Online Supplement for “Reward-based crowdfunding campaigns: informational value and access to venture capital”

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Appendix A

Derivation of the expected payoffs when the VC funds the project:

If $zR(n, x_i) - K \geq 0$, the VC funds the project and we assume that the entrepreneur and VC split the nonnegative future expected value of the project according to their relative bargaining positions δ and $1 - \delta$, respectively. Specifically, the entrepreneur can expect to receive a portion δ and the VC can expect to receive a portion $(1 - \delta)$ of the future expected value $[zR(n, x_i) - K]$. Here we show that contingent upon the entrepreneur's initial contribution to the development cost, her claim to future revenue $R(n, x_i)$ can be adjusted so that her expected payoff from the value of the project always reflects her bargaining position δ and, therefore, is equal to $\delta\,[zR(n, x_i) - K]$.

If the campaign is successful, the entrepreneur receives nr from backers. We first analyze the case when the entrepreneur contributes nr to cover part of the development cost. The VC contributes the remaining $K - nr$. Note that the entrepreneur at this point may have spent part of the campaign funds applying for patents or developing prototypes of the product, and such expenditure can be counted as part of her contribution to the total development cost. Let λ and $1 - \lambda$ denote the shares of the total revenue from selling to the mass market that will accrue to the entrepreneur and VC, respectively, if the product is successfully commercialized. With probability z, the entrepreneur succeeds in developing the full product and receives $\lambda R(n, x_i)$. With probability $1 - z$, the project fails in spite of being fully funded, in which case the entrepreneur receives nothing but has the obligation to provide each backer with a reward valued at βv_H. For the entrepreneur and VC to split the future expected value $[zR(n, x_i) - K]$ according to their relative bargaining positions δ and $1 - \delta$, respectively, we must have $\lambda = \delta - (\delta K - nr)/(zR(n, x_i))$, so that the entrepreneur's overall expected payoff when the VC funds the project following a successful campaign is

$$\lambda z R(n, x_i) - (1 - z)n\beta v_H + nr - nr = \delta\,[zR(n, x_i) - K] - (1 - z)\,\beta v_H n + nr. \tag{12}$$

Correspondingly, the VC's expected payoff is given by $(1 - \lambda)zR(n, x_i) - (K - nr) = (1 - \delta)\,[zR(n, x_i) - K]$.

The entrepreneur's expected payoff (12) suggests that in expectation, the entrepreneur receives a portion of the expected revenue $zR(n, x_i)$ and incurs the same portion of the total development cost

K, where the portion is determined by her bargaining power δ. Note that while the development costs have to be incurred with certainty in order for the project to proceed, the availability of revenues from the sale of the product is uncertain and realizes only with probability z. This is the reason that the probability measure z multiplies only the revenue $R(n, x_i)$ but not the cost K.

The expected payoff (12) would remain the same irrespective of the amount the entrepreneur contributes upfront for the project. To illustrate, if the entrepreneur contributed an amount s, $0 \leq s \leq nr$, to cover part of the development cost, the VC would contribute $K - s$. We can modify the share $\lambda = \delta - (\delta K - s)/(zR(n, x_i))$, so that λ is adjusted above (if $\delta K - s < 0$) or below (if $\delta K - s > 0$) the bargaining power δ to ensure that the players split the future expected value according to their relative bargaining powers, δ and $1 - \delta$.

In case that the campaign fails, the entrepreneur receives no contributions from backers. Consequently, she has no obligation to backers in case the project fails. If $zR(n, x_i) - K \geq 0$, the VC approves funding and contributes the entire development cost K. We can have $\lambda = \delta - (\delta K)/(zR(n, x_i))$, so that the entrepreneur's and VC's expected payoffs are $\lambda z R(n, x_i) = \delta \left[zR(n, x_i) - K\right]$ and $(1 - \lambda) zR(n, x_i) - K = (1 - \delta) \left[zR(n, x_i) - K\right]$, respectively, when the VC funds the project following a failed campaign.

To summarize, by carefully choosing the share of revenues that accrues to the entrepreneur, the entrepreneur and VC split the future expected value $[zR(n, x_i) - K]$ according to their relative bargaining power δ and $1 - \delta$, irrespective of the amount the entrepreneur contributes towards covering the development cost.

Proof of Lemma 1: We only provide the proof that $N^*_{\min} \geq N_H$ when $N_H \geq 0$ under Case 1. Proof of the other cases is either similar or trivial.

If $N_{\min} < N_H$, the fan's preference in favor of contributing in the campaign is valid if the following inequality holds:

$$\begin{aligned}
& -\left(r-\beta v_H\right)\left[p\frac{N_L-N_{\min}}{\overline{N}}+(1-p)\frac{N_H-N_{\min}}{\overline{N}}\right] \\
& +(zv_H+(1-z)\,\beta v_H-r)\left[p\frac{\overline{N}-N_L}{\overline{N}}+(1-p)\frac{\overline{N}-N_H}{\overline{N}}\right] \\
\geq\ & z(\gamma v_H-v_L)\left[p\frac{\overline{N}-N_L}{\overline{N}}+(1-p)\frac{\overline{N}-N_H}{\overline{N}}\right].
\end{aligned}$$

After simplification we obtain:

$$N_{\min}\geq\overline{N}-\frac{\left(\overline{N}-(1-p)\,N_H-pN_L\right)z\left[(1-\gamma-\beta)\,v_H+v_L\right]}{r-\beta v_H}. \tag{13}$$

The entrepreneur's problem, with $N_{\min}<N_H$, can be written as

$$\begin{aligned}
Max_{\{N_{\min},r\}}\ \pi_E^C\ =\ & \frac{1}{\overline{N}}\int_{N_{\min}}^{\overline{N}}nrdn-\frac{(1-p)}{\overline{N}}\int_{N_{\min}}^{N_H}\beta v_Hndn-\frac{p}{\overline{N}}\int_{N_{\min}}^{N_L}\beta v_Hndn \\
& +\frac{(1-p)}{\overline{N}}\int_{N_H}^{\overline{N}}\left(\delta[z\alpha v_L(hn+(1-h)x_H)-K]-(1-z)\,\beta v_Hn\right)dn \\
& +\frac{p}{\overline{N}}\int_{N_L}^{\overline{N}}\left(\delta[z\alpha v_L(hn+(1-h)x_H)-K]-(1-z)\,\beta v_Hn\right)dn \\
s.t.\quad 0\ \leq\ & N_{\min}\leq N_H, \\
N_{\min}\ \geq\ & \overline{N}-\frac{\left(\overline{N}-(1-p)\,N_H-pN_L\right)z\left[(1-\gamma-\beta)\,v_H+v_L\right]}{r-\beta v_H}.
\end{aligned}$$

The last two terms in the profit function π_E^C do not depend on decision variables. For the first three terms, the partial derivative with respect to $N_{\min}$ is negative, so the constraint (13) is binding with $N_{\min}=\overline{N}-\frac{\left(\overline{N}-(1-p)N_H-pN_L\right)z[(1-\gamma-\beta)v_H+v_L]}{r-\beta v_H}$. We rewrite this expression as $r=\beta v_H+\frac{\left(\overline{N}-(1-p)N_H-pN_L\right)z[(1-\gamma-\beta)v_H+v_L]}{\left(\overline{N}-N_{\min}\right)}$ and substitute it into the first term to obtain, after simplification,

$$\begin{aligned}
& \frac{1}{\overline{N}}\int_{N_{\min}}^{\overline{N}}nrdn-\frac{(1-p)}{\overline{N}}\int_{N_{\min}}^{N_H}\beta v_Hndn-\frac{p}{\overline{N}}\int_{N_{\min}}^{N_L}\beta v_Hndn \\
=\ & \frac{\overline{N}^2-(1-p)\,N_H^2-pN_L^2}{2\overline{N}}\beta v_H+\frac{\left(\overline{N}-(1-p)\,N_H-pN_L\right)z\left[(1-\gamma-\beta)\,v_H+v_L\right]}{2\overline{N}}\left(\overline{N}+N_{\min}\right),
\end{aligned}$$

which is increasing in $N_{\min}$. Thus $N_{\min}^*\geq N_H$. ■

Proof of Proposition 1: (i) When observing x_L is not necessarily fatal for the project (Case 1), by (P1) the entrepreneur's profit π_E^C decreases with $N_{\min}$. We first assume $N_{\min}$ achieves the lower bound in constraint (6), and then check if the corresponding solution satisfies $\max(N_H, 0) \leq N_{\min} \leq N_L$. Evaluating (6) as equality yields:

$$
\begin{aligned}
r\left[\left(\overline{N} - N_{\min}\right) - \beta p\left(N_L - N_{\min}\right)\right] &= \left[z(1-\gamma)v_H + zv_L\right]\left[(1-p)\left(\overline{N} - N_{\min}\right) + p\left(\overline{N} - N_L\right)\right] \\
r &= \beta v_H + \frac{z\left[(1-\gamma-\beta)v_H + v_L\right]\left[(1-p)\left(\overline{N} - N_{\min}\right) + p\left(\overline{N} - N_L\right)\right]}{\overline{N} - N_{\min}}
\end{aligned}
$$

Substituting it into the profit function, taking the derivative with respect to $N_{\min}$ and then setting it to zero yields the interior solution

$$
N_{\min}^{int} = \frac{\frac{p}{2(1-p)}\left[(1-\gamma-\beta)v_H + v_L\right]\left(\overline{N} - N_L\right) + \delta h \alpha v_L N_H (1-\xi)}{(1-\gamma)v_H + v_L + (1-\xi)\,\delta h \alpha v_L}. \tag{14}
$$

For this solution to satisfy the constraint $\max(N_H, 0) \leq N_{\min}^{int} \leq N_L$, we need:

$$
\begin{aligned}
N_{\min}^{int} &\geq \max(N_H, 0) \Rightarrow p \geq p_L = \begin{cases} \frac{2N_H[(1-\gamma)v_H + v_L]}{\left(2N_H + \overline{N} - N_L\right)[(1-\gamma)v_H + v_L] - \beta v_H\left(\overline{N} - N_L\right)} & \text{if } N_H \geq 0 \\ \frac{\delta(1-\xi)[z\alpha v_L(1-h)x_H - K]}{\frac{1}{2}z[(1-\gamma-\beta)v_H + v_L]\left(\overline{N} - N_L\right) + \delta(1-\xi)[z\alpha v_L(1-h)x_H - K]} & \text{if } N_H < 0 \end{cases}, \\
N_{\min}^{int} &\leq N_L \Rightarrow p \leq p_U = \frac{\left((1-\gamma)v_H + v_L\right)N_L + (1-\xi)\,\delta h \alpha v_L\left(N_L - N_H\right)}{\left((1-\gamma)v_H + v_L\right)\frac{\left(\overline{N} + N_L\right)}{2} + (1-\xi)\,\delta h \alpha v_L\left(N_L - N_H\right) - \beta v_H\left(\frac{\overline{N} - N_L}{2}\right)}.
\end{aligned}
$$

It is easy to verify $\overline{N} = N_L \Rightarrow p_L = p_U = 1$ and $0 \leq p_L < p_U \leq 1$ when $N_L \leq \overline{N}$. Therefore, $\max(N_H, 0) \leq N_{\min}^{int} \leq N_L \leq \overline{N} \Leftrightarrow p_L \leq p \leq p_U$ so that

$$
N_{\min}^{*} = \begin{cases} \max(N_H, 0) & \text{if } 0 \leq p < p_L \\ N_{\min}^{int} & \text{if } p_L < p < p_U \\ N_L & \text{if } p_U < p \leq 1 \end{cases}.
$$

Because the constraint (6) is binding at the optimum, we have

$$
r^{*} = \beta v_H + z\left[(1-\gamma-\beta)v_H + v_L\right]\left[1 - p + p\frac{\overline{N} - N_L}{\overline{N} - N_{\min}^{*}}\right].
$$

The highest pledge $r^{*} = \beta v_H + z\left[(1-\gamma-\beta)v_H + v_L\right]$ is reached when $p_U < p \leq 1$. The optimal goal is $G^{*} = r^{*}N_{\min}^{*}$. It is straightforward to derive the probability of launching the project.

(ii) If observing x_L kills the project (Case 2), then $N_L \geq \overline{N}$. By (P2), π_E^C decreases with $N_{\min}$ and increases with r. Thus, the optimal solution is reached at boundary with $N_{\min}^{*} = \max(N_H, 0)$, $r^{*} = \beta v_H + (1-p)\left[z(1-\gamma-\beta)v_H + zv_L\right]$, $G^{*} = r^{*}N_{\min}^{*}$. The VC will fund the project only when x_H realizes and $n \geq N_{\min}^{*}$ so that the probability of launching the project is $(1-p)\left(1 - \frac{\max(N_H,0)}{\overline{N}}\right)$.

■

Proof of Lemma 2: (i) These results can be directly derived from Equations (3) and (4).

