

## Electronic Companion to “Mandatory Standards and Organizational Information Security”

### Appendix A – Proofs of Lemmas and Propositions

**Proof of Lemma 1:** The proof goes in two steps.

*Step 1:* We temporarily ignore the boundary condition  $e_V \geq s_{SC}$ , and consider maximizing  $U_F$  over set  $(e_V, e_N) \in ([0,1], [0,1])$ . By jointly solving  $\partial U_F / \partial e_V = (1 - e_N)(1 - 2ke_V + k)D_F - c_V(e_V) = 0$  and  $\partial U_F / \partial e_N = (1 - e_V)(1 - ke_V)D_F - c_N(e_N) = 0$ , we get all possible critical points  $(\hat{e}_V, \hat{e}_N)$ :  $\hat{e}_V$  being the solution to  $(1 - r_N((1 - \hat{e}_V)(1 - k\hat{e}_V)D_F))(1 - 2k\hat{e}_V + k)D_F = c_V(\hat{e}_V)$  (i.e., it is the same as  $\hat{s}_{SC}$ ), and in turn  $\hat{e}_N$  being the solution to  $\hat{e}_N = r_N((1 - \hat{e}_V)(1 - k\hat{e}_V)D_F)$ .

Consider  $(1 - r_N((1 - \hat{e}_V)(1 - k\hat{e}_V)D_F))(1 - 2k\hat{e}_V + k)D_F = c_V(\hat{e}_V)$ . Its left-hand side is decreasing in  $\hat{e}_V$ , and its right-hand side is increasing in  $\hat{e}_V$ . Therefore  $\hat{e}_V$  is a unique solution. Also it is straightforward that  $0 < \hat{e}_V < 1$ . From  $\hat{e}_N = r_N((1 - \hat{e}_V)(1 - k\hat{e}_V)D_F)$  we then know  $\hat{e}_N$  is also unique. Therefore, there is *one and only one critical point*  $(\hat{e}_V, \hat{e}_N)$  over compact set  $([0,1], [0,1])$ .

We next prove that, over compact set  $([0,1], [0,1])$ ,  $U_F$  must be maximized at  $(\hat{e}_V, \hat{e}_N)$ . Given the uniqueness of this critical point, we only need to show that  $U_F$  cannot be maximized at any boundary point of the compact set  $([0,1], [0,1])$ . First note that  $U_F$  cannot be maximized at either  $e_V = 1$  or  $e_N = 1$  because costs at these boundary lines are infinity. At  $e_N = 0$ ,  $\partial U_F / \partial e_N|_{e_N=0} = (1 - e_V)(1 - ke_V)D_F > 0$ , thus  $U_F$  is not maximized. Similarly  $U_F$  is not maximized at  $e_V = 0$ .

*Step 2:* We now consider the boundary condition  $e_V \geq s_{SC}$ . The firm tries to maximize  $U_F$  over a compact set  $(e_V, e_N) \in ([s_{SC}, 1], [0,1])$ . Two possibilities as shown below.

First consider the case where the aforementioned critical point  $(\hat{e}_V, \hat{e}_N)$  is inside set  $([s_{SC}, 1], [0,1])$ , i.e.  $\hat{e}_V \geq s_{SC}$ . Since  $(\hat{e}_V, \hat{e}_N)$  is the unique maximum point for set  $([0,1], [0,1])$ , it is also the maximum point for the subset  $([s_{SC}, 1], [0,1])$ . This proves the case of  $s_{SC} \leq \hat{s}$  of this lemma.

Second consider the case where the aforementioned critical point  $(\hat{e}_V, \hat{e}_N)$  is outside of set

$([S_{SC}, 1], [0, 1])$ , i.e.,  $\hat{e}_V < s_{SC}$ . Then, there is no critical point over set  $([S_{SC}, 1], [0, 1])$ . Consequently,  $U_F$  can only be maximized at a boundary point of the compact set  $([S_{SC}, 1], [0, 1])$ .  $U_F$  cannot be maximized at either  $e_V = 1$  or  $e_N = 1$  because costs at these boundary lines is infinity. At  $e_N = 0$ ,  $\partial U_F / \partial e_N|_{e_N=0} = (1 - e_V)(1 - ke_V)D_F > 0$ , thus  $U_F$  is not maximized. Therefore  $U_F$  must be maximized at some boundary point  $(e_V = S_{SC}, e_N)$ . When  $e_V = S_{SC}$ , from  $\partial U_F / \partial e_N = 0$  we have  $e_N = r_N((1 - s_{SC})(1 - ks_{SC})D_F)$ . This completes this lemma. Q.E.D.

**Proof of Propositions 1 and 2:** For convenience, we suppress subscript SC in this proof. Let firm security be denoted as  $f(s) \equiv e_V^*(s) + (1 - e_V^*(s))(e_N^*(s))$ . If  $s \leq \hat{s}$ , from Lemma 1 we have  $f(s) = \hat{s} + (1 - \hat{s})r_N((1 - \hat{s})(1 - k\hat{s})D_F)$ , which is independent of  $s$ , thus part (i) of this proposition is proved.

Hereafter we only consider  $s > \hat{s}$ . From Lemma 1, we have  $f(s) = s + (1 - s)(r_N((1 - s)(1 - ks)D_F))$ .

