Audit Policies under the Sentinel Effect: Deterrence-Driven Algorithms

Online Appendix

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Appendix A: Supplemental Interviews - Instrument Description

Two research instruments were developed for this study targeting medical doctors and medical auditing experts. The open-ended questionnaires were developed based on key assumptions in our model. For content validity, pilot questionnaires were reviewed by medical and insurance experts and modified as needed.

Insurance Provider/MSO Interview Questionnaire

1. How many years of experience do you have in the medical claims processing industry?
2. What is your current position?
3. Do you think "waste and abuse" is an issue in healthcare? Why/why not?
4. How familiar are you with the audit of medical claims?
5. Do medical practitioners hear about audits of other practitioners? In what ways do they hear about audits of other practitioners?
6. Is there a greater awareness of audits of other practitioners when they are sanctioned or fined? If a colleague is sanctioned or fined would this have more of an impact on his peers when they hear about it.
7. If a practitioner is aware of a medical audit,
8. Would they share what they can about this information with other practitioners?
9. Are they more likely to share information with other colleagues if they are closer to the audit (i.e., know the doctor personally or even work with him or her)?
10. From your experience, do you think medical practitioners change their claims submission behavior when they themselves are audited? In what ways do they change?
11. Do you think service practitioners change their behavior (prescription/claim submission) in any way after they hear about others who were audited? In what ways do they change and would sanctions have more of an impact on change.

Physician Interview Questionnaire

1. How many years of experience do you have as a medical doctor?
2. What is your specialty?
3. Do you work at your own practice?
4. Do you think waste and abuse is an issue in healthcare? Why/why not?
5. Do you hear about audits of other practitioners? How do you hear about them?
6. Is there a greater awareness of audits of other practitioners when they are sanctioned or fined? Does information about audits with sanctions diffuse more to other providers than information about audits that end with no sanctions?
7. If you do become aware of a medical audit,

a. Would you share what you can about this with other practitioners?

b. Are you more likely to pass this audit information along the closer you are to the practitioner audited (i.e., you personally know or work with him or her)?

8. When you hear about practitioners who were audited, how well do you know the people who were audited? Please use a scale of 1(know extremely well) to 4 (know well, but do not personally know the person) and 5 (no idea who the person is, name anonymous) to rate medical practitioners of five recent audits that you had become aware of.
9. Do you think medical practitioners change their claims submission behavior when they themselves are audited? In what ways?
10. Do you think service practitioners change their behavior (prescription/claim submission) in any way after they hear about others who were audited? How do they change their behavior and does it matter if the audit they heard about resulted in a sanction or not?

Appendix B: Physician Survey Instrument

Please read the following scenario and select the likelihood of events (1: 0% likely, 100% likely):

Dr. Bill Smith, ophthalmologist, performed and billed for a Paretinal Laser Photocoagulation for his patient suffering from a diabetic retinopathy. Ten weeks later, the patient returned with the same condition and had the same procedure again. Last month, Dr. Smith got audited and denied reimbursement because his office evidently failed to check the 3-month eligibility period.

	1	2	3 (50%)	4	5
1. Bill will warn his colleagues at the same practice/hospital about what happened.					
2. Bill's friends will warn their colleagues about what happened.					
3. Bill is more likely to be flagged for suspicious behavior in the future.					
4. Bill will become more vigilant in the future.					
5. Practitioners who heard what happened through this word of mouth diffusion will become more vigilant in the future.					

Please read the following scenario and select the likelihood of events (1-least likely, 5-most likely):

In a news brief broadcasted last month from an insurance provider to all practitioners, about 4 audited issues were listed. One of which was described as follows: "Potential incorrect billing occurred when Panretinal (Scatter) Laser Photocoagulation (CPT code 67228) is paid more than once, per eye, within the global surgery period."

	1	2	3 (50%)	4	5
6. Practitioners receiving this broadcast will find the information useful.					
7. Practitioners receiving this broadcast will become more vigilant in the future					

8. Between receiving *this information* through word of mouth from a trusted friend/colleague or broadcast, which one is more effective.

Word of mouth Both Broadcast

9. Practitioners receiving this information through both word of mouth and broadcast are more likely to change than ones who receive it from broadcast only.

Yes No

10. In general, practitioners receiving billing and audit information through both word of mouth and broadcast are more likely to change than ones who receive it from broadcast only.

Yes No

Please read the following scenario and select the likelihood of events (1: 0% likely, 5: 100% likely):

Dr. Johnn Weber, general surgeon, performed and billed for a Bariatric Surgery for his patient who had a BMI of 40, a fasting blood sugar level of 90 mg/dL and no known co-morbidities. Last month, Dr. Weber got audited and denied reimbursement because his office failed to validate co-morbidity criteria.

	1	2	3 (50%)	4	5
11. John will warn his colleagues at the same practice/hospital about what happened.					
12. John's friends will warn their colleagues about what happened.					
13. John is more likely to be flagged for suspicious behavior in the future.					
14. John will become more vigilant in the future.					
15. Practitioners who heard what happened through this word of mouth diffusion will become more vigilant in the future.					

Please read the following scenario and select the likelihood of events (1: 0% likely, 5: 100% likely):

In a news brief broadcasted last month from an insurance provider to all practitioners, about 4 audited issues were listed. One of which was described as follows: "The surgical management for the treatment of morbid obesity is considered reasonable and necessary for Medicare beneficiaries who have a BMI > 35, have at least one co-morbidity related to obesity and have been previously unsuccessful with the medical treatment. "

	1	2	3 (50%)	4	5
16. Practitioners receiving this broadcast will find the information useful					
17. Practitioners receiving this broadcast will become more vigilant in the future					

18. Between receiving this information through word of mouth from a trusted friend/colleague or broadcast, which one is more effective.

Word of mouth Both Broadcast

19. Practitioners receiving this information through both word of mouth and broadcast are more likely to change than ones who receive it from broadcast only.

Yes No

20. In general, practitioners receiving billing and audit information through both word of mouth and broadcast are more likely to change than ones who receive it from broadcast only.

Yes No

21. What is your specialty?

22. How many years of experience do you have as a medical doctor?

5-10 10-15 15-20 20-25 25-30 30+

23. What is your type of practice?

Sole practice Group practice Large Institution Other

24. How many licensed medical practitioners are in your practice?

1-10 10-20 20-30 30-40 40-50 50+

25. Do you receive broadcast messages (clinical or legislative updates) from your insurance company?

	Never	Weekly	Bi-weekly	Monthly	Quarterly	Yearly
New legislation						
New procedure						
New Cases of fraud						
Common cases of waste and abuse						

26. How likely are to communicate (socially or professionally) with fellow licensed medical practitioners who share the same hospital privileges on a regular basis (1-least likely, 5-most likely)?

1 2 3 4 5

27. How likely are to communicate (socially or professionally) with fellow licensed medical practitioners within the same specialty on a regular basis (1-least likely, 5-most likely)?

1 2 3 4 5

28. How likely are to communicate (socially or professionally) with fellow licensed medical practitioners within the same area (zip code) on a regular basis (1-least likely, 5-most likely)?

1 2 3 4 5

29. How likely are to communicate (socially or professionally) with fellow licensed medical practitioners who teach at the same institution on a regular basis (1-least likely, 5-most likely)?

1 2 3 4 5

30. How likely are to communicate (socially or professionally) with fellow licensed medical practitioners who attended the same university at the same time on a regular basis (1-least likely, 5-most likely)?

1 2 3 4 5

Appendix C: Physician Survey Results

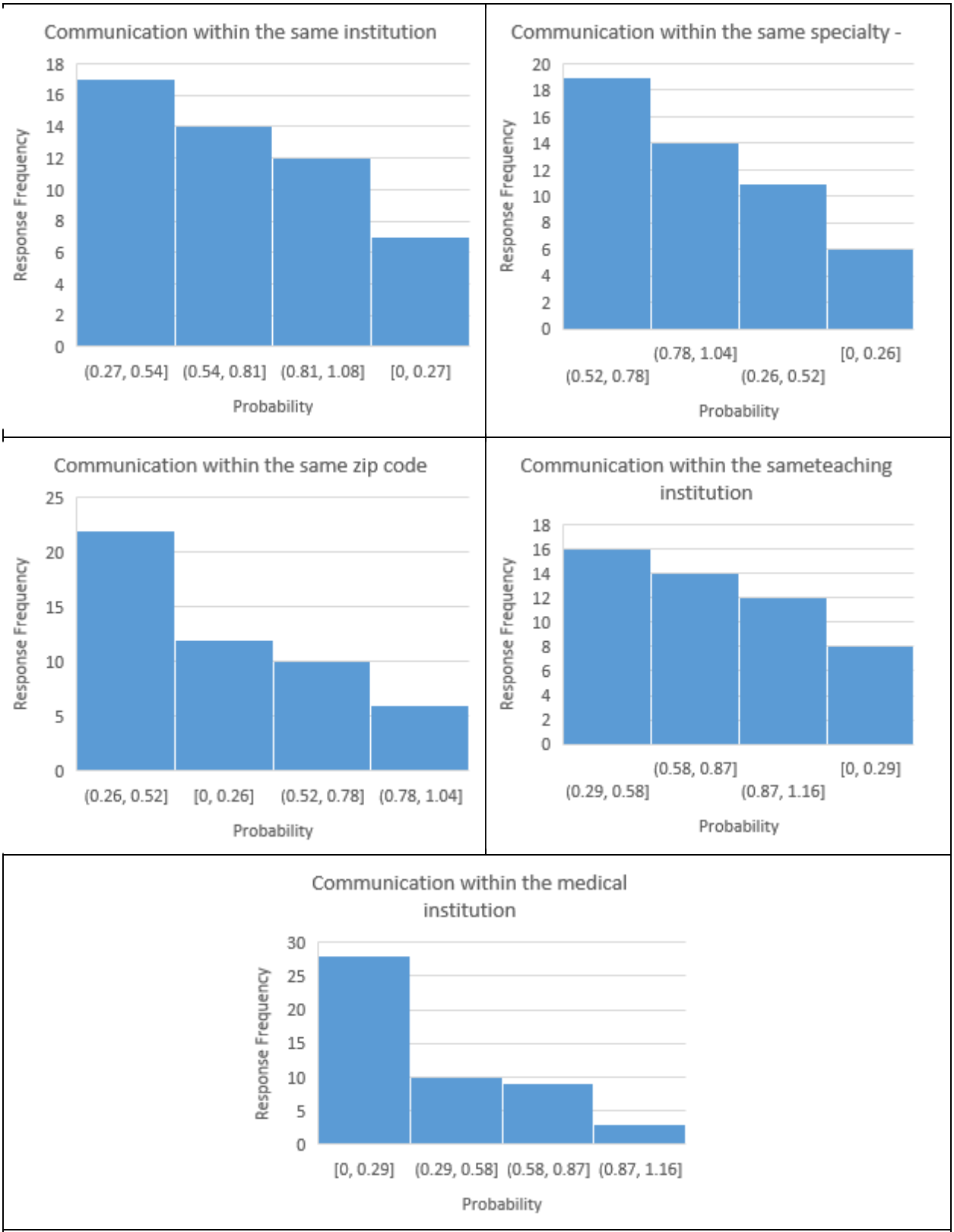

Figure 1: Physician Survey Responses- Physician Network Structure

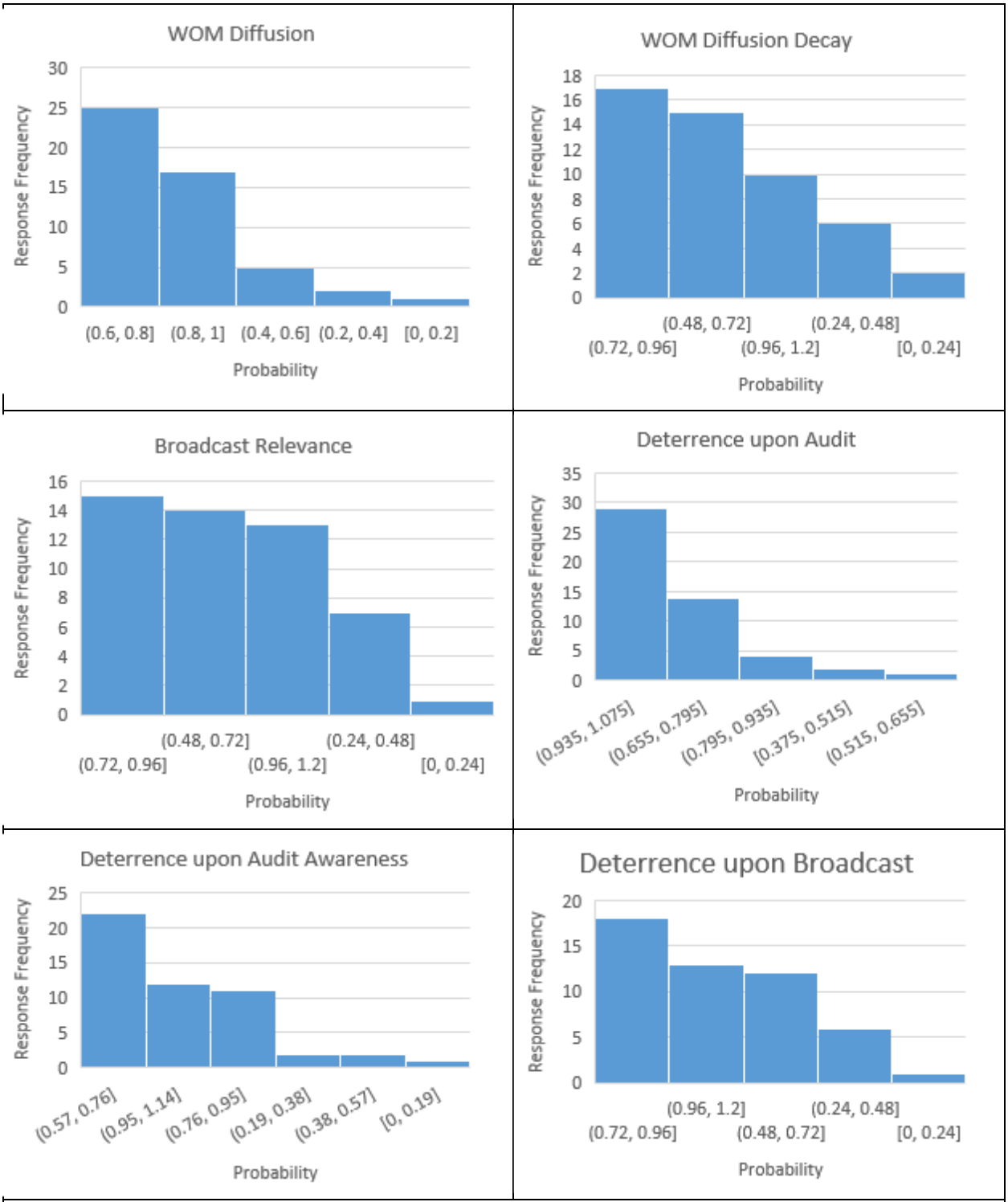

Figure 2: Physician Survey Responses- Audit Information Diffusion and Deterrence Probabilities