(ii) For the interior solution, taking the first order derivative with respect to h yields:

$$\frac{\partial N_{\min}^{int}}{\partial h} = \frac{\frac{1}{2}p\left[(1-\gamma-\beta)v_H + v_L\right](N_L - x_L)}{h(1-p)\left[(1-\gamma)v_H + v_L + (1-\xi)\,\delta\alpha v_L h\right]} + \frac{\delta\alpha v_L (1-\xi)\left(x_H - N_{\min}^{int}\right)}{(1-\gamma)v_H + v_L + (1-\xi)\,\delta\alpha v_L h}. \tag{15}$$

Setting it equal to zero yields a quadratic equation in h, which has at most one positive root

$$h^{int} = \frac{p\left[(1-\gamma)v_H + v_L\right]\sqrt{K - z\alpha v_L x_L}}{-(1-\xi)\,\delta\alpha v_L p\sqrt{K - z\alpha v_L x_L} + \alpha v_L\sqrt{\delta(1-\xi)\left[(1-\gamma)v_H + v_L\right]pH}},$$

where $H = zp(\overline{N} - x_L) + \frac{p\delta(1-\xi)(K - z\alpha v_L x_L)}{[(1-\gamma)v_H + v_L]} - 2(1-p)zx_H\frac{[(1-\gamma)v_H + v_L]}{[(1-\gamma-\beta)v_H + v_L]} + \frac{2\delta(1-p)(1-\xi)(K - z\alpha v_L x_H)}{[(1-\gamma-\beta)v_H + v_L]} \geq 0$. If $H < 0$, $N_{\min}^{int}$ increases with h always. We have verified that the second order derivative at h^{int} is negative and thus, h^{int} is a maximizer. Therefore, $N_{\min}^{int}$ either increases or first increases then decreases with h for $h > 0$. ■

Proof of Proposition 2: (i) When $0 \leq h \leq h_2$, we are in Case 2 where $N_{\min}^{*} = \max(N_H, 0)$ so that the optimal target number is non-decreasing with h. We now consider Case 1 where $h_2 < h \leq 1$.

(i.1) We first show that $N_{\min}^{int}$ and N_L cross at most once on $[h_2, 1]$ and $N_L > N_{\min}^{int}$ after they cross.

When $h > h_2$, we are in Case 1 where p_U strictly decreases with h because $N_L < \overline{N}$, N_L strictly decreases with h, and N_H strictly increases with h. As h increases from h_2 to 1, p_U decreases from 1 to $\frac{2K((1-\gamma)v_H + v_L)}{\overline{N}z\alpha v_L((1-\gamma-\beta)v_H + v_L) + K((1-\gamma+\beta)v_H + v_L)} < 1$.

Given any $p < 1$, if $p < \frac{2K((1-\gamma)v_H + v_L)}{\overline{N}z\alpha v_L((1-\gamma-\beta)v_H + v_L) + K((1-\gamma+\beta)v_H + v_L)}$, then the boundary solution N_L never arises. Otherwise, if $p \geq \frac{2K((1-\gamma)v_H + v_L)}{\overline{N}z\alpha v_L((1-\gamma-\beta)v_H + v_L) + K((1-\gamma+\beta)v_H + v_L)}$, then there exists a unique $h_U(p) \in [h_2, 1]$, such that $N_{\min}^{int} = N_L$ at $h = h_U(p)$ and the boundary solution N_L is valid on $h_U(p) \leq h \leq 1$. Therefore, $N_{\min}^{int}$ and N_L cross at most once and $N_{\min}^{*}$ strictly decreases with h after they cross.

(i.2) We next show that $N_{\min}^{int}$ and N_H cross at most once, and $N_{\min}^{int}$ is increasing with h when it crosses N_H. If $\left(\overline{N} - x_L\right)\left[(1-\gamma-\beta)v_H + v_L\right] - 2(1-p)\left[(1-\gamma)v_H + v_L\right]x_H/p \neq 0$,

$$N_{\min}^{int} = N_H \Leftrightarrow h = h_L = \frac{(K - z\alpha v_L x_L)\frac{[(1-\gamma-\beta)v_H + v_L]}{[(1-\gamma)v_H + v_L]} - 2(1-p)(z\alpha v_L x_H - K)/p}{z\alpha v_L\left[\left(\overline{N} - x_L\right)\frac{[(1-\gamma-\beta)v_H + v_L]}{[(1-\gamma)v_H + v_L]} - 2(1-p)\,x_H/p\right]}. \tag{16}$$

From (15), $\frac{\partial N_{\min}^{int}}{\partial h}|_{h=h_L} > 0$ because $N_L - x_L > 0$ and $x_H - N_H > 0$. If $\left(\overline{N} - x_L\right)\left[(1-\gamma-\beta)v_H + v_L\right] - 2(1-p)\left[(1-\gamma)v_H + v_L\right]x_H/p = 0$, then either N_H is dominated ($N_{\min}^{int} \geq N_H$ always) or $N_{\min}^{int}$ is dominated ($N_{\min}^{int} < N_H$ always). Therefore, $N_{\min}^{int}$ crosses N_H at most once and $N_{\min}^{int}$ is increasing with h when it crosses N_H.

Combining Lemma 2 and (i.1)-(i.2), we know that $N_{\min}^{*}$, if it ever strictly decreases with h, starts decreasing either at h^{int} or when $N_{\min}^{int}$ and N_L cross. In the former case, $N_{\min}^{*} = N_{\min}^{int} > \max(N_H, 0)$ at $h = h^{int}$. From Lemma 2, $N_{\min}^{int}$ decreases on $h > h^{int}$. From (i.2) $N_{\min}^{int}$ will not cross N_H on $h > h^{int}$. This implies that $N_{\min}^{*} = \min\{N_{\min}^{int}, N_L\}$ on $[h^{int}, 1]$ so that it strictly decreases with h on $[h^{int}, 1]$. In the latter case, from (i.1) we know $N_{\min}^{*} = N_L$ after $N_{\min}^{int}$ and N_L cross and therefore $N_{\min}^{*}$ will decrease afterwards.

Summarizing Case 2 and Case 1, the optimal target number is either a non-decreasing function of h over the entire interval $[0, 1]$, or is a single peak function of h. ■

Proof of Corollary 1: The result follows from Lemma 2, Proposition 1, and Proposition 2. ■

Proof of Proposition 3: For the interior solution, taking the first order derivative yields:

$$\begin{aligned}\frac{\partial N_{\min}^{int}}{\partial K} &= \frac{-\frac{p}{2(1-p)}\left[(1-\gamma-\beta)v_H + v_L\right] + \delta h \alpha v_L (1-\xi)}{hz\alpha v_L\left[(1-\gamma)v_H + v_L + (1-\xi)\,\delta h \alpha v_L\right]} < 0 \\ &\Leftrightarrow\ h < \frac{p}{2(1-\xi)(1-p)\delta\alpha v_L}\left[(1-\gamma-\beta)v_H + v_L\right].\end{aligned}$$

We observe that this inequality can hold for sufficiently small values of h.

Differentiating $N_{\min}^{int}$ with respect to p $(\overline{N})$, we verify that $N_{\min}^{int}$ increases with p and $\overline{N}$. To show $\frac{\partial N_{\min}^{int}}{\partial \delta} \leq 0$ we need $N_H(1-p) \leq \frac{1}{2}\frac{[(1-\gamma-\beta)v_H+v_L]}{[(1-\gamma)v_H+v_L]}p\left(\overline{N} - N_L\right)$,which holds when $N_H < 0$. When $N_H > 0$, this condition reduces to $\frac{2N_H[(1-\gamma)v_H+v_L]}{\left(2N_H+\overline{N}-N_L\right)[(1-\gamma)v_H+v_L]-\beta v_H\left(\overline{N}-N_L\right)} \leq p$. This is equivalent to the condition $p_L \leq p$ when $N_H > 0$. Therefore, $N_{\min}^{int}$ decreases with δ over the region that supports the interior solution. It is straightforward to see that $N_{\min}^{int}$ decreases with β.

$\frac{\partial N_{\min}^{int}}{\partial \gamma} = [\delta z \alpha v_L h]\,\frac{-\frac{1}{2}p\left(\overline{N}-N_L\right)((1-\xi)\delta h\alpha v_L+\beta v_H)+(1-p)\delta h \alpha v_L(1-\xi)N_H}{(1-p)z[w+v_L+\delta\alpha v_L h]^2} \leq 0$ always holds when $N_H \leq 0$. When $N_H > 0$, this inequality is equivalent to $p \geq p_L$, which holds when interior solution $N_{\min}^{int}$ arises as optimal. $\frac{\partial N_{\min}^{int}}{\partial v_H}$ might be positive or negative depending on whether

$$\begin{aligned} p &\geq \frac{2N_H(1-\gamma)(1-\xi)\,\delta\alpha h}{\left(2N_H + \overline{N} - N_L\right)(1-\gamma)(1-\xi)\,\delta\alpha h - \beta\left(1+\delta\alpha h\,(1-\xi)\right)\left(\overline{N} - N_L\right)} \\ \Leftrightarrow\ \beta &\leq \frac{p\left(2N_H + \overline{N} - N_L\right)(1-\gamma)(1-\xi)\,\delta\alpha h - 2N_H(1-\gamma)(1-\xi)\,\delta\alpha h}{p\left(1+\delta\alpha h\,(1-\xi)\right)\left(\overline{N} - N_L\right)}\end{aligned}$$

Therefore, $N_{\min}^{int}$ increases with v_H if β is sufficiently small.

When taking derivatives with respect to x_H and x_L, respectively, we obtain that $N_{\min}^{int}$ increases with x_L and decreases with x_H. Therefore, $N_{\min}^{int}$ decreases with the spread $x_H - x_L$ while keeping $\frac{x_H+x_L}{2}$ fixed. ■

The Model of No Crowdfunding

Let δ and $1-\delta$ be the shares of the expected total surplus of the new venture that accrue to each party. The expected profits of the entrepreneur and VC depend on the realization of the external signal.

i) if $X = x_H$, the entrepreneur's expected profit under no crowdfunding is

$$\pi_E^{NC}\big|_{X=x_H} = \frac{1}{\overline{N}}\int_0^{\overline{N}} \delta[z\alpha v_L(hn+(1-h)x_H)+zv_Ln-K]dn = \delta[z\alpha v_L(h\frac{\overline{N}}{2}+(1-h)x_H)+zv_L\frac{\overline{N}}{2}-K].$$

The superscript "NC" designates for the no crowdfunding option. Note that $\pi_E^{NC}\big|_{X=x_H} \geq 0$ when $\overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H$.

ii) If $X = x_L$, we have

$$\pi_E^{NC}\big|_{X=x_L} = \frac{1}{\overline{N}}\int_0^{\overline{N}} \delta[z\alpha v_L(hn+(1-h)x_L)+zv_Ln-K]dn = \delta[z\alpha v_L(h\frac{\overline{N}}{2}+(1-h)x_L)+zv_L\frac{\overline{N}}{2}-K].$$

Hence, $\pi_E^{NC}\big|_{X=x_L} \geq 0$ when $\overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$.

Therefore, three possible cases may arise:

a) if $\overline{N} > \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$, the VC always funds the project under no crowdfunding, which is unrealistic.

b) if $\left(\frac{2\alpha h}{\alpha h+1}\right) N_H \leq \overline{N} \leq \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$, with probability $1-p$, the good signal x_H realizes and the VC will fund the project. With probability p, the bad signal x_L realizes, and the VC will not fund the project. The total expected profit is:

$$\pi_E^{NC} + \pi_{VC}^{NC} = (1-p)\,[z\alpha v_L(h\frac{\overline{N}}{2} + (1-h)x_H) + zv_L\frac{\overline{N}}{2} - K], \tag{17}$$

which is split between the entrepreneur and VC according to their bargaining power δ and $1-\delta$, respectively.

c) if $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H$, the VC will never fund the project, and the expected profits of the entrepreneur and the VC are zero. ■

Proof of Lemma 3 and Proposition 4: We first prove Proposition 4 because its proof will be used to establish some results in Lemma 3.

When $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H$, the comparison is trivial because the profit is 0 under no crowdfunding.

When $\left(\frac{2\alpha h}{\alpha h+1}\right) N_H \leq \overline{N} \leq \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$, from (17) the entrepreneur's profit under no crowdfunding is

$$\pi_E^{NC} = \delta z \alpha v_L h(1-p)\left[\frac{\overline{N}}{2}\left(\frac{\alpha h+1}{\alpha h}\right) - N_H\right]. \tag{18}$$

The profit comparison depends on whether Case 2 or Case 1 arises under crowdfunding.

Case 2 arises under crowdfunding: In this case $N_L > \overline{N}$ and from Proposition 1 $N^*_{\min} = \max(N_H, 0)$. $N_L > \overline{N}$ is compatible with $\left(\frac{\alpha h}{\alpha h+1}\right) N_H \leq \frac{\overline{N}}{2} \leq \left(\frac{\alpha h}{\alpha h+1}\right) N_L$ if $N_L \geq 2\left(\frac{\alpha h}{\alpha h+1}\right) N_H$. From Problem (P2), the entrepreneur's profit under crowdfunding is

$$\pi_E^C = (1-p)\left[\frac{z\left[v_H(1-\gamma)+v_L\right]}{2\overline{N}}(\overline{N}^2 - (N^*_{\min})^2) + \frac{\delta z \alpha v_L h}{2\overline{N}}(\overline{N} - N^*_{\min})(\overline{N} + N^*_{\min} - 2N_H)\right], \tag{19}$$

so that $\pi_E^C > \pi_E^{NC}$ is equivalent to

$$\left[\frac{[v_H(1-\gamma)+v_L]}{2\delta\alpha v_L h}\right](\overline{N}^2 - (N^*_{\min})^2) > \frac{\overline{N}^2}{2}\left(\frac{1}{\alpha h}\right) - N^*_{\min}N_H + \frac{(N^*_{\min})^2}{2}.$$

Entrepreneur's profit comparison ($\delta + \gamma < 2$):

By (11), when $h < h_1$, we have $N_H < 0$, in which case $N^*_{\min} = 0$ and the above inequality can be simplified to

$$v_H(1-\gamma) + v_L(1-\delta) > 0, \tag{20}$$

which always holds for $\delta + \gamma < 2$.