Differentiating  $f(s)$  w.r.t.  $s$ , we get

$$f'(s) = 1 - r_N((1 - s)(1 - ks)D_F) - (1 - s)(1 + k - 2ks)r'_N((1 - s)(1 - ks)D_F)D_F.$$

Define  $\tau_1(s) \equiv r'_N((1 - s)(1 - ks)D_F)(1 - s)(1 - ks)D_F$  and

$$\tau_2(s) \equiv \frac{1 - r_N((1 - s)(1 - ks)D_F)}{r'_N((1 - s)(1 - ks)D_F)(1 - s)(1 - ks)D_F} - \frac{1 + k - 2ks}{1 - ks}. \text{ We then have } f'(s) = \tau_1(s)\tau_2(s). \text{ Notice that}$$

$\tau_1(s) > 0$  for any  $s \in [0, 1]$ . Therefore,  $f'(s)$  and  $\tau_2(s)$  have the same sign.

We now show that  $\tau_2(s)$  is a strictly increasing function. Let  $\sigma \equiv (1 - s)(1 - ks)D_F$ . We then have

$$\tau_2(s) \text{ as } \tau_2(s) = \frac{1 - r_N(\sigma)}{r'_N(\sigma)\sigma} - \left(2 - \frac{1 - k}{1 - ks}\right). \text{ Because } \sigma \text{ is decreasing in } s, \text{ from Assumption 1 we know}$$

$$\frac{1 - r_N(\sigma)}{r'_N(\sigma)\sigma} \text{ is weakly-increasing in } s. \text{ Also it is straightforward that } 2 - \frac{1 - k}{1 - ks} \text{ is strictly decreasing in } s.$$

Therefore,  $\tau_2(s)$  is *strictly increasing* in  $s$ . Let  $\underline{s}$  denote the solution to  $\tau_2(\underline{s}) = 0$  -- this solution is *unique* because of the strict monotonicity of  $\tau_2(s)$ .

We next check the boundary values of  $\tau_2(s)$ . When  $s$  approaches 1 (i.e., the standard requires near-

perfect security on control V), we have  $\lim_{s \rightarrow 1} \tau_2(s) = \lim_{\sigma \rightarrow 0} \frac{1 - r_N(\sigma)}{r'_N(\sigma)\sigma} - 2 + \lim_{s \rightarrow 1} \frac{1 - k}{1 - ks} \rightarrow \infty$ .

When  $s = 0$ ,  $\tau_2(0) = \frac{1 - r_N(D_F)}{r'_N(D_F)D_F} - 1 - k$ . We differentiate between two cases:

Case 1:  $D_F > \underline{D}_F$ . Recall that  $\underline{D}_F$  is the solution to  $\frac{1 - r_N(\underline{D}_F)}{r'_N(\underline{D}_F)\underline{D}_F} = 1$ , and that function  $\frac{1 - r_N(x)}{r'_N(x)x}$  is

weakly decreasing in  $x$  per Assumption 1. Therefore, condition  $D_F > \underline{D}_F$  is equivalent to  $\frac{1 - r_N(D_F)}{r'_N(D_F)D_F} < 1$ .

In this case,  $\tau_2(0) < 0$ . Therefore,  $0 < \underline{s} < 1$ . If  $s \in [0, \underline{s}]$ , then  $\tau_2(s) < 0$ , thus  $f'(s) < 0$ . This proves the sufficiency of the conditions in parts (ii) and (iii) of Proposition 1. If  $s \notin [0, \underline{s}]$ , then  $s \in [\underline{s}, 1)$ ,  $\tau_2(s) \geq 0$ , thus  $f'(s) \geq 0$ . This proves the necessity of the conditions in parts (ii) and (iii) of Proposition 1.

Case 2:  $D_F \leq \underline{D}_F$ , or equivalently  $\frac{1 - r_N(D_F)}{r'_N(D_F)D_F} \geq 1$ . In this case,  $\underline{s} > 0$ , or equivalently,  $\tau_2(0) < 0$ , if

and only if  $k > \frac{1 - r_N(D_F)}{r'_N(D_F)D_F} - 1$ . Given the last inequality, the rest of this proof for this case (Proposition 2)

is similar to that in Case 1. Q.E.D.

**Proof of Proposition 3.** Recall that  $\underline{s}$  is the solution to  $LHS = RHS$ , where

$$LHS \equiv \frac{1 - r_N((1 - \underline{s})(1 - k\underline{s})D_F)}{r'_N((1 - \underline{s})(1 - k\underline{s})D_F)(1 - \underline{s})(1 - k\underline{s})D_F} \text{ and } RHS \equiv 2 - \frac{1 - k}{1 - k\underline{s}}.$$

From Assumption 1, LHS is weakly decreasing in  $(1 - \underline{s})(1 - k\underline{s})D_F$ , thus in turn weakly decreasing in  $D_F$ .

Differentiating both sides of  $LHS = RHS$  w.r.t.  $D_F$  yields  $\frac{\partial LHS}{\partial \underline{s}} \frac{\partial \underline{s}}{\partial D_F} + \frac{\partial LHS}{\partial D_F} = \frac{\partial RHS}{\partial \underline{s}} \frac{\partial \underline{s}}{\partial D_F}$ . Recall

$\partial LHS / \partial \underline{s} \geq 0$  and  $\partial RHS / \partial \underline{s} < 0$  from the proof of Propositions 1 and 2. Since  $\partial LHS / \partial D_F \leq 0$ ,

$$\frac{\partial \underline{s}}{\partial D_F} = - \frac{\partial LHS}{\partial D_F} \left/ \left( \frac{\partial LHS}{\partial \underline{s}} - \frac{\partial RHS}{\partial \underline{s}} \right) \right. \geq 0. \text{ Q.E.D.}$$

**Proof of Lemma 2:** This proof is analogous to that for Lemma 1, thus we provide a more concise writing.

*Step 1:* We temporarily ignore the boundary condition  $e_V \geq s_{PC}$ , and consider maximizing  $U_F$  over set

$(e_V, e_N) \in ([0,1], [0,1])$ . By jointly solving  $\partial U_F / \partial e_V = 0$  and  $\partial U_F / \partial e_N = 0$ , we get the critical point  $(\hat{e}_V, \hat{e}_N)$ , where  $\hat{e}_V$  takes the same value as  $\hat{s}_{PC}$  that we defined. A similar technique as the one in the proof of Lemma 1 can show this critical point is unique over compact set  $([0,1], [0,1])$ , and furthermore this critical point is the global maximum as none boundary point can be the maximum.