Appendix D: Hillsborough County - Network Structure

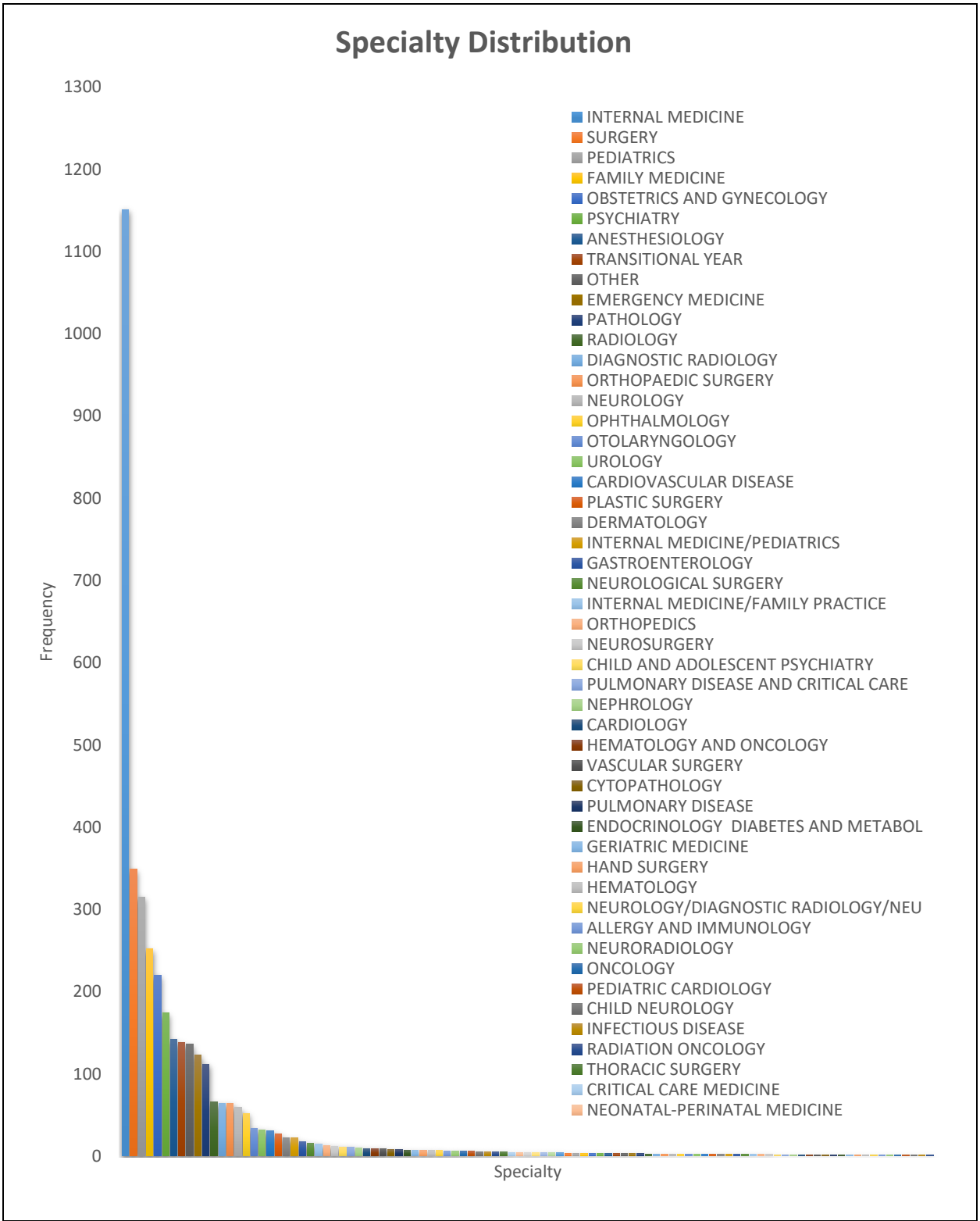

Figure 3: Hillsborough County Network – specialty distribution**
*** For space limitations, only the top 50 (out of 101) specialties were included in the legend*

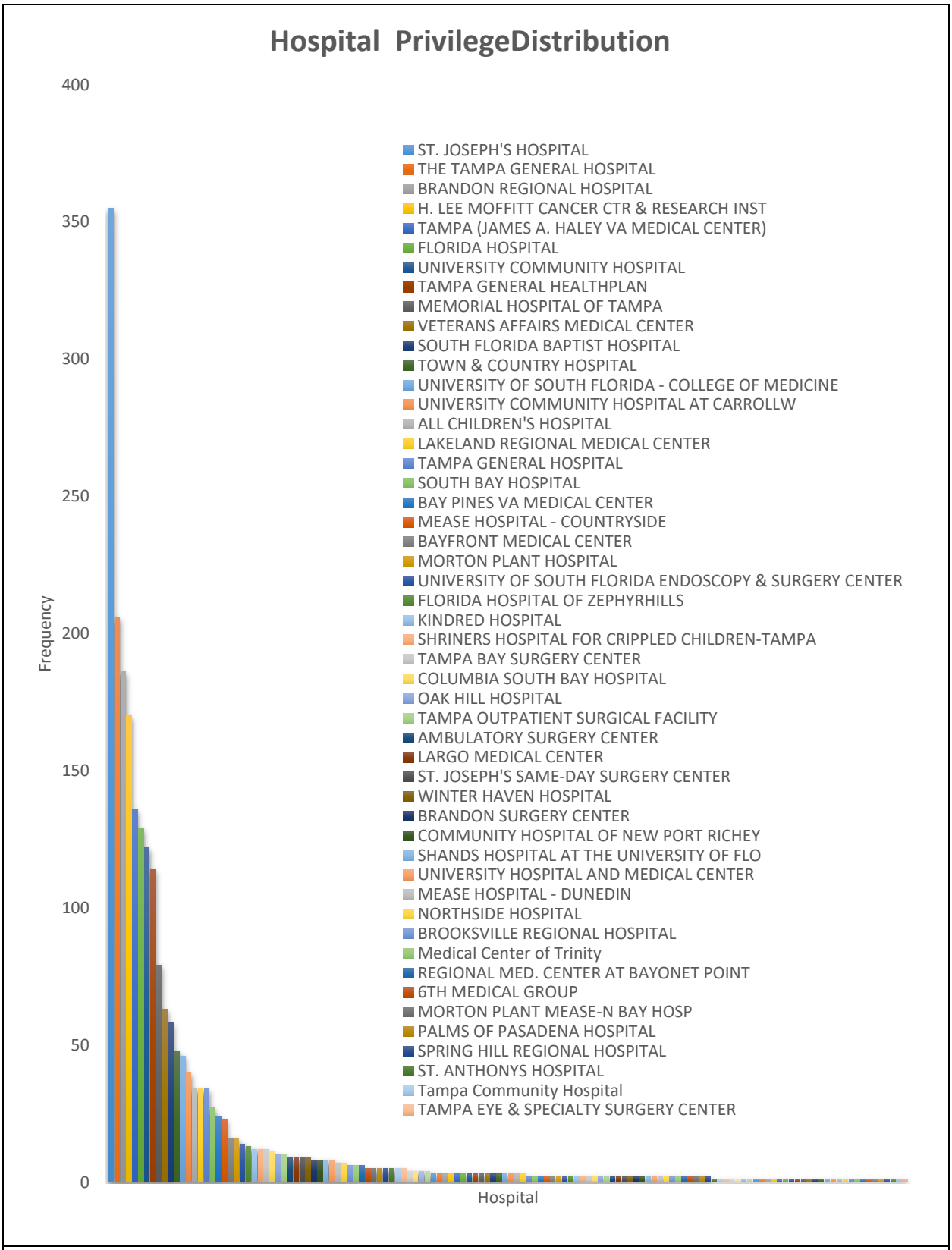

Figure 4: Hillsborough County Network – hospital privilege distribution**
*** For space limitations, only the top 50 (out of 187) hospitals were included in the legend*

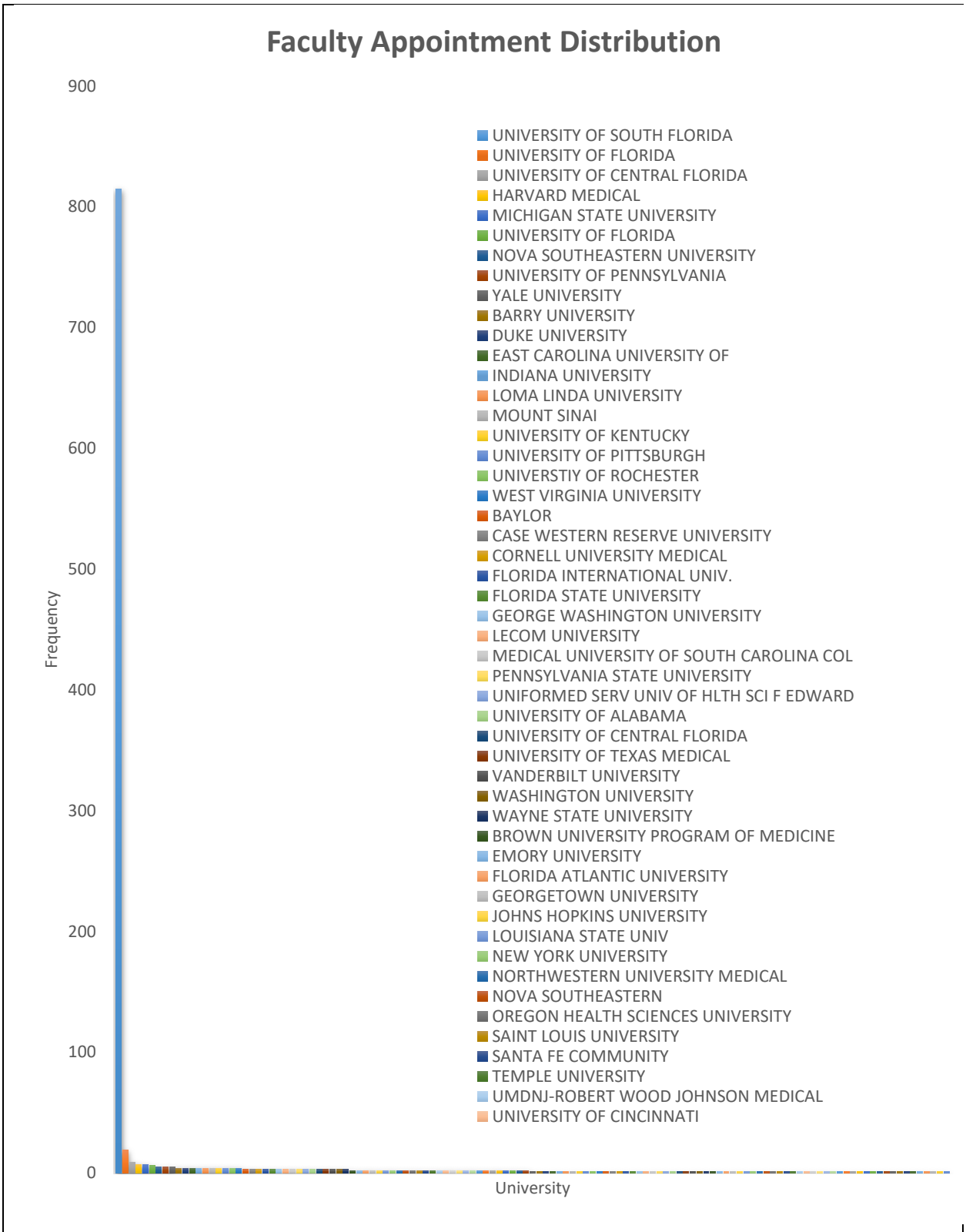

Figure 5: Hillsborough County Network – faculty appointment distribution**
*** For space limitations, only the top 50 (out of 125) institutions were included in the legend*

