Appendix A

Appendix A contains proofs for resubmission "Contracting Information Security in the Presence of Double Moral Hazard."

Proof of Lemma 1: Assume that, to the contrary, FBS efforts are achievable under a bilateral refund contract. Let $i, j = 1,2$ and $i \neq j$. Comparing (11) to (3), we have each firm's effort satisfy

$$\left.\frac{\partial P^i}{\partial e_F^i}\phi^i + \frac{\partial P^j}{\partial e_F^i}\right|_{E=E^*} = 0 \quad \text{(A1)}$$

$$\left.\frac{\partial P^j}{\partial e_F^j}\phi^j + \frac{\partial P^i}{\partial e_F^j}\right|_{E=E^*} = 0 \quad \text{(A2)}$$

Comparing (12) to (4), we have MSSP's effort for each firm satisfy

$$\left.\frac{\partial P^i}{\partial e_M^i}(1-\phi^i) + \frac{\partial P^j}{\partial e_M^i}(1-\phi^j)\right|_{E=E^*} = 0 \quad \text{(A3)}$$

$$\left.\frac{\partial P^j}{\partial e_M^j}(1-\phi^j) + \frac{\partial P^i}{\partial e_M^j}(1-\phi^i)\right|_{E=E^*} = 0 \quad \text{(A4)}$$

First suppose $\phi^i \neq 1$. If $|1-\phi^i| \geq |1-\phi^j|$, recall that we assume $\partial P^i(E)/\partial e_M^i > |\partial P^j(E)/\partial e_M^i|$ in the model (see page 9), it is then straightforward that (A3) cannot be true. Alternatively, if $|1-\phi^i| < |1-\phi^j|$, (A4) cannot be true. Therefore $\phi^i \neq 1$ leads to a contradiction. $\phi^j \neq 1$ will lead to a similar contradiction given the symmetry between notations *i* and *j*. Now suppose $\phi^i = \phi^j = 1$. (A1) cannot be true given our assumption that $\partial P^i(E)/\partial e_F^i > |\partial P^j(E)/\partial e_F^i|$. To summarize, no refund can lead to FBS efforts.

For the second half of the proposition: $\frac{\partial e_F^i}{\partial \phi} = \frac{\frac{\partial P^i(E)}{\partial e_F^i} d}{\frac{\partial^2 P^i(E)}{\partial (e_F^i)^2}(1-\phi)d - C^{F''}(e_F^i)} < 0$ and

$$\frac{\partial e_M^i}{\partial \phi} = \frac{-d\left(\frac{\partial P^i(E)}{\partial e_M^i} + \frac{\partial P^j(E)}{\partial e_M^i}\right)}{d\phi\left(\frac{\partial^2 P^i(E)}{\partial (e_M^i)^2} + \frac{\partial^2 P^j(E)}{\partial (e_M^i)^2}\right) - \frac{\partial^2 C(e_M^i, e_M^j)}{\partial (e_M^i)^2}} > 0 .$$ Q.E.D.

Proof of Lemma 2: We prove by contradiction. Suppose the MSSP's optimal offer of bilateral refund contracts are some $(f^1, f^2, \phi^1, \phi^2)$ where $(\phi^1, \phi^2) \neq (\hat{\phi}^1, \hat{\phi}^2)$. Contracts $(f^1, f^2, \phi^1, \phi^2)$ will induce stage 3 efforts $\hat{E}(\phi^1, \phi^2)$ and eventually result in payoff $U_M(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2))$ for the MSSP and payoff $U_F^i(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2))$ for firm *i*, $i = 1, 2$.

Our goal is to show that the MSSP can propose new contracts that are strictly better than $(f^1, f^2, \phi^1, \phi^2)$ in terms of payoffs for both firms and the MSSP -- i.e., Pareto-domination, thus establishing a contradiction. The new contracts we propose are $(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2)$, where $\hat{f}^i = f^i + \Delta_F^i - \varepsilon^i$. ε^i is a positive and small number (i.e., consider it infinitesimal). Δ_F^i is defined as follows:

$$\Delta_F^i = U_F^i(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) - U_F^i(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2)).$$

That is, Δ_F^i represents firm *i*'s payoff differential between contracts $(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2)$ and $(f^1, f^2, \phi^1, \phi^2)$, where the two sets of contracts are the same except for their refund clauses. Notice that by now we have introduced three sets of contracts: $(f^1, f^2, \phi^1, \phi^2)$, $(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2)$, and $(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2)$. Our goal is to compare $(f^1, f^2, \phi^1, \phi^2)$ and $(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2)$, and the introduction of $(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2)$ will facilitate this comparison. Under the new contracts $(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2)$, firm 1's expected payoff is

$$U_F^1(f^1 + \Delta_F^1 - \varepsilon^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2))$$

$$= U_F^1(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) - \Delta_F^1 + \varepsilon^1$$

$$= U_F^1(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) - U_F^1(f^1, f^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) + U_F^1(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2)) + \varepsilon^1$$

$$= U_F^1(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2)) + \varepsilon^1.$$

This is higher than firm 1's expected payoff under $(f^1, f^2, \phi^1, \phi^2)$ for any positive ε^1. Similarly, this new contract leads to a higher expected payoff for firm 2 than that under $(f^1, f^2, \phi^1, \phi^2)$ for any positive ε^2.

