Optimal Auction Design for WiFi Procurement

Online Appendix

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A Lemmas

Lemma 1 *In an $M/M/c$ queue, the expected waiting time is convex in service rate.*

Proof. The expected waiting time of an $M/M/c$ queue can be written as

$$W = \frac{1}{\mu} + \frac{A}{\mu^{c+1} - b\mu^c}$$

where both A and b are constant with $b < \mu$. Clearly, $W(\mu)$ is decreasing in μ because

$$\frac{d}{d\mu}\left(\frac{1}{\mu^{c+1} - b\mu^c}\right) = -\frac{(c+1)\mu^c - bc\mu^{c-1}}{\big(\mu^{c+1} - b\mu^c\big)^2} < 0.$$

For the convexity of $W(\mu)$, it suffices to show

$$\frac{d^2}{d\mu^2}\left(\frac{1}{\mu^{c+1} - b\mu^c}\right) > 0,$$

which is equivalent to

$$\begin{aligned}
\big(c(c+1)\mu^{c-1} - bc(c-1)\mu^{c-2}\big)\big(\mu^{c+1} - b\mu^c\big)^2 &< 2\big(\mu^{c+1} - b\mu^c\big)\big((c+1)\mu^c - bc\mu^{c-1}\big)^2 \\
\Leftrightarrow \big(c(c+1)\mu - bc(c-1)\big)(\mu - b) &< 2\big((c+1)\mu - bc\big)^2 \\
\Leftrightarrow (c+1)(c+2)\mu^2 - 2bc(c+2)\mu + b^2(c^2+c) &> 0. \\
\Leftrightarrow (c+1)(c+2)\big(\mu - \frac{bc}{c+1}\big)^2 + \frac{b^2c}{c+1} &> 0.
\end{aligned}$$

■

Lemma 2 *Let $\Theta \in \mathbb{R}$ be a closed interval and $f : \Theta \to \mathbb{R}$ be a function that is locally decreasing, that is, $\forall \theta \in \Theta$, there exists $\epsilon > 0$ such that $f(\theta)$ is monotone decreasing on $[\theta, \theta + \epsilon]$. If f is continuous, then f is monotone decreasign on Θ.*

Proof. Pick any $\theta_1, \theta_2 \in \Theta$ and assume $\theta_1 < \theta_2$. Let $m = \inf_{\theta \in \Theta} f(\theta)$ and $K = \{\theta \in [\theta_1, \theta_2] \big| f(\theta) = m\}$. By the Weierstrass theorem, K is non-empty. Let $\tilde{\theta} \equiv \sup(K)$, then

$\tilde{\theta} \in K$ because K is closed which is due to the continuity of f. Suppose $\tilde{\theta} \neq \theta_2$, then $\exists \epsilon > 0$ such that $\tilde{\theta} + \epsilon < \theta_2$ and

$$m = f(\tilde{\theta}) \geq f(\tilde{\theta} + \epsilon) \geq m.$$

Hence, $\tilde{\theta} + \epsilon \in K$. But $\tilde{\theta} \equiv \sup(K)$. Contradiction. Therefore, $\sup(K) = \theta_2$ and $f(\theta_2) = m \leq f(\theta_1)$. ■

B An Application with Simulation

Applying our proposed auction mechanism to the network data from one of the largest U.S. service providers, we address the following question in this section: Compared with the standard Vickrey-Clarke-Groves (VCG) auction for mobile data offloading suggested in the computer science literature (Dong et al. 2014), how much can our optimal procurement auction improve the cellular network's expected payoff? Since the VCG-type auction is a welfare maximizing mechanism, it is not surprising that our mechanism can outperform the standard VCG auction. However, our Monte Carlo simulation results demonstrate that the improvement is considerable: As compared with the standard VCG auction, our procurement auction significantly improves the cellular network's expected payoff and reduces procurement cost by more than 50%. We also evaluate the impact of the cellular capacity and the relative cost of deploying cellular resources on the performance difference between these two mechanisms.

