APPENDIX

The Proof of Lemma 1:

A perfectly informed Marketing department chooses a traffic level to maximize

$$\pi^M = \left(1 - \frac{\sqrt{\gamma}}{\sqrt{\beta}}\right)\alpha\lambda - \frac{1}{2}a\lambda^2.$$

$$\frac{\partial \pi^M}{\partial \lambda} = \alpha\left(1 - \frac{\sqrt{\gamma}}{\sqrt{\beta}}\right) - a\lambda = 0$$

Thus the optimal traffic level is

$$\lambda_d^* = \frac{\alpha}{a}\left(1 - \frac{\sqrt{\gamma}}{\sqrt{\beta}}\right),$$

where the subscript d stands for the decentralized case. The cost minimizing IT department will set the capacity

$$\mu_d^* = \gamma^{-1/2}(\sqrt{\beta} - \sqrt{\gamma})\lambda_d^*.$$

Thus the profit for the firm under this decentralized structure is

$$\Pi_d^* = \frac{(\sqrt{\beta} - \sqrt{\gamma})^2\alpha}{2a\beta}(2 - \alpha - 2\sqrt{\beta\gamma}),$$

where the subscript d stands for the *decentralized* case.

Recall that under centralized structure, we have $\lambda^* = a^{-1}(1 - \sqrt{\gamma})^2$, and $\mu^* = a^{-1}\gamma^{-1/2}\left(1 - \sqrt{\gamma}\right)^3$. Thus the profit for the firm under this decentralized structure is

$$\Pi_c^* = \frac{(1 - \sqrt{\gamma})^4}{2a}.$$

To have $\Pi_d^* = \Pi_c^*$, we can set $\alpha = 1 - \sqrt{\gamma}$, and $\beta = 1$. □

The Proof of Corollary 1:

The firm's total profit is $(1 - L)\lambda - a\lambda^2/2 - \gamma\mu$. Under decentralized structure, the optimal traffic level chosen by Marketing department is $(1 - \sqrt{\gamma})ka^{-1}$, and the optimal IT capacity level chosen by IT department is $(1 - \sqrt{\gamma})\lambda/\sqrt{\gamma}$. Therefore, the firm's total profit is $\frac{(1-\sqrt{\gamma})k}{a}((1 - \sqrt{\gamma})^2 - \frac{(1-\sqrt{\gamma})k}{2})$. Under centralized structure, the optimal traffic level chosen by Marketing department is $\lambda^* = a^{-1}(1 - \sqrt{\gamma})^2$, and the optimal IT capacity level chosen by IT department is $a^{-1}\gamma^{-1/2}(1 - \sqrt{\gamma})^3$. Therefore, the firm's total profit is $(1 - \sqrt{\gamma})^4/(2a)$. The difference of the firm's profits in centralized and decentralized cases is therefore $((1 - \sqrt{\gamma})^2 - (1 - \sqrt{\gamma})k)^2/(2a)$. □

The Proof of Lemma 2:

Substituting A_i in (5), the profit function for firm *i* can be rewritten as follows

$$\Pi_i = \frac{\mu_i\lambda_i}{\mu_i + \lambda_i} - \gamma_i\mu_i - \frac{a_i}{2(1+2b)^2}\left((1+b)\lambda_i + b\lambda_j\right)^2; \qquad \{i,j\} = \{1,2\}.$$

Thus, in equilibrium, we have the following FOCs.

$$\frac{\partial \Pi_i}{\partial \lambda_i} = \frac{\mu_i^2}{(\mu_i+\lambda_i)^2} - \frac{a_i(1+b)((1+b)\lambda_i + b\lambda_j)}{(1+2b)^2} = 0 \tag{A-1}$$

$$\frac{\partial \Pi_i}{\partial \mu_i} = \frac{\lambda_i^2}{(\mu_i+\lambda_i)^2} - \gamma_i = 0 \tag{A-2}$$