When $h \geq h_1$, we have $N^*_{\min} = N_H \geq 0$. From (18) and (19), $\pi_E^C > \pi_E^{NC}$ if:

$$\left[v_H(1-\gamma) + (1-\delta)v_L\right]\overline{N}^2 > \left[v_H(1-\gamma) + v_L - \delta\alpha v_L h\right](N_H)^2 \tag{21}$$

which always holds if $v_H(1-\gamma) + v_L - \delta\alpha v_L h < 0$ because $v_H(1-\gamma) + (1-\delta)v_L > 0$ for $\delta + \gamma < 2$. When $v_H(1-\gamma) + v_L - \delta\alpha v_L h \geq 0$, (21) reduces to:

$$\overline{N} > N_H\sqrt{\frac{v_H(1-\gamma) + v_L - \delta\alpha v_L h}{v_H(1-\gamma) + v_L - \delta v_L}}, \tag{22}$$

which holds when $h > 1/\alpha$. So the only region remains to be considered is $h \le 1/\alpha$ and $h \ge h_1$. Let

$$f(h) = \overline{N}\sqrt{v_H(1-\gamma) + v_L - \delta v_L} - N_H\sqrt{v_H(1-\gamma) + v_L - \delta\alpha v_L h}. \tag{23}$$

Then $f(h) \ge 0 \Leftrightarrow \pi_E^C \ge \pi_E^{NC}$. We verified that $f(h = 1/\alpha) > 0$ because $\overline{N} > N_H$, and $f(h = h_1) > 0$. Moreover, $f'(h) = 0$ is a quadratic with exactly one positive root

$$h_0 = \frac{-\left(\frac{z\alpha v_L x_H - K}{z x_H}\delta\right) + \sqrt{\left(\frac{z\alpha v_L x_H - K}{z x_H}\delta\right)^2 + 8\delta\left(\frac{z\alpha v_L x_H - K}{z x_H}\right)[v_H(1-\gamma) + v_L]}}{2\delta\alpha v_L} > 0.$$

The other root is negative, and therefore, is discarded. Taking the second order derivative with respect to h and evaluating it at h_0 yield

$$f''(h_0) = \left(2\frac{z\alpha v_L x_H - K}{h^3 z\alpha v_L}\right)[v_H(1-\gamma) + v_L] - \frac{z\alpha v_L x_H - K}{h^2 z}\frac{\delta}{2} > 0,$$

because we consider the case that $v_H(1-\gamma) + v_L - \delta\alpha v_L h \ge 0$. Therefore, $f(h)$ reaches the local minimum at $h_0 > 0$, with $f'(h) < 0$ on $0 < h < h_0$, and $f'(h) > 0$ on $h > h_0$. It is straightforward to verify $h_0 > h_1$. Because we only need to consider $h_1 \le h \le 1/\alpha$, there are two possible cases.

Case a) If $h_1 < 1/\alpha < h_0$, then $f(h) > 0$ on $[h_1, 1/\alpha]$ because $f(h)$ is decreasing over the region $[h_1, 1/\alpha]$ with both $f(h_1) > 0$ and $f(1/\alpha) > 0$. Then crowdfunding is always preferred.

Case b) If $h_1 < h_0 < 1/\alpha$, $f(h)$ is either positive on $h > 0$ or crosses 0 exactly twice on $[h_1, 1/\alpha]$.

Case b.1) $f(h) > 0$ on $h > 0$, then crowdfunding is always preferred.

Case b.2) $f(h)$ crosses 0 exactly twice. Let h_a and h_b be the two values of h such that $f(h_a) = f(h_b) = 0$. Then we must have $h_1 < h_a < h_0 < h_b < 1/\alpha$, such that $f(h) > 0$ except over the region $[h_a,\ h_b]$. A necessary condition for this case is $h_0 < 1/\alpha$, which can be simplified to $K > K_1$, where $K_1 = z v_L x_H\left[\alpha - \frac{\delta v_L}{2v_H(1-\gamma)+2v_L-\delta v_L}\right]$. Note that $K_1 > (\alpha - 1)z v_L x_H$ because $v_H(1-\gamma) + (1-\delta)v_L > 0$. Using the envelope theorem, we verify that $f(h_0(K), K)$ is decreasing in K because $\frac{\partial f(h_0(K),K)}{\partial K} = -\frac{\partial N_H}{\partial K}\sqrt{v_H(1-\gamma) + v_L - \delta\alpha v_L h} < 0$. Therefore, the entrepreneur may prefer no crowdfunding when $K > K_1$ and $h \in [h_a, h_b]$.

Next we show that h_a (h_b), if exists, will decrease (increase) with δ and γ. Because $h_1 < h_a < h_0 < h_b < 1/\alpha$, we know $f(h) > 0$ except over the region $[h_a,\ h_b]$. Therefore $f'(h_a) < 0$ and

$f'(h_b) > 0$. Because $f(h_b) = 0$, we have $\frac{\partial f(h_b)}{\partial \gamma} + f'(h_b)\frac{\partial h_b}{\partial \gamma} = 0$.

$$f(h_b) = 0 \Rightarrow \frac{\overline{N}}{\left(-\frac{z\alpha v_L x_H - K}{h_b z\alpha v_L} + x_H\right)} = \sqrt{\frac{v_H(1-\gamma) + v_L - \delta\alpha v_L h_b}{v_H(1-\gamma) + v_L - \delta v_L}}. \tag{24}$$

Because $h_b < 1/\alpha$, if follows that

$$\sqrt{\frac{v_H(1-\gamma) + v_L - \delta\alpha v_L h_b}{v_H(1-\gamma) + v_L - \delta v_L}} > 1 > \sqrt{\frac{v_H(1-\gamma) + v_L - \delta v_L}{v_H(1-\gamma) + v_L - \delta\alpha v_L h_b}}. \tag{25}$$

Note that $\frac{\partial f(h)}{\partial \gamma}|_{h=h_b} < 0$ requires $\frac{\overline{N}}{\left(-\frac{z\alpha v_L x_H - K}{h_b z\alpha v_L} + x_H\right)} > \sqrt{\frac{v_H(1-\gamma)+v_L-\delta v_L}{v_H(1-\gamma)+v_L-\delta\alpha v_L h_b}}$, which always holds given (24) and (25). Thus, $f'(h_b) > 0$ implies that $\frac{\partial h_b}{\partial \gamma} > 0$. Similarly, we have $\frac{\partial h_a}{\partial \gamma} < 0$. Therefore, the range $[h_a, h_b]$ expands as γ increases to 1.

The proof for $\frac{\partial h_b}{\partial \delta} > 0$ is similar. It suffices to show $\frac{\partial f(h)}{\partial \delta}|_{h=h_b} < 0$, which can be simplified to $\alpha h_b\sqrt{\frac{v_H(1-\gamma)+v_L-\delta v_L}{v_H(1-\gamma)+v_L-\delta\alpha v_L h_b}} < 1$. This always holds given (25) and $h_b < 1/\alpha$. Similarly we prove $\frac{\partial h_a}{\partial \delta} < 0$, so that $[h_a, h_b]$ expands as δ increases to 1.

Case 1 arises under crowdfunding:

In this case, we have $N_L \leq \overline{N}$, which is compatible with $\left(\frac{\alpha h}{\alpha h+1}\right) N_H \leq \frac{\overline{N}}{2} \leq \left(\frac{\alpha h}{\alpha h+1}\right) N_L$ only if $\alpha h \geq 1$. From Problem (P1) the entrepreneur's profit under crowdfunding is

$$\begin{aligned} \pi_E^C &= \frac{1}{\overline{N}}\int_{N_{\min}}^{\overline{N}} nr dn + \frac{(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} \left[\delta\left(zR(n,x_H) - K\right) - (1-z)\beta v_H n\right] dn \\ &\quad -\frac{p}{\overline{N}}\int_{N_{\min}}^{N_L} \beta v_H n dn + \frac{p}{\overline{N}}\int_{N_L}^{\overline{N}} \left[\delta\left[zR(n,x_L) - K\right] - (1-z)\beta v_H n\right] dn \\ &\quad +\xi\frac{(1-p)}{\overline{N}}\int_{\max(N_H,0)}^{N_{\min}} \delta\left[zR(n,x_H) - K\right] dn \end{aligned} \tag{26}$$

where $N_{\min}^*$ is given in Proposition 1. Next we show that $\pi_E^C \geq \pi_E^{NC}$ always holds for this case because $\alpha h \geq 1$.

It suffices to show 1) $\pi_E^C\left(N_{\min}^* = \max(N_H, 0)\right) \geq \pi_E^{NC}$ always, and 2) $\pi_E^C\left(N_{\min}^* = N_L\right) \geq \pi_E^{NC}$ on $p_U < p \leq 1$. This is because $\pi_E^C\left(N_{\min}^* = \max(N_H, 0)\right) \geq \pi_E^{NC}$ implies that $\pi_E^C\left(N_{\min}^* = N_{\min}^{int}\right) \geq \pi_E^{NC}$, given that the interior solution $N_{\min}^{int}$ outperforms the boundary solution $\max(N_H, 0)$.

1) When $N_{\min}^* = \max(N_H, 0)$, from (26) π_E^C is increasing in x_L, whereas π_E^{NC} does not depend on x_L. Hence, it suffices to show $\pi_E^C \geq \pi_E^{NC}$ at the minimum value of x_L, where $N_L = \overline{N}$. However,

this simply reduces to the comparison under Case 2, where we have shown that $\pi_E^C \geq \pi_E^{NC}$ as long as $\alpha h \geq 1$.

2) Consider $N_{\min}^* = N_L$, which is valid on $p_U < p \leq 1$. The entrepreneur's profit under crowdfunding is:

$$\begin{aligned}\pi_E^C\left(N_{\min}^* = N_L\right) &= \frac{(\overline{N}+N_L)(\overline{N}-N_L)}{2\overline{N}} z\left[(1-\gamma)v_H + v_L + \delta\alpha v_L h\right] \\ &+ \frac{\delta z \alpha v_L (1-h)}{\overline{N}}[(1-p)x_H + p x_L](\overline{N}-N_L) - \frac{\delta}{\overline{N}}K(\overline{N}-N_L) \\ &+ \xi\frac{(1-p)\delta z\alpha v_L h}{2\overline{N}}\left(N_L^2 - (\max(N_H,0))^2 - 2N_H N_L + 2N_H\max(N_H,0)\right).\end{aligned}$$

It is easy to show $\frac{\partial^2\left[\pi_E^C\left(N_{\min}^*=N_L\right)-\pi_E^{NC}\right]}{\partial p^2} = 0$, which implies that $\pi_E^C\left(N_{\min}^* = N_L\right) - \pi_E^{NC}$ is monotone in p on $p_U \leq p \leq 1$. Thus, it suffices to show $\pi_E^C\left(N_{\min}^* = N_L\right) \geq \pi_E^{NC}$ at both $p = p_U$ and $p = 1$.

At $p = p_U$, $N_{\min}^{int} = N_L$. Note that $\pi_E^C\left(N_{\min} = N_{\min}^{int}\right) \geq \pi_E^C\left(N_{\min} = \max(N_H, 0)\right)$ because $N_{\min}^{int}$ is the interior solution, and we have proved that $\pi_E^C\left(N_{\min} = \max(N_H, 0)\right) \geq \pi_E^{NC}$ on $p \in [0,1]$. Therefore, we have $\pi_E^C\left(N_{\min} = N_L\right) \geq \pi_E^C\left(N_{\min} = \max(N_H, 0)\right) \geq \pi_E^{NC}$ at $p = p_U$. At $p = 1$, $\pi_E^{NC} = 0 \leq \pi_E^C\left(N_{\min}^* = N_L\right)$. Thus, the entrepreneur's profit under crowdfunding is always higher in Case 1.

Entrepreneur's profit comparison ($\gamma = 1$ and $\delta = 1$):

If Case 1 arises, from the proof above, the entrepreneur always prefers crowdfunding.

If Case 2 arises, then $N_L > \overline{N}$, and therefore, $h < h_2$ from (11). We next show $1/\alpha < h_2$.

