Analyzing Software as a Service with Per-Transaction Charges

Dan Ma & Abraham Seidmann

Supplementary Materials

Analytical proofs and numerical examples to show that our major results are still valid when users' transaction volume follows other distributions.

In Section 4.3, we state, "*our outcomes are robust to the assumption about the distribution of users' transaction volumes.*" Here we provide the detailed analytical proof and numerical examples for this statement.

Distribution (1): Users' expected transaction volume d_i is in the range of [a, $a+1$], and each user i's actual transaction volume is uniformly distributed on [$d_i - \theta$, $d_i + \theta$] with $\theta < a$.

<Proof> When a user i considers the MOTS solution, its optimal capacity Q_i^* is given by Equation (1) and its expected total utility is given by Equation (2). The indifferent customer is denoted as d^*, as before. The MOTS provider now serves users in [d^*, $1+a$], with a market share of $1+a-d^*$, and the SaaS provider serves users in [a, d^*], with a market share of (d^*-a). They both maximize profit by choosing prices: $Max_{P_m}\Pi_{MOTS} = P_m(1+a-d^*)$; $Max_{p_s}\Pi_{SaaS} = \int_a^{d^*}(p_s - c_s)x dx = \frac{1}{2}(p_s - c_s)(d^{*2} - a^2)$; *s.t.:* $c_s \le p_s \le u-t$; $P_m \ge 0$; $a \le d^* \le 1+a$.

This problem could be solved the same way as before. There are constrained prices, due to binding constraints $P_m=0$, $p_s= u-t$, and $p_s= c_s$, leading to the critical values of t_1, t^*, and t_2 respectively. In addition, the unconstrained first order conditions for both providers are: $P_m^* = \frac{(1+a)(p_s^*+t-c-c_m)}{2} - \frac{C_{cust}}{2} - \frac{(\Delta u - c)c\theta}{2\Delta u}$ and $(P_m^* + C_{cust} + \frac{(\Delta u - c)c\theta}{\Delta u})^2(1 - \frac{2(p_s^*-c_s)}{p_s^*+t-c-c_m}) = \frac{a^2}{2}$.

Since a closed-form solution in this setting is intractable, we use numerical examples to show that when "lifting" the underlying parameter range from [*0,1*] to [*a, a+1*], our main results still hold.

For demonstration purpose, we take a =0.2. The other parameter values are: $C_{cust}=1$, $c=0.2$, $c_m=0.3$, $c_s=0.1$, $u=3$, $u_r=1$, $\theta=0.1$.

When the constraint $P_m \ge 0$ is binding, we get $P_m^* = 0$, $p_s^* = 0.5160$, and t_1=0.8323. When the constraint $p_s \le u-t$ is binding, we get $P_m^* = 0.991$, $p_s^* = 1.3438$, and t^*=1.6562. When the constraint $c_s \le p_s$ is binding, we get t_2=2.9.[1] Hence, we get the three critical values of t, which lead to four regimes similar to what we found in the paper:

(1) in Regime 1: $t < 0.8323$, only SaaS dominates the market;
(2) in Regime 2: $0.8323 \le t < 1.6562$, SaaS and MOTS coexist and compete with each other;
(3) in Regime 3: $1.6562 \le t \le 2.9$, two providers coexist but segment the market completely;
(4) in Regime 4: $t > 2.9$, SaaS fails and only MOTS exists as the monopoly.

The next table shows the two providers' price and market share (d^**-0.2* and *1.2-*d^* for the SaaS and MOTS provider respectively) at different values of t in each regime.

[1] If we calculate these t values in the supporting range $d_i \in [0,1]$, as in the main model of the paper, we get t_1=0.9009, t^*=1.65, and t_2=2.9.

Table A1. Software Providers' Price and Market Share

Lack of fit cost *t*	SaaS price	MOTS price	indifferent user *d**	SaaS market share	MOTS market share
0.43	0.13	na	1.20	100%	0%
0.53	0.22	na	1.20	100%	0%
0.63	0.32	na	1.20	100%	0%
0.73	0.42	na	1.20	100%	0%
***t1*= 0.832336**	0.52	na	1.20	100%	0%
0.93	0.62	0.12	1.09	89%	11%
1.03	0.72	0.24	1.01	81%	19%
1.13	0.82	0.36	0.95	75%	25%
1.23	0.92	0.48	0.91	71%	29%
1.33	1.02	0.60	0.88	68%	32%
1.43	1.12	0.72	0.85	65%	35%
1.53	1.22	0.84	0.83	63%	37%
***t**=1.632336**	1.32	0.96	0.81	61%	39%
1.76	1.24	0.99	0.80	60%	40%
1.86	1.14	0.99	0.80	60%	40%
1.96	1.04	0.99	0.80	60%	40%
2.06	0.94	0.99	0.80	60%	40%
2.16	0.84	0.99	0.80	60%	40%
2.26	0.74	0.99	0.80	60%	40%
2.36	0.64	0.99	0.80	60%	40%
2.46	0.54	0.99	0.80	60%	40%
2.56	0.44	0.99	0.80	60%	40%
2.66	0.34	0.99	0.80	60%	40%
2.76	0.24	0.99	0.80	60%	40%
***t2*=2.9**	na	0.99	0.80	0%	40%
3.00	na	0.99	0.80	0%	40%
3.10	na	0.99	0.80	0%	40%
3.20	na	0.99	0.80	0%	40%

In Table A1, "*na*" indicates the corresponding software provider exits the market, and so its price is not available. Three critical values of t are highlighted, and they form four regimes. We therefore conclude that main results in Propositions 1 to 5 still hold qualitatively when we use the parameter support range $[a, a+1]$ (at $a=0.2$), instead of the normalized line [0,1].