B.1 Derivations with Specific Functional Forms

To compute numerical examples, we first assume the following parameterization of $C(q, \theta_i)$: $C(q, \theta_i) = (0.5 + \theta_i)q^2$, which implies $c(q, \theta_i) = (1 + 2\theta_i)q$, $c_\theta(q, \theta_i) = 2q$.

In this case, we can explicitly solve for $g_i(\nu)$ as

$$g_i(\nu) = \frac{\nu}{\alpha\left(1 + 2\theta_i + 2H(\theta_i)\right)}.$$

We also assumed that

$$\omega_m(x) = \frac{\kappa_m}{x - \lambda_m},$$

where $\kappa_m > 0$ is the weight placed on region m. Hence, $\omega'_m(x) = -\kappa_m(x - \lambda_m)^{-2}$, and

$$\phi_m(x) = \lambda_m + \sqrt{-\frac{\kappa_m}{x}}.$$

So we have

$$\begin{aligned} \Phi(x) &= \sum_{m=1}^{M} \lambda_m + \sqrt{-\frac{1}{x}} \sum_{m=1}^{M} \sqrt{\kappa_m} \\ \Psi(x) &= -\frac{\left(\sum_{m=1}^{M} \sqrt{\kappa_m}\right)^2}{\left(x - \sum_{m=1}^{M} \lambda_m\right)^2} \end{aligned}$$

Therefore, q^* is determined by the following equation

$$\sum_{i=1}^{N} g_i \left(\frac{\left(\sum_{m=1}^{M} \sqrt{\kappa_m}\right)^2}{\left(\mu + q - \sum_{m=1}^{M} \lambda_m\right)^2} \right) = q.$$

Substituting the functional form of $g_i(\nu)$, we have q^* as the solution to the following cubic equation:

$$q \left(\mu + q - \sum_{m=1}^{M} \lambda_m \right)^2 = \left(\sum_{m=1}^{M} \sqrt{\kappa_m} \right)^2 \sum_{i=1}^{N} \frac{1}{\alpha \left(1 + 2\theta_i + 2H(\theta_i)\right)}.$$

q^* can be solved either explicitly or by using bisection to search in the interval $(0, \bar{q})$ where $\bar{q}$ is defined as

$$\bar{q} \equiv \sum_{i=1}^{N} g_i \left(-\Psi(\mu)\right) = \left(\sum_{m=1}^{M} \sqrt{\kappa_m} \right)^2 \frac{1}{(\mu - \sum_{m=1}^{M} \lambda_m)^2} \sum_{i=1}^{N} \frac{1}{\alpha \left(1 + 2\theta_i + 2H(\theta_i)\right)}.$$

B.2 VCG Auction

Before we do the comparison, we review the multi-unit VCG auction for procurement in our context. The VCG auction involves the following steps:

- Invite each hotspot to report its cost parameter θ. Denote the submitted cost parameters as $(\theta_1, \theta_2, \cdots, \theta_N)$.

- Under the VCG mechanism, the socially efficient allocation minimizes the sum of the expected congestion cost of the cellular service provider and the cost of hotspots. Hence, the minimization problem can be formalized as follows:

$$\begin{array}{ll} \min\limits_{q_1, q_2, \ldots, q_N} & \mathbb{E}\Big[J\left(\sum\limits_{i=1}^{N} q_i^*\right) + \sum\limits_{i=1}^{N} C(q_i^*, \theta_i)\Big] \\ \text{s.t.} & q_i \geq 0, \forall i = 1, 2, \cdots, N. \end{array}$$

- Let $\pi(\theta_1, \theta_2, \cdots, \theta_k)$ be the optimal value of the objective function, and let $(q_1^*, q_2^*, \cdots, q_n^*)$ be an optimal solution to the cost minimization problem. Let $\pi_{-i}(\theta_{-i})$ be the optimal value of the objective function with the additional constraint $q_i = 0$ (i.e., hotspot i does not participate in the auction).

- The cellular service provider will pay hotspot i according to the following:

$$P_i = \pi_{-i}(\theta_{-i}) - \pi(\theta_1, \theta_2, \cdots, \theta_N) + C(q_i^*, \theta_i)$$

  where $\pi_{-i}(\theta_{-i}) - \pi(\theta_1, \theta_2, \cdots, \theta_N)$ is the bonus payment to hotspot i, representing the positive externality that hotspot i is imposing on the cost minimization problem. The cellular service provider pays hotspot i its cost $C(q_i^*, \theta_i)$, plus its contribution to the cost minimization problem. This payment internalizes the externality.