Because the probability of getting funded by the VC without crowdfunding is less than 1, we have $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$. This implies that the development cost

$$K > z\alpha v_L\left(h\frac{\overline{N}}{2} + (1-h)x_L\right) + z v_L\frac{\overline{N}}{2}. \tag{27}$$

We next show that when this condition is satisfied, Case 2 always arises for $h < 1/\alpha$. That is, $N_L = \frac{K-(1-h)z\alpha v_L x_L}{hz\alpha v_L} > \overline{N}$ for $h < 1/\alpha$. From (27), it suffices to show $\frac{z\alpha v_L(h\frac{\overline{N}}{2}+(1-h)x_L)+zv_L\frac{\overline{N}}{2}-(1-h)z\alpha v_L x_L}{hz\alpha v_L} > \overline{N}$, which can be simplified to $(1-\alpha h)\frac{\overline{N}}{2} > 0$, a condition that always holds for $h < 1/\alpha$. This implies that when $h < 1/\alpha$, Case 2 arises so we must have $h < h_2$. Therefore, $1/\alpha < h_2$.

If $h < h_1$, then $N_H < 0$. From (20) we have $\pi_E^C = \pi_E^{NC}$ if both $\gamma = 1$ and $\delta = 1$. In this case, the campaign is not informative ($h < min\{h_1, h_2\}$), the entire profit accrues to the entrepreneur

($\delta = 1$) and the campaign does not price discriminate ($\gamma = 1$). The optimal target number is equal to zero, thus removing the risk of campaign failure. As a result, the entrepreneur is indifferent between crowdfunding and no crowdfunding.

If $h \geq h_1$, then $N_H \geq 0$ and (22) can be written as $0 > [1 - \alpha h] (N_H)^2 v_L$, which holds if $h > 1/\alpha$.

If $1/\alpha < h_1$ then $\pi_E^C \geq \pi_E^{NC}$ if $h \geq h_1 > 1/\alpha$. In this case, the entrepreneur is indifferent between crowdfunding and no crowdfunding for $h \leq \min\{h_1, h_2\}$ and prefers crowdfunding for $h > \min\{h_1, h_2\}$.

Otherwise, if $h_1 < 1/\alpha$, then $h_1 < 1/\alpha < h_2$ so that $\min\{h_1, h_2\} = h_1$. Then $\pi_E^C < \pi_E^{NC}$ if $h_1 < h < 1/\alpha$ and $\pi_E^C \geq \pi_E^{NC}$ if $h \geq 1/\alpha$. In this case, crowdfunding is strictly preferred when h is large, but is inferior to no crowdfunding for relatively low values of h. The condition $h_1 < 1/\alpha$ can also be written as $K > (\alpha - 1) z v_L x_H$, suggesting that when K is large, no crowdfunding is better for relatively low value of h.

Combining Cases 1 and 2, we know that if $h \leq \min\{h_1, h_2\}$, the entrepreneur is indifferent between crowdfunding and no crowdfunding. If the entrepreneur prefers no crowdfunding, it must be the case that $K > (\alpha - 1) z v_L x_H$ and $h_1 < h < 1/\alpha$. In all other cases, the entrepreneur prefers crowdfunding. ■

Proof of Proposition 5 (VC's Profit Comparison): When $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H$, the comparison is trivial because the profit is 0 under no crowdfunding. When $\left(\frac{2\alpha h}{\alpha h+1}\right) N_H \leq \overline{N} \leq \left(\frac{2\alpha h}{\alpha h+1}\right) N_L$, from (17) the VC's profit under no crowdfunding is

$$\pi_{VC}^{NC} = (1 - \delta)\, z \alpha v_L h\, (1 - p) \left[\frac{\overline{N}}{2} \left(\frac{\alpha h + 1}{\alpha h} \right) - N_H \right]. \tag{28}$$

Case 2. $N_L > \overline{N}$.

From (P2), the VC's profit under crowdfunding is $\pi_{VC}^C = \frac{z \alpha v_L h}{2\overline{N}} (1 - \delta)(1 - p)(\overline{N} - N_{\min}^*)(\overline{N} + N_{\min}^* - 2N_H)$. From (28), $\pi_{VC}^C > \pi_{VC}^{NC} \Leftrightarrow (2N_H - N_{\min}^*)\, N_{\min}^* > \frac{\overline{N}^2}{\alpha h}$, where $N_{\min}^* = \max(N_H, 0)$ because we are in Case 2. If $N_H \leq 0$, this condition is never satisfied so the VC never prefers crowdfunding. If $N_H > 0$, the condition can be simplified to $\alpha h > \left(\overline{N}/N_H\right)^2$. Thus, the VC's profit is higher under crowdfunding if $h > \left(\overline{N}/N_H\right)^2 / \alpha$. Because $\overline{N} \geq N_H$, the VC never prefers crowdfunding if $h < 1/\alpha$.

Case 1. $N_L \leq \overline{N}$.

$N_L \leq \overline{N}$ is compatible with $\left(\frac{\alpha h}{\alpha h+1}\right) N_H \leq \frac{\overline{N}}{2} \leq \left(\frac{\alpha h}{\alpha h+1}\right) N_L$ only if $\alpha h \geq 1$. Depending on the value of $N^*_{\min}$, there are three cases.

(I) $p \leq p_L$, so $N^*_{\min} = \max(N_H, 0)$. First consider $N_H \geq 0$. From (P1), the VC's profit under crowdfunding is: $\pi^C_{VC}\left(N^*_{\min} = N_H\right) = (1-\delta)\frac{z\alpha v_L h(1-p)(\overline{N}-N_H)^2}{2\overline{N}} + (1-\delta)\frac{pz\alpha v_L h}{2\overline{N}}(\overline{N}-N_L)^2$. From (28), $\pi^C_{VC}\left(N^*_{\min} = N_H\right) \geq \pi^{NC}_{VC}$ if and only if

$$N_H^2 - \frac{1}{\alpha h}\overline{N}^2 \geq p\left[N_H^2 - \frac{1}{\alpha h}\overline{N}^2 - \left(\overline{N}-N_L\right)^2\right].$$

If $\overline{N} \leq N_H\sqrt{\alpha h}$, this condition always holds. Otherwise, if $\overline{N} > N_H\sqrt{\alpha h}$, the VC prefers crowdfunding if $p > \frac{\overline{N}^2 - \alpha h N_H^2}{\overline{N}^2 - \alpha h N_H^2 + \alpha h\left(\overline{N}-N_L\right)^2}$.

When $N_H < 0$, then $N^*_{\min} = 0$. In this case, we have verified that when $1 - p - p\alpha h < 0$, crowdfunding leads to higher profit for the VC if $\overline{N}$ is sufficiently high, whereas under $1-p-p\alpha h \geq 0$, the VC always prefers no crowdfunding.

(II) $p \geq p_U$, so $N^*_{\min} = N_L$. From (P1), the VC's profit under crowdfunding is:

$$\begin{aligned}\pi^C_{VC}\left(N^*_{\min} = N_L\right) &= \frac{(1-\delta)\, z\alpha v_L h\left(\overline{N}-N_L\right)}{\overline{N}}\left[\frac{\left(\overline{N}+N_L\right)}{2} - \left(pN_L + (1-p)\, N_H\right)\right] \\ &\quad +\xi\frac{(1-p)\delta z\alpha v_L h}{2\overline{N}}\left(N^2_{\min} - \left(\max\left(N_H, 0\right)\right)^2 - 2N_H N_{\min} + 2N_H \max\left(N_H, 0\right)\right).\end{aligned}$$

From (28), we can simplify the inequality $\pi^C_{VC}\left(N^*_{\min} = N_L\right) \geq \pi^{NC}_{VC}$ to:

$$\begin{aligned}& p\left[\left(\overline{N}-N_L\right)^2 + \frac{\overline{N}^2}{\alpha h} + N_L^2 - 2N_H N_L - \xi(N_L - \max\left(N_H, 0\right))\left(N_L + \max\left(N_H, 0\right) - 2N_H\right)\right] \\ \geq\; & \frac{1}{\alpha h}\overline{N}^2 + (N_L - N_H)^2 - N_H^2 - \xi(N_L - \max\left(N_H, 0\right))\left(N_L + \max\left(N_H, 0\right) - 2N_H\right).\end{aligned}$$

If $\left(\overline{N}-N_L\right)^2 + \frac{1}{\alpha h}\overline{N}^2 + (N_L - N_H)^2 - N_H^2 - \xi(N_L - \max\left(N_H, 0\right))\left(N_L + \max\left(N_H, 0\right) - 2N_H\right) > 0$, the VC prefers crowdfunding if

$$p \geq \frac{\frac{1}{\alpha h}\overline{N}^2 + (N_L - N_H)^2 - N_H^2 - \xi(N_L - \max\left(N_H, 0\right))\left(N_L + \max\left(N_H, 0\right) - 2N_H\right)}{\left(\overline{N}-N_L\right)^2 + \frac{1}{\alpha h}\overline{N}^2 + (N_L - N_H)^2 - N_H^2 - \xi(N_L - \max\left(N_H, 0\right))\left(N_L + \max\left(N_H, 0\right) - 2N_H\right)}.$$

Otherwise, the VC always prefers crowdfunding.

(III) $p_L \leq p \leq p_U$, so that $N^*_{\min} = N^{int}_{\min}$. The inequality $\pi^C_{VC}\left(N^*_{\min} = N^{int}_{\min}\right) \geq \pi^{NC}_{VC}$ can be simplified to $\frac{p}{1-p}\left(\overline{N} - N_L\right)^2 - \frac{\overline{N}^2}{\alpha h} \geq \left(N^{int}_{\min} - N_H\right)^2 - N^2_H$. We have verified that it is possible for the VC to make higher or lower profit in this case.

Summarizing Cases 1 and 2, we conclude that when $\left(\frac{\alpha h}{\alpha h+1}\right) N_H \leq \frac{\overline{N}}{2} \leq \left(\frac{\alpha h}{\alpha h+1}\right) N_L$, the VC never prefers crowdfunding when $\alpha h < 1$, whereas his preference is ambiguous when $\alpha h \geq 1$. From Proposition 4, the entrepreneur always prefers crowdfunding when $\alpha h \geq 1$. ■

Appendix B

Appendix B provides the derivation and proofs of §5.

Appendix B.1 Correlation between signals X and N

Under the case of strong correlation between X and N, the underlying joint distribution of the random variables y, N and X is given by

$$f_C(y,n,x_i) = \begin{cases} 2\frac{(1-p)h}{\overline{N}} & y = \alpha n,\ x_i = x_H,\ n \in \left[\frac{\overline{N}}{2}, \overline{N}\right] \\ 2\frac{ph}{\overline{N}} & y = \alpha n,\ x_i = x_L,\ n \in \left[0, \frac{\overline{N}}{2}\right] \\ 2\frac{(1-p)(1-h)}{\overline{N}} & y = \alpha x_H,\ x_i = x_H,\ n \in \left[\frac{\overline{N}}{2}, \overline{N}\right] \\ 2\frac{p(1-h)}{\overline{N}} & y = \alpha x_L,\ x_i = x_L,\ n \in \left[0, \frac{\overline{N}}{2}\right] \end{cases}. \tag{29}$$

We first show that Lemma 1 still holds here, i.e., $\max(N_H, 0) \leq N^*_{\min} \leq \min\left(N_L, \overline{N}\right)$.

Case I. $\frac{\overline{N}}{2} \leq N_H < \min\left(N_L, \overline{N}\right)$

Suppose $N_{\min} > \min\left(N_L, \overline{N}\right)$, which can only happen if $N_L < \overline{N}$. Under this assumption, a fan's preference in favor of contributing in the campaign is valid if the following inequality holds:

$$\begin{aligned} & 2(1-p)(zv_H + (1-z)\,\beta v_H - r)\frac{\overline{N} - N_{\min}}{\overline{N}} + 2\xi(1-p)z(\gamma v_H - v_L)\frac{N_{\min} - N_H}{\overline{N}} \\ \geq\ & 2(1-p)z(\gamma v_H - v_L)\frac{\overline{N} - N_{\min}}{\overline{N}} + 2\xi(1-p)z(\gamma v_H - v_L)\frac{N_{\min} - N_H}{\overline{N}}. \end{aligned}$$

This simplifies to $r \leq \beta v_H + z\left[(1-\gamma-\beta)v_H + v_L)\right]$. The entrepreneur's problem is

$$\begin{aligned} Max_{\{N_{\min},r\}} \quad & \pi^C_E = \frac{2(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} \left[(r - (1-z)\,\beta v_H)\,n + \delta[z\alpha v_L(hn + (1-h)x_H) - K]\right] dn \\ & + \frac{2\xi(1-p)}{\overline{N}}\int_{N_H}^{N_{\min}} \delta[z\alpha v_L(hn + (1-h)x_H) - K] dn \\ s.t. \quad & N_{\min} \geq N_L \\ & r \leq \beta v_H + z\left[(1-\gamma-\beta)v_H + v_L)\right]. \end{aligned}$$

It is straightforward to show that π_E^C is decreasing in $N_{\min}$ and increasing in r. Therefore, $N_{\min} = N_L$ and $r = \beta v_H + z\left[(1-\gamma-\beta)v_H + v_L)\right]$, so this case is dominated by $N_{\min} \leq \min\left(N_L, \overline{N}\right)$.

