ONLINE APPENDIX A

MCMC Algorithm

Our MCMC algorithm is based on approach adopted by Rutz and Trusov (2011) and Rutz et al. (2012). The model can be written in the hierarchical form:

$\beta^k | X^{\beta}_{kat}, \omega, \Delta^{\beta}, V^{\beta}, \varepsilon^{\beta}_{kat}$

$\varepsilon^{\beta}_{kat} | \{\beta^k\}, X^{\beta}_{kat}, \Omega_0$

$V^{\beta} | \{\beta^k\}, \omega, \Delta^{\beta}, \nu, S$

$\Delta^{\beta} | \{\beta^k\}, \omega, A_0, \overline{\Delta^{\beta}}$

where X^{β}_{kat} are independent variables in equation 2. The endogenous variable position is estimated for each keyword advertiser combination using instruments as specified in equations 3 & 4. QualityRankAbove and QualityRankBelow are determined using estimated values of position and QualityRank for each advertiser keyword pair. In the robustness analysis, endogenous variable QualityRank is estimated using instruments as specified in equation 5.

We have used 0 as the initial value for elements of β^k, Δ^{β} and an identity matrix as an initial value for elements of V^{β}.

The MCMC algorithm is described below.

Step I: Draw β^k

We use random walk Metropolis-Hastings algorithm for sampling β_k (Rossi & Allenby, 2005)

The draws are accepted with a probability α where $\alpha = \min\left[\frac{\exp\left[-\frac{1}{2}\left(\beta^{k^{new}} - \Delta^{\beta}\omega\right)' A\left(\beta^{k^{new}} - \Delta^{\beta}\omega\right)\right] l(U^{new}_{kat})}{\exp\left[-\frac{1}{2}\left(\beta^{k^{old}} - \Delta^{\beta}\omega\right)' A\left(\beta^{k^{old}} - \Delta^{\beta}\omega\right)\right] l(U^{old}_{kat})}, 1\right]$

where $l(U_{kat})$ is the likelihood of clicks for all advertiser keyword combinations

$l(U_{kat}) = \prod_{ka=1}^{N} \prod_{t=1}^{T} (\Lambda_{kat})^{Clicks_{kat}} (1 - \Lambda_{kat})^{Impressions_{kat} - Clicks_{kat}}$

where N is the no of advertiser-keyword combinations, $\Lambda_{kat} = \frac{\exp(U_{kat})}{1+\exp(U_{kat})}$

$$U_{kat}^{new} = \beta^{k^{new}} X_{kat}^{\beta} + \varepsilon_{kat}^{\beta}$$

$$U_{kat}^{old} = \beta^{k^{old}} X_{kat}^{\beta} + \varepsilon_{kat}^{\beta}$$

$$A^{-1} = V^{\beta}$$

Step II: Draw $\varepsilon_{kat}^{\beta}$

We use random walk Metropolis-Hastings algorithm for sampling $\varepsilon_{kat}^{\beta}$

$$\varepsilon_{kat}^{\beta\ new} \sim N(0, \Omega)$$

The draws are accepted with a probability α where $\alpha = \min\left[\frac{\exp\left[-\frac{1}{2}\varepsilon_{kat}^{\beta\ new}\varepsilon_{kat}^{\beta\ new}\right] l(U_{kat}^{new})}{\exp\left[-\frac{1}{2}\varepsilon_{kat}^{\beta\ old}\varepsilon_{kat}^{\beta\ old}\right] l(U_{kat}^{old})}, 1\right]$

where $l(U_{kat})$ is the likelihood of clicks for advertiser keyword combinations and is same as defined in Step I

$$U_{kat}^{new} = \beta^{k} X_{kat}^{\beta} + \varepsilon_{kat}^{\beta\ new}$$

$$U_{kat}^{old} = \beta^{k} X_{kat}^{\beta} + \varepsilon_{kat}^{\beta\ old}$$

Step III: Draw Ω

$\Omega \sim IW(\nu_{\Omega} + N, \sum_{k=1}^{K} \sum_{t=1}^{T} Y_{kt}' Y_{kt} + S_{\Omega})$ where $Y_{kt} = \begin{bmatrix} U_{kat}^{CTR} - \beta^{k} X_{kat}^{\beta} \\ \log(Bid_{kat}) - \gamma^{k} X_{kat}^{\gamma} \\ log(Pos_{kat}) - \alpha^{k} X_{kat}^{\alpha} \\ \log(QualityRank_{kat}) - \theta^{k} X_{kat}^{\theta} \end{bmatrix}$, N = No of observations, $\nu_{\Omega} = 10$, $S_{\Omega} = 10I$