(2) Users' expected transaction volume d_i is in the range of [0,1] and each user i's actual transaction volume is uniformly distributed on $[(1-\theta)d_i, (1+\theta)d_i]$ with $\theta < 1$.

<Proof> When $d_i \sim [(1-\theta)d_i, (1+\theta)d_i]$, we need to resolve a user i's optimal capacity level Q_i^*.

$$\underset{Q_i}{Max} \quad E\left[(u-c_m)\min\{D_i,Q_i\}+(u_r-c_m)(D_i-Q_i)^+ - P_m - C_{cust} - cQ_i\right]$$

$= \text{Prob}\{D_i<=Q_i\} \cdot \text{E}[D_i / D_i <=Q_i] \cdot (u-c_m) + \text{Prob}\{D_i>Q_i\} \cdot \{ (\text{E}[D_i / D_i >Q_i] - Q_i) \cdot (u_r-c_m) + Q_i \cdot (u-c_m) \} - P_m - C_{cust} - cQ_i$

Since d_i is uniformly distributed on $[(1-\theta)d_i, (1+\theta)d_i]$, we have

$$\text{Prob}\{D_i<=Q_i\}=\frac{Q_i-(1-\theta)d_i}{2\theta d_i}; \quad \text{Prob}\{D_i>Q_i\}=\frac{(1+\theta)d_i-Q_i}{2\theta d_i};$$

$$E[D_i / D_i <= Q_i] = \frac{\int_{(1-\theta)d_i}^{Q_i} x\,dF(x)}{\frac{Q_i-(1-\theta)d_i}{2\theta d_i}} = \frac{\int_{(1-\theta)d_i}^{Q_i} x\,d\left(\frac{x-(1-\theta)d_i}{2\theta d_i}\right)}{\frac{Q_i-(1-\theta)d_i}{2\theta d_i}} = \frac{1}{2}(Q_i+(1-\theta)d_i)$$

$$E[D_i / D_i > Q_i] = \frac{\int_{Q_i}^{(1+\theta)d_i} x\,dF(x)}{\frac{(1+\theta)d_i-Q_i}{2\theta d_i}} = \frac{\int_{Q_i}^{(1+\theta)d_i} x\,d\left(\frac{x-(1-\theta)d_i}{2\theta d_i}\right)}{\frac{(1+\theta)d_i-Q_i}{2\theta d_i}} = \frac{1}{2}(Q_i+(1+\theta)d_i).$$

Plug in these values, we get:

$$\underset{Q_i}{Max} \quad E\left[(u-c_m)\min\{D_i,Q_i\}+(u_r-c_m)(D_i-Q_i)^+ - P_m - C_{cust} - cQ_i\right]$$

$$= \frac{(u-c_m)}{4\theta d_i}\{Q_i^2-(1-\theta)^2 d_i^2 + 2Q_i[(1+\theta)d_i-Q_i]\} + \frac{(u_r-c_m)}{4\theta d_i}\{(1+\theta)d_i-Q_i\}^2 - P_m - C_{cust} - cQ_i\}$$

Take derivative of Q_i, it gives

$$(u-c_m)\{(1+\theta)d_i-Q_i\}+(u_r-c_m)\{Q_i-(1+\theta)d_i\}-2c\theta d_i = 0.$$

The optimal Q_i^*, therefore, is given by $Q_i^* = d_i + \theta d_i\left(1-\frac{2c}{\Delta u}\right)$.

Compare this with Q_i^* in the main body of the paper, as in Equation (1). We see that the structure of the optimal Q_i^* is very similar. The only difference is that now Q_i^* deviates from the expected transaction volume d_i by $\theta d_i\left(1-\frac{2c}{\Delta u}\right)$, where the "deviation" also depends on d_i.

We plug Q_i^* back and get the expected utility for user i, if it uses the MOTS solution: $Eu_{MOTS}(d_i) = d_i(u-c-c_m) - P_m - C_{cust} - \frac{c(\Delta u - c)}{\Delta u}\theta d_i$. This expression again is similar to Equation (2) in the paper.

The Pre-SaaS Market

All analysis and derivation below follow the same logic as in the main body of the paper.