- Hotspot i provides capacity q_i^* and receives payment P_i.

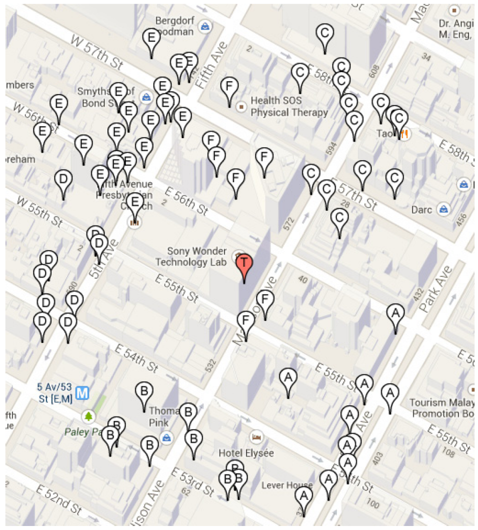

Figure 1: Area Map of A Typical Cell Sector

Note that the VCG auction is both truth-telling and socially efficient by standard arguments. All hotspots bid their cost parameters truthfully, irrespective of other hotspots' bids. The VCG mechanism guarantees the minimum total cost. However, it leads to an overpayment to hotspots that is shown in the simulation.

B.3 Simulation

In our simulations, we consider a typical urban neighborhood in New York City, as shown in Figure 1. We define a cell sector as the range of the cell tower. Our dataset consists of the location information of 14,576 cell towers from a large cellular provider in the U.S. In our simulation study, we pick a cell tower in New York City from the full list of cell towers and simulate the mobile data demand in this sector. In Figure 1, the cell tower is represented by the marker labelled with the letter "T", and the 69 WiFi hotspots in the given cell sector are represented by other markers.[1] We set the communication range for a cell tower as 250m, and set the communication range for Wi-Fi as 100m. The following steps describe the procedure of simulations:

[1]Locations of commercial WiFi hotspots are from http://wigle.net.

- Generate traffic demands in the given cell sector: To gain a sense of the population density in the coverage area of the cell tower, we use 2010 census data, which contains the land area coverage and population density of each zip code. Combining the market share of this service provider for the first quarter 2013,[2] we estimate the number of users in the given cell sector. On average, smartphone users consume about 1GB data per month, but the usage patterns of mobile data is highly uneven.[3] Paul et al. (2011) and Jin et al. (2012) found that a small number of heavy users contribute to a majority of data usage in the network. To consider the heterogeneity of data usage and the effects of peak hours, we simulate individual data usage from the byte distribution in Jin et al. (2012).[4]

- Generate WiFi regions in the cell sector: Dong et al. (2014) showed that the appropriate number of WiFi regions in a cell sector is six. Following their approach, we generate six WiFi regions by clustering the WiFi hotspots using k-means. In Figure 1, Region A, Region B, ... , and Region F indicate which region the WiFi hotspots belong to.

- Generate traffic demands in each WiFi region: We use two different methods to place users in the cell sector and assign them to the corresponding WiFi regions according to their locations. (1) All users are randomly placed in the cell sector. (2) All users are placed according to the densities of the hotspots.[5] After placing all the users, a nearest hotspot is calculated for each user location. If the distance between the nearest hotspot found and the user location is less than the hotspot range (100m), the user is counted

[2]See http://www.talkandroid.com/159929-t-mobile-loses-market-share-while-verizon-and-att-continue-to-dominate.

[3]See http://www.fiercewireless.com/special-report/average-android-ios-smartphone-data-use-across-tier-1-wireless-carriers-through.

[4]We obtain the quantiles of the byte distribution from Jin et al. (2012) and generate inidvidual usage using the Johnson System. We also adjust the usage by considering the effect of peak hours, see http://chitika.com/browsing-activity-by-hour.