Step IV: Draw V^{β}

$V^{\beta} \sim IW(\nu + N, \sum_{k=1}^{K} (\beta^k - \Delta^{\beta}\omega_k)'(\beta^k - \Delta^{\beta}\omega_k) + S)$

where N = No of keyword advertiser combinations, $\nu = 10, S = 10I$

Step V: Draw Δ^{β}

$\Delta^{\beta} \sim N(\widetilde{\Delta^{\beta}}, q_{\beta})$ where $q_{\beta} = [(\omega_k'\omega_k)^{-1} + A_0]^{-1}$ & $\widetilde{\Delta^{\beta}} = q_{\beta}[\omega_k'\beta^k + A_0\overline{\Delta^{\beta}}]$

$\overline{\Delta^{\beta}} = 0, A_0 = .01I$

ONLINE APPENDIX B

Robustness of Results

We outline several steps we have taken to evaluate the robustness of our results.

Holdout Sample Analysis: As one test of robustness, we have attempted to verify the prediction accuracy of our results using a holdout sample. To do this, we consider data for the first 60 days for each keyword as the estimation sample and data for the same keywords for the remaining 33 days as the holdout sample. We use mean absolute percentage error (MAPE) for daily CTR values at the aggregate level and at the keyword level. The error values are reported in Table B1 and indicate that the model prediction accuracy is similar for both the estimation and holdout samples. This suggests that our model estimates are robust.

Table B1: Prediction Accuracy for Estimation & Holdout Samples

Models	**CTR Fit (MAPE)**	
	Aggregate	Keyword
Estimation Sample	0.22	0.13
Holdout Sample	0.25	0.15

Aggregate MAPE is the average MAPE across all datapoints. Keyword MAPE is the average of the average MAPE for different keywords

Alternate Measure for the Quality of Competing Ads*:* We construct another measure where we weigh the quality rank of each competing ad with its distance from the focal ad along with impressions (see Table 2 for the definition) and consider the weighted average measure for ads appearing above and below. CTR results using this alternate definition are shown in Table B2 Panel A. Results remain qualitatively similar to that for our main model.

Alternate Measures for Ad Quality*:*

<u>Ad Quality using the actual value of fixed effect coefficient in the ranking equation</u>: We re-estimate our models using the actual values of the advertiser specific dummies for a keyword in equation 1 as the

measure of ad quality. While doing so we normalize the quality of advertisers for a keyword in each period with the maximum value of coefficients. Using actual values allows us to capture the effect of the magnitude of quality difference to some extent. Results are shown in Table B2, Panel A and are qualitatively similar to our main analysis. However, note that the model has a poor fit as compared to our main model as shown in Table 7.

Ad Quality based on GSP auction equilibrium: In our main analysis, we just consider the search engine ranking decision to estimate the quality of advertiser for each keyword. However, this does not consider the competitive bidding and potential equilibrium which imposes further restrictions on the possible quality rankings. For example, in equilibrium we can expect that the advertisers would not want to deviate from their allocated position as it provides the highest return on investment (Varian 2007; Edelman et al. 2007). In order to account for this, we consider the GSP auction equilibrium and infer the quality for each advertiser for a keyword based on the constraints imposed by the ranking mechanism and the monotonicity arising from the envy free equilibrium. Next, we describe the constraints based on GSP equilibrium and our estimation approach.