*Step 2:* We now consider the boundary condition  $e_V \geq s_{PC}$ . The firm tries to maximize  $U_F$  over a compact set  $(e_V, e_N) \in ([s_{PC}, 1], [0, 1])$ . If  $\hat{e}_V \geq s_{PC}$ ,  $(\hat{e}_V, \hat{e}_N)$  is inside set  $([s_{PC}, 1], [0, 1])$ . The global maximum for set  $([0, 1], [0, 1])$  must also be the global maximum for this subset  $([s_{PC}, 1], [0, 1])$ . If  $\hat{e}_V < s_{PC}$ , i.e.,  $(\hat{e}_V, \hat{e}_N)$  is outside of  $([s_{PC}, 1], [0, 1])$ , the global maximum must be on a boundary point since there is no critical point within. Similar to Lemma 1, it is straightforward to rule out all boundary points except for  $(e_V = s_{PC}, e_N = r_N(s_{PC}(1 - ks_{PC})D_F))$ . Q.E.D.

**Proof of Proposition 4:** For convenience and without causing ambiguity, we suppress subscript PC in this proof. If  $s \leq \hat{s}$ , from Lemma 2 we immediately have firm security be independent of  $s$ , thus part (i) of this proposition.

Hereafter we only consider  $s > \hat{s}$ . From Lemma 2, we can denote firm security under parallel configuration as  $g(s) = s \cdot r_N(s(1 - ks)D_F)$  under  $s > \hat{s}$ . Differentiating  $g(s)$  w.r.t.  $s$ , we have  $g'(s) = r_N(s(1 - ks)D_F) + s \cdot r'_N(s(1 - ks)D_F)(1 - 2ks)D_F$ .

When  $s \leq 1/(2k)$ , all three components in the above expression for  $g'(s)$  are positive, therefore  $g'(s) > 0$ . Therefore, for the rest of this proof, we only consider  $s > 1/(2k)$ .

Define  $\tau_1(s) \equiv r'_N(s(1 - ks)D_F)(s(1 - ks)D_F)$  and  $\tau_2(s) \equiv \frac{r_N(s(1 - ks)D_F)}{r'_N(s(1 - ks)D_F)(s(1 - ks)D_F)} + \frac{1 - 2ks}{1 - ks}$ . We then

have  $g'(s) = \tau_1(s)\tau_2(s)$ . Notice that  $\tau_1(s) > 0$  for any  $s \in [0, 1]$ . Therefore,  $g'(s)$  and  $\tau_2(s)$  have the same sign.

We now show that  $\tau_2(s)$  is a strictly decreasing function. Let  $\sigma \equiv s(1 - ks)D_F$ . We then have  $\tau_2(s)$

as  $\tau_2(s) = \frac{r_N(\sigma)}{r'_N(\sigma)\sigma} - \left(\frac{1}{1-ks} - 2\right)$ . From Assumption 1 we know  $\frac{r_N(\sigma)}{r'_N(\sigma)\sigma}$  is weakly-increasing in  $\sigma$ .

Given that we only consider  $s > 1/(2k)$ ,  $\sigma$  is decreasing in  $s$ . Therefore,  $\frac{r_N(\sigma)}{r'_N(\sigma)\sigma}$  is weakly-decreasing

in  $s$ . Also, clearly  $\frac{1}{1-ks} - 2$  is strictly increasing in  $s$ . Therefore,  $\tau_2(s)$  is *strictly decreasing* in  $s$ . Let  $\bar{s}$

denote the solution to  $\tau_2(\bar{s}) = 0$  -- this solution is *unique* because of the strict monotonicity of  $\tau_2(s)$ .

We next check the boundary values of  $\tau_2(s)$ .  $\lim_{s \rightarrow 0} \tau_2(s) = \lim_{\sigma \rightarrow 0} \frac{r_N(\sigma)}{r'_N(\sigma)\sigma} - \lim_{s \rightarrow 0} \frac{1}{1-ks} + 2 \rightarrow \infty$ .

$\lim_{s \rightarrow 1} \tau_2(s) = \lim_{\sigma \rightarrow (1-k)D_F} \frac{r_N(\sigma)}{r'_N(\sigma)\sigma} - \lim_{s \rightarrow 1} \frac{1}{1-ks} + 2 = \frac{r_N((1-k)D_F)}{r'_N((1-k)D_F)(1-k)D_F} - \frac{1}{1-k} + 2$ . Notice that  $\bar{k}$  (see main

text) is defined to equal the last expression to zero. Also notice that this last expression is decreasing in  $k$  because its first component is weakly decreasing in  $k$  (again because of Assumption 1) and its second component is strictly increasing in  $k$ . Therefore,  $\lim_{s \rightarrow 1} \tau_2(s) < 0$  if and only if  $k > \bar{k}$ .