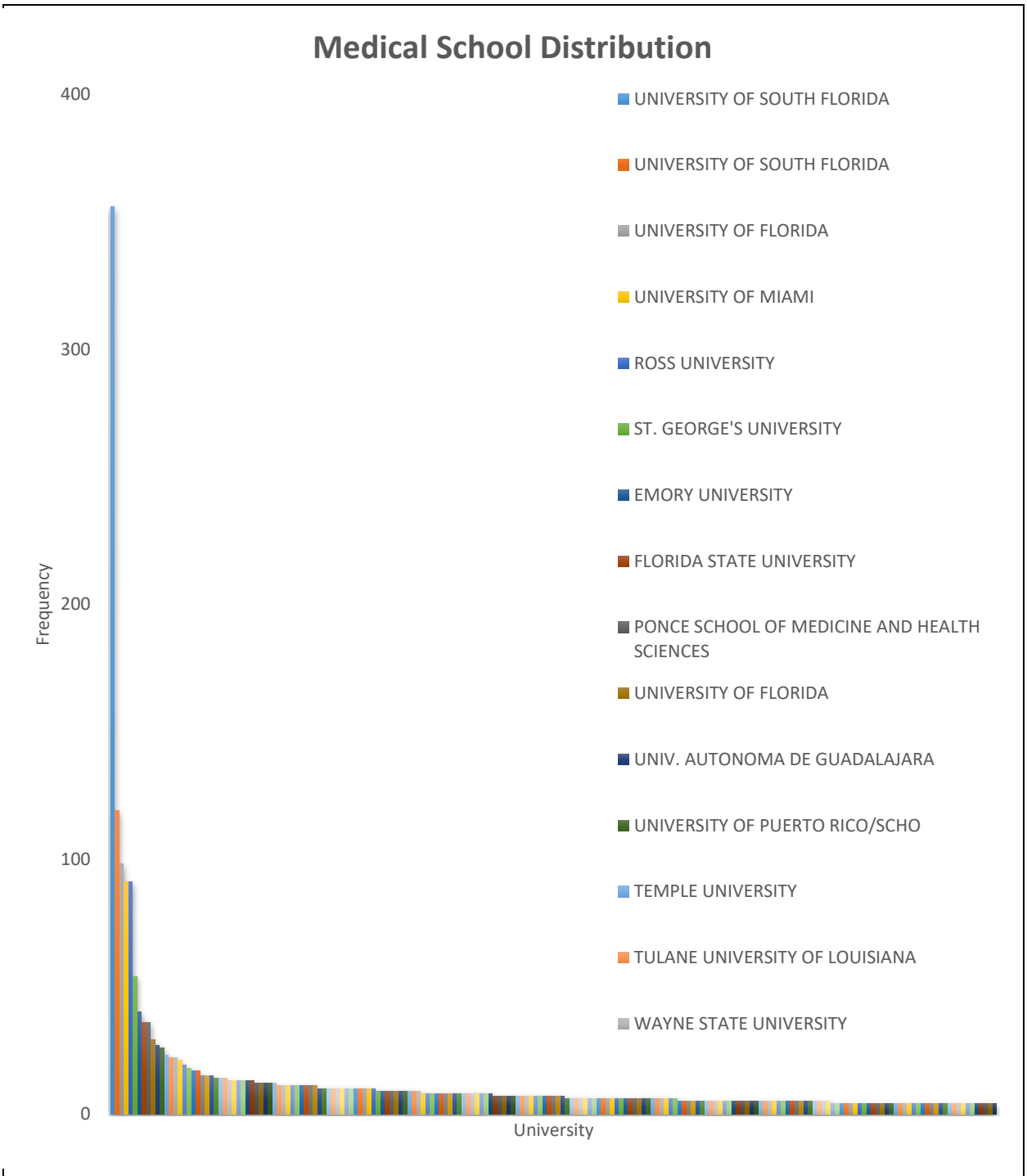

Figure 6: Hillsborough County Network – medical school distribution**
*** For space limitations, only the top 200 (out of 1495) hospitals were included in this chart; The top 15 are listed in the legend*

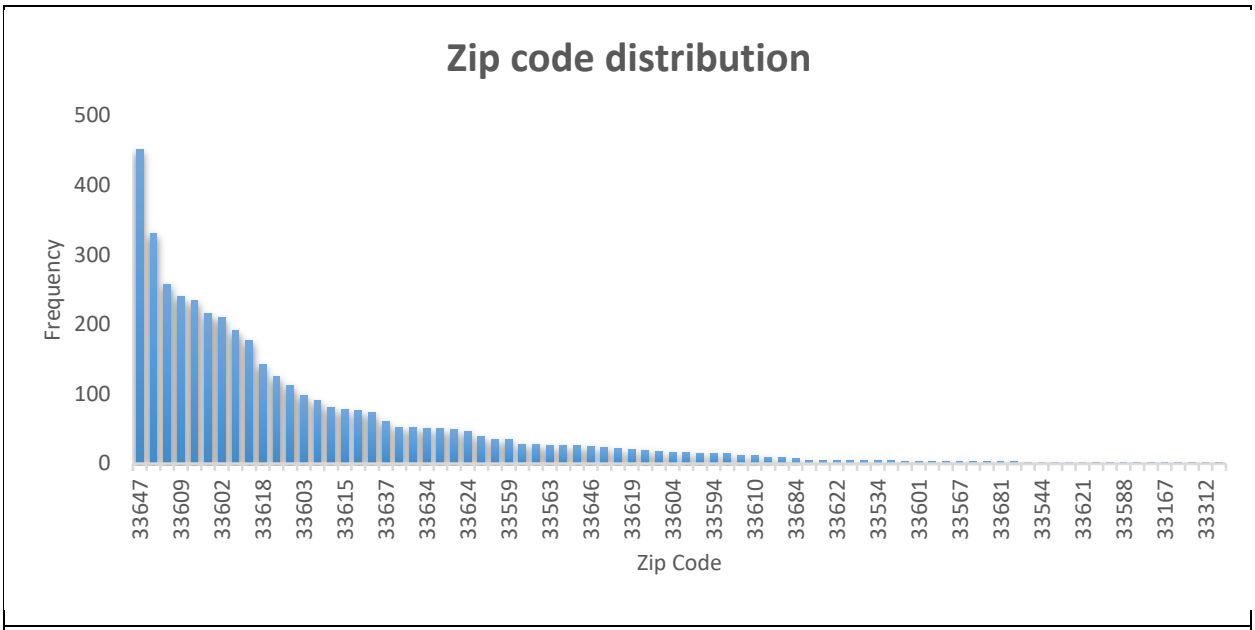

Figure 7: Hillsborough County Network – zip code distribution

Appendix E: Miami-Dade County - Network Topology

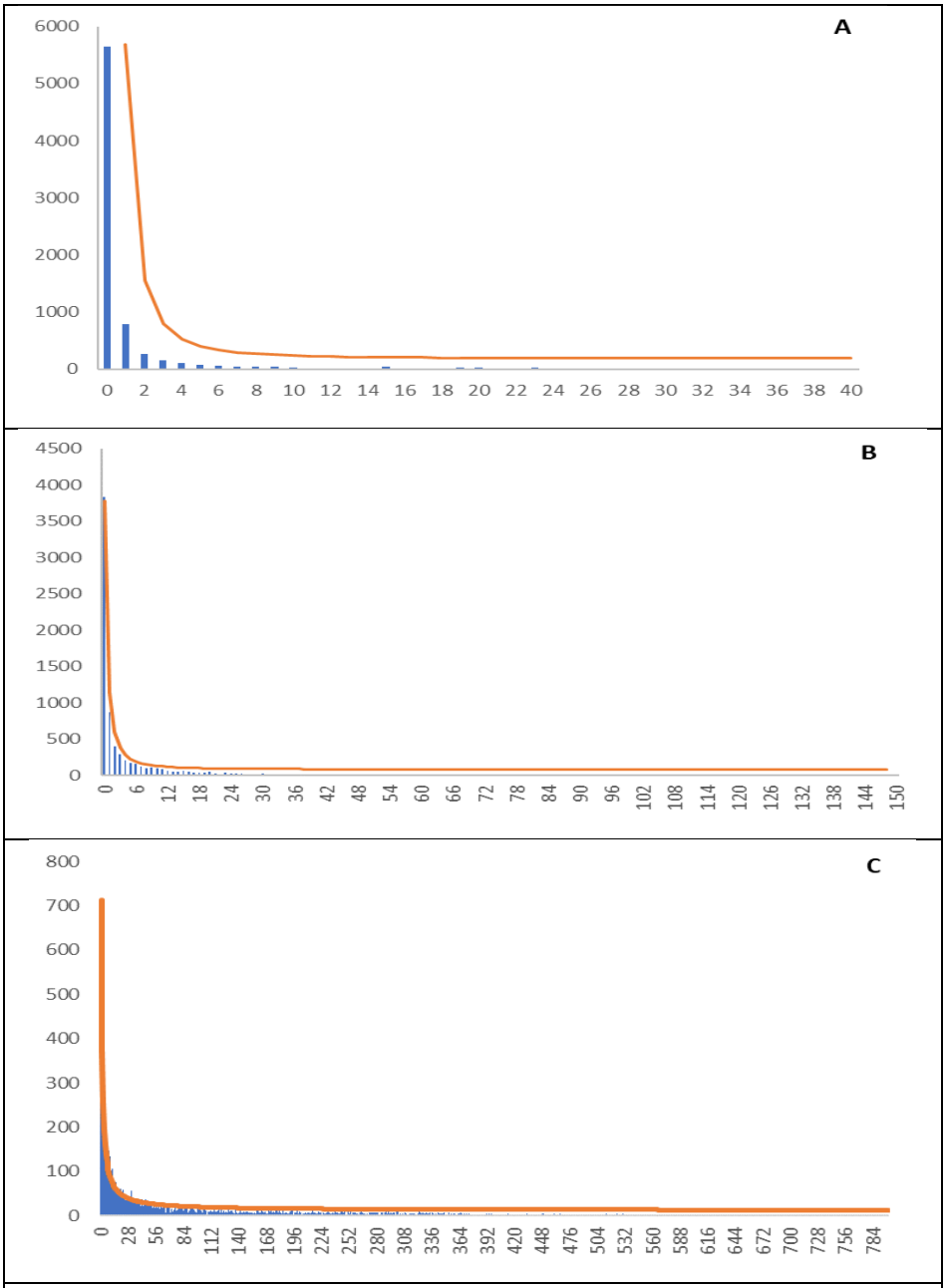

Figure 8: Degree distribution and power law approximation for Miami-Dade County's physician networks (A) 4 or more common attributes (B) 3 or more common attributes (C) 2 or more common attributes

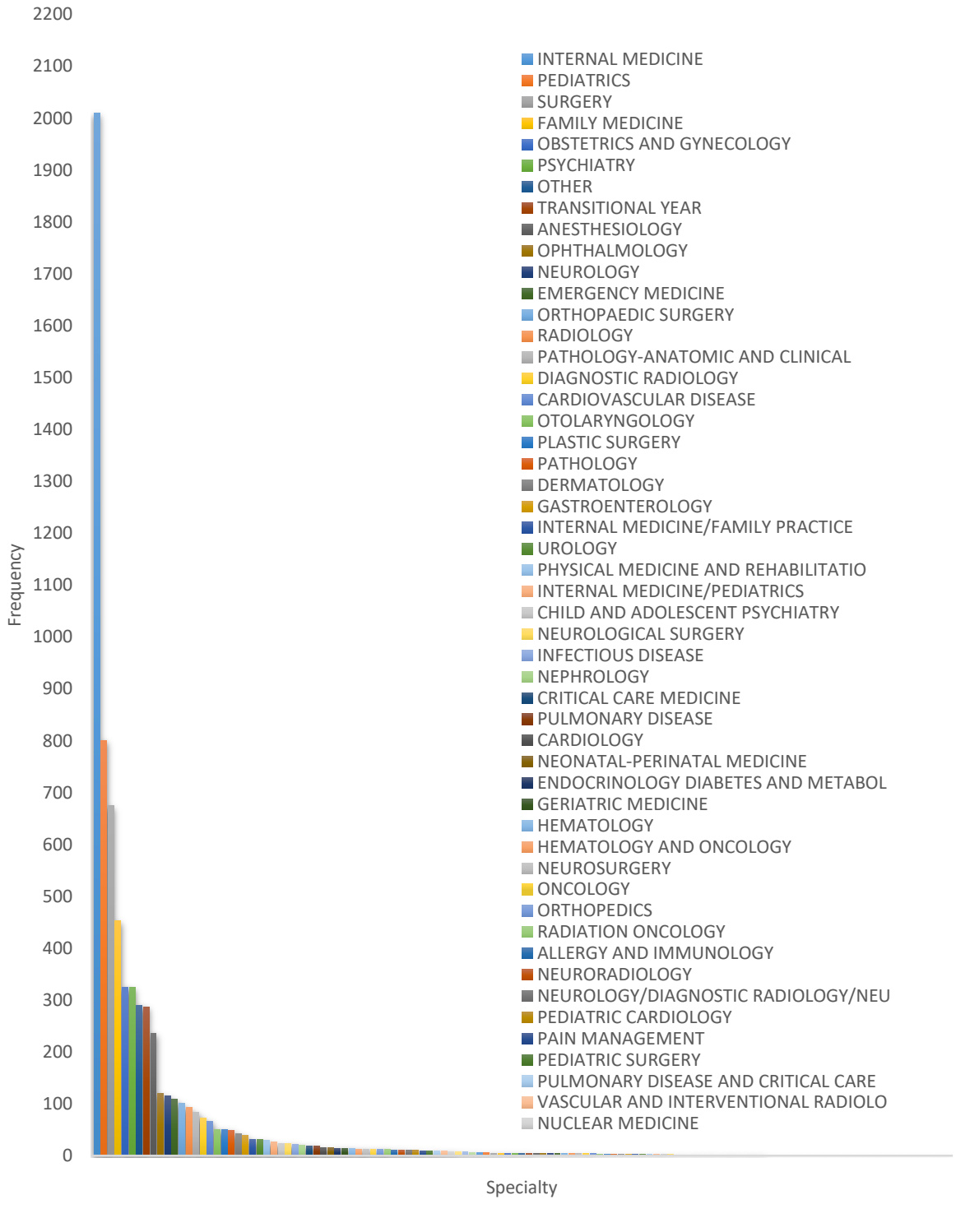

Figure 9: Miami-Dade County Network – specialty distribution**
*** For space limitations, only the top 50 (out of 121) specialties were included in the legend*