Because the new contracts contain refund clauses $(\hat{\phi}^1, \hat{\phi}^2)$, they maximize total profits of the firms and the MSSP given stage 3 equilibrium effort functions $\hat{E}()$. Therefore we have

$$\Delta_{SW} \equiv [U_M(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) + \sum_{i=1}^{2} U_F^i(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2))] - [U_M(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2)) +$$

$$\sum_{i=1}^{2} U_F^i (f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2))] > 0 .$$

Pick $\varepsilon^1 + \varepsilon^2 < \Delta_{SW}$, we have $U_M(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) - U_M(f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2))$

$$= \Delta_{SW} - \sum_{i=1}^{2} [U_F^i (\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2, \hat{E}(\hat{\phi}^1, \hat{\phi}^2)) - U_F^i (f^1, f^2, \phi^1, \phi^2, \hat{E}(\phi^1, \phi^2))] = \Delta_{SW} - \sum_{i=1}^{2} \varepsilon^i > 0 .$$

Therefore, the new contracts $(\hat{f}^1, \hat{f}^2, \hat{\phi}^1, \hat{\phi}^2)$ improve the MSSP's expected payoff as well. This contradicts the assumption that $(f^1, f^2, \phi^1, \phi^2)$ is the MSSP's optimal offer. Q.E.D.

Proof of Proposition 1: To analyze this generalized contract, we need to differentiate between two cases based on whether $\mu^i d$ is still counted as a part of the social welfare *SW*.

Case 1: $\mu^i d$ is included in the social welfare SW . Let U_{third} denote the expected utility of this third party, we then have

$$SW_G(E_G) = U_{MG} + U_{FG}^1 + U_{FG}^2 + U_{third} . \tag{A5}$$

Notice that it is straightforward from the proof of Lemma 2 that this lemma also applies to generalized bilateral refund contracts. Let $(\hat{\phi}_G^1, \hat{\phi}_G^2, \hat{\mu}_G^1, \hat{\mu}_G^2)$ represent the contingent payments in the MSSP's optimal contract offers in stage 1. From Lemma 2 we have that, in stage 1, the MSSP will only propose contracts that maximize the total expected utility of the two firms and itself (which does *not* equal to the social welfare):

$$(\hat{\phi}_G^1, \hat{\phi}_G^2, \hat{\mu}_G^1, \hat{\mu}_G^2) = \underset{(\phi_G^1, \phi_G^2, \mu_G^1, \mu_G^2)}{\arg\max}\ U_{MG} + U_{FG}^1 + U_{FG}^2 . \tag{A6}$$

Therefore, when $\mu^i d$ is included in the social welfare *SW*, the self-interested MSSP will not propose contracts that can maximize social welfare.

Case 2: $\mu^i d$ is excluded from the social welfare SW . In other words, given money burning upon security breach, $\mu^i d$, the expected social welfare is changed from equation (1) to

$$SW_G(E_G) = 2V - (1 - P^1(E_G))(1 + \mu^1)d - (1 - P^2(E_G))(1 + \mu^2)d - C_F(e_{FG}^1) - C_F(e_{FG}^2) - C_M(e_{MG}^1, e_{MG}^2) . \tag{A7}$$

Differentiating SW_G in (A7) with respect to each effort, and given that there is no externality, we know that FBS efforts satisfy

$$\frac{\partial P^i(E_G)}{\partial e^i_{FG}}(1+\mu^i)d - C'_F(e^i_{FG}) = 0 \tag{A8}$$

and

$$\frac{\partial P^i(E_G)}{\partial e^i_{MG}}(1+\mu^i)d - \frac{\partial C_M(e^i_{MG}, e^j_{MG})}{\partial e^i_{MG}} = 0 \tag{A9}$$

for $i, j = 1,2$ and $i \neq j$. Let $E^*_G = (e^{1*}_{FG}, e^{2*}_{FG}, e^{1*}_{MG}, e^{2*}_{MG})$ denote these FBS efforts. Observe that E^*_G is different from E^* (that we derived in Subsection 4.1) for any $\mu^i \neq 0$.