We now show the sufficiency of the condition (i.e.,  $k > \bar{k}$  and  $s > \max\{\bar{s}, 1/(2k)\}$ ) of Proposition 4. From  $k > \bar{k}$  we know  $\lim_{s \rightarrow 1} \tau_2(s) < 0$ . Therefore, from the monotonicity of function  $\tau_2(s)$  -- which was proved under the condition  $s > 1/(2k)$  -- we have  $0 < \bar{s} < 1$ . Then,  $\tau_2(s) < 0$  for any  $s \in (\bar{s}, 1)$ .

We next show the necessity of the condition. This condition has three pieces:  $k > \bar{k}$ ,  $s > \bar{s}$  and  $s > 1/(2k)$ . The necessity of the last one is apparent: otherwise, firm efforts on both controls increase in the standard. If  $k \leq \bar{k}$ ,  $\lim_{s \rightarrow 1} \tau_2(s) \geq 0$ , thus  $\tau_2(s)$  (and by inference,  $g'(s)$ ) is always nonnegative. Lastly,  $\tau_2(s)$  is nonnegative if  $s \leq \bar{s}$ . Thus all three pieces are necessary. Q.E.D.

**Proof of Proposition 5:** By the Envelope Theorem, we have  $\frac{d}{dk} U_{sw}(s_{SC}^*(k); k) = \frac{\partial}{\partial k} U_{sw}(s_{SC}^*(k); k)$

$$= (1 - s_{SC})^2 (-s_{SC}) D_F r'_N((1 - s_{SC})(1 - ks_{SC}) D_F) (D_{sw} - ((1 - ks_{SC}) D_F)) < 0.$$

Define  $\lambda_F \equiv D_F / D_{sw}$ , where  $0 < \lambda_F < 1$  from  $D_F < D_{sw}$ . The optimization problem (11) changes to

$$\max_{s_{SC}} U_{SW} = W_{SW} - (1 - s_{SC})(1 - r_N((1 - s_{SC})(1 - ks_{SC})\lambda_F D_{SW}))D_{SW} - C_V(s_{SC}) - C_N(r_N((1 - s_{SC})(1 - ks_{SC})\lambda_F D_{SW})).$$

Again by the Envelope Theorem, we have

$$\frac{dU_{SW}}{d\lambda_F} = \frac{\partial U_{SW}}{\partial \lambda_F} = (1 - s_{SC})^2 (1 - ks_{SC}) D_{SW}^2 r_N'((1 - s_{SC})(1 - ks_{SC})\lambda_F D_{SW})(1 - (1 - ks_{SC})\lambda_F) > 0. \quad \text{Q.E.D.}$$

**Proof of Proposition 6:** By the Envelope Theorem, we have  $\frac{d}{dk} U_{SW}(s_{PC}^*(k); k) = \frac{\partial}{\partial k} U_{SW}(s_{PC}^*(k); k) =$

$$(s_{PC})^2 (-s_{PC}) D_F r_N'(s_{PC}(1 - ks_{PC})D_F)(D_{SW} - (1 - ks_{PC})D_F) < 0.$$

Define  $\lambda_F \equiv D_F / D_{SW}$ , where  $0 < \lambda_F < 1$  from  $D_F < D_{SW}$ . The optimization problem (12) changes to

$$\max_{s_{PC}} U_{SW} = W_{SW} - (1 - s_{PC} r_N(s_{PC}(1 - ks_{PC})\lambda_F D_{SW}))D_{SW} - C_V(s_{PC}) - C_N(r_N(s_{PC}(1 - ks_{PC})\lambda_F D_{SW})).$$

Again by the Envelope Theorem, we have

$$\frac{dU_{SW}}{d\lambda_F} = \frac{\partial U_{SW}}{\partial \lambda_F} = s_{PC}^2 (1 - ks_{PC}) D_{SW}^2 \cdot r_N'(s_{PC}(1 - ks_{PC})\lambda_F D_{SW})(1 - (1 - ks_{PC})\lambda_F) > 0. \quad \text{Q.E.D.}$$

**Proof of Proposition 7:** We simplify notation  $s_{SC}$  to  $s$  in this proof. Given  $s > \hat{s}_{SC}$ , the firm's optimization problem can be simplified to  $\max_{e_N} U_F = W_F - (1 - s)(1 - e_N)(1 - ks)D_F - C_V(s) - C_N(e_N)$ .

Therefore, the firm's optimal effort on control N,  $e_N^*$ , satisfies:

$$\frac{\partial U_F}{\partial e_N} = (1 - s)(1 - ks)D_F - c_N(e_N) = 0. \quad (\text{A.1})$$

We then have:

$$\frac{\partial e_N^*}{\partial s} = -\frac{\partial^2 U_F}{\partial e_N \partial s} / \frac{\partial^2 U_F}{\partial e_N \partial e_N} = -\frac{(1 + k - 2ks)D_F}{c_N'(e_N)} < 0. \quad (\text{A.2})$$

Under the original model, the policy maker's utility function in (11) can be rewritten as:

$$U_{SW} = W_{SW} - (1 - s)(1 - e_N^*(s))D_{SW} - C_V(s) - C_N(e_N^*(s)) \quad \text{where} \quad e_N^*(s) = r_N((1 - s)(1 - ks)D_F). \quad \text{Therefore,}$$

$$\frac{\partial U_{SW}}{\partial s} = \left( (1 - s)e_N^{*'}(s) + 1 - e_N^*(s) \right) D_{SW} - c_V(s) - c_N(e_N^*(s))e_N^{*'}(s)$$

$= (1 - e_N^*(s))D_{SW} - c_V(s) + e_N^{*'}(s)((1-s)D_{SW} - c_N(e_N^*(s)))$ . We know  $e_N^{*'}(s) < 0$  from (A.2). Further,  $(1-s)D_{SW} - c_N(e_N^*(s)) > 0$  because of (A.1) and the fact that  $D_{SW} > (1-ks)D_F$ . Therefore,  $\frac{\partial U_{SW}}{\partial s} < (1 - e_N(s))D_{SW} - c_V(s) < D_{SW} - c_V(s)$ .