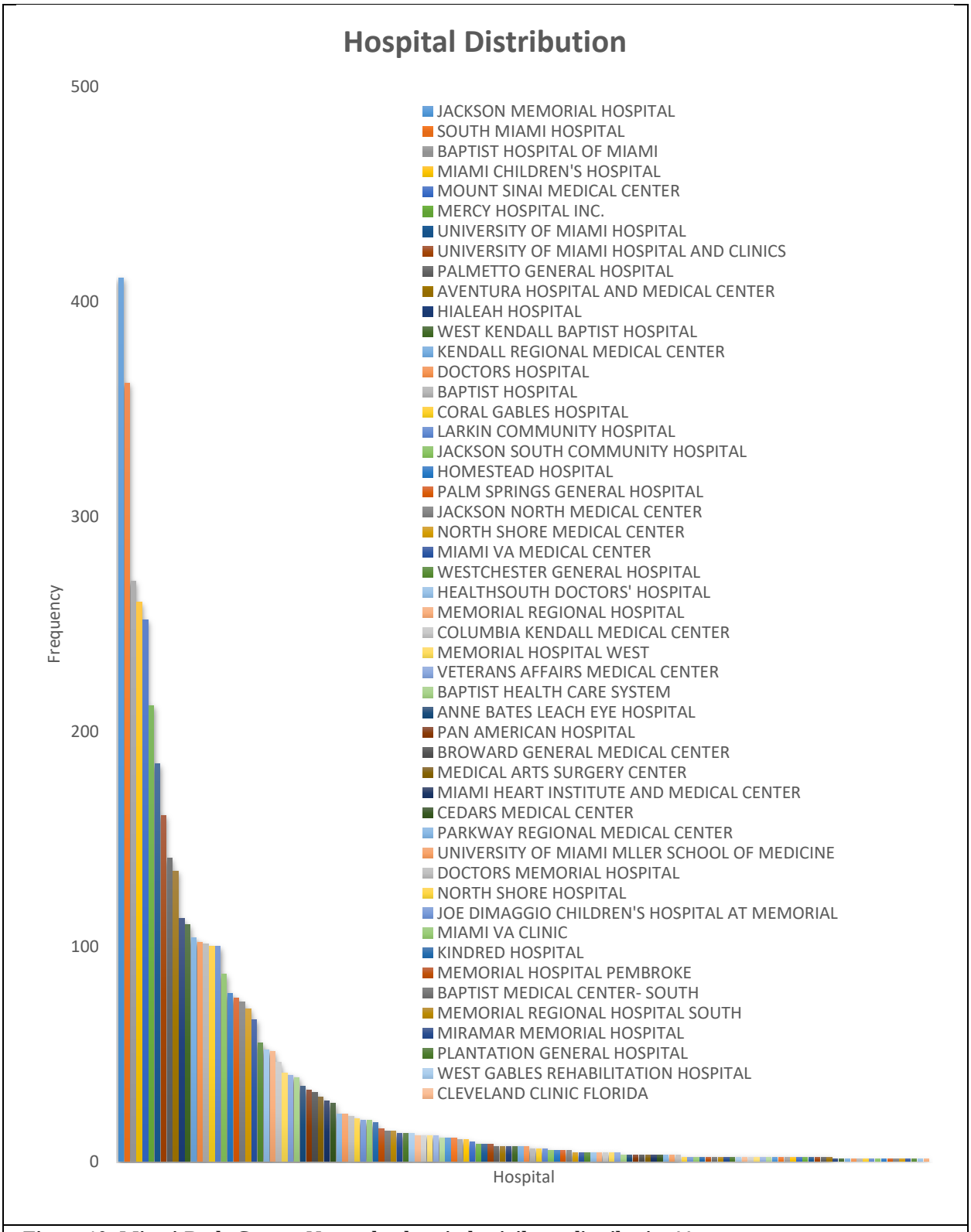

Figure 10: Miami-Dade County Network – hospital privileges distribution**

*** For space limitations, only the top 50 (out of 197) hospitals were included in the legend*

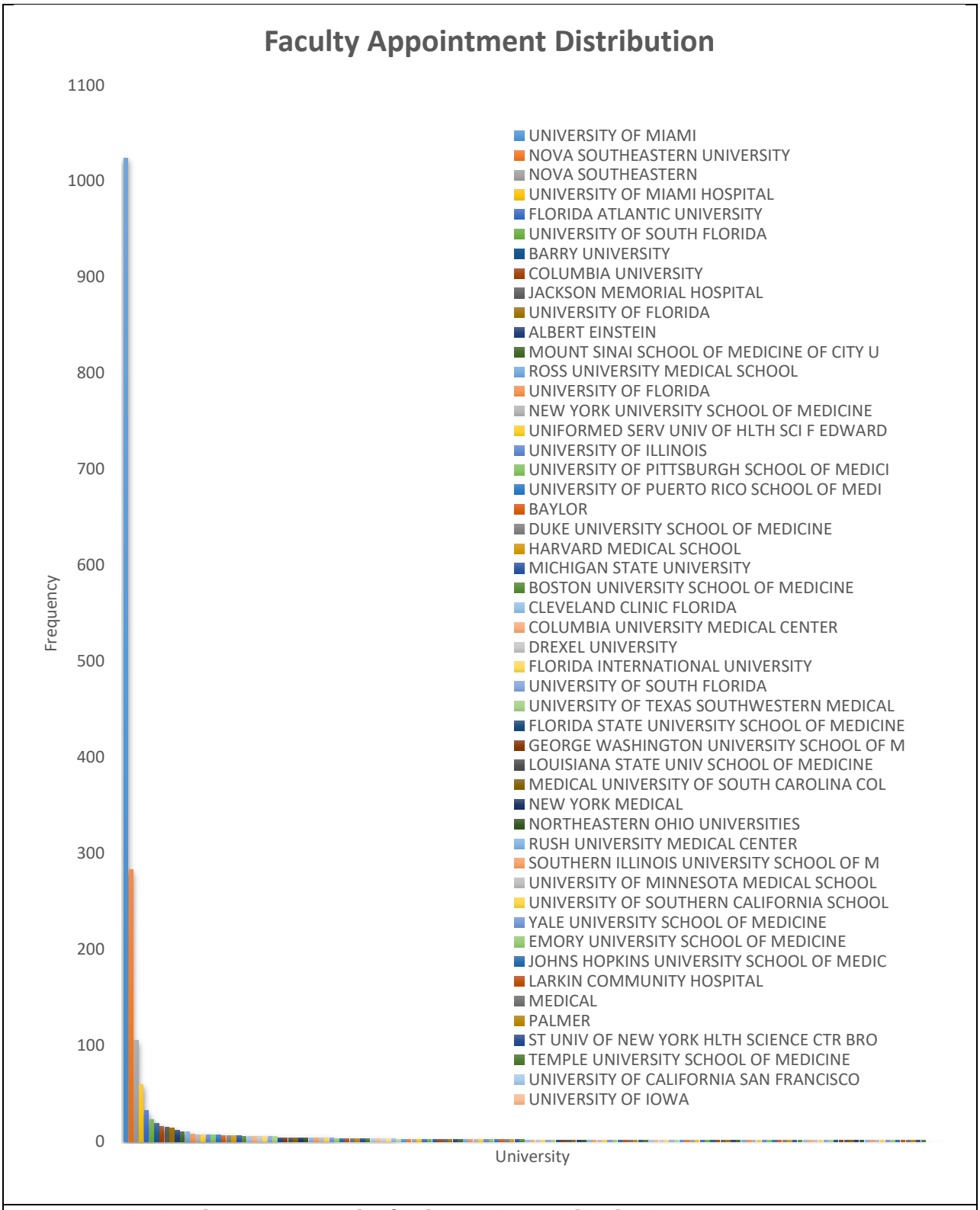

Figure 11: Miami-Dade County Network – faculty appointment distribution**
*****For space limitations, only the top 50 (out of 159) institutions were included in the legend*

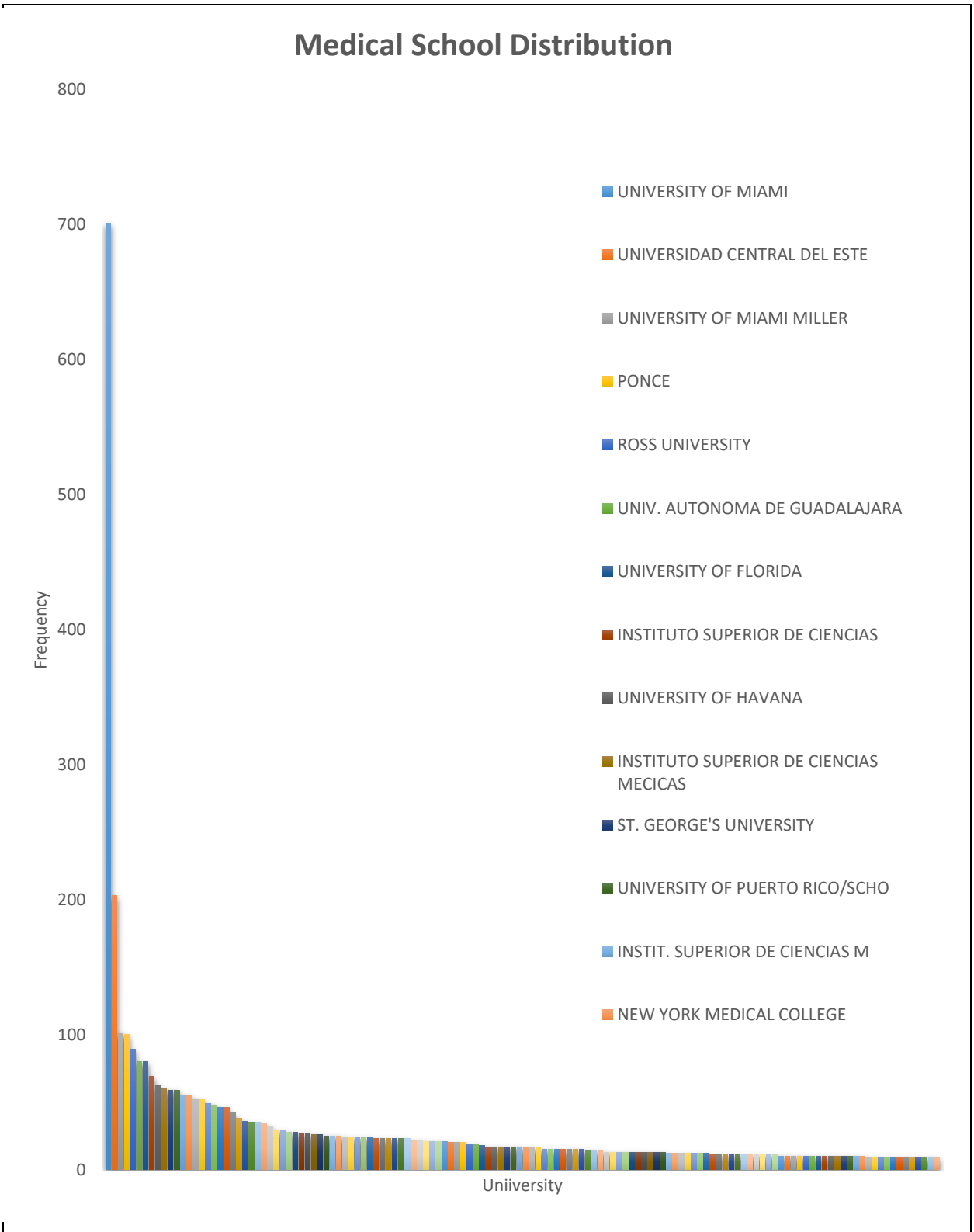

Figure 12: Miami-Dade County Network – medical school distribution**
*** For space limitations, only the top 200 (out of 2214) hospitals were included in this chart; The top 15 are listed in the legend*

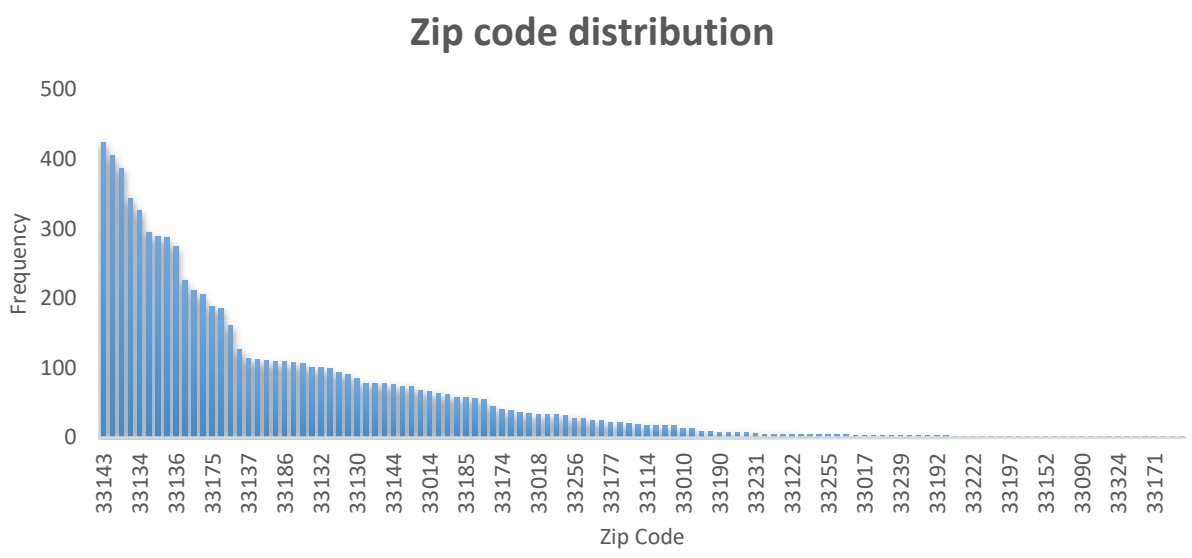

Figure 13: Miami-Dade County Network – zip code distribution

Appendix F: Supplemental Interviews - Report

In order to evaluate our model assumptions, we have conducted surveys with medical doctors and experts in medical auditing. Interview transcripts were coded and main concepts identified. Then, extracted quotes were classified in two different categories: diffusisson-deterrence model validation and diffusion-deterrence model extension. Complete interview transcripts will be made available through the online appendix.