Under generalized bilateral refund contracts, the new principal-agent problem, $\mathrm{R_G}$, is the same as Problem R except for a new expected payoff function for the MSSP:

$$U_{MG} = f^i + f^j - (1-P^i(E_G))(\phi^i + \mu^i)d - (1-P^j(E_G))(\phi^j + \mu^j)d - C_M(e^i_{MG}, e^j_{MG}) \tag{A10}$$

where $i, j \in \{1,2\}, i \neq j$. Therefore, given any (f^1, ϕ^1, μ^1) and (f^2, ϕ^2, μ^2) and without externality, the MSSP's individually optimal efforts in stage 3 satisfy

$$\frac{\partial P^i(E_G)}{\partial e^i_{MG}}(\phi^i+\mu^i)d - \frac{\partial C_M(e^i_{MG}, e^j_{MG})}{\partial e^i_{MG}} = 0\,. \tag{A11}$$

The condition for firm *i*'s individually optimal effort, as shown below, is analogous to equation (11):

$$\frac{\partial P^i(E_G)}{\partial e^i_{FG}}(1-\phi^i)d - C'_F(e^i_{FG}) = 0 \tag{A12}$$

where $i = 1,2$. Equations (A11) and (A12) jointly determine firm and MSSP efforts for any given generalized bilateral contracts (f^1, ϕ^1, μ^1) and (f^2, ϕ^2, μ^2).

By comparing (A11) to (A9), we know that the MSSP will exert FBS efforts if and only if $\phi^1 = \phi^2 = 1$. However, from (A12) it follows that $\phi^1 = \phi^2 = 1$ will result in zero efforts from the firms. *Therefore, when $\mu^i d$ is excluded from the social welfare SW, the generalized bilateral* r*efund contracts cannot lead to FBS efforts.*

To summarize, no matter $\mu^i d$ is considered as part of social welfare or not, generalized contracts do not lead to FBS efforts given that all players are self-interested. Q.E.D.

Proof of Lemma 3: Equilibrium refund level ϕ should maximize total profit $U_M + U^1_F + U^2_F$, i.e. the social welfare, subject to $\hat{e}_F^i = ad(1-\phi)/(2\alpha)$ and $\hat{e}_M^i = (b+\lambda_M)\phi d/(2\beta)$.

$$SW = V - 2d + (P^i + P^j)d - \beta((\hat{e}_M^i)^2 + (\hat{e}_M^j)^2) - \alpha((\hat{e}_F^i)^2 + (\hat{e}_F^j)^2)$$

$$= V - 2d + ((a\hat{e}_F^i + b\hat{e}_M^i + \lambda_F\hat{e}_F^j + \lambda_M\hat{e}_M^j) + (a\hat{e}_F^j + b\hat{e}_M^j + \lambda_F\hat{e}_F^i + \lambda_M\hat{e}_M^i))d - \beta((\hat{e}_M^i)^2 + (\hat{e}_M^j)^2) - \alpha((\hat{e}_F^i)^2 + (\hat{e}_F^j)^2)$$

$$= V - 2d + 2d(a(\frac{ad(1-\phi)}{2\alpha}) + b(\frac{(b+\lambda_M)d\phi}{2\beta}) + \lambda_F(\frac{ad(1-\phi)}{2\alpha}) + \lambda_M(\frac{(b+\lambda_M)d\phi}{2\beta}))$$

$-2\beta(\frac{(b+\lambda_M)d\phi}{2\beta})^2 - 2\alpha(\frac{ad(1-\phi)}{2\alpha})^2$. Therefore, equilibrium refund level is the solution to

$\frac{\partial SW}{\partial \phi} = \frac{d^2(-a\beta(a\phi+\lambda_F) + \alpha(1-\phi)(b^2+\lambda_M(2b+\lambda_M)))}{\alpha\beta} = 0$. Q.E.D.

Proof of Proposition 2: $\frac{\partial \phi^*}{\partial \lambda_F} = -\frac{a\beta}{a^2\beta + \alpha(b+\lambda_M)^2} < 0$ and $\frac{\partial \phi^*}{\partial \lambda_M} = \frac{2a\alpha\beta(a+\lambda_F)(b+\lambda_M)}{(a^2\beta+\alpha(b+\lambda_M)^2)^2} > 0$.

Substitute the expression of ϕ^* in Lemma 3 into effort expressions $\hat{e}_F^i = ad(1-\phi)/(2\alpha)$ and $\hat{e}_M^i = (b+\lambda_M)\phi d/(2\beta)$, we have

$\hat{e}_F^i = \frac{a^2\beta(a+\lambda_F)d}{2\alpha(a^2\beta+\alpha(b+\lambda_M)^2)}$ and $\hat{e}_M^i = \frac{(b+\lambda_M)(-a\beta\lambda_F + \alpha(b+\lambda_M)^2)d}{2\beta(a^2\beta+\alpha(b+\lambda_M)^2)}$.