Under the benchmark model, the policy maker's utility function is  $U_{SW}^{BM} = W_{SW} - (1-s)D_{SW} - C_V(s)$  s.t.  $s \geq \hat{s}^{BM}$ . Thus  $\frac{\partial U_{SW}^{BM}}{\partial s} = D_{SW} - c_V(s)$ . Therefore, in comparing the benchmark and the original models, we have

$$\frac{\partial U_{SW}^{BM}}{\partial s} > \frac{\partial U_{SW}}{\partial s} \text{ for any } s > \hat{s}_{SC}. \quad (\text{A.3})$$

The original standard satisfies  $\left. \frac{\partial U_{SW}}{\partial s} \right|_{s=s_{SC}^*} = 0$ , and the benchmark standard satisfies  $\left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s^{BM}} = 0$ .

Thus, from (A.3) we have  $\left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s_{SC}^*} > \left. \frac{\partial U_{SW}}{\partial s} \right|_{s=s_{SC}^*} = 0 = \left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s^{BM}}$ . Because  $U_{SW}^{BM}$  is a concave function

of  $s$ ,  $\left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s_{SC}^*} > \left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s^{BM}}$  implies that  $s_{SC}^* < s^{BM}$ . Q.E.D.

**Proof of Proposition 8:** We simplify notation  $s_{PC}$  to  $s$  in this proof. Given  $s > \hat{s}_{PC}$ , the firm's optimization problem is  $\max_{e_N} U_F = W_F - (1 - se_N)(1 - ks)D_F - C_V(s) - C_N(e_N)$ . Therefore, the firm's optimal effort on control N,  $e_N^*$ , satisfies:

$$\frac{\partial U_F}{\partial e_N} = s(1 - ks)D_F - c_N(e_N) = 0. \quad (\text{A.4})$$

We then have:

$$\frac{\partial e_N^*}{\partial s} = -\frac{\partial^2 U_F}{\partial e_N \partial s} / \frac{\partial^2 U_F}{\partial e_N \partial e_N} = \frac{(1 - 2ks)D_F}{c_N'(e_N)} \begin{cases} \leq 0 & \text{if } s \geq \frac{1}{2k} \\ > 0 & \text{otherwise} \end{cases}. \quad (\text{A.5})$$

Under the original model, the policy maker's utility function in (12) can be rewritten as:

$$U_{SW} = W_{SW} - (1 - se_N^*(s))D_{SW} - C_V(s) - C_N(e_N^*(s)) \text{ where } e_N^*(s) = r_N(s(1 - ks)D_F). \text{ Therefore,}$$

$$\frac{\partial U_{SW}}{\partial s} = \left( se_N^{*'}(s) + e_N^*(s) \right) D_{SW} - c_V(s) - c_N(e_N^*(s))e_N^{*'}(s) = e_N^*(s)D_{SW} - c_V(s) + e_N^{*'}(s)(sD_{SW} - c_N(e_N^*(s))).$$

Under the benchmark model, the policy maker's utility function is  $U_{SW}^{BM} = W_{SW} - (1 - s)D_{SW} - C_V(s)$

s.t.  $s \geq \hat{s}^{BM}$ . Thus  $\frac{\partial U_{SW}^{BM}}{\partial s} = D_{SW} - c_V(s)$ . Also notice that  $U_{SW}^{BM}$  is a concave function of  $s$ . Therefore, to

prove that  $s_{PC}^* < s^{BM}$ , it is necessary and sufficient to show that  $\left. \frac{\partial U_{SW}^{BM}}{\partial s} \right|_{s=s_{PC}^*} > \left. \frac{\partial U_{SW}}{\partial s} \right|_{s=s_{PC}^*}$ , i.e.,

$$\left\{ D_{SW} - c_V(s) \right\} \Big|_{s=s_{PC}^*} > \left\{ e_N^*(s)D_{SW} - c_V(s) + e_N^{*'}(s)(sD_{SW} - c_N(e_N^*(s))) \right\} \Big|_{s=s_{PC}^*}, \text{ i.e.,}$$

$$\left\{ (1 - e_N^*(s))D_{SW} \right\} \Big|_{s=s_{PC}^*} > \left\{ e_N^{*'}(s)(sD_{SW} - c_N(e_N^*(s))) \right\} \Big|_{s=s_{PC}^*}. \text{ Because } sD_{SW} - c_N(e_N^*(s)) \text{ is positive due to}$$

(A.4), the last inequality apparently holds if  $e_N^{*'}(s) \Big|_{s=s_{PC}^*} \leq 0$ , i.e.,  $s_{PC}^* \geq \frac{1}{2k}$  (from (A.5)). Q.E.D.