The search engine conducts a click weighted GSP auction. Let's say that there are K+N ads bidding for K positions to be displayed along with search results for a given keyword. If an ad k in position j has a quality score γ_{k_j} based on its expected click performance and a bid b_{k_j}, the bids are ranked in the order of the product $b_{k_j}\gamma_{k_j}$. In that case, bids and quality scores of ads in adjacent positions are related in the following manner

$$b_{k'_{j-1}}\gamma_{k'_{j-1}} > b_{k_j}\gamma_{k_j} > b_{k'_{j+1}}\gamma_{k'_{j+1}} \qquad \text{(b1)}$$

If the advertiser's ad is to remain in position j, she can be expected to pay the price p_{k_j} such that

$$p_{k_j} = \frac{b_{k'_{j+1}}\gamma_{k'_{j+1}}}{\gamma_{k_j}}$$

We can write the expected surplus of ad k from occupying position j as

$$CTR_{k_j}(v_{k_j} - p_{k_j})$$

where CTR_{k_j} is the expected clickthrough rate of ad in position j and v_{k_j} is the valuation of the ad k.

We can express CTR_{k_j}as a function of the quality score of advertiser γ_{k_j} and the position effect α_j. In that case, advertiser surplus can be restated as

$$\gamma_{k_j}\alpha_j(v_{k_j} - p_{k_j})$$

Note that the ad specific performance can vary with position even after controlling for the position effect. However, this assumption helps us to estimate an average score that can be uniquely associated with each ad. Further, the search engine auction mechanism is known to rely on such unique score to rank ads.

Previous literature (Edelman et al. 2007; Varian 2007) has characterized GSP auction using an envy free Nash equilibrium where each advertiser finds it unprofitable to deviate from the assigned position of her ad. Thus,

$$\gamma_{k_j}\alpha_j\left(v_{k_j} - p_{k_j}\right) \geq \gamma_{k_j}\alpha_{j+1}\left(v_{k_j} - p_{k_{j+1}}\right)$$

$$\gamma_{k_j}\alpha_j\left(v_{k_j} - p_{k_j}\right) \geq \gamma_{k_j}\alpha_{j-1}\left(v_{k_j} - p_{k_{j-1}}\right)$$

Substituting for the prices we get

$$\gamma_{k_j}\alpha_j\left(v_{k_j} - \frac{b_{k'_{j+1}}\gamma_{k'_{j+1}}}{\gamma_{k_j}}\right) \geq \gamma_{k_j}\alpha_{j+1}\left(v_{k_j} - \frac{b_{k'_{j+2}}\gamma_{k'_{j+2}}}{\gamma_{k_j}}\right)$$

$$\gamma_{k_j}\alpha_j\left(v_{k_j} - \frac{b_{k'_{j+1}}\gamma_{k'_{j+1}}}{\gamma_{k_j}}\right) \geq \gamma_{k_j}\alpha_{j-1}\left(v_{k_j} - \frac{b_{k'_{j-1}}\gamma_{k'_{j-1}}}{\gamma_{k_j}}\right)$$

These inequalities can be rewritten as

$$\frac{\alpha_{j-1}b_{k'_{j-1}}\gamma_{k'_{j-1}} - \alpha_j b_{k'_{j+1}}\gamma_{k'_{j+1}}}{\alpha_{j-1} - \alpha_j} \geq \gamma_{k_j}v_{k_j} \geq \frac{\alpha_j b_{k'_{j+1}}\gamma_{k'_{j+1}} - \alpha_{j+1}b_{k'_{j+2}}\gamma_{k'_{j+2}}}{\alpha_j - \alpha_{j+1}}$$

i.e.

$$\frac{\alpha_{j-1} b_{k'_{j-1}} \gamma_{k'_{j-1}} - \alpha_j b_{k'_{j+1}} \gamma_{k'_{j+1}}}{\alpha_{j-1} - \alpha_j} \geq \frac{\alpha_j b_{k'_{j+1}} \gamma_{k'_{j+1}} - \alpha_{j+1} b_{k'_{j+2}} \gamma_{k'_{j+2}}}{\alpha_j - \alpha_{j+1}}$$

or

$$\gamma_{k'_{j+1}} \leq \frac{(\alpha_j - \alpha_{j+1})\alpha_{j-1} b_{k'_{j-1}} \gamma_{k'_{j-1}} + (\alpha_{j-1} - \alpha_j)\alpha_{j+1} b_{k'_{j+2}} \gamma_{k'_{j+2}}}{(\alpha_{j-1} - \alpha_{j+1})\alpha_j b_{k'_{j+1}}} \qquad \text{(b2)}$$