When differentiating $\hat{e}_F^i$ and $\hat{e}_M^i$ with respect to λ_F and λ_M, we get

$\frac{\partial \hat{e}_F^i}{\partial \lambda_F} = \frac{a^2\beta d}{2\alpha(a^2\beta+\alpha(b+\lambda_M)^2)} > 0$, $\frac{\partial \hat{e}_F^i}{\partial \lambda_M} = -\frac{a^2 d\beta(a+\lambda_F)(b+\lambda_M)}{(a^2\beta+\alpha(b+\lambda_M)^2)^2} < 0$, $\frac{\partial \hat{e}_M^i}{\partial \lambda_F} = -\frac{ad(b+\lambda_M)}{2(a^2\beta+\alpha(b+\lambda_M)^2)} < 0$,

$\frac{\partial \hat{e}_M^i}{\partial \lambda_M} = \frac{d(a\beta\lambda_F(b^2\alpha - a^2\beta + \alpha\lambda_M(2b+\lambda_M)) + \alpha(b+\lambda_M)^2(b^2\alpha + 3a^2\beta + \alpha\lambda_M(2b+\lambda_M)))}{2\beta(a^2\beta+\alpha(b+\lambda_M)^2)^2} > 0$. Q.E.D.

Proof of Proposition 3: $e_F^{i*} - \hat{e}_F^i = \frac{d(a+\lambda_F)}{2\alpha} - \frac{a^2 d\beta(a+\lambda_F)}{2\alpha(a^2\beta+\alpha(b+\lambda_M)^2)}$ and

$e_M^{i*} - \hat{e}_M^i = \frac{d(b+\lambda_M)}{2\beta} - \frac{d(b+\lambda_M)(-a\beta\lambda_F + \alpha(b+\lambda_M)^2)}{2\beta(a^2\beta+\alpha(b+\lambda_M)^2)}$. By differentiating $e_F^{i*} - \hat{e}_F^i$ and $e_M^{i*} - \hat{e}_M^i$ with

respect to λ_F and λ_M, we have $\frac{\partial(e_F^{i*} - \hat{e}_F^i)}{\partial \lambda_F} = \frac{d(b+\lambda_M)^2}{2(a^2\beta+\alpha(b+\lambda_M)^2)} > 0$,

$\frac{\partial(e_F^{i*} - \hat{e}_F^i)}{\partial \lambda_M} = \frac{a^2 d\beta(a+\lambda_F)(b+\lambda_M)}{(a^2\beta+\alpha(b+\lambda_M)^2)^2} > 0$, $\frac{\partial(e_M^{i*} - \hat{e}_M^i)}{\partial \lambda_F} = \frac{ad(b+\lambda_M)}{2(a^2\beta+\alpha(b+\lambda_M)^2)} > 0$, and

$\frac{\partial(e_M^{i*}-\hat{e}_M^i)}{\partial\lambda_M}=\frac{ad(a+\lambda_F)(a^2\beta-\alpha(b+\lambda_M)^2)}{2(a^2\beta+\alpha(b+\lambda_M)^2)^2}$. The last partial derivative is positive if and only if

$\lambda_M\in(-a\sqrt{\beta/\alpha}-b,a\sqrt{\beta/\alpha}-b)$. Q.E.D.

Proof of Lemma 4: This proof is analogous to the proof of Lemma 2, and thus we provide a concise analysis here. Similar to Lemma 2, we prove by contradiction. Suppose the MSSP's optimal offer of multilateral contracts are some (f,Φ) where $\Phi\neq\tilde{\Phi}$. Now we construct a new contract $(\tilde{f},\tilde{\Phi})$ where $\tilde{f}=f+\Delta_F-\varepsilon$. ε is a positive and small number. Δ_F is defined as follows:

$$\Delta_F=U_F^i(f,\tilde{\Phi},\tilde{E}(\tilde{\Phi}))-U_F^i(f,\Phi,\tilde{E}(\Phi)).$$

We then have, for firm 1, $U_F^1(\tilde{f},\tilde{\Phi},\tilde{E}(\tilde{\Phi}))=U_F^1(f,\Phi,\tilde{E}(\Phi))+\varepsilon$. Therefore, the new contract improves firm 1's expected payoff for any positive ε. Similarly, this new contract leads to a higher expected payoff for firm 2 for any positive ε.

Because the new contracts contain refund clauses $\tilde{\Phi}$, they maximize total profits of the firms and the MSSP given Stage 3 equilibrium effort functions $\tilde{E}(\bullet)$. Therefore we have

$$\Delta_{SW}\equiv[U_M(\tilde{f},\tilde{\Phi},\tilde{E}(\tilde{\Phi}))+\sum_{i=1}^{2}U_F^i(\tilde{f},\tilde{\Phi},\tilde{E}(\tilde{\Phi}))]-[U_M(f,\Phi,\tilde{E}(\Phi))+\sum_{i=1}^{2}U_F^i(f,\Phi,\tilde{E}(\Phi))]>0.$$

Pick $\varepsilon<\Delta_{SW}/2$, we have

$$U_M(\tilde{f},\tilde{\Phi},\tilde{E}(\tilde{\Phi}))-U_M(f,\Phi,\tilde{E}(\Phi))=\Delta_{SW}-2\varepsilon>0.$$

Therefore, this new contract also improves the MSSP's expected payoff. This contradicts the assumption that (f,Φ) is the MSSP's optimal offer. Q.E.D.