**Proof of Proposition 9:** For convenience and without causing ambiguity, we simplify notation  $s_{PC}$  to  $s$  in this proof. Under contingent liability reduction, the firm's optimization problem is:

$$\begin{aligned} \max_{e_V, e_N} U_F = & V_F - ((1 - e_V)(e_N)((1 - \mu)(1 - k_H e_V) + \mu(1 - k_0 e_V)) + (e_V)(1 - e_N)((1 - \mu) + \mu(1 - k_0 e_V)) \\ & + (1 - e_V)(1 - e_N)((1 - \mu)(1 - k_H e_V) + \mu(1 - k_0 e_V)))D_F - C_V(e_V) - C_V(e_N) \text{ s.t., } e_V \geq s \end{aligned}$$

We consider only  $s > \hat{s}_{PC}$ . The optimization problem can be re-written as:

$$\begin{aligned} \max_{e_N} U_F = & V_F - ((1 - s)(e_N)((1 - \mu)(1 - k_H s) + \mu(1 - k_0 s)) + (s)(1 - e_N)((1 - \mu) + \mu(1 - k_0 s)) \\ & + (1 - s)(1 - e_N)((1 - \mu)(1 - k_H s) + \mu(1 - k_0 s)))D_F - C_V(s) - C_V(e_N) \end{aligned}$$

Solving  $\partial U_F / \partial e_N = 0$  in terms of  $e_N$ , we get  $e_N^* = r_N(s((1 - \mu) + \mu(1 - k_0 s))D_F)$ . Therefore,  $e_N^*(s)$  is

a decreasing function of  $s$  if and only if  $s > \frac{1}{2k_0\mu}$ .

Denote firm security in contingent liability reduction under parallel configuration with  $h(s) = s \cdot (r_N(s((1-\mu) + \mu(1-k_0s))D_F))$ . Differentiating  $h(s)$  with respect to  $s$ , we have

$$h'(s) = r_N(s((1-\mu) + \mu(1-k_0s))D_F) + s \cdot r_N'(s((1-\mu) + \mu(1-k_0s))D_F)(1-2k_0\mu s)D_F.$$

When  $s \leq 1/(2k_0\mu)$ , all three components in the above expression for  $h'(s)$  are positive, therefore  $h'(s) > 0$ . Therefore, for the rest of this proof, we only consider  $s > 1/(2k_0\mu)$ .

Define  $\eta_1(s) \equiv r_N'(s((1-\mu) + \mu(1-k_0s))D_F)s((1-\mu) + \mu(1-k_0s))D_F$  and

$$\eta_2(s) \equiv \frac{r_N(s((1-\mu) + \mu(1-k_0s))D_F)}{r_N'(s((1-\mu) + \mu(1-k_0s))D_F)s((1-\mu) + \mu(1-k_0s))D_F} + 2 - \frac{1}{1-k_0\mu s}. \quad \text{We then have}$$

$h'(s) = \eta_1(s)\eta_2(s)$ . Notice that  $\eta_1(s) > 0$  for any  $s \in [0,1)$ . Therefore,  $h'(s)$  and  $\eta_2(s)$  have the same sign.

We now show that  $\eta_2(s)$  is a strictly decreasing function. Let  $\sigma \equiv s((1-\mu) + \mu(1-k_0s))D_F$ . We then

have  $\eta_2(s)$  as  $\eta_2(s) = \frac{r_N(\sigma)}{r_N'(\sigma)\sigma} - (\frac{1}{1-k_0\mu s} - 2)$ . From Assumption 1 we know  $\frac{r_N(\sigma)}{r_N'(\sigma)\sigma}$  is weakly-

increasing in  $\sigma$ . Given that we only consider  $s > 1/(2k_0\mu)$ ,  $\sigma$  is decreasing in  $s$ . Therefore,  $\frac{r_N(\sigma)}{r_N'(\sigma)\sigma}$  is

weakly-decreasing in  $s$ . Also, clearly  $\frac{1}{1-k_0\mu s} - 2$  is strictly increasing in  $s$ . Therefore,  $\eta_2(s)$  is *strictly*

*decreasing* in  $s$ . Let  $\bar{s}$  denote the solution to  $\eta_2(\bar{s}) = 0$  -- this solution is *unique* because of the strict monotonicity of  $\eta_2(s)$ .

We next check the boundary values of  $\eta_2(s)$ .  $\lim_{s \rightarrow 0} \eta_2(s) = \lim_{\sigma \rightarrow 0} \frac{r_N(\sigma)}{r_N'(\sigma)\sigma} - \lim_{s \rightarrow 0} \frac{1}{1-k_0\mu s} + 2 \rightarrow \infty$ .

$$\lim_{s \rightarrow 1} \eta_2(s) = \lim_{\sigma \rightarrow (1-k_0\mu)D_F} \frac{r_N(\sigma)}{r_N'(\sigma)\sigma} - \lim_{s \rightarrow 1} \frac{1}{1-k_0\mu s} + 2 = \frac{r_N((1-k_0\mu)D_F)}{r_N'((1-k_0\mu)D_F)(1-k_0\mu)D_F} - \frac{1}{1-k_0\mu} + 2.$$

Notice first that  $\bar{k}_0$  (see main text) is defined to equal the last expression to zero. This last expression is decreasing in  $k_0$  because its first component is weakly decreasing in  $k_0$  (again because of Assumption 1) and its second component is strictly increasing in  $k_0$ .  $\lim_{s \rightarrow 1} \eta_2(s) < 0$  if and only if  $k_0 > \bar{k}_0$

For contingent liability reduction, we additionally need to check the feasibility of  $k_0$  (i.e., whether

$k_0 \in (0,1)$  ). We check the boundary values of  $\lim_{s \rightarrow 1} \eta_2(s)$  :  $\lim_{\substack{s \rightarrow 1 \\ k_0 \rightarrow 0}} \eta_2(s) \equiv \frac{r_N(D_F)}{r'_N(D_F)D_F} + 1 > 0$  .