We have checked that the Hessian is negative semi-definite, and therefore Π_i is a concave function of (λ_i, μ_i). From Equations (A-1) and (A-2), for firm $i, i = 1, 2$, we get the response function, in terms of demand, as:

$$\lambda_i = \frac{1}{a_i}(1-\sqrt{\gamma_i})^2\left(\frac{1+2b}{1+b}\right)^2 - \frac{b}{1+b}\lambda_j. \tag{A-3}$$

The equilibrium demand levels are further obtained as

$$\lambda_{i,c,c}^* = (1+2b)\left(\frac{(1-\sqrt{\gamma_i})^2}{a_i} - (1-\sqrt{\gamma_j})^2\frac{b}{a_j(1+b)}\right).$$

With the equilibrium demand levels, the optimal profits for the two firms facing inter-firm competition follow.

$$\Pi_{i,c,c}^* = \frac{(1-\sqrt{\gamma_i})^2(1+2b)}{2}\left(\frac{(1-\sqrt{\gamma_i})^2(1+2b+2b^2)}{a_i(1+b)^2} - \frac{2b(1-\sqrt{\gamma_j})^2}{a_j(1+b)}\right),$$

where the subscript c in $\Pi_{i,c,c}^*$ and $\lambda_{i,c,c}^*$ stands for *Centralized*.

The sufficient and necessary condition for $\Pi_{i,c,c}^*$ to be positive is

$$\frac{(1-\sqrt{\gamma_i})^2}{(1-\sqrt{\gamma_j})^2} > \frac{2a_i b(1+b)^2}{\sqrt{a_j}(1+b)(1+2b+2b^2)}; \qquad \{i,j\} = \{1,2\}.$$

Under this condition, we can also ensure that $\lambda_{i,c,c}^*$ is positive. □

The Proof of Lemma 3:

The profit function of Department *M* of firm *i* can be rewritten as follows.

$$\pi_i^M = \alpha_i k_i \lambda_i - \frac{a_i}{2(1+2b)^2}\Big((1+b)\lambda_i + b\lambda_j\Big)^2; \qquad \{i,j\} = \{1,2\}.$$

Thus, at equilibrium, we have the following FOCs,

$$\frac{\partial \pi_i^M}{\partial \lambda_i} = \alpha_i k_i - \frac{a_i(1+b)((1+b)\lambda_i + b\lambda_j)}{(1+2b)^2} = 0. \tag{A-4}$$

From (A-4), we get the response function, in terms of demand, as:

$$\lambda_i = \frac{\alpha_i k_i}{a_i}\left(\frac{1+2b}{1+b}\right)^2 - \frac{b}{1+b}\lambda_j.$$

This response function maximizes π_i^M since $\partial^2\pi_i^M/\partial\lambda_i^2 < 0$. Thus, in the presence of competition, if both firms make decentralized decisions on advertising and IT planning, then the equilibrium demand levels at firm $i,\ i = 1, 2$, can be obtained as below.

$$\lambda_i^* = (1+2b)\left(\frac{\alpha_i k_i}{a_i} - \frac{b\alpha_j k_j}{a_j(1+b)}\right) \tag{A-5}$$

We write the cost function for Department *IT* of firm *i* as below.

$$C_i^{IT} = \lambda_i L_i + \gamma_i\mu_i = \frac{\lambda_i^2}{\mu_i + \lambda_i} + \gamma_i\mu_i$$

Since $\partial^2 C_i^{IT}/\partial\mu_i^2 = 2\lambda_i^2/(\mu_i+\lambda_i)^3 > 0$, from $\partial C_i^{IT}/\partial\mu_i = 0$, we have the minimum $\mu_i^* = (1/\sqrt{\gamma_i} - 1)\lambda_i^*$. Thus, $C_i^{IT*} = (2\sqrt{\gamma_i} - \gamma_i)\lambda_i^*$. The optimal profits for the two firms facing competition follow.

$$\Pi_{i,d,d}^* = \frac{(1+2b)(1-\sqrt{\gamma_i})^2}{2}\left(\frac{2(1-\sqrt{\gamma_i})k_i(1+b)^2 - k_i^2(1+2b)}{a_i(1+b)^2} - \frac{2b(1-\sqrt{\gamma_j})k_j}{(1+b)a_j}\right),$$

where the subscript *d* in $\Pi_{i,d,d}$ stands for *decentralized*.