Now suppose $N_{\min} < N_H$, then there are two subcases: $N_{\min} < \frac{\overline{N}}{2}$ and $N_{\min} \geq \frac{\overline{N}}{2}$. When $N_{\min} < \frac{\overline{N}}{2} \leq N_H$, a fan prefers to pledge in the campaign if the following inequality holds:

$$
\begin{aligned}
& 2(1-p)(zv_H + (1-z)\,\beta v_H - r)\frac{\overline{N}-N_H}{\overline{N}} - \frac{2\,(r-\beta v_H)\left[p\left(\frac{\overline{N}}{2}-N_{\min}\right)+(1-p)\left(N_H-\frac{\overline{N}}{2}\right)\right]}{\overline{N}} \\
\geq \;& 2(1-p)z(\gamma v_H - v_L)\frac{\overline{N}-N_H}{\overline{N}},
\end{aligned}
$$

which reduces to $N_{\min} \geq \frac{\overline{N}}{2p} - \frac{\left(\overline{N}-N_H\right)z(1-p)[(1-\gamma-\beta)v_H+v_L]}{p(r-\beta v_H)}$. The entrepreneur's problem can be written as:

$$
\begin{aligned}
Max_{\{N_{\min},r\}} \quad \pi_E^C = \quad & \frac{2p}{\overline{N}}\int_{N_{\min}}^{\frac{\overline{N}}{2}}(r-\beta v_H)\,ndn + \frac{2\,(1-p)}{\overline{N}}\int_{\frac{\overline{N}}{2}}^{\overline{N}} nrdn - \frac{2\,(1-p)}{\overline{N}}\int_{\frac{\overline{N}}{2}}^{N_H}\beta v_H ndn \\
& + \frac{2(1-p)}{\overline{N}}\int_{N_H}^{\overline{N}}\left[\delta\left(z\alpha v_L(hn+(1-h)x_H)-K\right)\right]-(1-z)\,\beta v_H n]\,dn \\
s.t. \qquad\qquad & 0 \leq N_{\min} \leq \frac{\overline{N}}{2} \\
& N_{\min} \geq \frac{\overline{N}}{2p} - \frac{\left(\overline{N}-N_H\right)z\,(1-p)\left[(1-\gamma-\beta)\,v_H+v_L\right]}{p\,(r-\beta v_H)}.
\end{aligned}
$$

The last two terms in the profit function π_E^C do not depend on decision variables. The first two terms are decreasing in $N_{\min}$, so $N_{\min} = \frac{\overline{N}}{2p} - \frac{\left(\overline{N}-N_H\right)z(1-p)[(1-\gamma-\beta)v_H+v_L]}{p(r-\beta v_H)}$. We rewrite this expression as $r = \beta v_H + \frac{\left(\overline{N}-N_H\right)z(1-p)[(1-\gamma-\beta)v_H+v_L]}{\frac{\overline{N}}{2}-pN_{\min}}$ and substitute it into the first two terms to obtain, after simplification,

$$
\begin{aligned}
& \frac{2p}{\overline{N}}\int_{N_{\min}}^{\frac{\overline{N}}{2}}(r-\beta v_H)\,ndn + \frac{2\,(1-p)}{\overline{N}}\int_{\frac{\overline{N}}{2}}^{\overline{N}} nrdn \\
= \quad & \frac{\left(\overline{N}-N_H\right)zp\,(1-p)\left[(1-\gamma-\beta)\,v_H+v_L\right]}{\overline{N}\left(\frac{\overline{N}}{2}-pN_{\min}\right)}\left(\frac{\overline{N}^2}{4} - N_{\min}^2 + \frac{(1-p)\,3\overline{N}^2}{4p}\right) + \beta v_H\,(1-p)\,\frac{3\overline{N}}{4}
\end{aligned}
$$

which is increasing in $N_{\min}$ as long as $(3-2p)\,\overline{N}^2 - 4\overline{N}N_{\min} + 4pN_{\min}^2 > 0$. Note that this expression is monotone with p and it is positive at both $p = 0$ and $p = 1$ when $N_{\min} \leq \frac{\overline{N}}{2}$. As a result, the entrepreneur's profit is increasing in $N_{\min}$ so that $N_{\min}^* \geq \frac{\overline{N}}{2}$.

It remains to prove that $\frac{\overline{N}}{2} \leq N_{\min} \leq N_H$ is not optimal either. In this case, a fan prefers to pledge in the campaign if the following inequality holds:

$$
\begin{aligned}
& -2(1-p)\left(r-\beta v_H\right)\left(\frac{N_H-N_{\min}}{\overline{N}}\right)+2(1-p)(z v_H+(1-z)\,\beta v_H-r)\frac{\overline{N}-N_H}{\overline{N}} \\
\geq\;& 2(1-p)z(\gamma v_H-v_L)\frac{\overline{N}-N_H}{\overline{N}}.
\end{aligned}
$$

After simplification we obtain $N_{\min} \geq \overline{N}-\frac{\left(\overline{N}-N_H\right)z(1-p)[(1-\gamma-\beta)v_H+v_L]}{r-\beta v_H}$. The entrepreneur's problem can be written as:

$$
\begin{aligned}
Max_{\{N_{\min},r\}} \quad \pi_E^C =\;& \frac{2\,(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} nrdn-\frac{2\,(1-p)}{\overline{N}}\int_{N_{\min}}^{N_H}\beta v_H ndn \\
& +\frac{2(1-p)}{\overline{N}}\int_{N_H}^{\overline{N}}\left[\delta\left(z\alpha v_L(hn+(1-h)x_H)-K\right)\right]-(1-z)\,\beta v_H n]\,dn \\
s.t. \quad & \frac{\overline{N}}{2}\leq N_{\min}\leq N_H \\
& N_{\min}\geq \overline{N}-\frac{\left(\overline{N}-N_H\right)z\,(1-p)\left[(1-\gamma-\beta)\,v_H+v_L\right]}{r-\beta v_H}.
\end{aligned}
$$

The last term in the profit function π_E^C does not depend on decision variables. The first two terms are decreasing in $N_{\min}$, so that $N_{\min}=\overline{N}-\frac{\left(\overline{N}-N_H\right)z(1-p)[(1-\gamma-\beta)v_H+v_L]}{r-\beta v_H}$. We rewrite this expression as $r=\beta v_H+\frac{\left(\overline{N}-N_H\right)z[(1-\gamma-\beta)v_H+v_L]}{\overline{N}-N_{\min}}$ and substitute it into the first two terms to obtain, after simplification,

$$
\begin{aligned}
& \frac{2\,(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} nrdn-\frac{2\,(1-p)}{\overline{N}}\int_{N_{\min}}^{N_H}\beta v_H ndn \\
=\;& \frac{(1-p)\left(\overline{N}-N_H\right)z\left[(1-\gamma-\beta)\,v_H+v_L\right]}{\overline{N}}\left[\overline{N}+N_{\min}\right]+\frac{(1-p)}{\overline{N}}\beta v_H\left(\overline{N}^2-N_H^2\right),
\end{aligned}
$$

which is increasing in $N_{\min}$. Thus $N_{\min}\geq N_H$.

In summary, Lemma 1 holds in Case I when the joint distribution is given by (29). The proof of Case II with $\max\left(N_H,0\right)<\frac{\overline{N}}{2}<\min\left(N_L,\overline{N}\right)$ is similar and is omitted.

Proof of Proposition 6: We next derive the optimal campaign instruments, knowing that $\max\left(N_H,0\right)\leq N_{\min}^*\leq\min\left(N_L,\overline{N}\right)$. There are two cases, which we analyze in turn.

Case I. $\frac{\overline{N}}{2}\leq N_H<\min\left(N_L,\overline{N}\right)$

Given the joint distribution (29), the VC never funds the project if $n<\frac{\overline{N}}{2}$, whereas when $n\geq\frac{\overline{N}}{2}$, he funds the project if $n\geq N_H$ (in case of campaign failure he considers funding with probability

ξ). Following the same logic in Sections 3.1 and 3.2, fans choose to pledge in the campaign if the following inequality holds:

$$
\begin{aligned}
& (zv_H + (1-z)\,\beta v_H - r)(1-p)2\frac{\overline{N}-N_{\min}}{\overline{N}} + z(\gamma v_H - v_L)2\xi(1-p)\frac{N_{\min}-N_H}{\overline{N}} \\
\geq \;\; & z(\gamma v_H - v_L)2(1-p)\frac{\overline{N}-N_{\min}}{\overline{N}} + z(\gamma v_H - v_L)2\xi(1-p)\frac{N_{\min}-N_H}{\overline{N}},
\end{aligned}
$$

which reduces to $r \leq \beta v_H + z[(1-\gamma-\beta)v_H + v_L]$. The entrepreneur's problem is:

$$
\begin{aligned}
Max_{\{N_{\min},r\}} \;\; \pi_E^C \;\; = \;\; & \frac{2(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} [nr + \delta[z\alpha v_L(hn+(1-h)x_H) - K] - (1-z)\,\beta v_H n]\,dn \\
& + \frac{2\xi(1-p)}{\overline{N}}\int_{N_H}^{N_{\min}} \delta[z\alpha v_L(hn+(1-h)x_H) - K]dn \\
s.t. \qquad & N_H \leq N_{\min} \leq \min\left(N_L, \overline{N}\right) \\
& r \leq \beta v_H + z[(1-\gamma-\beta)v_H + v_L].
\end{aligned}
$$

Therefore, $N_{\min}^* = N_H$ and $r^* = \beta v_H + z[(1-\gamma-\beta)v_H + v_L]$.

Case II. $\max(N_H, 0) \leq \frac{\overline{N}}{2} \leq \min\left(N_L, \overline{N}\right)$

Given the joint distribution (29), the VC never funds the project if $n < \frac{\overline{N}}{2}$. Otherwise, if $n \in [\frac{\overline{N}}{2}, \overline{N}]$, then x_H will realize so that the project is funded by the VC (in case of campaign failure this happens with probability ξ). Fans pledge in the campaign if the following inequalities hold:

$$
\begin{aligned}
& -(r-\beta v_H)\,2p\left(\frac{\frac{\overline{N}}{2}-N_{\min}}{\overline{N}}\right) + (zv_H + (1-z)\,\beta v_H - r)2(1-p)\frac{\overline{N}-\frac{\overline{N}}{2}}{\overline{N}} \\
& \qquad \geq z(\gamma v_H - v_L)2(1-p)\frac{\overline{N}-\frac{\overline{N}}{2}}{\overline{N}} \qquad\qquad \text{if } N_{\min} \leq \frac{\overline{N}}{2}, \\
& (zv_H + (1-z)\,\beta v_H - r)2(1-p)\frac{\overline{N}-N_{\min}}{\overline{N}} + z(\gamma v_H - v_L)2\xi(1-p)\frac{N_{\min}-\frac{\overline{N}}{2}}{\overline{N}} \\
& \qquad \geq z(\gamma v_H - v_L)2(1-p)\frac{\overline{N}-N_{\min}}{\overline{N}} + z(\gamma v_H - v_L)2\xi(1-p)\frac{N_{\min}-\frac{\overline{N}}{2}}{\overline{N}} \qquad \text{if } N_{\min} > \frac{\overline{N}}{2},
\end{aligned}
$$

which reduce to, respectively:

$$
\begin{aligned}
N_{\min} \;\; & \geq \;\; \frac{\overline{N}\left(r - \beta v_H - z\,(1-p)\left[(1-\gamma-\beta)\,v_H + v_L\right]\right)}{2p\,(r-\beta v_H)} \qquad \text{if } N_{\min} \leq \frac{\overline{N}}{2}, \\
r \;\; & \leq \;\; \beta v_H + z\left[v_H(1-\gamma-\beta) + v_L\right)] \qquad \text{if } N_{\min} > \frac{\overline{N}}{2}.
\end{aligned}
$$

Hence, if $N_{\min} \leq \frac{\overline{N}}{2}$, the entrepreneur's problem can be written as:

$$\begin{aligned}
Max_{\{N_{\min},r\}} \ \pi_E^C \ &= \ \frac{2p}{\overline{N}} \int_{N_{\min}}^{\frac{\overline{N}}{2}} (r - \beta v_H)\, ndn + \frac{2(1-p)}{\overline{N}} \int_{\frac{\overline{N}}{2}}^{\overline{N}} nrdn \\
&\quad + \frac{2(1-p)}{\overline{N}} \int_{\frac{\overline{N}}{2}}^{\overline{N}} (\delta[z\alpha v_L(hn + (1-h)x_H) - K] - (1-z)\,\beta v_H n)\, dn \\
s.t. \quad & \max(N_H, 0) \leq N_{\min} \leq \frac{\overline{N}}{2} \\
& N_{\min} \geq \frac{\overline{N}\,(r - \beta v_H - z(1-p)\,[(1-\gamma-\beta)\, v_H + v_L])}{2p\,(r - \beta v_H)}.
\end{aligned}$$

The last term in π_E^C does not depend on decision variables. The first two terms are decreasing in $N_{\min}$ so that $N_{\min} = \frac{\overline{N}(r-\beta v_H - z(1-p)[(1-\gamma-\beta)v_H+v_L])}{2p(r-\beta v_H)}$. We rewrite this expression as $r = \beta v_H + \frac{\overline{N}z(1-p)[(1-\gamma-\beta)v_H+v_L]}{\overline{N}-2pN_{\min}}$ and substitute it into the first two terms to obtain, after simplification,

$$\begin{aligned}
& \frac{2p}{\overline{N}} \int_{N_{\min}}^{\frac{\overline{N}}{2}} (r - \beta v_H)\, ndn + \frac{2(1-p)}{\overline{N}} \int_{\frac{\overline{N}}{2}}^{\overline{N}} nrdn - \frac{2(1-p)}{\overline{N}} \int_{\frac{\overline{N}}{2}}^{\overline{N}} (1-z)\,\beta v_H ndn \\
= \ & \frac{\overline{N}z(1-p)\,[(1-\gamma-\beta)\, v_H + v_L]}{\overline{N}\left(\overline{N} - 2pN_{\min}\right)} \left[p\left(\frac{\overline{N}^2}{4} - N_{\min}^2\right) + (1-p)\left(\overline{N}^2 - \frac{\overline{N}^2}{4}\right)\right] \\
& + \frac{(1-p)\, z\beta v_H}{\overline{N}} \left(\overline{N}^2 - \frac{\overline{N}^2}{4}\right),
\end{aligned}$$

which is increasing with $N_{\min}$ as long as $(3-2p)\,\overline{N}^2 - 4\overline{N}N_{\min} + 4pN_{\min}^2 > 0$. This expression is monotone with p and it is positive at both $p = 0$ and $p = 1$ given that $N_{\min} \leq \frac{\overline{N}}{2}$ here. As a result, the entrepreneur's profit is increasing in $N_{\min}$, and thus $N_{\min}^* = \frac{\overline{N}}{2}$ and $r^* = \beta v_H + z[(1-\gamma-\beta)v_H + v_L]$.