[5]To calculate the densities of the hotspots for different locations, we divide the square circumscribing the cell sector into a 20 by 20 array of grids. By default, each grid has a weight of 1, except the grids whose centers are not in the range of the tower. The grid's weight is increased by the number of hotspots whose locations are inside the grid. Then, a list of grid indices is created according to the weight of each grid. Finally, for each user, a grid index is first uniformly chosen from the list, and then the location of the user is uniformly chosen from the range of the grid with the grid index just picked.

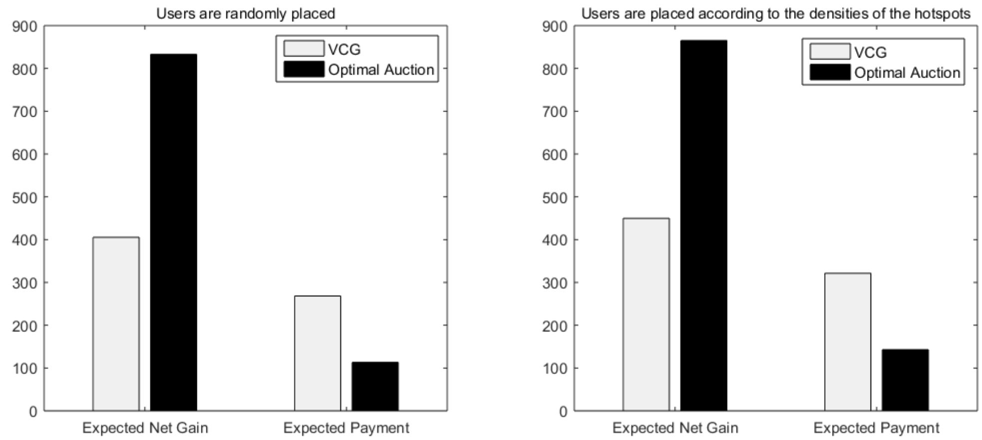

Figure 2: Performance Comparison of the Procurement Mechanisms for the Service Provider

as one of the regional population according to the WiFi region; otherwise, the user is considered as in the region with no hotspots (region 0). We run 1,000 simulations to generate traffic demands in each WiFi region.

- Generate cell tower capacity: The cell tower capacity is set to three times 3.84 MHz (Dong et al. 2014). Data spectral efficiency varies across towers from 0.5 to 2 bps/Hz.[6] We set spectral efficiency to be 1 by default and then vary the spectral efficiency to evaluate its impact. Note that when the user demand for mobile data is below 80% of the cell tower capacity, the cellular service provider faces no congestion cost.

We conduct a variety of simulations to compute the corresponding allocation under the VCG mechanism and under our optimal mechanism. The relative cost of deploying cellular resources as compared with WiFi resources affects the bandwidth allocation result. Joseph et al. (2004) assumed that the relative cost of deploying cellular resources as compared with WiFi resources is 4:1. We follow their assumptions and set the parameter values: $\omega_m\left(\mu_m + y_m\right) = 0.5a\left(\frac{1}{\mu_m + y_m - \lambda_m}\right)$ where a is set to 4 and $C\left(Q, \theta_i\right) = \left(0.5 + \theta_i\right) Q^2$. In the simulation, we vary the relative cost parameter a and find that the results are robust. A hotspot's private cost parameters θ_i is drawn from a uniform distribution $U[0,1]$ for 1,000

[6]See http://www.rysavy.com/Articles/2011_05_Rysavy_Efficient_Use_Spectrum.pdf.