Note that this can also be derived from the monotone condition and the equilibrium proposed by Edelman et al. (2007) such that

$$\gamma_{k_j} v_{k_j} \geq \frac{\alpha_j b_{k'_{j+1}} \gamma_{k'_{j+1}} - \alpha_{j+1} b_{k'_{j+2}} \gamma_{k'_{j+2}}}{\alpha_j - \alpha_{j+1}} = \gamma_{k_{j+1}} v_{k_{j+1}}$$

The above equation imposes addition restriction on the bids and quality scores of the adjacent ads in the presence of an envy-free equilibrium.

Once, we know the bids of advertisers, the distribution of their quality scores for the keyword can be estimated using system of inequalities obtained from equations b1 and b2. While doing so we assume that the bids represent the equilibrium condition. This is similar to the approach taken by Yang et al. (2013) who use the bids to determine the advertiser valuation. Note that they do not consider ad quality score in their setup.

Estimation: In our setup, we know the bids and impressions of each advertiser keyword combination for different positions on a daily basis. However, we don't know the exact ordering of ads for each search result. So we determine the relative ordering and bids of advertisers for a keyword for a period using the impression weighted positions and bids for the past 7 days. We normalize the quality score of the ad in the top position to 1. We determine the quality for each ad for every keyword a rolling basis, using the following procedure

Estimation Steps

1) Estimate the position specific effects using equation (6) in the paper

2) *Initialization*: For advertiser k in position j (j > 1) in each period, draw γ_{k_j} such that $\gamma_{k_j} \in (0, \frac{b_{k'_{j-1}}\gamma_{k'_{j-1}}}{b_{k_j}})$

3) *Gauss Seidel Iterative Approach*: For advertiser k in position j (j > 1), draw $\gamma_{k_j}{}^{(i+1)}$ such that $\gamma_{k_j}{}^{(i+1)} \in$

$$\left(\frac{b_{k'_{j+1}}\gamma_{k'_{j+1}}{}^{(i)}}{b_{k_j}}, \min\left(\frac{b_{k'_{j-1}}\gamma_{k'_{j-1}}{}^{(i)}}{b_{k_j}}, \frac{(\alpha_{j-1}-\alpha_j)\alpha_{j-2}b_{k'_{j-2}}\gamma_{k'_{j-2}}{}^{(i)}+(\alpha_{j-2}-\alpha_{j-1})\alpha_j b_{k'_{j+1}}\gamma_{k'_{j+1}}{}^{(i)}}{(\alpha_{j-2}-\alpha_j)\alpha_{j-1}b_{k'_j}}\right)\right)$$

4) Repeat step 3 until we find an equilibrium allocation satisfying the inequalities[1]

5) Repeat steps 2 to 4 to determine a distribution of quality values for ads for each keyword

Table B2, Panel B shows the results using the quality estimated based on the GSP equilibrium. These are qualitatively similar to our main results.[2] Table 7 shows this approach has a poor model fit as compared to our main model.

Ad Quality using overall CTR: We also conduct our analysis using a measure of ad quality derived from the click performance. The click performance of an ad for a keyword depends on the ad quality and the ad position. Similar to the approach adopted by Jeziorski and Segal (2015), we assume the quality of an ad does not change during the 93 day period and any time based variation is captured by the error term. In that case we can use the following equation to determine ad quality as a constant term.