Similarly, if $N_{\min} > \frac{\overline{N}}{2}$, the entrepreneur's problem is:

$$\begin{aligned}
Max_{\{N_{\min},r\}} \quad \pi_E^C \ &= \frac{2(1-p)}{\overline{N}} \int_{N_{\min}}^{\overline{N}} [nr + \delta[z\alpha v_L(hn + (1-h)x_H) - K] - (1-z)\,\beta v_H n]\, dn \\
&\quad + \frac{2\xi(1-p)}{\overline{N}} \int_{\frac{\overline{N}}{2}}^{N_{\min}} \delta[z\alpha v_L(hn + (1-h)x_H) - K] dn \\
s.t. \quad & \frac{\overline{N}}{2} \leq N_{\min} \leq \min\left(N_L, \overline{N}\right) \\
& r \leq \beta v_H + z\,[v_H(1-\gamma-\beta) + v_L)]\,.
\end{aligned}$$

It is straightforward to show that the profit function is decreasing in $N_{\min}$ and increasing in r. Therefore, $N_{\min}^* = \frac{\overline{N}}{2}$ and $r^* = \beta v_H + z\,[v_H(1-\gamma-\beta) + v_L)]$.

Summing up Case I and Case II, we have the optimal campaign instruments under the joint distribution (29) as:

$$N^*_{\min} = \max(N_H, \frac{\overline{N}}{2}) \text{ and } r^* = \beta v_H + z[(1-\gamma-\beta)v_H + v_L]. \tag{30}$$

It is straightforward to determine the probability of VC funding in the two cases. ■

Derivation of no crowdfunding under correlation

We now derive the entrepreneur's expected profit under no crowdfunding:

i) if $X = x_H$, the entrepreneur's expected profit under no crowdfunding is $\pi_E^{NC}\big|_{X=x_H} = \delta[z\alpha v_L(h\frac{3\overline{N}}{4} + (1-h)x_H) + zv_L\frac{3\overline{N}}{4} - K]$. Note that $\pi_E^{NC}\big|_{X=x_H} \geq 0$ when $\overline{N} \geq \frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H$.

ii) If $X = x_L$, we have $\pi_E^{NC}\big|_{X=x_L} = \delta[z\alpha v_L(h\frac{\overline{N}}{4} + (1-h)x_L) + zv_L\frac{\overline{N}}{4} - K]$. Hence, $\pi_E^{NC}\big|_{X=x_L} \geq 0$ when $\overline{N} \geq 4\left(\frac{\alpha h}{\alpha h+1}\right)N_L$.

Therefore, three possible cases may arise:

a) If $\overline{N} > 4\left(\frac{\alpha h}{\alpha h+1}\right)N_L$, the VC always funds the project under no crowdfunding, which is unrealistic.

b) If $\frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H \leq \overline{N} \leq 4\left(\frac{\alpha h}{\alpha h+1}\right)N_L$, the VC funds the project only if the good signal x_H realizes. The total expected profit is $(1-p)\left[z\alpha v_L(h\frac{3\overline{N}}{4} + (1-h)x_H) + zv_L\frac{3\overline{N}}{4} - K\right]$, which is split between the entrepreneur and VC according to their bargaining power δ and $1-\delta$, respectively.

c) If $\overline{N} < \frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H$, the VC will never fund the project, and the expected profits of the entrepreneur and the VC are zero.

Because case a) never arises, we restrict to the range $\overline{N} \leq 4\left(\frac{\alpha h}{\alpha h+1}\right)N_L$. Under correlation, the entrepreneur's expected profit without crowdfunding is:

$$\pi_E^{NC} = \begin{cases} (1-p)\,\delta z\alpha v_L h\left(\frac{3\overline{N}(\alpha h+1)}{4\alpha h} - N_H\right) & \text{if } \frac{4\alpha h N_H}{3(\alpha h+1)} \leq \overline{N} \leq \frac{4\alpha h N_L}{\alpha h+1} \\ 0 & \text{if } \overline{N} < \frac{4\alpha h N_H}{3(\alpha h+1)} \end{cases}. \tag{31}$$

Proof of Proposition 7: From (30), we derive the entrepreneur's optimal profit under crowdfunding:

$$\pi_E^C = \begin{cases} \frac{(1-p)z[(1-\gamma)v_H+v_L]}{\overline{N}}\left(\overline{N}^2 - N_H^2\right) + \frac{(1-p)\delta z\alpha v_L h}{\overline{N}}\left(\overline{N} - N_H\right)^2 & \text{if } \frac{\overline{N}}{2} \leq N_H < \min\left(N_L, \overline{N}\right), \\ z(1-p)\overline{N}\left[\frac{3}{4}[(1-\gamma)v_H + v_L] + \delta\alpha v_L h\left(\frac{3}{4} - \frac{N_H}{\overline{N}}\right)\right] & \text{if } \max\left(N_H, 0\right) < \frac{\overline{N}}{2} \leq \min\left(N_L, \overline{N}\right) \end{cases} \tag{32}$$

Note that $\max(N_H, 0) < \frac{\overline{N}}{2}$ holds for any value of h when $\frac{\overline{N}}{2} > x_H$, given that $x_H \geq N_H$ from Equation (4). If $\frac{\overline{N}}{2} \leq x_H$, then $\max(N_H, 0) \leq \frac{\overline{N}}{2}$ if $h \leq h_c = \frac{2(z\alpha v_L x_H - K)}{z\alpha v_L\left(2x_H - \overline{N}\right)}$.

Note that $2\max(N_H, 0) \geq \frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right) N_H$ always holds. Therefore, we can summarize the comparison in two cases: $\frac{\overline{N}}{2} > x_H$ and $\frac{\overline{N}}{2} \leq x_H$.

If $\frac{\overline{N}}{2} > x_H$, then $\max(N_H, 0) < \frac{\overline{N}}{2}$. Comparing (31) and (32), we know that the entrepreneur makes higher profit under crowdfunding if $(1-\gamma)v_H + v_L \geq \delta v_L$, which always holds when $\gamma+\delta < 2$. If $\gamma + \delta = 2$, the entrepreneur is indifferent between crowdfunding and no crowdfunding.

If $\frac{\overline{N}}{2} \leq x_H$, then as long as $h \leq h_c$, the comparison is the same as in the case $\frac{\overline{N}}{2} > x_H$. If $h > h_c$, the entrepreneur's expected profit under crowdfunding is $\pi_E^C = \frac{(1-p)z[(1-\gamma)v_H + v_L]}{\overline{N}}\left(\overline{N}^2 - N_H^2\right) + \frac{(1-p)\delta z\alpha v_L h}{\overline{N}}\left(\overline{N} - N_H\right)^2$ and her profit under no crowdfunding is given by (31). We consider the case $\gamma = \delta = 1$, where the two additional benefits of crowdfunding are removed and the informational value is the only advantage. We first consider that the entrepreneur's expected profit under no crowdfunding is strictly positive, i.e., $\frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right) N_H \leq \overline{N}$. This occurs if $\frac{3\overline{N}}{4} \geq x_H$ or $h \leq h_d = \frac{4z\alpha v_L x_H + 3zv_L\overline{N} - 4K}{zv_L a\left(4x_H - 3\overline{N}\right)}$. Note that $h_d \geq h_c$ always holds. After some algebra, the comparison yields the condition $\left(\overline{N} - 2N_H\right)\left((ah+1)\overline{N} - 2(ah-1)N_H\right) \geq 0$, under which the entrepreneur receives higher profit under crowdfunding.

Note that $\overline{N} \geq 2N_H$ never arises because $h > h_c$ implies $\overline{N} < 2N_H$. Therefore, the entrepreneur prefers crowdfunding if $(ah+1)\overline{N} - 2(ah-1)N_H \leq 0$, which can be simplified to $h \geq h_e$, where

$$h_e = \frac{x_H + \frac{\overline{N}}{2} + \alpha x_H - \frac{K}{zv_L}}{\alpha\left(2x_H - \overline{N}\right)}\left[1 + \frac{\sqrt{\left(x_H + \frac{\overline{N}}{2}\right)^2 + \left(\alpha x_H - \frac{K}{zv_L}\right)^2 + \left(3\overline{N} - 2x_H\right)\left(\alpha x_H - \frac{K}{zv_L}\right)}}{x_H + \frac{\overline{N}}{2} + \alpha x_H - \frac{K}{zv_L}}\right].$$

The other root is always lower than h_c and thus is discarded. h_e always exists because $\left(x_H + \frac{\overline{N}}{2}\right)^2 + \left(\alpha x_H - \frac{K}{zv_L}\right)^2 + \left(3\overline{N} - 2x_H\right)\left(\alpha x_H - \frac{K}{zv_L}\right) \geq 0$ always holds (solving this inequality in $\overline{N}$ yields two negative roots, implying that the inequality always holds). Note that $h_e \geq h_c$ always holds. Also $h_e \leq h_d$ as long as $0 \leq h_d \leq 1$ (recall $0 \leq h \leq 1$) because $h_e > h_d$ and $0 \leq h_d \leq 1$ contradict $z\alpha v_L\overline{N} - K \geq 0$, which must hold. Therefore, the entrepreneur makes higher profit under crowdfunding if $h \geq h_e$. However, when $K < \frac{z\alpha v_L(\alpha+1)\overline{N}}{2(\alpha-1)}$, $h_e > 1$, which implies that crowdfunding is never preferable for the entire range $h_c < h \leq 1$ without violating any condition.

This implies that the entrepreneur's profit is higher under no crowdfunding if $h_c < h \le \min[h_e, 1]$, whereas it is higher under crowdfunding if $\min[h_e, 1] \le h \le 1$, with the latter region being empty when $K < \frac{z\alpha v_L(\alpha+1)\overline{N}}{2(\alpha-1)}$.

When $\frac{3\overline{N}}{4} < x_H$ it is possible that the entrepreneur's expected profit is zero under no crowdfunding. Specifically, this occurs when $h > h_d$. However, given that $h_e \le h_d$ in the sensible range, the case $\frac{\overline{N}}{2} \le x_H$ when $h > h_c$ can be consolidated as follows. The entrepreneur prefers crowdfunding if $\min[h_e, 1] \le h \le 1$, whereas she prefers no crowdfunding if $h_c < h \le \min[h_e, 1]$.

We have already shown that, if the two additional benefits of crowdfunding are introduced, i.e., $\gamma + \delta < 2$, there is no longer equivalence of crowdfunding and no crowdfunding when $\frac{\overline{N}}{2} > x_H$ and when $\frac{\overline{N}}{2} \le x_H$ but $h < h_c$, and the entrepreneur strictly prefers crowdfunding in these cases. Similar to the proof of Proposition 5, we have verified that when the two additional benefits of crowdfunding are added back, i.e., $\gamma + \delta < 2$, the region under which no crowdfunding dominates shrinks as γ or δ decreases. ■

Proof of Proposition 8 (VC's preference under correlation): The VC's expected profit under crowdfunding is:

$$\pi_{VC}^{C} = \begin{cases} \frac{(1-p)(1-\delta)z\alpha v_L h}{\overline{N}}\left(\overline{N} - N_H\right)^2 & \text{if } \frac{\overline{N}}{2} \le N_H < \min\left(N_L, \overline{N}\right) \\ (1-p)(1-\delta)z\alpha v_L h\left(\frac{3}{4}\overline{N} - N_H\right) & \text{if } \max(N_H, 0) < \frac{\overline{N}}{2} \le \min\left(N_L, \overline{N}\right) \end{cases}.$$

The VC's expected profit under no crowdfunding is:

$$\pi_{VC}^{NC} = \begin{cases} (1-p)(1-\delta)z\alpha v_L h\left(\frac{3}{4}\overline{N}\left(\frac{\alpha h+1}{\alpha h}\right) - N_H\right) & \text{if } \frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H \le \overline{N} \le 4\left(\frac{\alpha h}{\alpha h+1}\right)N_L \\ 0 & \text{if } \overline{N} < \frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H. \end{cases}.$$

Similarly to the case of the entrepreneur, we can summarize the comparison in two cases: $\frac{\overline{N}}{2} > x_H$ and $\frac{\overline{N}}{2} \le x_H$.