Proof of Proposition 4: Given any Φ, in stage 1 the MSSP will propose fixed payment (that it collects from firm *i* upon signing a contract) such that firm *i* is indifferent between signing up for the contract and taking the outside option $\underline{U}_F$.[1] In other words, the fixed payment will be $f=U_F^i(0,\Phi,\tilde{\mathrm{E}}(\Phi))-\underline{U}_F$. Therefore, the MSSP's expected payoff in stage 1 is $U_M=U_M(0,\Phi,\tilde{E}(\Phi))+2f$

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[1] Strictly speaking, to induce participation by the firm, the contract the MSSP proposed should make the firm better off by an infinitesimal amount (as compared to the outside option).

$= U_M(\cdot) + U_F^1(\cdot) + U_F^2(\cdot) - 2\underline{U}_F = SW(\cdot) - 2\underline{U}_F$, where $SW(\cdot)$ is the total profit of all three players, i.e. the social welfare. Therefore, the MSSP's incentive in stage 1 is aligned with social welfare maximization. In other words, the MSSP will pick any Φ that can induce FBS efforts.

We next solve for the optimal Φ that can induce FBS efforts. For $i, j = 1, 2$, $i \neq j$, firm *i*'s expected payoff is $U_F^i = -f + V - (1-P^i)d + (1-P^i)(1-P^j)d\phi_{bb} + (1-P^i)P^j d\phi_{bn} + P^i(1-P^j)d\phi_{nb} - C_F(e_F^i)$. Thus firm *i*'s equilibrium effort satisfies

$$\frac{\partial U_F^i}{\partial e_F^i} = \Omega_{FNE}^i + d\left(\frac{\partial P^i}{\partial e_F^i}\right) - C_F^{'}(e_F^i) = 0 \qquad \text{(A13)}$$

where $\Omega_{FNE}^i \equiv [(\phi_{bb} - \phi_{bn} - \phi_{nb})P^j - \phi_{bb} + \phi_{nb}]\frac{\partial P^i}{\partial e_F^i} d$.

A MSSP's expected payoff is

$U_M = 2f - C^M(e_M^i, e_M^j) - (1-P^i)(1-P^j)2d\phi_{bb} - \left((1-P^i)P^j + P^i(1-P^i)\right)(d\phi_{bn} + d\phi_{nb})$. Thus the MSSP's equilibrium effort satisfies

$$\frac{\partial U_M}{\partial e_M^i} = \Omega_{MNE}^i + d\left(\frac{\partial P^i}{\partial e_M^i}\right) - \frac{\partial C_M(e_M^i, e_M^j)}{\partial e_M^i} = 0 \qquad \text{(A14)}$$

where $\Omega_{MNE}^i \equiv 2[(\phi_{bb} - \phi_{bn} - \phi_{nb})(1-P^j) + (\phi_{bn} + \phi_{nb} - 1)/2]\frac{\partial P^i}{\partial e_M^i} d$.

For $i, j = 1, 2$, $i \neq j$, comparing (A13) to (3), and (A14) to (4), a multilateral contract can induce FBS efforts if $\Omega_{FNE}^i\big|_{E=E^*} = 0$ and $\Omega_{MNE}^i\big|_{E=E^*} = 0$, i.e. if the following two conditions all hold for any *j*:

$$(\phi_{bb} - \phi_{bn} - \phi_{nb})P^j\big|_{E=E^*} - \phi_{bb} + \phi_{nb} = 0, \qquad \text{(A15)}$$

$$(\phi_{bb} - \phi_{bn} - \phi_{nb})(1 - P^j\big|_{E=E^*}) + (\phi_{bn} + \phi_{nb} - 1)/2 = 0. \qquad \text{(A16)}$$

The only Φ that can satisfy conditions (A15) and (A16) regardless of the value of $P^j\big|_{E=E^*}$ are solutions to $\phi_{bb} - \phi_{bn} - \phi_{nb} = 0$, $\phi_{bb} - \phi_{nb} = 0$ and $\phi_{bn} + \phi_{nb} - 1 = 0$. That is, $\Phi = \Phi_{NE}^* = (\phi_{bn} = 0, \phi_{nb} = 1, \phi_{bb} = 1)$.

Note that we have not checked second order conditions -- it is straightforward to verify that they hold under Φ_{NE}^*. A rigorous discussion on second order conditions is provided next in the proof of Proposition

5; it also applies to Proposition 4 as this is a special case of Proposition 5. Q.E.D.

Proof of Proposition 5: This proof is analogous to the proof of Proposition 4. Given any Φ, in stage 1 the MSSP will propose fixed payment $f = U_F^i(0,\Phi,\tilde{\mathrm{E}}(\Phi)) - \underline{U}_F$. Therefore, the MSSP's expected payoff in stage 1 is $U_M = U_M(0,\Phi,\tilde{E}(\Phi)) + 2f = U_M(\cdot) + U_F^1(\cdot) + U_F^2(\cdot) - 2\underline{U}_F = SW(\cdot) - 2\underline{U}_F$, where $SW(\cdot)$ is the social welfare. Consequently, the MSSP will pick any Φ that can induce FBS efforts.