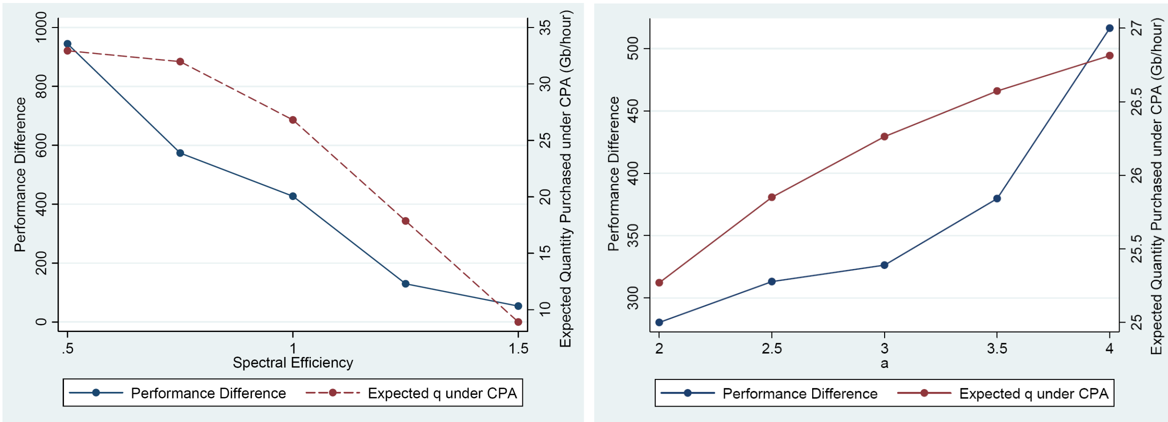

Figure 3: Performance Difference and Cell Tower Capacity (Left); Performance Difference and Relative Cost of Deploying Cellular Resources (Right)

times.

The simulation result of the performance comparison is shown in Figure 2. In the left panel, the users are randomly placed in the cell sector. In the right panel, the users are placed according to the densities of the hotspots. The two panels show similar results: our proposed procurement auction significantly outperforms the VCG mechanism in terms of the expected net gain of the cellular service provider (the expected net gain = the reduction of congestion cost - the payment to hotspots).

Data spectral efficiency varies across cell towers using different wireless technologies. An increase in spectral efficiency significantly contributes to tower capacity. The left panel of Figure 3 evaluates the impact of spectral efficiency (cell tower capacity) on the performance difference, which is defined as the difference between the service provider's expected net gain under the proposed mechanism and the gain under the VCG mechanism.[7] Note that the unit of the performance difference is normalized, and we are only interested in the trend. As the cellular capacity increases, the advantage of our proposed mechanism, in comparison with the VCG mechanism, decreases. This is because the bandwidth purchased from the WiFi hotspots also decreases as cellular capacity increases, as is indicated by the dashed line

[7] The simulation results are similar when the users are randomly placed or are placed according to the densities of the hotspots, so here we only present the result when the users are randomly placed.

in the left panel of Figure 3. The service provider is less willing to purchase WiFi resources when it owns a relatively large cellular capacity, and the overpayment problem in the VCG mechanism is thus less severe.

The right panel of Figure 4 shows that as a increases, the advantage of our mechanism as compared with the VCG mechanism increases, which is expected because, with congestion being more costly, the service provider is more willing to procure from the WiFi hotspots, thereby exacerbating the overpayment problem in the VCG mechanism.

C Cellular Technology and Broadband Technology

We elaborate the differences between broadband technology and cellular technology from the two perspectives.

(1) Broadband Technology

With the rapid deployment of fiber optics, broadband capacity constraints are gradually becoming less of an issue, even as the consumption of online content continues to grow at a rapid pace. More specifically, our assumption was guided by the fact that over the past years, broadband providers have increased capacity, and thanks to rapid advances in fiber technology (whose rate of growth is even faster than Moore's Law in semiconductors; at the same time, the networking equipment have been getting cheaper by around 25-40% every year following Moore's Law), broadband providers have been able to increase capacity at a very low cost, even as consumers have increased their consumption for online content. The cost of provisioning the marginal customer at large broadband providers today is less than \$1/month: about half of that cost is till the point of peering (the "backhaul" cost, in industry terminology), and the other half is incurred while carrying the data from the point of peering to the local exchange. Thus, broadband capacity has not been a bottleneck even as consumption for data has increased. Choi et al. (2014) highlighted the difference between fixed and mobile networks: Mobile networks encounter technical and physical constraints in expanding capacity due to the limited availability of spectrum. The Federal Communications Commission (FCC) also stated the difference: "Mobile broadband is an earlier-stage platform than fixed broadband, ..., Mobile broadband speeds, capacity, and penetration are typically much lower than for fixed broadband. ... In addition, existing mobile networks present operational constraints that fixed broadband networks do not typically encounter." (FCC Order, page 94-95).