$\lim_{\substack{s \rightarrow 1 \\ k_0 \rightarrow 1}} \eta_2(s, k) = \frac{r_N((1-\mu)D_F)}{r'_N((1-\mu)D_F)(1-\mu)D_F} - \frac{1}{1-\mu} + 2$  . Notice first that  $\mu$  (see main text) is defined to equal

the last expression to zero. This last expression is also decreasing in  $\mu$  because its first component is weakly decreasing in  $\mu$  and its second component is strictly increasing in  $\mu$  . Therefore,  $\lim_{\substack{s \rightarrow 1 \\ k_0 \rightarrow 0}} \eta_2(s, k) < 0$

if and only if  $\mu > \bar{\mu}$  .

We now show the sufficiency of the condition (i.e.,  $\mu > \bar{\mu}$  ,  $k_0 > \bar{k}_0$  and  $s > \max\{\bar{s}, 1/(2k_0\mu)\}$  ) .

From  $\mu > \bar{\mu}$  we know  $\lim_{\substack{s \rightarrow 1 \\ k_0 \rightarrow 1}} \eta_2(s, k) < 0$  and from  $k_0 > \bar{k}_0$  we know  $\lim_{s \rightarrow 1} \eta_2(s) < 0$  . Therefore, from the

monotonicity of function  $\eta_2(s)$  -- which was proved under the condition  $s > 1/(2k_0\mu)$  -- we have

$0 < \bar{s} < 1$  . Then,  $\eta_2(s) < 0$  for any  $s \in (\bar{s}, 1)$  .

We next show the necessity of the condition. This condition has four pieces:  $\mu > \bar{\mu}$  and  $k_0 > \bar{k}_0$  ,  $s > \bar{s}$  and  $s > 1/(2k_0\mu)$  . The necessity of the last one is apparent: otherwise, firm efforts on both controls increase in the standard. If  $\mu \leq \bar{\mu}$  ,  $\lim_{\substack{s \rightarrow 1 \\ k_0 \rightarrow 1}} \eta_2(s) \geq 0$  . If  $k_0 \leq \bar{k}_0$  ,  $\lim_{s \rightarrow 1} \eta_2(s) \geq 0$  , thus  $\eta_2(s)$  (and by inference,

$h'(s)$ ) is always nonnegative. Lastly,  $\eta_2(s)$  is nonnegative if  $s \leq \bar{s}$  . Thus all four pieces are necessary.

Q.E.D.

## Appendix B – Existence of Feasible Regions where Firm Security Decreases in Security Standard

In Appendix B we study the feasibility of part (ii) of Proposition 1 and part (ii) of Proposition 2, where we have the interesting results that firm security decreases in security standard for serial and parallel configurations.

### B.1. Analytical Analysis of Feasible Region under Serial Configuration

**Assumption B.1:** There exists  $\tilde{D}_F > 0$  such that, for all  $D_F > \tilde{D}_F$ ,  $\partial \hat{s}_{SC} / \partial D_F > 0$ .

**Theorem B.1:** Under serial configuration, given Assumption B.1, there exists  $D_{F0}$  such that part (ii) of Proposition 1 is feasible for any  $D_F > D_{F0}$ .

**Proof of Theorem B.1:** Part (ii) of Proposition 1 is feasible if  $\underline{s} > \hat{s}_{SC}$ . Recall that  $\underline{s}$  is the solution of  $x$  in equation

$$\frac{1 - r_N((1-x)(1-kx)D_F)}{r'_N((1-x)(1-kx)D_F)(1-x)(1-kx)D_F} - 2 + \frac{1-k}{1-kx} = 0, \quad (\text{B.1})$$

and  $\hat{s}_{SC}$  is the solution of  $x$  in equation

$$1 - r_N((1-x)(1-kx)D_F) - \frac{c_V(x)}{(1-2kx+k)D_F} = 0. \quad (\text{B.2})$$

Denote the solution of  $x$  in terms of  $D_F$  from equation (B.1) as  $x_1(D_F)$ . Thus  $\underline{s} = x_1(D_F)$ . From Assumption 1 we know  $\frac{1 - r_N(\sigma)}{r'_N(\sigma)\sigma}$  weakly decreases in  $\sigma$ . Therefore, the left-hand-side of (B.1) weakly decreases in  $D_F$  and strictly increases in  $x$ . Therefore, by the Implicit Function Theorem,  $x_1(D_F)$  is a strictly increasing function. Therefore, the inverse function of  $x_1(D_F)$  exists, and denote this inverse function as  $D_{F1}(x)$ .

Similarly, denote the solution of  $x$  in terms of  $D_F$  from equation (B.2) as  $x_2(D_F)$ . Thus  $\hat{s}_{SC} = x_2(D_F)$ . Assumption (B.1) states that, for any  $D_F > \tilde{D}_F$ ,  $\hat{s}_{SC}$  monotonically increases in  $D_F$ . Thus the inverse function of  $x_2(D_F)$  exists for any  $D_F > \tilde{D}_F$ . Denote this inverse function as  $D_{F2}(x)$ . Hereafter we only consider  $D_F > \tilde{D}_F$ .

Define new variable  $y = (1-x)D_F$ , and then replace  $D_F$  with  $y/(1-x)$  in equations (B.1) and (B.2).

We then have  $\underline{s}$  as the solution of  $x$  in equation:

$$\frac{1 - r_N((1-kx)y)}{r'_N((1-kx)y)(1-kx)y} - 2 + \frac{1-k}{1-kx} = 0, \quad (\text{B.3})$$

and  $\hat{s}_{SC}$  as the solution of  $x$  in equation:

$$1 - r_N((1-kx)y) - \frac{(1-x)c_V(x)}{(1-2kx+k)y} = 0. \quad (\text{B.4})$$

We first consider equation (B.3). Solving (B.3) for  $y$  as a function of  $x$  -- denote it as  $y_1(x)$ .