We next solve for the optimal Φ that can induce FBS efforts. For $i, j = 1,2$, $i \neq j$, firm *i*'s expected payoff is $U_F^i = -f + V - (1-P^i)d + (1-P^i)(1-P^j)d\phi_{bb} + (1-P^i)P^j d\phi_{bn} + P^i(1-P^j)d\phi_{nb} - C^F(e_F^i)$. Thus firm *i*'s equilibrium effort satisfies

$$\frac{\partial U_F^i}{\partial e_F^i} = \Omega_F^i + d\left(\frac{\partial P^i}{\partial e_{Fi}} + \frac{\partial P^j}{\partial e_{Fi}}\right) - C_F^{'}(e_F^i) = 0 \qquad \text{(A17)}$$

where $\Omega_F^i \equiv d\frac{\partial P^i}{\partial e_F^i} + (-\frac{\partial P^i}{\partial e_F^i}(1-P^j) - \frac{\partial P^j}{\partial e_F^i}(1-P^i))d\phi_{bb} + (-\frac{\partial P^i}{\partial e_F^i}P^j + \frac{\partial P^j}{\partial e_F^i}(1-P^i))d\phi_{bn}$

$$+(\frac{\partial P^i}{\partial e_F^i}(1-P^j) - \frac{\partial P^j}{\partial e_F^i}P^i)d\phi_{nb} - d\left(\frac{\partial P^i}{\partial e_F^i} + \frac{\partial P^j}{\partial e_F^i}\right).$$

A MSSP's expected payoff is

$U_M = 2f - C^M(e_M^i, e_M^j) - (1-P^i)(1-P^j)2d\phi_{bb} - \left((1-P^i)P^j + P^i(1-P^i)\right)(d\phi_{bn} + d\phi_{nb})$. Thus the MSSP's equilibrium effort satisfies

$$\frac{\partial U_M}{\partial e_M^i} = \Omega_M^i + d\left(\frac{\partial P^i}{\partial e_M^i} + \frac{\partial P^j}{\partial e_M^i}\right) - \frac{\partial C_M(e_M^i, e_M^j)}{\partial e_M^i} = 0 \qquad \text{(A18)}$$

where $\Omega_M^i \equiv (\frac{\partial P^i}{\partial e_M^i}(1-P^j) + \frac{\partial P^j}{\partial e_M^i}(1-P^i))2d\phi_{bb}$

$$-(-\frac{\partial P^i}{\partial e_M^i}P^j + \frac{\partial P^j}{\partial e_M^i}(1-P^i) + \frac{\partial P^i}{\partial e_M^i}(1-P^j) - \frac{\partial P^j}{\partial e_M^i}P^i)d(\phi_{bn} + \phi_{nb}) - d\left(\frac{\partial P^i}{\partial e_M^i} + \frac{\partial P^j}{\partial e_M^i}\right).$$

For $i, j = 1,2$, $i \neq j$, comparing (A17) to (3), and (A18) to (4), a multilateral contract can induce FBS efforts if $\Omega_F^i\big|_{E=E^*} = 0$, $\Omega_M^i\big|_{E=E^*} = 0$ and the following two second-order conditions hold:

$$\left.\frac{\partial\Omega_M^i}{\partial e_F^i}\right|_{E=E^*} \le -d\left(\frac{\partial^2 P^i}{\partial(e_F^i)^2}+\frac{\partial^2 P^j}{\partial(e_F^i)^2}\right)\Bigg|_{E=E^*}, \text{ and} \tag{A19}$$

$$\left.\frac{\partial\Omega_M^i}{\partial e_M^i}\right|_{E=E^*} \le -d\left(\frac{\partial^2 P^i}{\partial(e_M^i)^2}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}\right)\Bigg|_{E=E^*}. \tag{A20}$$

Note that $\frac{\partial\Omega_F^i}{\partial e_F^i}=\left[-d\frac{\partial^2 P^j}{\partial(e_F^i)^2}-\frac{\partial^2 P^i}{\partial(e_F^i)^2}d\phi_{bb}-\frac{\partial^2 P^j}{\partial(e_F^i)^2}d\phi_{bb}+\frac{\partial^2 P^j}{\partial(e_F^i)^2}d\phi_{bn}+\frac{\partial^2 P^i}{\partial(e_F^i)^2}d\phi_{nb}\right]$