An independent verification that broadband capacity is not a bottleneck comes from empirical observations. FCC comes out with an annual state of the broadband report every year (called "Measuring Broadband America"), and the latest report that is available currently is for the year 2014. In this report, one of the performance metrics that the Commission measures is the "24 Hour versus Peak Performance Variation by Technology." The data shows that there is hardly any dip in performance during peak periods (for example, for fiber, during peak periods, the performance drops from an average 115% of advertised speeds to 112% of the advertised speed during peak hours; for cable, the drop is from 105% of advertised speeds to 101%; and for DSL, the drop is from 95% to 91%), and these numbers have arguably become better since (currently, the FCC has the raw data available for the 2015 report on its website). Therefore, broadband providers are gradually becoming more able to handle their peak load without any degradation in speed of delivery.

(2) Cellular Technology

The cellular capacity is determined by amount of spectrum, number of cell towers, and spectral efficiency of technology, as is illustrated in the following figure which is from Rysavy-Research (2014).

Spectrum is a limited and finite resource for mobile networks (Rysavy-Research 2014). In the U.S., cellular systems use roughly 500 MHz, although an individual operator's access to spectrum is much smaller and is subject to spectrum aggregation rules. On the other hand, wired network can access far more frequencies in the mediums (e.g., coax cable, fiber-optic cable, etc.) they use, and they can carry their spectrum within the physical medium with near-complete control. Once the capacity of one cable is exhausted, another one can be placed alongside. This is in stark contrast with wireless networks which rely on the propagation of signals through the air and the same frequencies cannot be used without interference until some distance away.

Because of the finiteness of the spectrum resource, obtaining radio spectrum is very costly: In the United States, the Federal Communications Commission (FCC) conducts competitive auctions of licenses for electromagnetic spectrum. Since July 1994, the FCC has conducted 87 spectrum auctions, which raised over $60 billion for the U.S. Treasury (Cramton et al. 2002). Therefore, additional radio spectrum is not always available and obtaining it from spectrum auctions is expensive. Additionally, due to antitrust concerns in the wireless industry, several influential economists suggested that FCC should place limits on how much spectrum AT&T and Verizon are allowed to buy (Cramton et al. 2007). Such concern is reflected in the action taken by the FCC to block the merger between AT&T and T-Mobile in 2011. Due to these regulatory constraints, it is very difficult for cellular service providers to acquire additional spectrum resources.

Given limited spectrum, the cellular industry has been using sophisticated modulation and encoding methods to extract as much capacity as possible from available spectrum to meet the growing demand from mobile users. However, today's networks already operate at close to maximum theoretical spectral efficiency constrained by the laws of physics. It is also far more challenging to increase efficiency in radio technology than to increase efficiency in wire or fiber cables because radio connections in open environment have more noise than shielded wires.

Although building more cell towers can also increase wireless capacity, building a new cell tower is very expensive and time consuming. In the United States, the number of cell towers increased from 12,824 in 1993 to 304,360 in 2013. However, the increased number of cell towers has not allowed capacity to come even close to matching the capacity of wired network. Some industry expert estimates that it will cost at least $150,000 to construct a tower.

Moreover, there are health concerns on the radiation from cell towers. Many governmental bodies require that cellular service providers share cell towers so as to decrease environmental and cosmetic impact. This issue is an influential factor of rejection of installation of new cell towers in communities. For example, in February 2009, the French telecom company Bouygues Telecom was ordered to take down a cell tower due to uncertainty about its effect on health. Residents in the commune Charbonnieres in the Rhone department had sued the company claiming adverse health effects from the radiation emitted by the 19-meter-tall cell tower.

Because of the unique characteristics of the information and communications technology (ICT) industries, broadband capacity has not been a constraining factor in the past several years. In contrast, cellular capacity is limited by the finite amount of radio spectrum and the inherent limitations of radio as a medium. Even with a breakthrough in cellular technologies in the near future, cellular capacity will still be limited by various regulatory constraints, which is less of an issue for broadband because it does not rely on radio spectrum and cell towers.

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