If $\frac{\overline{N}}{2} > x_H$, the VC's expected profit under crowdfunding is $\pi_{VC}^{C} = (1-p)(1-\delta)z\alpha v_L h\left(\frac{3}{4}\overline{N} - N_H\right)$, whereas under no crowdfunding is $\pi_{VC}^{NC} = (1-p)(1-\delta)z\alpha v_L h\left(\frac{3}{4}\overline{N}\left(\frac{\alpha h+1}{\alpha h}\right) - N_H\right)$. Thus, the VC is never better off under crowdfunding.

If $\frac{\overline{N}}{2} \le x_H$, then as long as $h \le h_c$, the comparison is the same as in the case $\frac{\overline{N}}{2} > x_H$. If $h > h_c$, the optimal VC's expected profit under crowdfunding is $\pi_{VC}^{C} = \frac{(1-p)(1-\delta)z\alpha v_L h}{\overline{N}}\left(\overline{N} - N_H\right)^2$, whereas either profit expression can arise under no crowdfunding. We first consider that the VC's expected profit under no crowdfunding is strictly positive, i.e., $\frac{4}{3}\left(\frac{\alpha h}{\alpha h+1}\right)N_H \leq \overline{N}$. This occurs if $\frac{3\overline{N}}{4} \geq x_H$ or $h \leq h_d = \frac{4z\alpha v_L x_H + 3zv_L\overline{N} - 4K}{zv_L a\left(4x_H - 3\overline{N}\right)}$. After some algebra, the comparison yields higher VC's expected profit under crowdfunding if $(ah-3)\,\overline{N}^2 - 4ah\overline{N}N_H + 4ahN_H^2 \geq 0$. Solving the inequality, we know that the VC prefers crowdfunding if:

$$
\begin{aligned}
h \quad \geq \quad h_f = & \frac{zv_L\left(3\overline{N}^2 - 4\alpha x_H\overline{N} + 8\alpha\left(x_H\right)^2\right) - 4K\left(2x_H - \overline{N}\right)}{2z\alpha v_L\left(2x_H - \overline{N}\right)^2} \\
& + \frac{\overline{N}\sqrt{(3zv_L)\left(zv_L\left(3\overline{N}^2 - 8\alpha x_H\overline{N} + 16\alpha\left(x_H\right)^2\right) - 8K\left(2x_H - \overline{N}\right)\right)}}{2z\alpha v_L\left(2x_H - \overline{N}\right)^2}.
\end{aligned}
$$

The other root is lower than h_c and is discarded.

The solution h_f exists because $zv_L\left(3\overline{N}^2 - 8\alpha x_H\overline{N} + 16\alpha\left(x_H\right)^2\right) - 8K\left(2x_H - \overline{N}\right) \geq 0$ always holds in the feasible range. Note that $h_f \geq h_c$ always holds, also $h_f \leq h_d$ as long as $0 \leq h_d \leq 1$ (recall $0 \leq h \leq 1$) because $h_f > h_d$ and $0 \leq h_d \leq 1$ contradict $z\alpha v_L\overline{N} - K \geq 0$, which must hold. Therefore, the VC makes higher profit under crowdfunding if $h \geq h_f$. However, when $K < \frac{zv_L\overline{N}\left(\alpha+\sqrt{3\alpha}\right)}{2}$, $h_f > 1$, which implies that crowdfunding is never preferable to the VC for the entire range $h_c < h \leq 1$ without violating any condition. This implies that the optimal VC's profit is higher under no crowdfunding if $h_c < h \leq \min\left[h_f, 1\right]$, whereas it is higher under crowdfunding if $\min\left[h_f, 1\right] \leq h \leq 1$, with the latter region being empty when $K < \frac{zv_L\overline{N}\left(\alpha+\sqrt{3\alpha}\right)}{2}$ (the other root is negative and is discarded).

When $\frac{3\overline{N}}{4} < x_H$ it is possible that the VC's expected profit is zero under no crowdfunding. Specifically, this occurs when $h > h_d$. However, given that $h_f \leq h_d$ in the sensible range, the case $\frac{\overline{N}}{2} \leq x_H$ when $h > h_c$ can be consolidated as follows. The VC prefers crowdfunding when $\min\left[h_f, 1\right] \leq h \leq 1$, whereas he prefers no crowdfunding if $h_c < h \leq \min\left[h_f, 1\right]$.

It is then straightforward that the VC is less likely to prefer crowdfunding than the entrepreneur. If $h \leq h_c$, we have shown that the VC never prefers crowdfunding whereas the entrepreneur is indifferent between crowdfunding and no crowdfunding when $\delta = \gamma = 1$ and strictly prefers crowdfunding when $\delta + \gamma < 2$. If $h > h_c$, to show that the region under which the VC prefers crowdfunding is smaller it suffices to show $h_f > h_e$ (as long as they are in the range $[0, 1]$). Indeed,

$h_f > h_e$ holds because the profit difference between crowdfunding and no crowdfunding computed at h_f is zero for the VC and is strictly positive for the entrepreneur. Given that both the profit difference for the VC and the profit difference for the entrepreneur cross zero only once if they cross and the condition for the profit difference to cross is more stringent for the VC, it follows that $h_f > h_e$. ■

Appendix B.2 The entrepreneur has private information about z

For simplicity, we focus on fully separating equilibrium when the entrepreneur chooses crowdfunding. Therefore, we restrict attention to $N_H > 0$, because otherwise the entrepreneur may choose $N^*_{\min} = 0$ over a region of different values of z, resulting in a pooling equilibrium. We assume that the backers and the VC believe that z is distributed uniformly over $[0, z_1]$ where $z_1 = \min\left(1, \frac{K}{(1-h)\alpha v_L x_H}\right)$ to ensure that $N_H > 0$. With crowdfunding, two cases may arise.

Case 2: $N_L > \overline{N}$ (Observing x_L kills the project)

It is easy to prove that Lemma 1 holds here, so that $N_H \leq N_{\min} \leq \overline{N}$. While the entrepreneur may strategically distort the value of $N_{\min}$ to take advantage of her private information, the VC and backers can anticipate this distortion by observing $N_{\min}$, as long as the optimal $N_{\min}$ is a continuous and invertible function of z, namely if $N_{\min} = f(z)$, then $z = f^{-1}(N_{\min})$. Given that under common knowledge Case 2 yields $N^*_{\min} = N_H$, which is a decreasing function of z, we conjecture that under private information $f^{-1}(N_{\min})$ is a decreasing function of $N_{\min}$.

The model can be formulated as in Section 3, with the only difference that z is replaced with $f^{-1}(N_{\min})$, whenever backers and VC draw inferences about the value of z. As fans do not know the value of z, they draw inferences about z from $N_{\min}$. Therefore, they choose to pledge in the campaign instead of purchasing the product if it becomes available if the following inequality holds:

$$\begin{aligned}
& \left[-r + (1-p)(f^{-1}(N_{\min})\, v_H + \left(1 - f^{-1}(N_{\min})\right) \beta v_H)\right] \frac{\overline{N} - N_{\min}}{\overline{N}} \\
& +(1-p) f^{-1}(N_{\min})\, (\gamma v_H - v_L) \left(\xi \frac{(N_{\min} - N_H(N_{\min}))}{\overline{N}}\right) \\
\geq \; & (1-p) f^{-1}(N_{\min})\, (\gamma v_H - v_L) \left[\frac{\overline{N} - N_{\min}}{\overline{N}} + \xi \frac{(N_{\min} - N_H(N_{\min}))}{\overline{N}}\right].
\end{aligned}$$

Given that the VC does not know the value of z, N_H and N_L can only be inferred by observing

$N_{\min}$. Specifically, $N_H = x_H - \frac{x_H}{h} + \frac{K}{f^{-1}(N_{\min})h\alpha v_L}$ and $N_L = x_L - \frac{x_L}{h} + \frac{K}{f^{-1}(N_{\min})h\alpha v_L}$, which are increasing in $N_{\min}$ when $f^{-1}(N_{\min})$ is decreasing as we conjectured. The above inequality reduces to $r \leq \beta v_H + (1-p)f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H + v_L\right]$. The entrepreneur's solves the problem:

$$\begin{aligned} Max_{\{N_{\min},r\}} \ \pi_E^C \quad &= \quad \xi\frac{(1-p)}{\overline{N}}\int_{N_H(N_{\min})}^{N_{\min}} \delta[z\alpha v_L(hn+(1-h)x_H)-K]dn + \frac{1}{\overline{N}}\int_{N_{\min}}^{\overline{N}} (r-p\beta v_H)\,ndn \\ &\quad +\frac{(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}} \left[\delta\left(z\alpha v_L(hn+(1-h)x_H)-K\right)-(1-z)\,\beta v_H n\right]dn \\ s.t. \qquad & N_H(N_{\min}) \leq N_{\min} \leq \overline{N} \\ & r \leq \beta v_H + (1-p)f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H + v_L\right]. \end{aligned}$$

Since the entrepreneur knows the true value of z, this value appears in the terms of the above problem that are not related to backers' and VC's inferences. Because $f^{-1}(N_{\min})$ is decreasing and $\xi \leq 1$, the profit function is decreasing in $N_{\min}$ and increasing in r so that $N_{\min}^* = N_H(N_{\min}^*)$ and $r^* = (1-p)f^{-1}(N_{\min})\left[v_H(1-\gamma-\beta)+v_L)\right]$. We need to solve the equation $N_{\min}^* = N_H(N_{\min}^*)$ where $f^{-1}(N_{\min}^*)$ is unknown:

$$N_{\min}^* = x_H - \frac{x_H}{h} + \frac{K}{f^{-1}\left(N_{\min}^*\right)h\alpha v_L}.$$

The above equation has a unique solution for $N_{\min}^*$ as a function of z, namely $N_{\min}^* = x_H - \frac{x_H}{h} + \frac{K}{zh\alpha v_L}$, which is decreasing in z. This is consistent with our conjecture that $f^{-1}(N_{\min})$ is decreasing in $N_{\min}$. That is, the entrepreneur under Case 2 does not distort the optimal target number and pledge level in the presence of private information on the probability z. The optimal pledge is $r^* = (1-p)z\left[v_H(1-\gamma-\beta)+v_L)\right]$ and the entrepreneur's optimal profit under crowdfunding is:

$$\pi_E^C = (1-p)\left[\frac{z\left[v_H(1-\gamma)+v_L\right]}{2\overline{N}}(\overline{N}^2 - N_H^2) + \frac{\delta z\alpha v_L h}{2\overline{N}}(\overline{N}-N_H)^2\right]. \tag{33}$$

Case 1: $N_L \leq \overline{N}$ (Observing x_L does not kill the project)

We verify that Lemma 1 still holds in this case. Hence, it is optimal to set $N_{\min}$ between $N_H(N_{\min})$ and $N_L(N_{\min})$ because $N_L(N_{\min}) \leq \overline{N}$ under Case 1. Under common knowledge Case 1 yields an interior solution $N_{\min}^{int}$ that is monotone in z but can be either decreasing or increasing depending on the sign of $\frac{p}{2(1-p)}\left[(1-\gamma-\beta)v_H + v_L\right] - (1-\xi)\,\delta h\alpha v_L$. That is, if the benefits from the future profit are higher than the benefits from charging a high pledge to backers, $N_{\min}^{int}$ is decreasing, otherwise it is increasing. As both cases are possible, we conjecture that under private information $f^{-1}(N_{\min})$ can be either a decreasing or increasing function of $N_{\min}$.