(6) $$\mathrm{CTR}_{\mathrm{kat}} = \alpha_0^{\mathrm{k}} + \sum_{\mathrm{a}} \alpha_{\mathrm{a}}^{\mathrm{k}}\delta_{\mathrm{a}} + \sum_{pos} \alpha_{\mathrm{pos}}^{\mathrm{k}}\delta_{pos} + \varepsilon_{\mathrm{kat}}^{\rho}$$

where δ_{a} and δ_{pos} are advertiser specific dummies and position specific dummies for the keyword, and $\alpha_{\mathrm{a}}^{\mathrm{k}}$ and $\alpha_{\mathrm{pos}}^{\mathrm{k}}$ are the corresponding keyword coefficients. We can identify the quality of an advertiser for every keyword up to a constant as every advertiser for a keyword appears in several different positions

[1] This allows us to do a random search to get a distribution of quality values and avoid concentration on a particular type of allocation

[2] We use the actual quality and not the quality rank for this quality measure

and every position is occupied at different times by different advertisers. Results using this quality measure are shown in Table B2, Panel A and are qualitatively similar to our main analysis.

Fixed Effect Model: We also use a fixed effects approach to control for keyword and advertiser specific effects using the following linear model

$$\mathrm{CTR}_{\mathrm{kat}} = \rho_0 + \rho_1 \log(\mathrm{QualityRank}_{\mathrm{kat}}) + \rho_2 \log(\mathrm{QualityRankAbove}_{\mathrm{kat}}) + \rho_3 \log(\mathrm{QualityRankBelow}_{\mathrm{kat}}) + \rho_4(\log(\mathrm{QualityRankAbove}_{\mathrm{kat}}) \times \log(\mathrm{Pos}_{\mathrm{kat}})) + \rho_5(\log(\mathrm{QualityRankAbove}_{\mathrm{kat}}) \times \mathrm{Specificity}_k) + +\rho_6(\log(\mathrm{Pos}_{\mathrm{kat}}) \times \mathrm{Specificity}_k) + \rho_7(\log(\mathrm{QualityRankAbove}_{\mathrm{kat}}) \times \log(\mathrm{Pos}_{\mathrm{kat}}) \times \mathrm{Specificity}_k) + \rho_8 \mathrm{Time}_{\mathrm{kt}} + \sum_{\mathrm{k}} \rho_{\mathrm{k}} \delta_{\mathrm{k}} + \sum_{\mathrm{a}} \rho_{\mathrm{a}} \delta_{\mathrm{a}} + \varepsilon^{\rho}_{\mathrm{kat}}$$

where δ_k and δ_a are keyword and advertiser specific dummies and ρ_k and ρ_a are the corresponding coefficients. Following Animesh, Viswanathan and Agarwal (2011) we estimate the model using OLS where observations are weighted by impressions. In order to account for possible correlation across unobservables, we obtain robust standard errors clustered at keyword level. Results are included in Table B2, Panel A and are qualitatively similar to our main results.

Table B2: Parameter Estimates using other Models

Panel A.

	Without Interaction Term	**Quality using ranking of fixed effect coefficients weighted by impressions and position**	**Quality using the actual value of fixed effect coefficient in the ranking equation**	**Quality using overall CTR**	**Fixed Effects with Clusters Errors for Every Keyword**
Constant	-1.15 (0.25)***	-1.66 (0.18)***	-3.41 (0.12)***	0.54 (0.16)***	0.087 (0.024)***
Quality Rank	-0.5 (0.09)***	-0.51 (0.08)***	-0.28 (0.1)***	-0.65 (0.05)***	-0.002 (0.003)
QualityRankAbove	-1.57 (0.16)***	-1.84 (0.14)***	-0.54 (0.13)***	-1.12 (0.17)***	-0.10 (0.014)***
QualityRankBelow	0.42(0.2)**	0.53 (0.14)***	0.8 (0.11)***	-0.16 (0.15)	0.032 (0.013)**
QualityRankAbove X Position		-0.19 (0.08)**	0.66 (0.15)***	-0.6 (0.2)***	-0.027 (0.010)**
Specificity	-0.47 (0.18)***	-0.21 (0.12)*	-0.01 (0.1)	0.42 (0.11)***	
Quality Rank x Specificity	0.04 (0.07)	0.08 (0.07)	0.01 (0.09)	0.01 (0.04)	
QualityRankAbove x Specificity	0.22 (0.11)*	-0.06 (0.09)	-0.15 (0.09)*	-0.16 (0.13)	0.034 (0.008)***
QualityRankBelow x Specificity	0.13 (0.13)	0.22 (0.12)*	0.13 (0.07)*	0.03 (0.12)	
QualityRankAbove X Position x Specificity		-0.44 (0.06)***	-0.21 (0.09)**	-0.39 (0.15)***	-0.018 (0.009)**