To ensure that $\Pi_{i,d,d}^*$ is positive, we need to have $k_i^2 a_j(1+b)(1+2b)/\left(2(1+b)\left(k_i a_j(1-\sqrt{\gamma_i})(1+b)^2 - k_j a_i b(1-\sqrt{\gamma_j})(1+b)\right)\right) < 1$ and $k_i a_j(1-\sqrt{\gamma_i})(1+b)^2 - k_j a_i b(1-\sqrt{\gamma_j})(1+b) > 0$. The latter condition also ensures that λ_i^* in (A-5) is positive. □

The Proof of Proposition 1:

1) *Only One Firm Adopts Centralization*

If only one of the two firms, say firm 1, adopts centralization, then the profit function that firm 1 maximizes is

$$\Pi_{1,c,d} = \lambda_1 - A_1 - L_1\lambda_1 - \gamma_1\mu_1,$$

while the Marketing department of firm 2 maximizes $\pi_{2,c,d}^M = (1-\sqrt{\gamma_2})k_2\lambda_2 - A_2$.

Substituting A_1 in $\Pi_{1,c,d}$, from $\partial\Pi_{1,c,d}/\partial\lambda_1 = 0$ and $\partial\Pi_{1,c,d}/\partial\mu_1 = 0$, we get the response function of firm 1, in terms of traffic, as:

$$\lambda_1 = \frac{1}{a_1}(1-\sqrt{\gamma_1})^2\left(\frac{1+2b}{1+b}\right)^2 - \frac{b}{1+b}\lambda_2.$$

The response function of firm 2 is

$$\lambda_2 = \frac{(1-\sqrt{\gamma_2})k_2}{a_2}\left(\frac{1+2b}{1+b}\right)^2 - \frac{b}{1+b}\lambda_1.$$

Thus, we have

$$\lambda_1^* = (1+2b)\left(\frac{(1-\sqrt{\gamma_1})^2}{a_1} - \frac{b(1-\sqrt{\gamma_2})k_2}{a_2(1+b)}\right),$$

and

$$\lambda_2^* = (1+2b)\left(\frac{(1-\sqrt{\gamma_2})k_2}{a_2} - \frac{b(1-\sqrt{\gamma_1})^2}{a_1(1+b)}\right).$$

Under this circumstance, optimal profits become,

$$\Pi_{1,c,d}^* = \frac{(1-\sqrt{\gamma_1})^2(1+2b)}{2(1+b)}\left(\frac{(1-\sqrt{\gamma_1})^2(1+2b+2b^2)}{a_1(1+b)} - \frac{2b(1-\sqrt{\gamma_2})k_2}{a_2}\right).$$

To ensure that $\Pi_{1,c,d}^*$ is positive, we need to have

$$(1-\sqrt{\gamma_1})^2 > \frac{2a_1 b(1-\sqrt{\gamma_2})k_2(1+b)}{a_2(1+2b+2b^2)},$$

which also ensures that λ_1^* is positive.

Note that firm 2's profit is $\Pi_{2,c,d} = (1-L_2)\lambda_2 - A_2 - \gamma_2\mu_2$, where $L_2 = \frac{\lambda_2}{\lambda_2+\mu_2}$. Since IT department of firm 2 is trying to minimize

$$C_2^{IT} = \lambda_2^* L_2 + \gamma_2\mu_2,$$

we have $\mu_2^* = (1-\sqrt{\gamma_2})\lambda_2^*/\sqrt{\gamma_2}$. Thus, the optimal profits of firm 2 become,

$$\Pi_{2,c,d}^* = \frac{(1+2b)(1-\sqrt{\gamma_2})^2}{2(1+b)}\left(\frac{2(1-\sqrt{\gamma_2})k_2(1+b)^2 - k_2^2(1+2b)}{a_2(1+b)} - \frac{2b(1-\sqrt{\gamma_1})^2}{a_1}\right).$$

