0$. Hence if $d\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/d\mu_S < 0$, we must have $\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/\partial p_{Lc} < 0$. Further, since by (A.22), $\Pi_L([S\langle C\rangle L], p_L, p_C)$ is concave in p_L, it follows that if $d\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/d\mu_S < 0$, then p_{Lc} maximizes $\Pi_L([S\langle C\rangle L], p_L, p_C)$ over $[p_{Lc}, v_v]$. Now again by (A.22) and (A.57), $d\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/d\mu_S < 0$ if

$$p_{Lc} \geq \beta\mu_S/\lambda\,. \tag{A.63}$$

Finally by (A.18)-(A.21) we know that $\Pi_L([S\langle C\rangle\langle L\rangle], p_L, p_C)$ is decreasing for $p_L \geq v_v$. We also have $\Pi_L([S\langle C\rangle L], v_v, p_C) = \Pi_L([S\langle C\rangle\langle L\rangle], v_v, p_C)$ with $\partial\Pi_L([S\langle C\rangle L], p_{Lc}, p_C)/\partial p_{Lc} < 0$ and since $\Pi_L([S\langle C\rangle L], p_L, p_C)$ is concave in p_L, it follows that $\Pi_L([S\langle C\rangle L], p_L, p_C)$ is decreasing for $p_L \geq p_{Lc}$. Therefore, under (A.56), (A.60), (A.63) and when $p_{Lc} < \beta\mu_S$, Π_L is maximized at $p_L = p_{Lc}$ and $\max_{\{0 \leq p_L \leq 1\}} \Pi_L$ is decreasing in μ_S. In order these four conditions to hold it is sufficient that

$$\frac{\beta\mu_S}{\lambda} \leq p_{Lc} < \min\{\beta\mu_S, (1 + \frac{\sqrt{2}\sqrt{8-6\lambda+\lambda^2}}{4-\lambda})\frac{\beta\mu_S}{\lambda}\}\,. \tag{A.64}$$

Then, since $\lambda > 1$ and by (A.57), for any $f > 0$, there exist $0 < \mu_C < \mu_S$ such that (A.64) is satisfied for $\alpha > 0$. This completes the proof. ■

B Applicability Regions for Propositions

In this section, we derive the applicability regions in the parameter space for Propositions 1-3. These regions are also demonstrated in Figure 5.

B.1 Applicability Region for Proposition 1

Corollary B.1

(i) For any $0 < F < 1/64$*, in the parameter region* $(1-\sqrt{1-64F})/2 \leq \beta\mu_S \leq (1+\sqrt{1-64F})/2$ *there exist* $\alpha \in (0,1)$*, such that for small enough* μ_C*,* μ_S*,* Π_L *is increasing in* α *in equilibrium.*

(ii) Define $S_1 \subset [0,1]\times[0,1]$ *as the subset of the parameter space where for* $(F, \beta\mu_S) \in S_1$*,* Π_L *is increasing for some* $\alpha \in (0,1)$ *as described in Proposition 1. Then for* $(F, \beta\mu_S) \in S_1$*, (a) as* F *increases,* S_1 *contracts; (b) as* $\beta\mu_S$ *increases* S_1 *expands in the region where* $\beta\mu_S < 1/2$ *and contracts otherwise.*

Proof: First, $\max_{\alpha,\, \beta\mu_S \in [0,1]} \overline{F}(\alpha, \beta\mu_S) = 1/64$ and hence, for any $0 < F < 1/64$, by (A.35) when

$$\frac{1-\sqrt{1-64F}}{2} < \beta\mu_S < \frac{1+\sqrt{1-64F}}{2}\,, \tag{B.1}$$

there exists $\alpha \in (0,1)$, such that $F < \overline{F}(\alpha, \beta\mu_S)$. Second, note that by (A.34) and (A.35), both $\underline{F}(\alpha, \beta\mu_S)$ and $\overline{F}(\alpha, \beta\mu_S)$ are continuous in α and approach zero as $\alpha \to 1$. Therefore, for any $0 < F < 1/64$, there exists an $\alpha \in (0,1)$, and β and $\mu_C < \mu_S$ small enough and satisfying (B.1) such that Π_L is increasing in α in equilibrium. Part (ii) follows directly from (A.35) and (B.1). ■