Before the SaaS entered the market, the marginal user, who gets zero expected utility from using the MOTS solution, is located at $d' = \frac{P_m + C_{cust}}{u-c-c_m-\frac{c(\Delta u - c)}{\Delta u}\theta}$ (by setting the utility expression $Eu_{MOTS}(d_i) = d_i(u-c-c_m) - P_m - C_{cust} - \frac{c(\Delta u - c)}{\Delta u}\theta d_i = 0$).

The MOTS provider's pricing problem is formulated the same as before: $\underset{P_m}{Max} \quad \Pi_{MOTS} = P_m \cdot (1-d')$.

The optimal price is $P_m' = \frac{(u-c-c_m)}{2} - \frac{1}{2}C_{cust} - \frac{(\Delta u - c)c}{2\Delta u}\theta$. Consequently, the marginal user is located at $d' = \frac{1}{2} + \frac{C_{cust}}{2\{(u-c_m-c)-\frac{(\Delta u - c)c}{\Delta u}\theta\}} > \frac{1}{2}$.

Comparing these two values to Equations (4) and (5) in the paper, we find (1) the optimal monopoly price P_m' remains the same, and (2) the characteristics of the marginal user d' remain the same, though the concrete value of d' is different. As a result, the conclusion in Proposition 1 still holds.

The Market with MOTS-SaaS Competition

Again, all analysis and derivation below follow the same logic as in the main body of the paper. The indifferent user's location, denoted as d^*, is given by $Eu_{SaaS}(d^*) = Eu_{MOTS}(d^*)$: $d^* = \dfrac{P_m + C_{cust}}{p_s + t - c - c_m - \dfrac{c(\Delta u - c)\theta}{\Delta u}}$. The MOTS and SaaS optimization problems also are the same as in the paper: $\underset{P_m}{Max}\ \Pi_{MOTS} = P_m(1 - d^*)$; $\underset{p_s}{Max}\ \Pi_{SaaS} = \int_0^{d^*} (p_s - c_s)\, x\, dx$, *s.t.:* $c_s \leq p_s \leq u - t$; $P_m \geq 0$; $0 \leq d^* \leq 1$. Solving them gives the following results:

(1) When $t < t_1$: the SaaS provider charges $p_s^* = 2c_s + t - c - c_m - \dfrac{(\Delta u - c)c\theta}{\Delta u}$ and serves all clients. The MOTS provider is priced out of the market.

(2) When $t_1 \leq t < t^*$: the two providers coexist and compete in the market, with equilibrium prices $\{P^* = c_s + t - c - c_m - \dfrac{C_{cust}}{2} - \dfrac{(\Delta u - c)c\theta}{\Delta u}; p_s^* = 2c_s + t - c - c_m - \dfrac{(\Delta u - c)c\theta}{\Delta u}\}$.

(3) When $t^* \leq t \leq t_2$: the two providers coexist. The equilibrium prices are $\{P^* = \dfrac{(u - c - c_m)}{2} - \dfrac{1}{2}C_{cust} - \dfrac{(\Delta u - c)c}{2\Delta u}\theta;\ p_s^* = u - t\}$. They segment the market with no direct competition.

(4) When $t > t_2$: the SaaS provider is not able to survive. Only the MOTS provider is in the market, with the monopolistic price $P^* = \dfrac{(u - c - c_m)}{2} - \dfrac{1}{2}C_{cust} - \dfrac{(\Delta u - c)c}{2\Delta u}\theta$.

These are just the same four regimes described in the paper, as in Figure 4.

The critical values of t are: $t_1 = \dfrac{c(\Delta u - c)\theta}{\Delta u} + \dfrac{C_{cust}}{2} + c + (c_m - c_s)$;
$t^* = \dfrac{u - 2c_s + c + c_m}{2} + \dfrac{c(\Delta u - c)\theta}{2\Delta u}$; and $t_2 = u - c_s$. It is easy to verify that we still have $t_1 \leq t^* \leq t_2$.

Comparing these critical values, we can see that under this new distribution of d_i, t_1 and t^* are larger by a magnitude of $\dfrac{c(\Delta u - c)\theta}{2\Delta u}$ (compared to these two critical values when $d_i \sim [d_i - \theta, d_i + \theta]$ as in the paper), meaning that the SaaS provider is in a more advantageous position in the competition —Regime 1 (SaaS dominates the market) and Regime 2 (SaaS can compete aggressively and take large-volume users) expand. This is intuitively correct, because when d_i is uniformly distributed on [$(1-\theta)d_i$, $(1+\theta)d_i$], users' demand volatility is proportional to the expected transaction volume. So volatility is higher for high-end users under this distribution. Note that high-end users are those who should be relatively more likely to choose the MOTS model due to their large usage, and who are considered “more profitable” users by the SaaS provider, also due to their large usage. With higher volatility, they now will find SaaS more attractive for its easy scalability, making it more competitive in the market.

To summarize, we find the key results are robust when user *i*’s demand is uniformly distributed on [$(1-\theta)d_i$, $(1+\theta)d_i$]. We identify four similar regimes, and the pattern of competition remains. As a result, the conclusions in Propositions 1 to 5 still hold, though the concrete math expressions might be different.