To ensure that $\Pi_{2,c,d}^*$ is positive, we need to have

$$\frac{a_1 k_2^2(1+2b)}{2(1+b)(k_2 a_1(1+b)(1-\sqrt{\gamma_2}) - b a_2(1-\sqrt{\gamma_1})^2)} < 1,$$

and $a_1 k_2(1-\sqrt{\gamma_2})(1+b) - b a_2(1-\sqrt{\gamma_1})^2 > 0$. The latter condition also ensures that λ_2^* is positive.

Note that we have

$$\Pi_{2,c,d}^* - \Pi_{2,c,c}^* = \frac{(1-\sqrt{\gamma_2})^2(1+2b)}{2a_2(1+b)^2} * (k_2 - (1-\sqrt{\gamma_2}))\left((1-\sqrt{\gamma_2})(1+2b+2b^2) - k_2(1+2b)\right).$$

When $k_2 > \zeta(1-\sqrt{\gamma_2})$ or $k_2 < 1-\sqrt{\gamma_2}$, we have $\Pi_{2,c,d}^* < \Pi_{2,c,c}^*$; when $1-\sqrt{\gamma_2} < k_2 \leq \zeta(1-\sqrt{\gamma_2})$, we have $\Pi_{2,c,d}^* \geq \Pi_{2,c,c}^*$. Thus, given that firm 1 chooses centralization, firm 2 chooses decentralization if its Marketing department's estimate of IT service quality is between its lower and upper structure thresholds, otherwise, it chooses centralization.

Similarly, we have

$$\Pi_{1,c,c}^* - \Pi_{1,d,c}^* = \frac{(1-\sqrt{\gamma_1})^2(1+2b)}{2a_1(1+b)^2} * (k_1 - (1-\sqrt{\gamma_1}))\left(k_1(1+2b) - (1-\sqrt{\gamma_1})(1+2b+2b^2)\right)$$

When $k_1 > \zeta(1-\sqrt{\gamma_1})$ or $k_1 < 1-\sqrt{\gamma_1}$, we have $\Pi_{1,c,c}^* > \Pi_{1,d,c}^*$; when $1-\sqrt{\gamma_1} < k_1 \leq \zeta(1-\sqrt{\gamma_1})$, we have $\Pi_{1,c,c}^* \leq \Pi_{1,d,c}^*$. Thus, given that firm 2 chooses centralization, firm 1 chooses decentralization if its

Marketing department's estimate of IT service quality is between its lower and upper structure thresholds; otherwise, it chooses centralization.

2) *Both Firms Adopt Decentralization*

From Lemma 3, when both firms adopt decentralization, we have

$$\Pi^*_{i,d,d} = \frac{(1+2b)(1-\sqrt{\gamma_i})^2}{2(1+b)}\left(\frac{2k_i(1+b)^2(1-\sqrt{\gamma_i})-k_i^2(1+2b)}{a_i(1+b)} - \frac{2bk_j(1-\sqrt{\gamma_j})}{a_j}\right),$$

where the subscript *d* in $\Pi_{i,d,d}$ stands for *decentralized*.

Note that we have

$$\Pi^*_{1,c,d} - \Pi^*_{1,d,d} = \frac{(1-\sqrt{\gamma_1})^2(1+2b)}{2a_1(1+b)^2} * (k_1 - (1-\sqrt{\gamma_1}))\left(k_1(1+2b) - (1-\sqrt{\gamma_1})(1+2b+2b^2)\right).$$