Upon observation of the pledge level r and $N_{\min}$ (and thus, the goal G) selected by the entrepreneur, fans choose to participate in the campaign if the following inequality holds:

$$\begin{aligned}
& -(r-\beta v_H)\,p\frac{N_L(N_{\min})-N_{\min}}{\overline{N}}+f^{-1}(N_{\min})(\gamma v_H-v_L)\left[\xi\frac{(1-p)(N_{\min}-N_H(N_{\min}))}{\overline{N}}\right] \\
& +(f^{-1}(N_{\min})v_H+\left(1-f^{-1}(N_{\min})\right)\beta v_H-r)\left[\frac{(1-p)\left(\overline{N}-N_{\min}\right)}{\overline{N}}+\frac{p\left(\overline{N}-N_L(N_{\min})\right)}{\overline{N}}\right] \\
\geq\ & f^{-1}(N_{\min})(\gamma v_H-v_L)\frac{(1-p)\left(\overline{N}-N_{\min}\right)+p\left(\overline{N}-N_L(N_{\min})\right)+\xi(1-p)(N_{\min}-N_H(N_{\min}))}{\overline{N}},
\end{aligned}$$

which can be reduced to:

$$N_{\min}\geq\frac{(r-\beta v_H)\,\overline{N}-f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right](\overline{N}-pN_L(N_{\min}))}{r-\beta v_H-(1-p)f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]}.$$

The entrepreneur's problem is to maximize her expected profit under crowdfunding π_E^C:

$$\begin{aligned}
Max_{\{N_{\min},r\}}\ \pi_E^C \ =\ & \frac{1}{\overline{N}}\int_{N_{\min}}^{\overline{N}}nrdn-\frac{p}{\overline{N}}\int_{N_{\min}}^{N_L(N_{\min})}\beta v_H n dn \\
& +\frac{(1-p)}{\overline{N}}\int_{N_{\min}}^{\overline{N}}\left[\delta\left(z\alpha v_L(hn+(1-h)x_H)-K\right)-(1-z)\,\beta v_H n\right]dn \\
& +\frac{p}{\overline{N}}\int_{N_L(N_{\min})}^{\overline{N}}\left[\delta\left(z\alpha v_L(hn+(1-h)x_L)-K\right)-(1-z)\,\beta v_H n\right]dn \\
& +\xi\frac{(1-p)}{\overline{N}}\int_{N_H(N_{\min})}^{N_{\min}}\delta[z\alpha v_L(hn+(1-h)x_H)-K]dn \\
s.t.\quad & N_H(N_{\min})\leq N_{\min}\leq N_L(N_{\min}) \\
& N_{\min}\geq\frac{(r-\beta v_H)\,\overline{N}-f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right](\overline{N}-pN_L(N_{\min}))}{r-\beta v_H-(1-p)f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]}.
\end{aligned}$$

In the problem above the true value of z appears in all the terms that are unrelated to backers' and VC's inferences, whereas $f^{-1}(N_{\min})$ appears only in terms related to backers' and VC's inferences. If $f^{-1}(N_{\min})$ is a decreasing function of $N_{\min}$, then it is straightforward to show that the above profit function decreases with $N_{\min}$. If $f^{-1}(N_{\min})$ is an increasing function of $N_{\min}$, the above profit function may no longer be decreasing with $N_{\min}$. This is because $N_H(N_{\min})$ and $N_L(N_{\min})$ decrease with $N_{\min}$ when $f^{-1}(N_{\min})$ is increasing with $N_{\min}$. However, the above profit function still decreases with $N_{\min}$, if we make the reasonable assumption that the direct effect of $N_{\min}$ in the above profit function is stronger than the indirect effect via the inferences appearing in $N_H(N_{\min})$ and $N_H(N_{\min})$. As a result, irrespective of whether $f^{-1}(N_{\min})$ decreases or increases with $N_{\min}$, the entrepreneur extracts all the surplus from backers under crowdfunding and $N_{\min} = \frac{(r-\beta v_H)\overline{N} - f^{-1}(N_{\min})[(1-\gamma-\beta)v_H+v_L](\overline{N}-pN_L)}{r-\beta v_H-(1-p)f^{-1}(N_{\min})[(1-\gamma-\beta)v_H+v_L]}$. This can be rewritten as:

$$r = \beta v_H + \frac{f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]\left[(1-p)\left(\overline{N}-N_{\min}\right)+p\left(\overline{N}-N_L\right)\right]}{\overline{N}-N_{\min}}.$$

Substituting it into the profit function yields:

$$\begin{aligned}
\pi_E^C \;=\;& \frac{\beta v_H + f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]\begin{bmatrix} (1-p)\left(\overline{N}-N_{\min}\right) \\ +p\left(\overline{N}-N_L(N_{\min})\right)\end{bmatrix}}{2\overline{N}}\left(\overline{N}+N_{\min}\right) \\
& -p\frac{\beta v_H}{2\overline{N}}\left(N_L(N_{\min})^2 - N_{\min}^2\right) - \frac{(1-z)\beta v_H(1-p)}{2\overline{N}}\left(\overline{N}^2 - N_{\min}^2\right) \\
& -\frac{(1-z)\beta v_H p}{2\overline{N}}\left(\overline{N}^2 - N_L(N_{\min})^2\right) + \frac{p\delta z\alpha v_L h}{2\overline{N}}\begin{pmatrix} \overline{N}^2 - (N_L(N_{\min}))^2 \\ -2N_L\left(\overline{N}-N_L(N_{\min})\right)\end{pmatrix} \\
& +\frac{(1-p)\delta z\alpha v_L h}{2\overline{N}}\left(\overline{N}^2 - N_{\min}^2 - 2\overline{N}N_H + 2N_H N_{\min}\right) \\
& +\xi\frac{(1-p)\delta z\alpha v_L h}{2\overline{N}}\left(N_{\min}^2 - (N_H(N_{\min}))^2 - 2N_H\left(N_{\min}-N_H(N_{\min})\right)\right)
\end{aligned}$$

Taking the first order derivative yields the following differential equation:

$$\begin{aligned}
& -\frac{2\beta v_H N_{\min}}{2\overline{N}} - \frac{2(1-p)f^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]N_{\min}}{2\overline{N}} + \frac{2(1-z)\beta v_H(1-p)N_{\min}}{2\overline{N}} \\
& +\frac{pf^{-1}(N_{\min})\left[(1-\gamma-\beta)v_H+v_L\right]\left(\overline{N}-N_L(N_{\min})\right)}{2\overline{N}} + \frac{2p\beta v_H N_{\min}}{2\overline{N}}\left(N_L(N_{\min})^2 - N_{\min}^2\right) \\
& -\frac{2(1-p)(1-\xi)\delta z\alpha v_L h(N_{\min}-N_H)}{2\overline{N}}\frac{\partial f^{-1}(N_{\min})}{\partial N_{\min}}\begin{bmatrix} \frac{[(1-\gamma-\beta)v_H+v_L](1-p)\left(\overline{N}^2-N_{\min}^2\right)}{2\overline{N}} \\ +\frac{p[(1-\gamma-\beta)v_H+v_L]\left(\overline{N}-N_L(N_{\min})\right)\left(\overline{N}+N_{\min}\right)}{2\overline{N}}\end{bmatrix} \\
& -\frac{\partial N_L}{\partial N_{\min}}\begin{bmatrix} \frac{f^{-1}(N_{\min})p[(1-\gamma-\beta)v_H+v_L]\left(\overline{N}+N_{\min}\right)}{2\overline{N}} \\ +\frac{zp\beta v_H 2N_L(N_{\min})}{2\overline{N}} + \frac{2zp\delta\alpha v_L h}{2\overline{N}}\left(N_L(N_{\min})-N_L\right)\end{bmatrix} \\
& -\frac{\partial N_H}{\partial N_{\min}}\left[\frac{2\xi(1-p)z\alpha v_L h}{2\overline{N}}\left(N_H(N_{\min})-N_H\right)\right].
\end{aligned}$$

At the equilibrium $N_L(N_{\min}^*) = N_L$ and $N_H(N_{\min}^*) = N_H$ as $f^{-1}(N_{\min}^*) = z$. The last three terms represent the distortion in the entrepreneur's choice of strategy arising due to the existence of private information. The remaining terms are instead those present also in the case of common knowledge. If $\frac{\partial f^{-1}(N_{\min})}{\partial N_{\min}} < 0$, it follows that $\frac{\partial N_L}{\partial N_{\min}} > 0$ and $\frac{\partial N_H}{\partial N_{\min}} > 0$, therefore it is straightforward that the additional terms are negative. This implies that, when $f^{-1}(N_{\min})$ decreases with $N_{\min}$, the entrepreneur distorts the optimal $N^*_{\min}$ downward. That is, the optimal $N^*_{\min}$ in the presence of private information is smaller than the optimal $N^*_{\min}$ in the case of common knowledge. If $\frac{\partial f^{-1}(N_{\min})}{\partial N_{\min}} > 0$, it follows that $\frac{\partial N_L}{\partial N_{\min}} < 0$ and $\frac{\partial N_H}{\partial N_{\min}} < 0$, therefore it is straightforward to show that the additional terms are positive. This implies that, when $f^{-1}(N_{\min})$ increases with $N_{\min}$, the entrepreneur distorts the optimal $N^*_{\min}$ upward. That is, the optimal $N^*_{\min}$ in the presence of private information is bigger than the optimal $N^*_{\min}$ in the case of common knowledge.

To summarize, if Case 2 arises, the entrepreneur has no incentive to distort her target number. If Case 1 arises, at the separating equilibrium, the entrepreneur may distort $N_{\min}$ and the direction of distortion depends on whether $N_{\min}$ increases or decreases with z.

Proof of Proposition 9: Because Case 1 cannot be solved analytically, we consider only Case 2 when comparing the profitability of crowdfunding with no crowdfunding. We restrict to $z \in [0, z_1]$ and $z_1 \alpha v_L[h\overline{N} + (1-h)x_L] \leq K$ to ensure that $N_H > 0$ and Case 2 arises for all values of z on $[0, z_1]$. These assumptions are compatible with the other assumptions of the model, as long as h is not too close to 1.

Under no crowdfunding, the VC cannot unequivocally infer the value of z and can only decide whether to fund the project based on expectation over z values consistent with the entrepreneur's decision of not running the campaign.

To support the possibility that crowdfunding transmits useful information to the VC about the value of z, we consider an equilibrium where entrepreneurs facing relatively high z choose to run a campaign and those facing low z choose against it. Specifically, there exists a threshold z_0, so that if $z \geq z_0$, the entrepreneur chooses crowdfunding, and if $z < z_0$ she approaches the VC directly. If such an equilibrium exists, the VC infers that z is below the threshold z_0, and, on average, is equal to $z_0/2$ when the entrepreneur approaches the VC directly.

Consistent with our main model, we exclude the case that the project is always funded under no crowdfunding because this is unrealistic. This implies that $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_L^{NC}$, where $N_L^{NC} =$

$x_L + \frac{K - \frac{z_0}{2}\alpha v_L x_L}{h\frac{z_0}{2}\alpha v_L}$. There are two cases to consider under no crowdfunding:

a) the VC funds the project when x_H realizes as long as the expected profit from the project is nonnegative, i.e., $\overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC}$ where $N_H^{NC} = x_H + \frac{K - \frac{z_0}{2}\alpha v_L x_H}{h\frac{z_0}{2}\alpha v_L}$. Note that when $z = z_0$ this condition is more demanding than the corresponding condition derived under common knowledge, namely $\overline{N} \geq \frac{2\alpha h}{\alpha h+1} N_H(z_0)$ with $N_H(z_0) = x_H + \frac{K - z_0 \alpha v_L x_H}{h z_0 \alpha v_L}$.

b) the VC never funds the project if $\overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC}$.

To find the threshold z_0 we need to compare the entrepreneur's profit under crowdfunding with that under no crowdfunding. We have shown that in Case 2 the entrepreneur sets the optimal target number and pledge without distorting their values, and the VC and backers can infer the exact value of z as long as $N_H > 0$. Therefore, in Case 2 the entrepreneur's profit is the same as that under common knowledge if she chooses crowdfunding.

Given that the entrepreneur of type z knows her true type but the VC's decision is based upon expectation over the region $[0, z_0]$, when she chooses against crowdfunding, her expected profit under private information, $\Pi_{EP}^{NC}(z)$, can be calculated as:

$$\Pi_{EP}^{NC}(z) = \begin{cases} (1-p)\,\delta[z\alpha v_L(h\frac{\overline{N}}{2} + (1-h)x_H) + zv_L\frac{\overline{N}}{2} - K] & \text{if } \overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC} \\ 0 & \text{if } \overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC} \end{cases}. \tag{34}$$

The "P" in the subscript designates private information. In particular, for the entrepreneur of type z_0, her profit is $\Pi_{EP}^{NC}(z_0)$ and she is indifferent between crowdfunding and no crowdfunding.

From (17), in an environment with common knowledge the same entrepreneur of type z who chooses against crowdfunding has expected profit:

$$\pi_E^{NC}(z) = \begin{cases} (1-p)\,\delta[z\alpha v_L(h\frac{\overline{N}}{2} + (1-h)x_H) + zv_L\frac{\overline{N}}{2} - K] & \text{if } \overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H(z) \\ 0 & \text{if } \overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H(z) \end{cases}. \tag{35}$$

Note that $N_H(z)$ is a function of z and $N_H(z_0) < N_H^{NC}$. Based on (34) and (35) there are two possibilities:

1. When $\overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC}$, we have $\pi_E^{NC}(z = z_0) = \Pi_{EP}^{NC}(z = z_0) > 0$ because $N_H(z_0) < N_H^{NC}$. In this case, entrepreneurs of type $z > z_0$ prefer crowdfunding and those of type $z < z_0$ prefer no crowdfunding under common knowledge or under private information. That is, the entrepreneur's

decision of running a crowdfunding campaign is the same no matter whether she has public or private information about z. It is then straightforward to compute z_0 from Equation (21) and verify $z_0 < z_1$.

2. When $\left(\frac{2\alpha h}{\alpha h+1}\right) N_H(z_0) \leq \overline{N} < \left(\frac{2\alpha h}{\alpha h+1}\right) N_H^{NC}$, from (34) we have $\Pi_{EP}^{NC}(z) = 0$. This implies that for an entrepreneur with private information about z, the VC never funds her in the absence of crowdfunding. In contrast, under common knowledge, there may still exist a range over which no crowdfunding is preferred because $\pi_E^{NC}(z) > 0$ when $\overline{N} \geq \left(\frac{2\alpha h}{\alpha h+1}\right) N_H(z)$. It follows that the likelihood of crowdfunding is higher if the entrepreneur has private information about z. ■