Model Validation

The practitioner network - Closeness measure in the medical network

- Specialty:

		Quote	Assumption Support
MD Transcripts	Transcript 1	"People within the same specialty are also likely to impact each other because people within the same specialty are likely to experience the same issues."	Yes
	Transcript 2	None	N/A
	Transcript 3	"If someone was audited for some procedure code, and that happened to be a gastroenterologist code, then of course that will affect my behavior." "And if it is a friend in the same field, he'll make us aware of what happened."	Yes
	Transcript 4	"Specialty definitely matters."	Yes
	Transcript 5	None	N/A
	Transcript 6	None	N/A
	Transcript 7	None	N/A
	Transcript 8	None	N/A
Insurance Transcripts	Transcript 1	"For example, we've seen cases where a certain specialty was being audited, and the leadership within that specialty would send out messages to all the facilities even if only one or two of the facilities were the center of the issue."	Yes
	Transcript 2	"Individual doctors turn to other doctors in their specialty that are being audited because they exchange information amongst themselves."	Yes
	Transcript 3	None	N/A

- Hospital:

		Quote	Assumption Support
MD Transcripts	Transcript 1	"It [the interaction] could be with the hospital medical staff."	Yes
	Transcript 2	None	N/A
	Transcript 3	"If, for example, I know a doctor who works in the same hospital has been audited, even if he is not close to me, I would still want to check my own practice and see if we are doing things right or not."	Yes
	Transcript 4	"Mainly at the hospital"	Yes
	Transcript 5	None	N/A

	Transcript 6	None	N/A
	Transcript 7	None	N/A
	Transcript 8	None	N/A
Insurance Transcripts	Transcript 1	None	N/A
	Transcript 2	None	N/A
	Transcript 3	None	N/A

- Practice:

		Quote	Assumption Support
MD Transcripts	Transcript 1	"The interaction could be in the same place where my practice is located."	Yes
	Transcript 2	None	N/A
	Transcript 3	"We do, especially if one member of the group has been involved [in audit] of course we get aware of it."	Yes
	Transcript 4	None	N/A
	Transcript 5	None	N/A
	Transcript 6	None	N/A
	Transcript 7	I would say [that I share audit information with] people that are in my practice."	Yes
	Transcript 8	None	N/A
Insurance Transcripts	Transcript 1	None	N/A
	Transcript 2	None	N/A
	Transcript 3	None	N/A

- Social/Professional community:

		Quote	Assumption Support
MD Transcripts	Transcript 1	"It [interaction] could also be in the community where we have community gatherings"	Yes
	Transcript 2	None	N/A
	Transcript 3	None	N/A
	Transcript 4	"You know there is spread in the medical community."	Yes
	Transcript 5	None	N/A
	Transcript 6	"In meetings, we talk to each other."	Yes
	Transcript 7	None	N/A
	Transcript 8	None	N/A
Insurance Transcripts	Transcript 1	"There is always word of mouth that can come just from socializing or you know participating or being involved in communities of practice."	Yes
	Transcript 2	None	N/A
	Transcript 3	None	N/A

Audit Information Diffusion

- Word of Mouth Diffusion:

		Quote	Assumption Support
MD Transcripts	Transcript 1	"During a hallway conversation with colleagues, someone mentions xyz was audited."	Yes
	Transcript 2	[Do you hear about audits of other practitioners? How do you hear about them?] "Word of mouth, media"	Yes
	Transcript 3	"Some people talk among themselves. They would say that they have been audited and talk about what happened."	Yes
	Transcript 4	"word of mouth" "Most of the time it's from the physicians themselves, from colleagues."	Yes
	Transcript 5	"No I don't hear [about audits]. Doctors don't talk about audits."	No
	Transcript 6	"From colleagues. In meetings, we talk to each other. We complain to each other about it [audit]."	Yes
	Transcript 7	"I mean if I hear about that [audit], we [physicians in the practice] will generally discuss what happened."	Yes
	Transcript 8	"No I don't. Even 3 or 4 people removed. It doesn't come up in the conversation, like 'hey did you hear about doctor so and so from two years before us'? I never hear anything like that at all."	No
Insurance Transcripts	Transcript 1	"There is always word of mouth that can come just from socializing or participating or being involved in communities of practice." "When a member of your profession is experiencing negative effects from their common business practices, folks are interested in learning more. If for nothing else than to just avoid the potential pitfalls." "Look, when it happens to a friend at minimum, that's kitchen table conversation at home and certainly water cooler conversation at the office."	Yes
	Transcript 2	"So the answer is yes. I do believe that when doctors or healthcare service providers are audited, that information gets conveyed to other providers and that's a good thing." "Yes, any time a doctor gets notified that he or she individually or the hospital is being audited some way or another, they usually communicate that to other providers that are similar."	Yes
	Transcript 3	"If a particular plan is performing audits, practitioners frequently go to their peers and mention that XYZ health plan sent them a letter and want to audit a percentage of their claims; or if CMS is doing that as well… So they talk about it with their peers. All the time."	Yes

- **Deterrence – Behavioral Change**

  - Audit Effect:

	Quote	Assumption Support

MD Transcripts	Transcript 1	"I do think that the practitioners do change because the audit experience is not exactly a pleasant one."	Yes
	Transcript 2	"It may permanently change their behavior, depending upon the penalty."	Yes
	Transcript 3	"Well I think if after auditing, they are convinced they were not coding properly, they change their behavior. If on the other hand, they were clear that nothing wrong was found then they continue their old behavior."	Yes
	Transcript 4	"Definitely if there are sanctions, they will be more careful going forward; otherwise they can lose their license or get more sanctions."	Yes
	Transcript 5	None	N/A
	Transcript 6	"Yes, they learn from their mistakes. Basically what they do is learn how to game the system. They say OK, they got me this time but they're not going to get me next time"	Yes
	Transcript 7	"I would hope so. I think that I would generally say yes [change behavior after an audit]."	Yes
	Transcript 8	"I guess it would depend on the result of the audit; if they got in trouble or they were fined then of course I would think they would change their ways but if the audit comes through and says hey everything looks good, I would say I would see no reason for them to change their ways at all."	Yes
Insurance Transcripts	Transcript 1	"Absolutely they do [change after audit]."	Yes
	Transcript 2	"When a doctor is audited and found to have a problem, we educate that doctor, we expect that doctor [to] change. And most of the time they do. If they don't then we take action."	Yes
	Transcript 3	"Well like I said, it depends on the audit. So let's say primary care physicians went through an E&M code audit, and they know that they were sanctioned because of a repeated specific issue like copying and pasting elements from the EMR system, consistent with a higher level E&M, they will certainly change their behavior. So they will stop the copying and pasting from the system and that will likely lower their level of E&M coding."	Yes

- Sentinel Effect:

		Quote	Assumption Support
MD Transcripts	Transcript 1	[Now as opposed to somebody who has been audited, if somebody hears about the audit, do you think they will change behavior?] "So I would say it really depends upon the person. Most of the practitioners will try to prevent being in a situation where they are audited."	Yes
	Transcript 2	"Change of behavior will occur only if they are personally charged and penalized."	No
	Transcript 3	"Just hearing about the audit definitely affects people." "I think seeing that someone is audited in the community is enough to change your behavior."	Yes
	Transcript 4	[After hearing about an audit, would you change behavior?] I think so because, as I said before, you don't want the audit coming to your own office. So, I myself would change my pattern if I'm unintentionally doing something wrong then I will correct it."	Yes
	Transcript 5	None	N/A
	Transcript 6	"Yes, depending on the information they get. They'll say, oh I do that too, or oh I don't do that, I'd better do it."	Yes
	Transcript 7	"I think, well it depends. If they are doing things right to begin with	Yes

		then they probably don't have to change their behavior. But if they're billing you know… if they're billing fraudulently I would think that clearly that would make them think twice, but I think that the biggest single thing is impetus for proper documentation."	
	Transcript 8	"But just the fact that someone got audited doesn't mean anything to me because as I understand it, and I could be wrong, but as I understand it the audits are random. It's like a tax audit, they just pick a name. It's not because they see something wrong; they just do a random audit to make sure everything is fine."	No
Insurance Transcripts	Transcript 1	"We see changes in operational practices or behaviors when they know we're looking. There is a legal quote, and I forget who coined it, but it goes something along the lines of 'sunlight is the best disinfectant.' So, when an inappropriate practice comes to light, organizations will usually correct those practices, at least temporarily, as long as we have a program that's looking at it."	Yes
	Transcript 2	"I hope so. That's the reason that people do audits. The intent is not so much to address that one claimant, the one provider who is submitting claims that are of concern, but actually to have a larger effect on other providers who are submitting claims so they question themselves." "The other way is that we announce to large group of providers, for example a group orthopedic doctors, that we are auditing all orthopedic claims, or doing a random audit of orthopedic claims over the next six months. Then what happens, is just by virtue of making the announcement that we are going to be doing it and that we're going to be looking, we usually get individual doctors to practice differently, and be more careful about submitting claims. We call that the sentinel effect."	Yes
	Transcript 3	"If they hear from a reliable source, let's say a specialty or a medical society, a recognized expert in the field or a colleague that they truly have a mentor relationship with, then they would likely change their behavior based on the fact that they trust the source providing the information."	Yes

Model Extension

- **Broadcast**

		Quote
MD Transcripts	Transcript 1	"Medical societies are repeatedly providing information regarding that [audits]." "The CMS weekly update or the periodic updates for the state."
	Transcript 2	Media
	Transcript 3	None
	Transcript 4	None
	Transcript 5	None
	Transcript 6	None
	Transcript 7	"We see more of them [audits] in the newspaper than I hear personally."
	Transcript 8	"The only audits I ever hear about are the same audits everyone hears about in the news."
Insurance Transcripts	Transcript 1	"That's something we've been in the news for, the over-prescription of opioids."
	Transcript 2	"At my health plan, we call it 'provider update.' Different health plans have different ways of communicating. That's a monthly conversation. That's actually an e-mailed newsletter. In that newsletter, every month, we are simply relaying the fact that we are doing audits in

		various positions throughout hospitals or some individual provider related to a particular topic, to make them aware of the fact that we are looking and checking the claims that have been billed to verify that they're accurate."
	Transcript 3	None

Appendix G: Additional Simulation Results

Fairness Scenarios

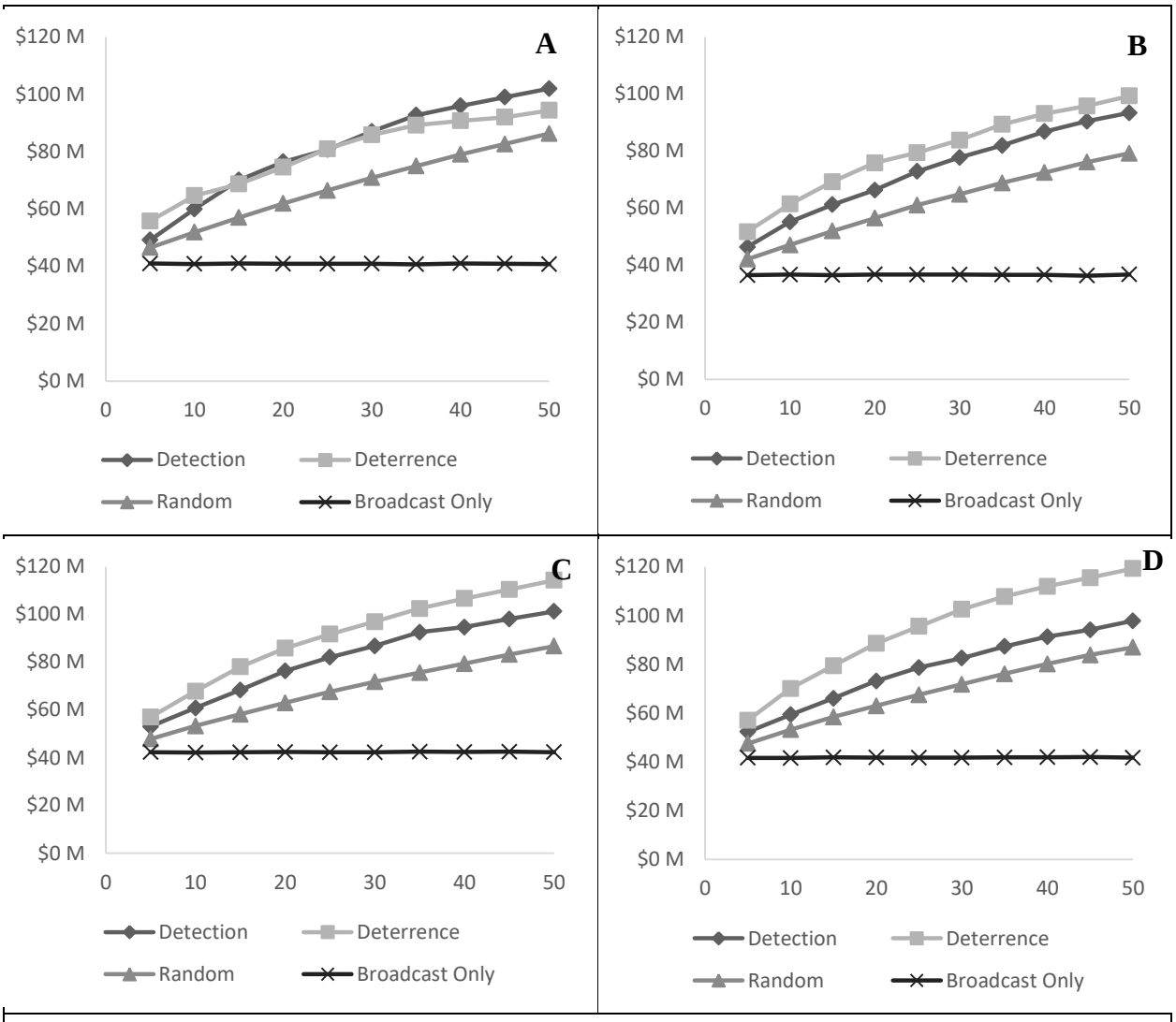

Figure 14: Y-axis: Network Deterrence Amount X-axis: Number of Audited Practitioners
(A) Fairness – 6% (B) Fairness – 16.5% (C) Fairness – 20% (D) Fairness – 30%