Corollary B.1 states that Proposition 1 has a wide range of applicability as can also be seen in panel (a) of Figure 5, which depicts the applicable region S_1 as described in the corollary. Specifically part (i) of the corollary states that for any "entry risk" level (F) less than 1/64, there is a large band of β values for which the legal publisher's profits are increasing with α for some range of $\alpha \in (0,1)$.This band starts with the entire range of $(0, 1/\mu_S)$ for F close to 0, decreases in width as F increases as stated in part (ii)(a) of the corollary and vanishes as F approaches 1/64 as can also be seen in Figure 5. This is because as the entry risk for the commercial pirate increases, the commercial pirate is less likely to produce illegal copies and consequently change the market structure from $[SL]$ to $[S\langle C\rangle L]$ as shown in Figure 3. From another viewpoint, for a given level of entry risk for the commercial pirate, for very small user risk levels, i.e., small $\beta\mu_S$ and $\beta\mu_C$ values, the legal publisher would not be able to keep the commercial pirate out of the market and set a sufficiently high price at the same time to benefit from the increase in the population of technologically savvy users. Therefore, for small $\beta\mu_S$ values, an increase in $\beta\mu_S$ increases the region over which the results are valid. On the other hand, when $\beta\mu_S$ and $\beta\mu_C$ become too large, piracy becomes less of an attractive option for the users, and the region over which there is a market structure shift from $[SL]$

to $[S\langle C\rangle L]$ diminishes. As a result, beyond a certain point, an increase in $\beta\mu_S$ decreases the parameter region over which there is an increase in legal publisher's profits with increased α.

B.2 Applicability Region for Proposition 2

Corollary B.2

(i) For any $0 < F < 1/64$, *in the parameter region* $(1-\sqrt{1-64\frac{F}{\beta\mu_S}})/2 \le \frac{\mu_C}{\mu_S} \le (1+\sqrt{1-64\frac{F}{\beta\mu_S}})/2$ *such that for small enough* μ_C, μ_S, Π_L *is decreasing in* μ_S *in equilibrium as described in Proposition 2.*

(ii) For a given $\beta\mu_S \in (0,1)$, *define* $S_2(\beta\mu_S) \subset [0,1]\times[0,1]$ *as the subset of the parameter space where for* $(F, \frac{\mu_C}{\mu_S}) \in S_2(\beta\mu_S)$, Π_L *is decreasing for some* $\mu_S \in (0,1)$ *as described in Proposition 2. Then for* $(F, \frac{\mu_C}{\mu_S}) \in S_2(\beta\mu_S)$, *(a) as* F *increases,* $S_2(\beta\mu_S)$ *contracts; (b) as* μ_C/μ_S *increases,* $S_2(\beta\mu_S)$ *expands if* $\mu_C/\mu_S < 1/2$ *and shrinks otherwise; (c) as* $\beta\mu_S$ *increases* $S_2(\beta\mu_S)$ *expands.*

Proof: Rearranging (A.49), the upper bound for F is

$$\frac{\beta\mu_S}{16}\frac{\mu_C}{\mu_S}(1-\frac{\mu_C}{\mu_S}), \tag{B.2}$$

which obtains a maximum value of $1/64$ over μ_C/μ_S, $\beta\mu_S \in [0,1]$. Given any $0 < F < 1/64$, by (B.2), for any $\beta\mu_S \in (0,1)$ and

$$\frac{1-\sqrt{1-64\frac{F}{\beta\mu_S}}}{2} \le \frac{\mu_C}{\mu_S} \le \frac{1+\sqrt{1-64\frac{F}{\beta\mu_S}}}{2}, \tag{B.3}$$

there exists F, such that for small enough $\mu_C < \mu_S$, Π_L is decreasing in μ_S. This proves part (i). Part (ii) follows from (B.3) . ■