$+\left[d(\phi_{bb}-\phi_{bn}-\phi_{nb})(\frac{\partial^2 P^j}{\partial(e_F^i)^2}P^i+\frac{\partial P^j}{\partial e_F^i}\frac{\partial P^i}{\partial e_F^i}+\frac{\partial^2 P^j}{\partial(e_F^i)^2}P^j+\frac{\partial P^i}{\partial e_F^i}\frac{\partial P^j}{\partial e_F^i})\right]$, which is dependent on the values of P^i, P^j, $\partial P^i/\partial e_F^i$ and $\partial P^j/\partial e_F^i$, and thus (A19) is not guaranteed true when $\phi_{bb}-\phi_{bn}-\phi_{nb}\neq 0$. However, if we impose the condition that $\phi_{bb}-\phi_{bn}-\phi_{nb}=0$, shortly we will see that both (A19) and (A20) are true for generic P^i and P^j functions. Therefore hereafter we only consider the case where $\phi_{bb}=\phi_{bn}+\phi_{nb}$. We then have

$\frac{\partial\Omega_F^i}{\partial e_F^i}=-d\frac{\partial^2 P^j}{\partial(e_F^i)^2}-\frac{\partial^2 P^i}{\partial(e_F^i)^2}d\phi_{bb}-\frac{\partial^2 P^j}{\partial(e_F^i)^2}d\phi_{bb}+\frac{\partial^2 P^j}{\partial(e_F^i)^2}d\phi_{bn}+\frac{\partial^2 P^i}{\partial(e_F^i)^2}d\phi_{nb}$. (A19) is then simplified to

$$-\frac{\partial^2 P^j}{\partial(e_F^i)^2}\phi_{nb}\le-\frac{\partial^2 P^i}{\partial(e_F^i)^2}(1-\phi_{bn}). \tag{A21}$$

Similarly, given $\phi_{bb}=\phi_{bn}+\phi_{nb}$, we have

$$\begin{aligned}\frac{\partial\Omega_M^i}{\partial e_M^i}&=\left[\frac{\partial^2 P^i}{\partial(e_M^i)^2}2d\phi_{bb}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}2d\phi_{bb}-\frac{\partial^2 P^j}{\partial(e_M^i)^2}d(\phi_{bn}+\phi_{nb})-\frac{\partial^2 P^i}{\partial(e_M^i)^2}d(\phi_{bn}+\phi_{nb})-d\left(\frac{\partial^2 P^i}{\partial(e_M^i)^2}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}\right)\right]\\&\quad-\left[2d(\phi_{bb}-\phi_{bn}-\phi_{nb})(\frac{\partial^2 P^i}{\partial(e_M^i)^2}P^i+\frac{\partial P^i}{\partial e_M^i}\frac{\partial P^i}{\partial e_M^i}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}P^j+\frac{\partial P^j}{\partial e_M^i}\frac{\partial P^j}{\partial e_M^i})\right]\\&=\frac{\partial^2 P^i}{\partial(e_M^i)^2}2d\phi_{bb}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}2d\phi_{bb}-\frac{\partial^2 P^j}{\partial(e_M^i)^2}d(\phi_{bn}+\phi_{nb})-\frac{\partial^2 P^i}{\partial(e_M^i)^2}d(\phi_{bn}+\phi_{nb})-d\left(\frac{\partial^2 P^i}{\partial(e_M^i)^2}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}\right)\\&=\left(d\phi_{bb}-d\right)\left(\frac{\partial^2 P^i}{\partial(e_M^i)^2}+\frac{\partial^2 P^j}{\partial(e_M^i)^2}\right)\end{aligned}$$

, which is non-positive (a condition stronger than (A20)) if

$$\phi_{bb}\ge 1. \tag{A22}$$

From $\Omega_F^i\big|_{E=E^*}=0$, $\Omega_M^i\big|_{E=E^*}=0$ and $\phi_{bb}=\phi_{bn}+\phi_{nb}$ we have optimal multilateral contract payments as:

$$\phi_{bb}^* = 1\,,\ \phi_{bn}^* = -2\frac{\partial P^j/\partial e_F^i}{\partial P^i/\partial e_F^i - \partial P^j/\partial e_F^i}\Bigg|_{E=E^*},\ \text{and}\ \phi_{nb}^* = \frac{\partial P^i/\partial e_F^i + \partial P^j/\partial e_F^i}{\partial P^i/\partial e_F^i - \partial P^j/\partial e_F^i}\Bigg|_{E=E^*}.$$

Finally we check the second-order conditions: (A22) is apparently true; (A21) is equivalent to $\left|\frac{\partial^2 P^j}{\partial e_F^i}\right| \le \left|\frac{\partial^2 P^i}{\partial e_F^i}\right|$, which is our model assumption. Q.E.D.

Proof of Proposition 6: This proof is analogous to the proof of Propositions 4 and 5. Given any Φ_N, in stage 1 the MSSP will propose fixed payment $f_N = U_{FN}^i(0, \Phi_N, \tilde{E}_N(\Phi_N)) - \underline{U}_F$. Therefore, the MSSP's expected payoff in stage 1 is $U_{MN} = U_{MN}(0, \Phi_N, \tilde{E}_N(\Phi_N)) + 2f_N = U_{MN}(\cdot) + U_{FN}^1(\cdot) + U_{FN}^2(\cdot) - 2\underline{U}_F$ $= SW_N(\cdot) - 2\underline{U}_F$, where $SW_N(\cdot)$ is the social welfare under $N \ge 2$. Consequently, the MSSP, though self-interested, will pick any Φ_N in stage 1 that can induce FBS efforts.