Diffusion Level Scenarios

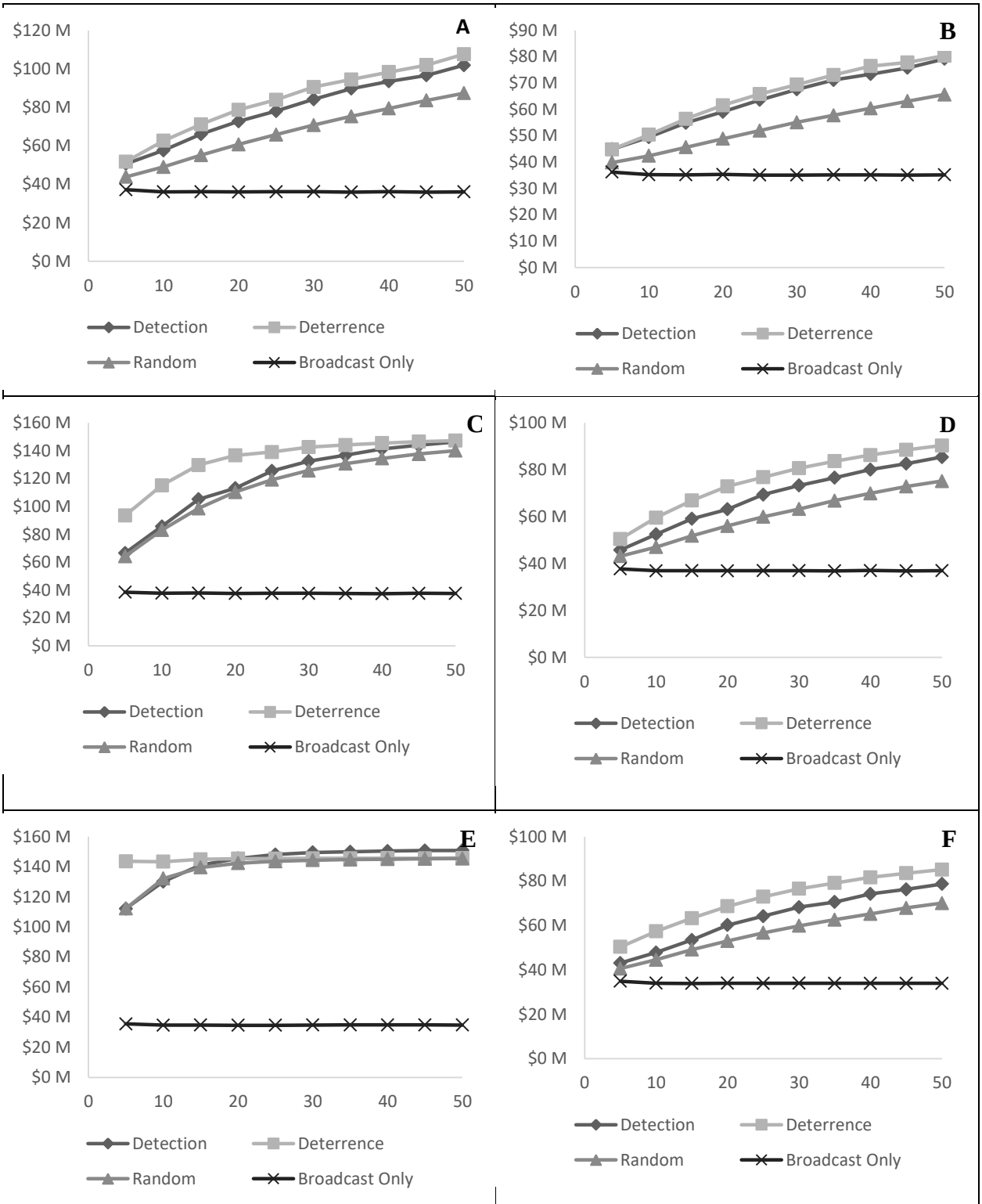

Figure 15: Y-axis: Network Deterrence Amount X-axis: Number of Audited Practitioners
(A) Two-level Diffusion, Low Decay (B) Two-level Diffusion, High Decay
(C) Three-level Diffusion, Low Decay (D) Three-level Diffusion, High Decay

(E) Four-level Diffusion, Low Decay **(F) Four-level Diffusion, High Decay**

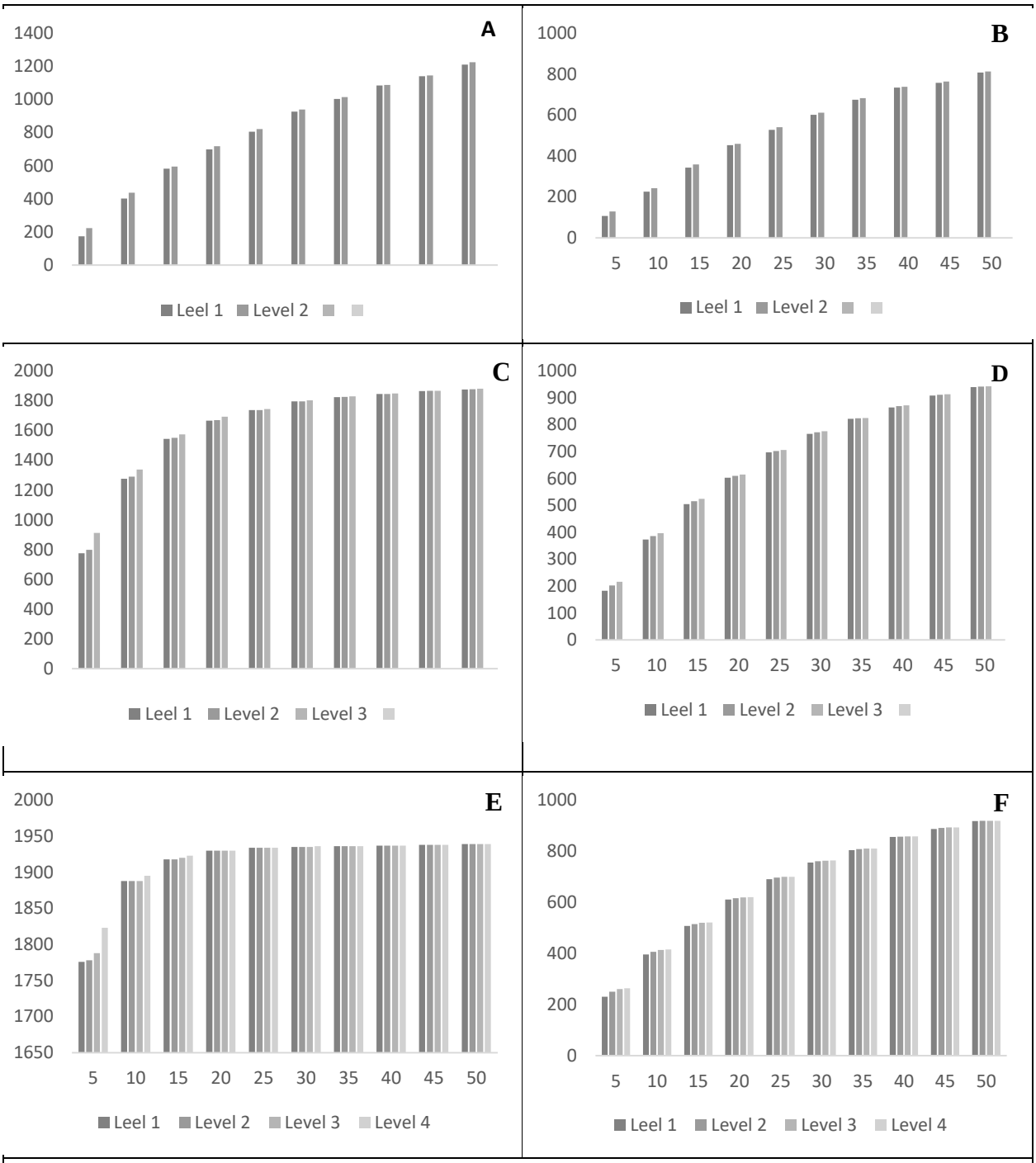

Figure 16: Y-axis: Number of Aware Practitioners X-axis: Number of Audited Practitioners Practitioners selected for audit using the Greedy Deterrence Heuristic Algorithm
(A) Two-level Diffusion, Low Decay **(B) Two-level Diffusion, High Decay**
(C) Three-level Diffusion, Low Decay **(D) Three-level Diffusion, High Decay**
(E) Four-level Diffusion, Low Decay **(F) Four-level Diffusion, High Decay**

Multi-Period Auditing

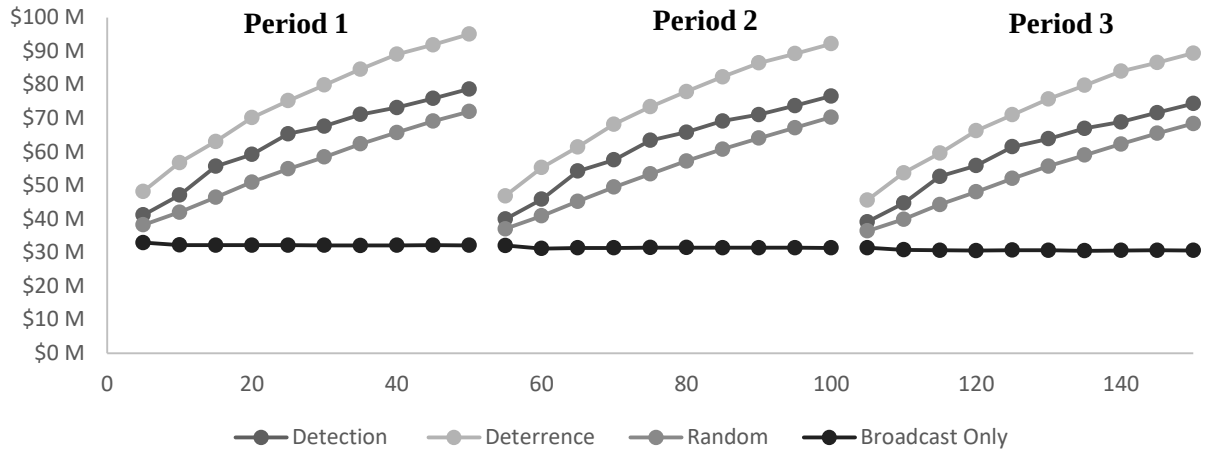

Figure 17: Multi Period Auditing

Appendix H: Analytical Results

Grounding our work on prior game theory literature (Cavusoglu, Raghunathan, and Cavusoglu 2009), we set up and analyze an audit game model. We identify two players, the Insurance Provider (IP), and the Service Practitioner (SP).

Among all practitioners within the network, ε practitioners are influential (with a number of neighboring practitioners exceeding a threshold predefined by the insurance provider), while the rest are not. Our model is similar to the "IDS-Firewall model" (Cavusoglu, Raghunathan, and Cavusoglu 2009) in that the two players represent the firm and user, where the user elects to hack or not, and the firm decides to audit or not, based on the outputs from an Intrusion Detection System (IDS) and a firewall.

For modeling purposes, we consider the case of a particular set of an IP and a SP. The SP could elect to defraud with a probability ψ. Hence, the total claims amount can include an amount of fraudulent claims ρ (mark-up). Once audited and if found to be fraudulent, the SP is imposed a penalty γ. Therefore we identify the practitioner's expected utility in Table 1.

Table 1. Service Practitioner's Payoff Table

		Service Practitioner's strategies	
		Defraud	Don't Defraud
Insurance Provider's Strategies	Audit	$\rho - \gamma$	0
	Don't Audit	ρ	0

The IP handles claims submitted by the SP. In order to investigate the legitimacy of the claims, the IP incurs the cost of audit (c). To select practitioners for audit we follow a two-step process (Figure 3). First, a detection algorithm is used to filter out all genuine practitioners (practitioners with *detection values* below a specific threshold). These are practitioners who have low prior fraud probabilities and submitted claims amounts within the industry's norms. Pre-selected likely fraudulent practitioners are then presented to the greedy deterrence heuristic which selects the group of practitioners with the highest *Expected Network Deterrence Value* for audit.

By combining both algorithms in this manner, the insurance provider is able to incorporate fairness in its auditing policy, thereby obtaining the benefit of targeting both fraudulent and influential practitioners. Combining the detection and deterrence algorithms also assures the adherence to some industry regulations such as those that outlaw the random audit of practitioners without a likelihood of sustained or high level of payment error (Medicare Prescription Drug Improvement and Modernization Act, 2003). The IP therefore could choose to audit 1) practitioners *targeted* by the deterrence algorithm with a probability p_1, or 2) practitioners *not targeted* by the deterrence algorithm with a probability p_2.

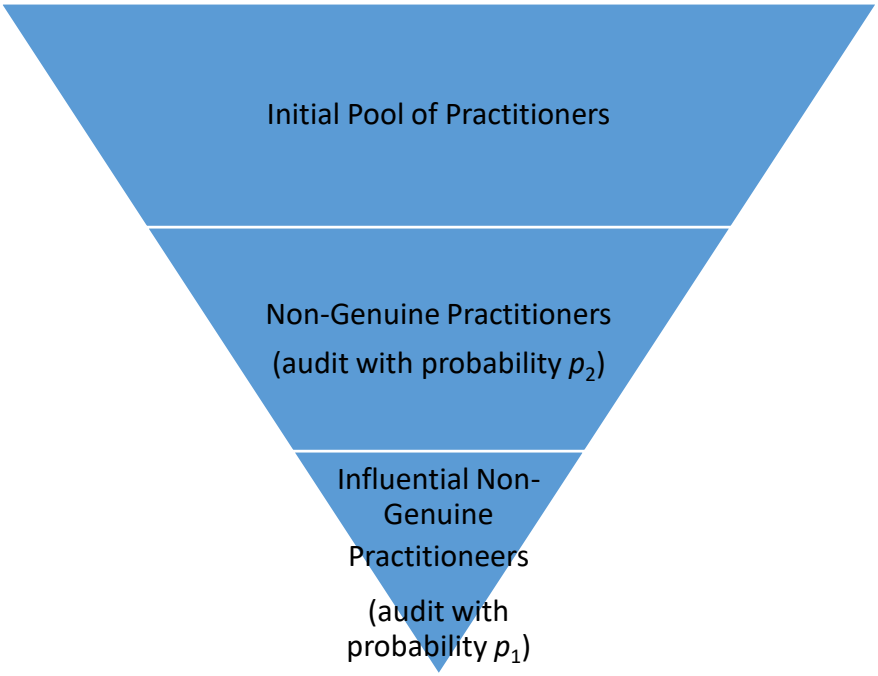

Figure 18: The Deterrence Algorithm

Within the network we differentiate between two groups of practitioners: influential and non-influential. While all audited practitioners are expected to change their behavior after audit, influential nodes which are highly connected nodes, are anticipated to trigger a larger diffusion of the audit information in the network, thereby generating more deterrence in the network.

Therefore, we denote the additional benefit from auditing an *influential fraudulent* practitioner Φ_1 and the additional benefit from auditing an *influential non-fraudulent* practitioner committing waste and abuse Φ_2. The deterrence algorithm aims at selecting influential practitioners with the highest *Expected*

Network Deterrence Value (DV) for audit, thereby maximizing the insurance's payoff. The insurance provider's payoffs for the different scenarios are expressed in Table 2.

Table 2. Insurance Provider's Payoff table

		Insurance Provider's Strategies			
		Don't Audit		Audit	
		Non-Influential Practitioner	Influential Practitioner	Non-Influential Practitioner	Influential Practitioner
Service Practitioner's Strategies	Defraud	-ρ	-ρ	$-\rho + \gamma - c$	$-\rho + \gamma + \Phi_1 - c$
	Don't Defraud	0	0	$-c$	$\Phi_2 - c$

In the quest for utility maximization, a service practitioner can elect to defraud, waste and abuse, or not, while the insurance provider can elect to either audit the practitioners selected by the detection algorithm or audit the practitioner who were not tagged by the deterrence algorithm. The game is summarized in strategic format in Figure 4.