As shown in the above equation, firm 1's decision on its organizational structure when firm 2 chooses decentralization does not depend on k_2. When $k_1 > \zeta(1-\sqrt{\gamma_1})$ or $k_1 < 1-\sqrt{\gamma_1}$, we have $\Pi^*_{1,c,d} > \Pi^*_{1,d,d}$; when $1-\sqrt{\gamma_1} < k_1 \le \zeta(1-\sqrt{\gamma_1})$, we have $\Pi^*_{1,c,d} \le \Pi^*_{1,d,d}$. Thus, given that firm 2 chooses decentralization, firm 1 chooses decentralization if its Marketing department's estimate of IT service quality is between its lower and upper structure thresholds, otherwise, it chooses centralization. Recall that, when firm 2 chooses centralization, we have the same result (i.e., firm 1 chooses decentralization if its Marketing department's estimate of IT service quality is between its lower and upper structure thresholds, otherwise, it chooses centralization). Thus, for a given value of k_i, centralization (or decentralization) is a dominant strategy for firm i (i.e., a firm chooses a structure regardless of the structure chosen by the other firm).

Similarly, when $k_2 > \zeta(1-\sqrt{\gamma_2})$ or $k_2 < 1-\sqrt{\gamma_2}$, it holds that $\Pi^*_{2,d,c} > \Pi^*_{2,d,d}$; when $1-\sqrt{\gamma_2} < k_2 \le \zeta(1-\sqrt{\gamma_2})$, it holds that $\Pi^*_{2,d,c} \le \Pi^*_{2,d,d}$. Thus, given that firm 1 chooses decentralization, firm 2 chooses decentralization if its Marketing department's estimate of IT service quality is between its lower and upper structure thresholds; otherwise, it chooses centralization. □

The Proof of Corollary 2:

It is easy to show that $\partial\zeta/\partial b > 0$. As b increases to $\hat{b}$, the upper threshold $\zeta(1-\sqrt{\gamma_i})$ for centralization (via intra-firm coordination) increases to $\hat{\zeta}(1-\sqrt{\gamma_i})$. When the competition level is b, if $k_i > \zeta(1-\sqrt{\gamma_i})$, then firm *i* chooses centralization at equilibrium; however, as *b* increases to $\hat{b}$, it is possible that $1-\sqrt{\gamma_i} < k_i < \hat{\zeta}(1-\sqrt{\gamma_i})$, which implies that firm *i* may not choose centralization at equilibrium. If $k_i < 1-\sqrt{\gamma_i}$, then firm *i* chooses centralization at equilibrium, and will continue to choose centralization as *b* increases. This is because the inequality $k_i < 1-\sqrt{\gamma_i}$ does not depend on *b*.

It holds that $\partial\zeta/\partial a_1 > 0$ and $\partial\zeta/\partial a_2 > 0$.

When $k_i > \zeta(1-\sqrt{\gamma_i})$, as γ_i reduces, it could hold that $(1-\sqrt{\gamma_i}) < k_i < \zeta(1-\sqrt{\gamma_i})$; when $1-\sqrt{\gamma_i} < k_i < \zeta(1-\sqrt{\gamma_i})$, as γ_i reduces, it could hold that $k_i < 1-\sqrt{\gamma_i}$. □

The Proof of Lemma 4:

Under the symmetric setting where $a_1 = a_2 = a, k_1 = k_2 = k, \gamma_1 = \gamma_2 = \gamma$, the following equations hold.

$$\Pi_{1,c,c} + \Pi_{2,c,c} - (\Pi_{1,d,d} + \Pi_{2,d,d}) = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1-\sqrt{\gamma}))}{a(1+b)^2} \quad \text{(A-6)}$$

$$\Pi_{1,c,c} + \Pi_{2,c,c} - (\Pi_{1,c,d} + \Pi_{2,c,d}) = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1-\sqrt{\gamma}))}{2a(1+b)^2} \quad \text{(A-7)}$$

$$\Pi_{2,c,c} - \Pi_{2,c,d} = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1+2b+2b^2)(1-\sqrt{\gamma}))}{2a(1+b)^2} \quad \text{(A-8)}$$

$$\Pi_{1,c,c} - \Pi_{1,c,d} = \frac{(1-\sqrt{\gamma})^3 b(1+2b)(k-(1-\sqrt{\gamma}))}{a(1+b)} \quad \text{(A-9)}$$