We next solve for the optimal Φ_N that can induce FBS efforts. Note that we already solved for γ that can induce FBS efforts prior to this proposition. Consider the case where n ($1 \le n \le N$) firms are breached. Suppose firm i is not one of the breached firms. Then, for each breach, the MSSP distributes $(d + \gamma d)/(N-1)$ to firm i. Therefore the total firm i receives is $n(d + \gamma d)/(N-1)$.

Suppose firm i is one of the breached firms. Then it pays out γd for its own breach. But it also receives $(d + \gamma d)/(N-1)$ because of each of the other $n-1$ breached firms. Therefore it pays in total $\gamma d - (n-1)(d + \gamma d)/(N-1)$. Q.E.D.

Appendix B. Bargaining over Multilateral Contracts

Here we analyze one variation of the principal-agent model in which we allow bargaining between the MSSP and the firms with any arbitrary sharing of market power. Recall that in the main text of this paper our analysis is based on the timeline in Figure 1, in which firms are Stackelberg followers and can only accept or reject the offers by the MSSP. Understandably, such a game structure implies a much greater market power on the MSSP side than that on the firm side. Consequently, in Propositions 4 and 5 we see that the MSSP uses the fixed payment f to squeeze as much surplus as possible out of the contractual relationship to the extent that firm payoffs are merely matching the values of their outside options.

In practice, however, negotiation over security contracts -- as one type of business-to-business contracts -- can be more complicated than that in Figure 1. Instead of having to terminate the game after rejecting the MSSP's offer in stage 2, firms may counter-offer and thus extend the negotiation process before any final deal is reached. The details of such bargaining processes are out of the scope of this paper. Rather, in this appendix we take each side's market power as given and analyze how a given market power structure affects the optimal multilateral contracts.

Formally, for any given N and under optimal multilateral contracts, the social welfare is $SW_N^* = SW_N(E_N^*)$. Note that, given the outside option $\underline{U}_F$, this firm will not accept a multilateral contract if the contract results in an expected firm payoff less than $\underline{U}_F$. Similarly, the MSSP will not serve a firm if this service results in a value for the MSSP less than $\underline{U}_M$. We thus define market power in the following way: given a multilateral contract (f_N, Φ_N) , the MSSP is said to have market power σ if

$$U_{MN}(f_N, \Phi_N, E_N(\Phi_N)) = N\underline{U}_M + \sigma(SW_N(\Phi_N) - N\underline{U}_M - N\underline{U}_F)\,. \tag{A23}$$

We say the MSSP has full market power (or firms have zero market power) if $\sigma = 1$: in this case, a firm's expected payoff equals its outside option value. This is what we have analyzed so far. Alternatively, the MSSP has zero market power (or firms have full market power) if $\sigma = 0$: in this case, the MSSP's expected payoff equals its outside option value. σ thus has a feasible region [0,1].

Proposition A1: *Given any $\sigma \in [0,1]$, the multilateral contracts bargained and agreed by all parties have the following properties:*

i. *the contingent payment plan is always Φ_N^* (i.e. same as the one in Proposition 6);*

ii. *for any $1 \le i \le N$ and upon signing of a contract, firm i pays the MSSP a fixed payment.*

$$f_N \equiv -U_{MN}(0, \Phi_N^*, E_N^*)/N + \underline{U}_M + \sigma(SW_N^*/N - \underline{U}_M - \underline{U}_F)\,.$$

This multilateral contract induces FBS efforts from all players regardless of σ .

Part (i) of this proposition is straightforward given Lemma 4: regardless of how the social welfare is shared, in stage 1 the self-interested MSSP has incentive to only propose contracts that are Pareto-efficient (same about any counter-offers by firms), which requires a contingent payment plan Φ_N^* .

Similarly, if in stage 2 a firm counter-offers any contract that is inconsistent with Φ_N^* , the MSSP can then counter back with a new and Pareto-dominant contract with Φ_N^* . Recall that the proof of Lemma 4 provides details on how to construct such a Pareto-dominant contract with Φ_N^* .

Part (ii) shows that the fixed payment from a firm to the MSSP consists of three elements. The first element is to compensate for the costs incurred on the MSSP side for security services. The second element incentivizes the MSSP against its outside option. The third element reflects the MSSP's market power in bargaining.

Proposition A1 shows that, in addition to the principal-agent setup, our proposed multilateral contracts can also induce FBS efforts under a bargaining setup with any arbitrary division of market power between the MSSP side and the firm side. Therefore our research also applies to business scenarios where security outsourcing contracts are determined by back-and-forth negotiations rather than take-it-or-leave-it one-time offers.