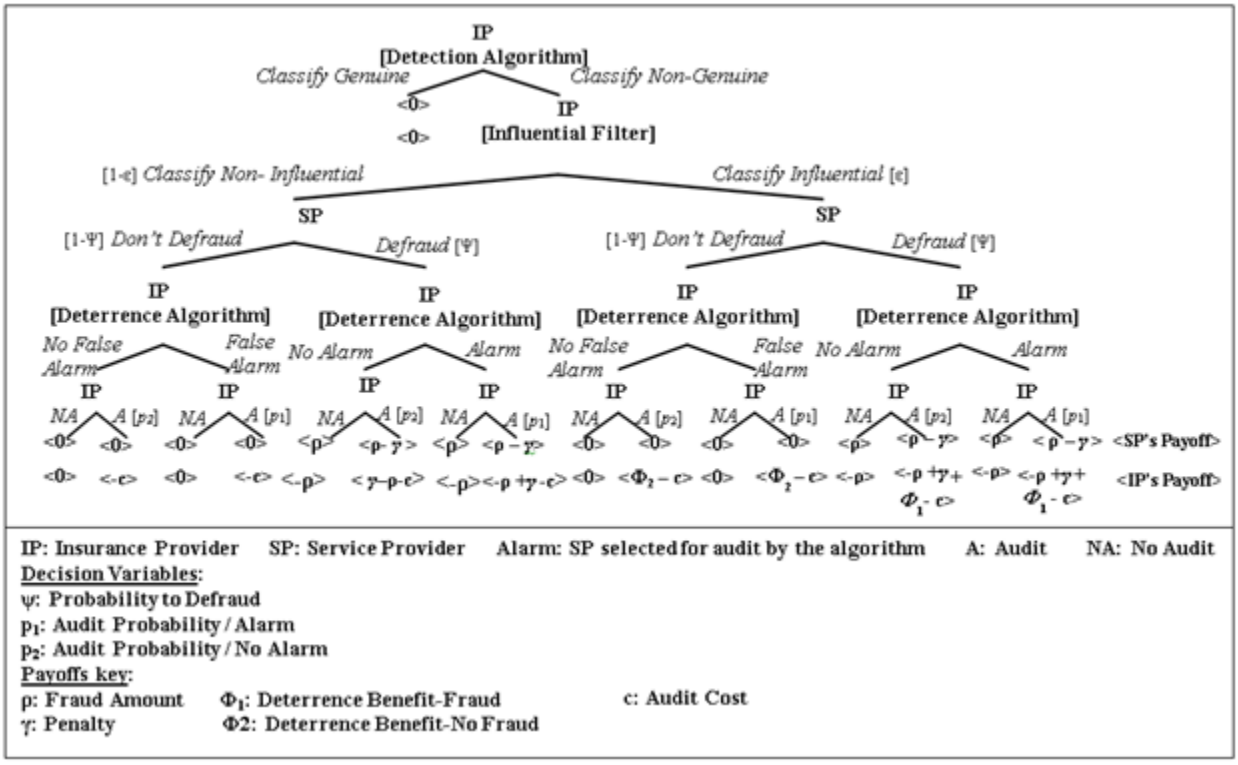

Figure 19: Game Tree

We represent the performance of the deterrence algorithm through the probabilities of true and false positives. We therefore define the following metrics:

P_F^I: The probability that the **deterrence** algorithm generates an alarm for a **fraudulent influential** practitioner.

P_F^{NI}: The probability that the **deterrence** algorithm generates an alarm for a **fraudulent non-influential** practitioner.

P_{NF}^I: The probability that the **deterrence** algorithm generates an alarm for a **non-fraudulent influential** practitioner.

P_{NF}^{NI}: The probability that the **deterrence** algorithm generates an alarm for a **non-fraudulent non-influential** practitioner.

In order to derive the Nash equilibrium for the game, we calculate both parties' expected payoffs (F) as follows:

$$F_{SP} = P_{\text{Audit/Fraud}}.((\rho-\gamma).\psi) + P_{\text{No Audit/Fraud}}.(\rho.\psi) \quad (10)$$

The Insurance Provider (IC)'s payoff is derived as follows:

$$F_{\text{IC/Alarm}} = -(1-p_1).(\rho).P_{\text{NI-Fraud/Alarm}} - p_1.(\rho-\gamma).P_{\text{NI-Fraud/Alarm}} - (1-p_1).(\rho).P_{\text{I-Fraud/Alarm}} - p_1.(\rho-\gamma-\Phi_1).P_{\text{I-Fraud/Alarm}} - p_1.(-\Phi_2).P_{\text{I-NFraud/Alarm}} - p_1.c \quad (11)$$

$$F_{\text{IC/No Alarm}} = -(1-p_2).(\rho).P_{\text{NI-Fraud/NoAlarm}} - p_2.(\rho-\gamma).P_{\text{NI-Fraud/NoAlarm}} - (1-p_2).(\rho).P_{\text{I-Fraud/NoAlarm}} - p_2.(\rho-\gamma-\Phi_1).P_{\text{I-Fraud/NoAlarm}} - p_2.(-\Phi_2).P_{\text{I-NFraud/NoAlarm}} - p_2.c \quad (12)$$

Where p_1 = P (Audit/Alarm) and p_2 = P (Audit/No Alarm)

The mixed strategy Nash equilibrium derived is represented by the following:

$$\psi_1^* = \frac{(P_{NF}^{NI}.(1-\varepsilon)+P_{NF}^{I}.\varepsilon)c - P_{NF}^{I}.\varepsilon.\Phi_2}{P_F^{NI}(1-\varepsilon)(\gamma-c) + P_F^{I}.\varepsilon.(\gamma+\Phi 1-c) - (P_{NF}^{NI}.(-c)(1-\varepsilon)+P_{NF}^{I}.\varepsilon(\Phi_{2-c}))}$$

$$p_1^* = \frac{\rho}{(\gamma.(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}))} \quad \text{given} \quad p_2 = 0 \quad \text{if } \rho < (\gamma.(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}))$$

$$\psi_2^* = \frac{((c-\varepsilon.\Phi_2) - P_{NF}^{NI}.c(1-\varepsilon) - P_{NF}^{I}.\varepsilon(c-\Phi_2))}{(\gamma+\varepsilon.(\Phi_1-\Phi_2) + P_F^{NI}(1-\varepsilon)(c-\gamma) - P_{NF}^{NI}.c(1-\varepsilon) + P_F^{I}.\varepsilon(c-\gamma-\Phi_1) - P_{NF}^{I}.\varepsilon(c-\Phi_2))}$$

$$p_2^* = \frac{(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}).\lambda - \rho}{(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}).\lambda - \lambda} \quad \text{given} \quad p_1 = 1 \text{ if } \rho > (\gamma.(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}))$$

We provide all probabilities and calculations in online Appendix E.

In both scenarios, corresponding to the two strategies above, the insurance firm uses a combination of detection and deterrence algorithms. The detection algorithm first selects a pool of presumed non-genuine practitioners. Afterwards, the deterrence algorithm targets influential practitioners for audit.

The first scenario describes the case where the insurance provider elects to audit only non-genuine practitioners, a choice which generates an alarm through the deterrence algorithm. This strategy is applicable only when the amount of fraud is less than the penalty imposed on audited fraudulent practitioners.

$$(\rho < \gamma.(\varepsilon.\mathrm{P}_{\mathrm{F}}^{\mathrm{I}} + (1-\varepsilon).\mathrm{P}_{\mathrm{F}}^{\mathrm{NI}})$$

In the case when the fraud amount exceeds the penalty imposed upon fraud, the insurance provider has to use the alternative strategy.

The second strategy consists of the insurance firm auditing all the service practitioners selected by the deterrence algorithm (p_1 = 1), in addition to auditing practitioners who are not targeted by the deterrence algorithm with a probability p_2.

In the alarm case, the optimal probability to defraud (ψ_1^*) is nil when the insurance provider's expected cost of auditing non-fraudulent practitioners equals the expected deterrence benefit from auditing non-fraudulent *influential* practitioners $\left(\mathrm{P}_{\mathrm{NF}}^{\mathrm{NI}}.(1-\varepsilon) + \mathrm{P}_{\mathrm{NF}}^{\mathrm{I}}.\varepsilon\right)\mathrm{c} = \mathrm{P}_{\mathrm{NF}}^{\mathrm{I}}.\varepsilon.\Phi_2$.

The optimal fraud probability is at a maximum when the insurance provider's expected payoff from auditing fraudulent practitioners is nil, i.e., $(\mathrm{P}_{\mathrm{F}}^{\mathrm{NI}}(1-\varepsilon)(\gamma-\mathrm{c}) + \mathrm{P}_{\mathrm{F}}^{\mathrm{I}}.\varepsilon.(\gamma+\Phi_1-\mathrm{c})) = 0$

That means that 1) the cost of audit (c) equals the penalty collected upon auditing fraudulent practitioners (γ) and 2) the deterrence benefit from auditing influential practitioners (Φ_1) is nil ($\gamma = \mathrm{c}$ and $\Phi_1 = 0$).

As intuitively expected, the optimal probability to audit is a function of the loss incurred by the insurance providers for unaudited fraud cases (ρ), the penalty imposed on fraudulent practitioners (γ), and

the algorithm's positive rate ($\varepsilon.P_F^I + (1- \varepsilon).P_F^{NI}$). The optimal probability to audit is expected to increase as the loss incurred from fraud increases.

Similarly, the more efficient the algorithm is at detecting fraud, and the higher the penalties imposed, the lower the optimal probability is to audit.

Special Case Scenarios – Analytical Results

Given a combination of both the detection and deterrence algorithms, the insurance provider needs to select a strategy for auditing practitioners. To avoid auditing genuine practitioners we first use the detection algorithm to filter out genuine practitioners. We then manipulate the deterrence algorithm for various profiles. It is to be noted that we set the algorithm to generate an alarm targeting different segments of practitioners. This does not necessarily mean auditing all the targeted practitioners. We examine the insurance provider's payoff values at equilibrium as derived through solving the game above.

One of the insurance provider's alternatives is to not take the sentinel effect into consideration, and target the entire pool of non-genuine practitioners selected by the detection algorithm. This scenario could be achieved by setting the deterrence algorithm to target all (non-genuine) practitioners. We therefore set both the true positive and false positive rates to 1($P_F^I = P_F^{NI} = P_{NF}^I = P_{NF}^{NI} = 1$).

With the deterrence idea in mind, the insurance provider could elect to target all influential non-genuine practitioners. After filtering out the genuine practitioners, the insurance provider could use the deterrence algorithm to target influential practitioners. This setting aims at diffusing the audit information and deterrence of neighboring practitioners. In that case, $P_F^I = P_{NF}^I = 1$ and $P_F^{NI} = P_{NF}^{NI} = 0$.

Given that a considerable amount of fraud occurs though Home Health Care practitioners who are not connected to the rest of the service practitioner community (US Department of Justice 2013), the insurance provider could elect to target specifically non-influential practitioners. In our game setup, we set the deterrence algorithm to generate an alarm for all non-influential non-genuine practitioners. This scenario is represented by the following: $P_F^{NI} = P_{NF}^{NI} = 1$ and $P_F^I = P_{NF}^I = 0$.

For each scenario, we first compute the optimal probability to audit in both the alarm and no-alarm cases (Table 3).

In the first scenario, an alarm is generated by the deterrence algorithm for all types of practitioners (influential/ non-influential, fraudulent/ non-fraudulent). Therefore, no practitioners fall under the "No alarm" category.

As illustrated in Table 3 the optimal defraud probability in this case increases as the cost of audit exceeds the deterrence benefit from auditing non-fraudulent influential practitioners ($c > \varepsilon.\Phi_2$). The same probability decreases as 1) the penalty imposed on fraudulent practitioners increases, and 2) the deterrence benefit from auditing influential fraudulent practitioners exceeds the deterrence benefit from auditing influential non-fraudulent practitioners ($\varepsilon.\Phi_1 > \varepsilon.\Phi_2$). It is important to note that in this scenario, the ratio of influential practitioners in the network affects the defraud probabilities.

In the second scenario, the deterrence algorithm generates an alarm for all influential (non-genuine) practitioners ($\varepsilon = 1$). Practitioners targeted by the alarm are therefore expected to defraud as long as 1) the cost of audit is larger than the deterrence benefit from auditing non-fraudulent practitioners in the pool and 2) the deterrence benefit from auditing fraudulent practitioners exceeds the deterrence benefit from auditing non-fraudulent practitioners. The last scenario targets all non-influential practitioners for alarm. Thus, practitioners in the alarm pool do not take into consideration the deterrence benefit, and defraud with a probability ($\frac{c}{\gamma}$). The defraud probability in this case increases with the rise of audit cost (c), and decreases with the rise of the penalty imposed upon detecting fraud (γ).

Table 3. Service Practitioner's Optimal Defraud Probabilities

	Algorithm's Profile	Optimal Defraud Probability Alarm Case (ψ_1^*)	Optimal Defraud Probability No Alarm Case (ψ_2^*)
Scenario 1	**Generate alarm for *all non-genuine* practitioners)** $P_F^I = P_F^{NI} = 1$ & $P_{NF}^I = P_{NF}^{NI} = 1$	$\frac{c - \varepsilon.\Phi_2}{(\gamma + \varepsilon(\Phi_1 - \Phi_2))}$	Not Applicable (An alarm is generated for *all non-genuine* practitioners ➔ There are no practitioners that fall in the No Alarm Case)

Scenario 2	**Generate alarm for *all non-genuine influential* practitioners** $P_F^I = P_{NF}^I = 1$ & $P_F^{NI} = P_{NF}^{NI} = 0$	$\frac{(c - \Phi_2)}{(\gamma + \Phi_1 - \Phi_2)}$	$\frac{c}{\gamma}$
Scenario 3	**Generate alarm for *all non-genuine non-influential* practitioners** $P_F^{NI} = P_{NF}^{NI} = 1$ & $P_F^I = P_{NF}^I = 0$	$\frac{c}{\gamma}$	$\frac{(c - \Phi_2)}{(\gamma + \Phi_1 - \Phi_2)}$

Using the optimal defraud probability set above, we calculate the expected insurance provider payoff in each of the three scenarios.

$$F = P_{Alarm} \cdot F_{IC/Alarm} + P_{NoAlarm} \cdot F_{IC/No\ Alarm}$$

We summarize our findings in Table 4 below.