Part (i) of Corollary B.2 states that as long as $F \le 1/64$, there is a wide band of μ_C/μ_S values in $(0,1)$ in the parameter space such that when μ_C and μ_S are small enough and lie in this band, increasing the detection and prosecution rate for individual piracy will decrease the legal publisher's profit. Panel (b) of Figure 5 demonstrates the region in the parameter space as a function of $\beta\mu_S$ and the ratio μ_C/μ_S. The rim of the surface $\overline{F}(\alpha,\beta\mu_S)$ corresponds to the boundary of the region $S_2(\beta\mu_S)$ as defined in Corollary B.2. As it can be seen from the figure and as stated in part (ii) of the corollary, as F increases, $S_2(\beta\mu_S)$ shrinks for all $\beta\mu_S$. This is because when the risk of entry is high for the commercial pirate, it becomes harder for him to enter the market to change the market structure from $[SL]$ to $[SCL]$, and hence the region where the legal publisher can obtain strategic benefit by holding the legal publisher outside of the market with a decrease in the detection rate of individual piracy will decline. On the other hand, an increase in the opportunity cost of being detected and prosecuted for the consumers ($\beta\mu_S$) will decrease the appeal of piracy for the consumers and expand the opportunity set of the legal publisher to strategically increase her price while still being able to keep the commercial piracy out of the market. For small values of μ_C/μ_S, the commercial pirate's product will be much more attractive for the consumers compared to individual piracy, and therefore, the legal publisher will have less power to use individual piracy against the commercial pirates. As μ_C/μ_S increases, individual piracy will be attractive enough for the consumers so that the legal publisher can utilize it effectively as a deterrent weapon combined with strategic pricing for a larger parameter region. On the other hand, when μ_C/μ_S becomes too large, the commercial pirate's product will start becoming less competitive in the marketplace and, as a result, individual piracy will become the main competitor for the legal publisher. In such cases, the parameter region on which the legal publisher can benefit from an increase in the detection rate of individual piracy will shrink, as can also be seen in Figure 5.

B.3 Applicability Region for Proposition 3

Corollary B.3

(i) For any $0 < \alpha < 1/2$, in the parameter region $0 < \beta\mu_S < (2-\alpha)(1-2\alpha)/(2(1-\alpha)^2)$, for small enough μ_C and μ_S as described in Proposition 3, Π_L is decreasing in μ_S.

(ii) Define $S_3 \subset [0,1]\times[0,1]$ as the subset of the parameter space where for $(\alpha, \beta\mu_S) \in S_3$, Π_L is decreasing for some $\mu_S \in (0,1)$ as described in part (i) and Proposition 3. Then for $(\alpha, \beta\mu_S) \in S_3$, (a) as α increases, or (b) as $\beta\mu_S$ increases S_3 contracts.

Proof: For part (*i*), notice that the right hand side of (A.56) is strictly decreasing in $\beta\mu_S$ and maximized at $\beta\mu_S = 0$ with value 1/2. Further, inverting (A.56) for the positive root, for any $\alpha \in (0, 1/2)$, we obtain the necessary condition

$$0 < \beta\mu_S < \frac{(2-\alpha)(1-2\alpha)}{2(1-\alpha)^2}\,. \tag{B.4}$$

Then by (A.57) and (A.64), for small enough F and μ_C and μ_S as described in Proposition 3 Π_L is decreasing in μ_S. Part (*ii*) directly follows from (A.56) and (B.4). This completes the proof. ■

As part (*i*) of Corollary B.3 states, as long as $\alpha \le 1/2$, there is a range of $\beta\mu_S$, over which, for small enough μ_C and μ_S values, increasing μ_S will decrease the legal publisher's profits as described in Proposition 2. This region of applicability (S_3) is depicted in panel (c) of Figure 5. As can be seen from the figure and as part (*ii*) of Corollary B.3 states, as α increases, S_3 contracts. This is because when the percentage of technologically savvy consumers increases, it will be more attractive for the commercial pirate to price lower to target the entire consumer population, and hence the region, where the legal publisher can strategically increase her unit price to keep the commercial pirate from aggressively lowering his price as described in Proposition 3, shrinks. Similarly, an increase in the consumers' opportunity cost of being detected for piracy weakens the commercial pirate. As a result, a higher $\beta\mu_S$ means that the legal publisher is compelled to concede market share to the commercial pirate by increasing her unit price for a smaller set of parameter values. Therefore, increasing $\beta\mu_S$ shrinks the region on which the legal publisher can benefit from decreased detection and prosecution of individual piracy, as can be seen from Figure 5 and as also stated in part (*ii*) of Corollary B.3.