Immediately we have  $y_1(x) = (1-x)D_{F1}(x)$ . Also, again because  $\frac{1-r_N(\sigma)}{r'_N(\sigma)\sigma}$  weakly decreases in  $\sigma$ , we

know the left-hand-side of (B.3) weakly decreases in  $y$  and strictly increases in  $x$ . Therefore, by the Implicit Function Theorem,  $y_1(x)$  is a strictly increasing function.

Now consider equation (B.4). Solving (B.4) for  $y$  as a function of  $x$  -- denote it as  $y_2(x)$ .

Immediately we have  $y_2(x) = (1-x)D_{F2}(x)$ .

We next study the limit case of equations (B.3) and (B.4) when  $x \rightarrow 1$ . Define  $\tilde{y}_1 = \lim_{x \rightarrow 1} y_1(x)$  and

$\tilde{y}_2 = \lim_{x \rightarrow 1} y_2(x)$ . By applying operation  $\lim_{x \rightarrow 1}$  to both sides of equation (B.3), we get:

$$\frac{1 - r_N((1-k)\tilde{y}_1)}{r'_N((1-k)\tilde{y}_1)(1-k)\tilde{y}_1} - 1 = 0,$$

which implies

$$1 - r_N((1-k)\tilde{y}_1) = r'_N((1-k)\tilde{y}_1)(1-k)\tilde{y}_1 > 0,$$

or simply

$$1 - r_N((1-k)\tilde{y}_1) > 0. \quad (\text{B.5})$$

Therefore,  $\tilde{y}_1 < \infty$ .

By applying operation  $\lim_{x \rightarrow 1}$  to both sides of equations (B.4), we get:

$$1 - r_N((1-k)\tilde{y}_2) = 0. \quad (\text{B.6})$$

Therefore,  $\tilde{y}_2 = \infty$ .

We can then conclude that  $\tilde{y}_1 < \tilde{y}_2$ , i.e.,  $\lim_{x \rightarrow 1} y_1(x) < \lim_{x \rightarrow 1} y_2(x)$ . In other words, there exists  $\tilde{x} \in [0,1)$  such that, for all  $x > \tilde{x}$ ,  $y_1(x) < y_2(x)$ . Since  $y_1(x) = (1-x)D_{F_1}(x)$  and  $y_2(x) = (1-x)D_{F_2}(x)$ , we have for all  $x > \tilde{x}$ ,  $D_{F_1}(x) < D_{F_2}(x)$ .

Recall that both  $D_{F_1}(x)$  and  $D_{F_2}(x)$  are strictly increasing functions with inverse functions  $x_1(D_F)$  and  $x_2(D_F)$ , respectively. Therefore, from  $\underline{s} = x_1(D_F)$  we have  $D_F = D_{F_1}(\underline{s})$ , from  $\hat{s}_{SC} = x_2(D_F)$  we have  $D_F = D_{F_2}(\hat{s}_{SC})$ . Therefore  $D_{F_1}(\underline{s}) = D_{F_2}(\hat{s}_{SC})$ .

Denote  $D_{F_0} \equiv \max\{\tilde{D}_F, D_{F_1}(\tilde{x}), D_{F_2}(\tilde{x})\}$ . Then, for any  $D_F > D_{F_0}$ , we have  $D_{F_1}(\underline{s}) = D_{F_2}(\hat{s}_{SC}) > D_{F_1}(\hat{s}_{SC})$ . Thus  $\underline{s} > \hat{s}_{SC}$ . Q.E.D.

## B.2. Analytical Analysis of Feasible Region under Parallel Configuration

**Theorem B.2:** *Under parallel configuration, part (ii) of Proposition 4 is feasible if*

$$D_F > \lim_{\sigma \rightarrow 0^+} r_N(\sigma) / r'_N(\sigma).$$

**Proof of Theorem B.2:** Part (ii) of Proposition 4 is feasible if all of the following conditions are true:

$$\bar{k} < 1, \hat{s}_{PC} < 1 \text{ and } \bar{s} < 1.$$

Recall that  $\hat{s}_{PC}$  is the solution to  $kD_F + r_N(\hat{s}_{PC}(1-k\hat{s}_{PC})D_F)(1-2k\hat{s}_{PC})D_F = c_V(\hat{s}_{PC})$ , thus  $\hat{s}_{PC} < 1$  always holds, as otherwise the right-hand-side of the above equation goes to infinity while the left-hand-side is bounded. In the second-to-the-last paragraph of the proof of Proposition 4, we already show that  $\bar{s} < 1$  for any  $k > \bar{k}$ . Therefore, we only need to analyze when condition  $\bar{k} < 1$  can hold.

Recall that  $\bar{k}$  is the solution to  $\frac{r_N((1-\bar{k})D_F)}{r'_N((1-\bar{k})D_F)(1-\bar{k})D_F} = \frac{1}{1-\bar{k}} - 2$ , which can be rewritten as  $r_N((1-\bar{k})D_F) / r'_N((1-\bar{k})D_F) = D_F(2\bar{k} - 1)$ . By Assumption 1, the left-hand-side strictly decreases in  $\bar{k}$  and the right-hand-side strictly increases in  $\bar{k}$ . When  $\bar{k} \rightarrow 1$ , the left-hand-side equals  $\lim_{\sigma \rightarrow 0^+} r_N(\sigma) / r'_N(\sigma)$  and the right-hand-side equals  $D_F$ . By the opposite monotonicity of the two sides, if  $\lim_{\sigma \rightarrow 0^+} r_N(\sigma) / r'_N(\sigma) < D_F$ , it must be true that only some  $\bar{k} < 1$  can equal both sides. Q.E.D.