Table 4. Insurance Provider's Expected Payoff at Equilibrium (F)

Algorithm's Profile		**Strategy 1** ($p_1 = p_1^*$, $p_2 = 0$ and $\psi = \psi_1^*$)	**Strategy 2** ($p_1 = 1$, $p_2 = p_2^*$ and $\psi = \psi_2^*$)
Scenario 1	**Generate alarm for *all non-genuine* practitioners)** $P_F^I = P_F^{NI} = 1$ & $P_{NF}^I = P_{NF}^{NI} = 1$	$F = F_{Alarm}$ $\frac{\rho.(\varepsilon.\Phi_2 - c)}{(\gamma + \varepsilon.(\Phi_1 - \Phi_2))}$	Not Applicable (An alarm is generated for *all non-genuine* practitioners ➔ There are no practitioners that fall in the No Alarm Case)
Scenario 2	**Generate alarm for *all non-genuine influential* practitioners** $P_F^I = P_{NF}^I = 1$ & $P_F^{NI} = P_{NF}^{NI} = 0$	$\frac{\rho.(\Phi_2 - c)}{(\gamma + \Phi_1 - \Phi_2)}$	$\frac{c.\varepsilon.(\Phi_1 - \Phi_2) + \varepsilon.\gamma.\Phi_2 + \varepsilon.\rho(\Phi_2 - c) - \rho.c}{(\rho + \gamma)}$
Scenario 3	**Generate alarm for *all non-genuine non-influential* practitioners** $P_F^{NI} = P_{NF}^{NI} = 1$ & $P_F^I = P_{NF}^I = 0$	$F = F_{Alarm} = F_{NoAlarm}$ $\frac{-\rho(c)}{((\gamma))}$	$\frac{\rho(\Phi_2 - c) - (1-\varepsilon)(c.\rho + \gamma.\Phi_2 + c.(\Phi_1 - \Phi_2))}{(\gamma + \rho + \Phi_1 - \Phi_2)}$

To better illustrate the benefits of applying our algorithm in practice, we perform a sensitivity analysis using results from our game theory model (Online Appendix E). These scenarios show the variance in the savings amount depending on the cost of audit, the penalty imposed on fraudulent practitioners and the amount of fraud. We consider a network similar to the one used in previous sections composed of 1,000 practitioners for whom the total amount of claims submitted had a mean of $500,000 and a standard deviation of $100,000. Within the network, 10% of practitioners are set to be highly connected (having

more than 10 immediate neighboring practitioners). Therefore, the fraction of influential practitioners in the network (ε) is 0.1. As per the CMS (Center of Medicare and Medicaid Services), the rate of improper billing for the year 2012 (CERT) was 8.6%. We use the CERT rate in the simulation as the rate of waste and abuse in the network. We calculate the benefit from auditing an influential fraudulent practitioner (Φ1), as well as the benefit from auditing an influential non-fraudulent practitioner (Φ2). By varying the expected amount of fraud (ρ) and cost of audit (c), we look at the expected insurance provider payoff in two different worlds, namely high diffusion/low decay, and low diffusion/high decay.

In addition to providing strategies for audit, these results and sensitivity analyses can even be used by insurance providers to set important factors such as penalties for fraud, waste and abuse. The analytical results reinforce some conclusions from the agent-based simulation studies, such as the value of auditing highly influential/connected practitioners in certain cases.

These are also complementary and provide new findings since they do take a different and important perspective. In the game, the setting is one where both parties are making strategic decisions given the information available. In the agent-based simulation, the agents are assumed to be fraudulent or not and determine their billings, and the algorithm then works to determine practitioners for audit. Both perspectives are useful, and in this case, point to the value of taking the sentinel effects into account in audit algorithms.

Appendix I: Game Theory Model Description and Calculations

SP: Service Practitioner - Practitioner (e.g. service practitioner)

IP: Insurance Provider

p_1: Probability to audit practitioners *targeted* by the deterrence algorithm

p_2: Probability to audit practitioners not *targeted* by the deterrence algorithm

Ψ: SP's fraud probability

ρ: Mark-up included in fraudulent claims

γ: Penalty imposed on audited-fraudulent practitioners

ε: Proportion of influential service practitioners

Φ_1: The additional benefit from auditing an *influential fraudulent* practitioner

Φ_2: The additional benefit from auditing an *influential non-fraudulent* practitioner performing waste and abuse

P_F^I: Probability that the **deterrence** algorithm generates an alarm for a **fraudulent influential** practitioner

P_F^{NI}: Probability that the **deterrence** algorithm generates an alarm for a **fraudulent non-influential** practitioner

P_{NF}^I: Probability that the **deterrence** algorithm generates an alarm for a **non-fraudulent influential** practitioner

P_{NF}^{NI}: Probability that the **deterrence** algorithm generates an alarm for a **non-fraudulent non-influential** practitioner.

P_D^T: Probability that the **deterrence** algorithm generates an *alarm in case of fraud*

P_F^T: Probability that the **deterrence** algorithm generates an *alarm in case of no fraud*

$$P_D^T = \varepsilon . P_F^I + (1-\varepsilon) . P_F^{NI}$$

$$P_F^T = \varepsilon . P_{NF}^I + (1-\varepsilon) . P_{NF}^{NI}$$

P_{Alarm}: Probability that the **deterrence** algorithm *generates* an alarm

$P_{No\ Alarm}$: Probability that the **deterrence** algorithm *doesn't generate* an alarm

$$P_{Alarm} = \psi . P_D^T + (1-\psi) P_F^T = P_F^T + \psi . (P_D^T - P_F^T)$$

$P_{\text{No Alarm}} = 1 - P_{\text{Alarm}}$

$P_{\text{NI-Fraud/Alarm}}$: Given an alarm, probability that a **non-influential** practitioner **defrauds**

$P_{\text{I-Fraud/Alarm}}$: Given an alarm, probability that an **influential** practitioner **defrauds**

$P_{\text{I-NFraud/Alarm:}}$ Given an alarm, probability that an **influential** practitioner **does not defraud**

$$P_{\text{NI-Fraud/Alarm}} = \frac{(1-\varepsilon).P_F^{NI}.\psi}{P_{Alarm}}$$

$$P_{\text{I-Fraud/Alarm}} = \frac{\varepsilon.P_F^{I}.\psi}{P_{Alarm}}$$

$$P_{\text{I-NFraud/Alarm}} = \frac{\varepsilon.P_{NF}^{I}.(1-\psi)}{P_{Alarm}}$$

$P_{\text{NI-Fraud/NoAlarm}}$: Given no alarm, probability that a **non-influential** practitioner **defrauds**

$P_{\text{I-Fraud/NoAlarm}}$: Given no alarm, probability that an **influential** practitioner **defrauds**

$P_{\text{I-NFraud/NoAlarm}}$: Given no alarm, probability that an **influential** practitioner **does not defraud**

$$P_{\text{NI-Fraud/NoAlarm}} = \frac{(1-\varepsilon).\psi.(1-P_F^{NI})}{P_{NoAlarm}}$$

$$P_{\text{I-Fraud/NoAlarm}} = \frac{\varepsilon.\psi.(1-P_F^{I})}{P_{NoAlarm}}$$

$$P_{\text{I-NFraud/NoAlarm}} = \frac{\varepsilon.(1-\psi).(1-P_{NF}^{I})}{P_{NoAlarm}}$$

$P_{\text{Audit/Fraud}}$: Probability of audit given fraud

$P_{\text{NoAudit/Fraud}}$: Probability of no audit given fraud

$P_{\text{Audit/Fraud}} = (p_1.P_D^{T}) + (p_2.(1-P_D^{T}))$

$P_{\text{NoAudit/Fraud}} = 1 - P_{\text{Audit/Fraud}}$

F_{SP}: Payoff of service practitioner

$F_{SP} = \psi.[P_{Audit/Fraud}.(\rho-\gamma) + P_{NoAudit/Fraud}.(\rho)]$

$\frac{\partial F_{SP}}{\partial \psi} = -\rho.(p_1.(P_F^I.\varepsilon - P_F^{NI}.(\varepsilon - 1)) + p_2.(P_F^{NI}.(\varepsilon - 1) - P_F^I.\varepsilon + 1) - 1) - (\gamma - \rho).(p_1.(P_F^I.\varepsilon - P_F^{NI}.(\varepsilon - 1)) + p_2.(P_F^{NI}.(\varepsilon - 1) - P_F^I.\varepsilon + 1))$

Solve $\frac{\partial F_{SP}}{\partial \psi} = 0$ for p_1 given $p_2 = k$

p_1^*: Probability to audit practitioners *targeted* by the deterrence algorithm at equilibrium

p_2^*: Probability to audit practitioners not *targeted* by the deterrence algorithm at equilibrium

$p_1^* = \frac{\rho + k.(1-\gamma)}{(\gamma.(\varepsilon.P_F^I + (1-\varepsilon).P_F^{NI}))}$

Solve $\frac{\partial F_{SP}}{\partial \psi} = 0$ for p_2 given $p_1 = 1$

$p_2^* = \frac{-(\rho - P_F^{NI}.(1-\varepsilon).\lambda - P_F^I.\varepsilon.\lambda)}{(\lambda.(P_F^{NI}(1-\varepsilon) + P_F^I.\varepsilon - 1))}$

F_{Alarm} : Payoff of insurance provider given in case of alarm

$F_{NoAlarm}$: Payoff of insurance provider given in case of no alarm

$F_{Alarm} = -(1-p_1).(\rho).P_{NI\text{-}Fraud/Alarm} - p_1.(\rho-\gamma).P_{NI\text{-}Fraud/Alarm} - (1-p_1).(\rho).P_{I\text{-}Fraud/Alarm}$
$- p_1.(\rho-\gamma-\Phi_1).P_{I\text{-}Fraud/Alarm} - p_1.(-\Phi_2).P_{I\text{-}NFraud/Alarm} - p_1.c$

$F_{NoAlarm} = -(1-p_2).(\rho).P_{NI\text{-}Fraud/NoAlarm} - p_2.(\rho-\gamma).P_{NI\text{-}Fraud/NoAlarm} - (1-p_2).(\rho).P_{I\text{-}Fraud/NoAlarm}$
$- p_2.(\rho-\gamma-\Phi_1).P_{I\text{-}Fraud/NoAlarm} - p_2.(-\Phi_2).P_{I\text{-}NFraud/NoAlarm} - p_2.c$

$\frac{\partial_{FAlarm}}{\partial p1} = \frac{-(P_{NF}^I.\varepsilon(c-\Phi_2) + P_{NF}^{NI}.c(1-\varepsilon) + P_F^{NI}.\psi(1-\varepsilon)(c-\gamma) - P_{NF}^{NI}.c.\psi(1-\varepsilon) + P_F^I.\varepsilon.\psi(c-\gamma-\Phi_1) - P_{NF}^I.\varepsilon.\psi(c-\Phi_2))}{(P_{NF}^{NI}(1-\varepsilon) + P_{NF}^I.\varepsilon(1-\psi) + P_F^{NI}.\psi(1-\varepsilon) - P_{NF}^{NI}.\psi(1-\varepsilon) + P_F^I.\varepsilon.\psi)}$

$$\frac{\partial F_{NoAlarm}}{\partial p2} = \frac{-(-c+ P_{NF}^{NI}.c\,(1-\varepsilon)(1-\psi)+ \varepsilon.\Phi_2(1-\psi)+ \gamma.\psi+ P_{NF}^{I}.\varepsilon\,(c\ -\Phi_2)(1-\psi)+ P_{F}^{NI}.c.\psi(1-\varepsilon)- P_{F}^{NI}.\gamma.\psi(1-\varepsilon)+ \varepsilon.\Phi_1.\psi +P_{F}^{I}.\varepsilon.\psi(c-\gamma-\Phi_1))}{(P_{NF}^{NI}\,(1-\varepsilon)+ P_{NF}^{I}.\varepsilon\ (1-\psi)+ P_{F}^{NI}.\psi\,(1-\varepsilon)- P_{NF}^{NI}.\psi(1-\varepsilon)+ P_{F}^{I}.\varepsilon.\psi\ \ -1)}$$

Solve $\frac{\partial F_{Alarm}}{\partial p1}$=0 and $\frac{\partial F_{NoAlarm}}{\partial p2}$=0 for ψ ➔ Empty Set

We can verify that $\frac{\partial_{FAlarm}}{\partial p1} > \frac{\partial F_{NoAlarm}}{\partial p2}$. Therefore, at equilibrium, $\frac{\partial FAlarm}{\partial p1}$=0 and $\frac{\partial F_{NoAlarm}}{\partial p2} < 0$, or $\frac{\partial FNoAlarm}{\partial p2}$=0 and $\frac{\partial F_{Alarm}}{\partial p1} > 0$

Ψ_1^*: SP's fraud probability at equilibrium – Strategy 1

Ψ_2^*: SP's fraud probability at equilibrium – Strategy 2

Solve $\frac{\partial FAlarm}{\partial p1}$=0 for ψ given $\frac{\partial F_{NoAlarm}}{\partial p2} < 0$

➔ $\psi_1^* = \frac{(P_{NF}^{NI}.c(1-\varepsilon)+ P_{NF}^{I}.\varepsilon(c-\Phi_2)}{(P_{NF}^{NI}.c(1-\varepsilon)+ P_{F}^{NI}(1\ -\varepsilon)(-c+\gamma)+ P_{NF}^{I}.\varepsilon(c-\Phi_2)+ P_{F}^{I}.\varepsilon.(-c+\gamma+\Phi 1)}$

Solve $\frac{\partial FNoAlarm}{\partial p2}$=0 for ψ given $\frac{\partial F_{Alarm}}{\partial p1} > 0$

➔ $\psi_2^* = \frac{((c-\varepsilon.\Phi_2)- P_{NF}^{NI}.c(1-\varepsilon)- P_{NF}^{I}.\varepsilon(c-\Phi_2))}{(\gamma+\varepsilon.(\Phi_1-\Phi_2)+ P_{F}^{NI}\,(1-\varepsilon)(c-\gamma)- P_{NF}^{NI}.c\,(1-\varepsilon)+ P_{F}^{I}.\varepsilon(c\ -\gamma-\Phi_1)- P_{NF}^{I}.\varepsilon(c\ -\Phi_2)\ )}$

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Cavusoglu, H., S. Raghunathan, and H. Cavusoglu. 2009. "Configuration of and Interaction between Information Security Technologies: The Case of Firewalls and Intrusion Detection Systems." *Information Systems Research*, 20(2), 198-217.