*,**,*** Statistically significant at 10%, 5%, and 1% respectively

Panel B.

	Quality using the GSP Auction Equlibrium
Constant	-4.85 (0.14)***
Quality	0.14 (0.06)**
QualityAbove	0.28 (0.12)**
QualityBelow	1.1 (0.8)
QualityAbove X Position	1.21 (0.13)***
Specificity	-0.45 (0.09)***
Quality x Specificity	-0.04 (0.05)
QualityAbove x Specificity	0.16 (0.08)**
QualityBelow x Specificity	0.3 (0.07)***
QualityAbove X Position x Specificity	0.15 (0.07)**

*,**,*** Statistically significant at 10%, 5%, and 1% respectively

ONLINE APPENDIX C

Analysis for Base-Keywords

Table C1: **Parameter Estimates for Base-keywords**

Variable	Estimate
Constant	-1.67 (0.46)***
Quality Rank	-1.27 (0.49)***
QualityRankAbove	-0.21 (0.51)
QualityRankBelow	0.24 (0.45)
QualityRankAbove X Position	-1.36 (0.59)**

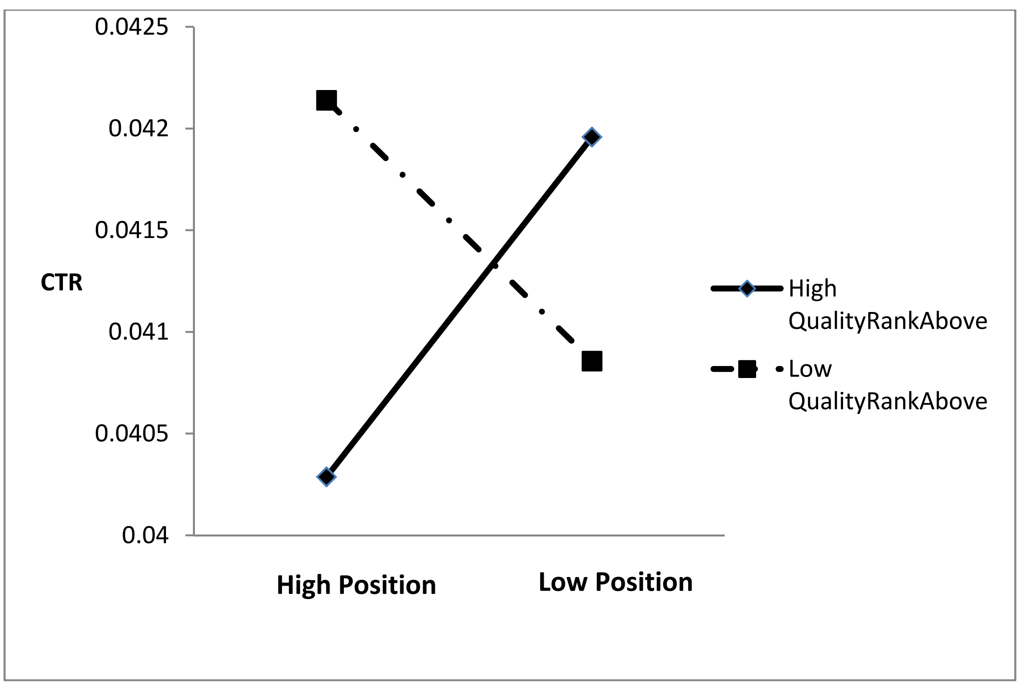

Figure C1: **Impact of Competition on CTR for Base-keywords**

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