$$\Pi_{1,c,d} - \Pi_{1,d,d} = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1+2b+2b^2)(1-\sqrt{\gamma}))}{2a(1+b)^2} \quad \text{(A-10)}$$

$$\Pi_{2,c,c} - \Pi_{2,c,d} = \Pi_{1,c,d} - \Pi_{1,d,d} \quad \text{(A-11)}$$

When $k > \zeta(1-\sqrt{\gamma})$, where $\zeta = (1+2b+2b^2)/(1+2b)$, from Proposition 1, the only equilibrium is (c,c). Since $\zeta > 1$, from Equations (A-8) and (A-9), we have $\Pi_{1,c,c} - \Pi_{1,c,d} > 0$, and $\Pi_{2,c,c} - \Pi_{2,c,d} > 0$. Similarly, we have $\Pi_{1,c,c} - \Pi_{1,d,c} > 0$, and $\Pi_{2,c,c} - \Pi_{2,d,c} > 0$. From (A-6), we have $\Pi_{i,c,c} > \Pi_{i,d,d} > 0,\ i = 1,2$. Hence, equilibrium (c,c) Pareto-dominates all the other structure combinations, which leads to further conclusion that (*c*, *c*) is socially optimal. □

The Proof of Lemma 5:

When $1-\sqrt{\gamma} < k \leq \zeta(1-\sqrt{\gamma})$, from Proposition 1, the only equilibrium is (d,d). From Equations (A-6) and (A-7), we conclude that (c,c) is the social optimal. Furthermore, under the symmetric setting of model parameters, (c,c) Pareto-dominates (d,d). Therefore, (d,d) is a Prisoner's dilemma equilibrium, which is Pareto-dominated by (c,c). □

The Proof of Corollary 3:

The proof follows immediately from Lemmas 4 and 5. □

The Proof of Proposition 2:

When $(1-\sqrt{\gamma})/(1+2b) < k < 1-\sqrt{\gamma}$, the following inequalities hold.

$$\Pi_{2,c,c} - \Pi_{2,c,d} = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1+2b+2b^2)(1-\sqrt{\gamma}))}{2a(1+b)^2} > 0$$

$$\Pi_{2,d,c} - \Pi_{2,d,d} = \frac{(1+2b)(1-\sqrt{\gamma})^2(k-(1-\sqrt{\gamma}))((1+2b)k-(1+2b+2b^2)(1-\sqrt{\gamma}))}{2a(1+b)^2} > 0$$

Similarly, it holds that $\Pi_{1,c,c} - \Pi_{1,d,c} > 0$ and $\Pi_{1,c,d} - \Pi_{1,d,d} > 0$. Thus, Centralization is the *dominant* strategy for both firms. And from Proposition 1, the only equilibrium is (c,c), which is also the dominant strategy equilibrium.

According to the definition of Kuhn (2014), under the symmetric settings, the game we are studying is a Prisoner's Dilemma game in the strong sense, if the following chain inequalities hold for the game: $\Pi_{2,d,c} > \Pi_{2,d,d} > \Pi_{2,c,c} > \Pi_{2,c,d}$. As shown above, we have proved that $\Pi_{2,d,c} > \Pi_{2,d,d}$ and $\Pi_{2,c,c} > \Pi_{2,c,d}$. Under the symmetric settings, from (A-6), when $(1-\sqrt{\gamma})/(1+2b) < k < 1-\sqrt{\gamma}$, we have $\Pi_{2,c,c} < \Pi_{2,d,d}$. Thus, we conclude that the chain inequalities $\Pi_{2,d,c} > \Pi_{2,d,d} > \Pi_{2,c,c} > \Pi_{2,c,d}$ are satisfied, and therefore the game we are studying can be characterized as a Prisoner's Dilemma game in the strong sense.

Under the symmetric settings, when $(1-\sqrt{\gamma})/(1+2b) < k < 1-\sqrt{\gamma}$, from (A-6) and (A-7), we have $\Pi_{i,c,c} < \Pi_{i,d,d},\ i = 1,2$, and $\Pi_{1,c,c} + \Pi_{2,c,c} < \Pi_{1,c,d} + \Pi_{2,c,d} < \Pi_{1,d,d} + \Pi_{2,d,d}$. Thus, (*d*, *d*) is the socially optimal outcome, and it pareto-dominates the Prisoner's Dilemma equilibrium (*c*, *c*). □

The Proof of Lemma 6:

When $k < (1-\sqrt{\gamma})/(1+2b)$, it also holds that $k < 1-\sqrt{\gamma}$. From Proposition 1, the only equilibrium is (*c*, *c*). From (A-6) and (A-7), we have $\Pi_{1,c,c} + \Pi_{2,c,c} > \Pi_{1,c,d} + \Pi_{2,c,d} > \Pi_{1,d,d} + \Pi_{2,d,d}$. Thus, (*c*, *c*) is socially optimal, and it, under the symmetric parameter settings, Pareto-dominates (*d*, *d*). □

Consumer Choice Model for Firms' Traffic

To calculate the traffic for each firm in a duopoly, we consider consumers to be heterogeneous in terms of their intrinsic affinity for a particular firm (i.e., prior to being affected by any advertisement). This affinity is captured by a *preference parameter* x_p, uniformly distributed over $[0,1]$. Specifically, $x_p = 0$ implies that consumer p has the highest possible preference for firm 1, while $x_p = 1$ represents the highest possible preference for firm 2.

The *Marketing Influence* of firm i exerted on consumer p is denoted by $M_i(x_p), i = 1,2$, and is composed of two effects: (1) a positive effect (i.e., a "promoting/attracting" effect) caused by the advertising of firm i, and (2) a negative effect (i.e., a "canceling effect") caused by the advertising of firm j. The positive effect increases with the advertising level of the focal firm, whereas the magnitude of the negative effect increases with the intensity of advertising competition and the advertising level of the rival firm. Combining the positive and negative effects, we define $M_1(x_p) = (1-x_p)\sqrt{2A_1/a_1} - b\sqrt{2A_2/a_2}$, and $M_2(x_p) = x_p\sqrt{2A_2/a_2} - b\sqrt{2A_1/a_1}$, respectively. The preference parameter x_p measures the extent to which consumer p would respond to the "promoting/attracting" effect. The parameter b, on the other hand, measures the extent to which the advertising of the rival firm would negatively influence a consumer. Consumer p's choice of the firm to visit depends on the *relative* Marketing Influence of the two firms imposed on her. That is, consumer p chooses firm 1 if $M_1(x_p) > M_2(x_p)$, and firm 2 otherwise.[1] A consumer with preference $\tilde{x}$ would be indifferent between the two firms if $M_1(\tilde{x}) = M_2(\tilde{x})$. Therefore, we have

$$\tilde{x} = \frac{\sqrt{2A_1/a_1} + b(\sqrt{2A_1/a_1} - \sqrt{2A_2/a_2})}{\sqrt{2A_1/a_1} + \sqrt{2A_2/a_2}}.$$

[1] Here, for simplification, we assume that a specific consumer only visits one firm before making the final purchase. However, it can be shown that all the main findings in this study hold when this assumption is relaxed.

Because the consumers attracted by firm 1 are in the region [0, $\tilde{x}$], the traffic attracted by firm 1 is given by $\tilde{x}$ times the total traffic. From (3), the total traffic is given by $\sqrt{2A_1/a_1} + \sqrt{2A_2/a_2}$. Hence, the traffic attracted by firm 1 is given by $\lambda_1 = \sqrt{2A_1/a_1} + \sqrt{2}b(\sqrt{A_1/a_1} - \sqrt{A_2/a_2})$. In general, the traffic attracted by firm i is,

$$\lambda_i = \sqrt{2/a_i}\sqrt{A_i} + \sqrt{2}b(\sqrt{A_i/a_i} - \sqrt{A_j/a_j}); \qquad \{i, j\} = \{1, 2\}, \tag{A-12}$$

where $\lambda_i > 0$.