Online Appendix to "The Effectiveness of Regulations on the Dual-Role Retailer's Data Use for Sellers"

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Appendix A: Proofs of the Main Results

Table S1 Summary of Notation

Parameters	Definition
q_j	demand for product $j \in \{s, r\}$
a	fixed base demand
θ	uncertain base demand, $Var[\theta] = \sigma^2$
σ	standard deviation of uncertain base demand, i.e., $Var[\theta] = \sigma^2$
Y	signal about the uncertain demand
β	signal accuracy
γ	substitutability between products s and r
f	commission fee
π_j	profit of firm $j \in \{s, r\}$
Π_j	expected (i.e., ex-ante) profit of firm $j \in \{s, r\}$
p_j	retail price of product $j \in \{s, r\}$; decision variable
w_s	wholesale price of product s; decision variable

Following the literature (e.g., Shang et al. 2016), we use Bayesian Nash equilibrium as the solution concept for deriving equilibrium outcomes, a widely used approach for static games with incomplete information. In this framework, a firm without market information (i.e., the signal Y) forms conjectures about the decisions of the other firm when the other firm uses market information. Our analysis considers two data-use policies, S and RS, in addition to the benchmark case (R), under both agency (AG) and wholesale (WH) contracts. For example, AG-R (WH-R) denotes the benchmark case with no data regulation under agency (wholesale) contracts.

Proof of Lemma 1

Under ***AG-R***, the retailer and the seller choose their optimal prices to maximize their profits as follows:

$$\begin{aligned}
\max_{p_r} \pi_r &= p_r E[q_r|Y] + f p_s E[q_s|Y] \\
&= \frac{p_r}{1+\gamma}\left(a + \beta Y - \frac{1}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}p_s\right) + \frac{f p_s}{1+\gamma}\left(a + \beta Y - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}p_r\right), \qquad \text{(S1)} \\
\max_{p_s} \pi_s &= (1-f) p_s E[q_s] = \frac{(1-f)p_s}{1+\gamma}\left(a - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}E[p_r]\right).
\end{aligned}$$

This retailer's (seller's) objective function is concave in p_r (p_s), and it is sufficient to solve its first-order conditions (FOCs) to derive the best response. Thus, solving the FOCs gives the retailer's (seller's) best response as $p_r = \frac{a(1-\gamma)}{2} + \frac{\gamma(f+1)}{2}p_s + \frac{Y\beta(1-\gamma)}{2}$ and $p_s = \frac{a(1-\gamma)}{2} + \frac{\gamma}{2}E[p_r]$. Solving their best responses gives the optimal retail prices as follows:

$$p_r = \frac{a(1-\gamma)(\gamma+\gamma f+2)}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)Y}{2}, \text{ and } p_s = \frac{a(1-\gamma)(2+\gamma)}{4-\gamma^2(f+1)}.$$

Substituting p_r and p_s into Equation (S1), their profits can be derived as

$$\begin{aligned}
\pi_r &= \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{a\beta(\gamma-1)Y(\gamma+2(\gamma+1)f+2)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)} + \frac{\beta^2(1-\gamma)Y^2}{4(\gamma+1)},\\
\pi_s &= \frac{a^2(\gamma-1)(\gamma+2)^2(f-1)}{(\gamma+1)(\gamma^2(f+1)-4)^2}.
\end{aligned}$$

Taking the expectation over the distribution of Y, the expected profits for the retailer and the seller can be derived as follows:

$$\begin{aligned}
\Pi_r^{AG-R} = E[\pi_r] &= \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} := \Pi_r^{AG} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)},\\
\Pi_s^{AG-R} = E[\pi_s] &= \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} := \Pi_s^{AG}.
\end{aligned}$$

Under ***AG-S***, both firms choose their optimal prices to maximize their profits as follows:

$$\begin{aligned}
\max_{p_r} \pi_r = \quad & p_r E[q_r] + f p_s E[q_s]\\
= \quad & \frac{p_r}{1+\gamma}\left(a - \frac{1}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}E[p_s]\right) + \frac{fE[p_s]}{1+\gamma}\left(a - \frac{1}{1-\gamma}E[p_s] + \frac{\gamma}{1-\gamma}p_r\right),\\
\max_{p_s} \pi_s = \quad & (1-f)p_s E[q_s|Y] = \quad \frac{(1-f)p_s}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}p_r\right).
\end{aligned} \tag{S2}$$

Similar to AG-R, solving the FOCs of each objective function gives their best response as $p_r = \frac{a(1-\gamma)}{2} + \frac{\gamma(f+1)}{2}E[p_s]$, and $p_s = \frac{a(1-\gamma)}{2} + \frac{\gamma p_r}{2} + \frac{Y\beta(1-\gamma)}{2}$. Then, solving these best responses gives the optimal retail prices as follows:

$$p_r = \frac{a(1-\gamma)(\gamma+\gamma f+2)}{4-\gamma^2(f+1)} \text{ and } p_s = \frac{a(1-\gamma)(2+\gamma)}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)Y}{2}.$$

Substituting p_r and p_s into Equation (S2), their profits can be derived as

$$\begin{aligned}
\pi_r &= \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2},\\
\pi_s &= \frac{a^2(\gamma-1)(\gamma+2)^2(f-1)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} - \frac{a\beta(\gamma-1)(\gamma+2)(f-1)Y}{(\gamma+1)\left(\gamma^2(f+1)-4\right)} + \frac{\beta^2(\gamma-1)(f-1)Y^2}{4(\gamma+1)}.
\end{aligned}$$

Taking the expectation over the distribution of Y, the expected profits for both the retailer and the seller can be derived as follows:

$$\begin{aligned}
\Pi_r^{AG-S} = E[\pi_r] &= \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} := \Pi_r^{AG},\\
\Pi_s^{AG-S} = E[\pi_s] &= \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)} := \Pi_s^{AG} + \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)}.
\end{aligned}$$

Under ***AG-RS***, they choose their optimal prices to maximize their profits as follows:

$$\begin{aligned}\max_{p_r}\ \pi_r =& p_r E[q_r|Y] + f p_s E[q_s|Y]\\ =& \frac{p_r}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}p_s\right) + \frac{fp_s}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}p_r\right),\\ \max_{p_s}\ \pi_s =& (1-f)p_s E[q_s|Y] = \quad \frac{(1-f)p_s}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S3}$$

Similarly, solving the FOCs of each objective function gives their best response. Then, solving these best responses gives the optimal retail prices as follows:

$$p_r = \frac{a(1-\gamma)(\gamma+\gamma f+2)}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)(\gamma+\gamma f+2)Y}{4-\gamma^2(f+1)} \text{ and } p_s = \frac{a(1-\gamma)(2+\gamma)}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)(\gamma+2)Y}{4-\gamma^2(f+1)}.$$

Substituting p_r and p_s into Equation (S3), given a signal Y, their profits can be derived as

$$\begin{aligned}\pi_r =& \quad \frac{(\gamma-1)\left(-(\gamma+2)^2+(\gamma+1)\gamma^2 f^2+\left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)(a+\beta Y)^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2},\\ \pi_s =& \quad \frac{(\gamma-1)(\gamma+2)^2(f-1)(a+\beta Y)^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.\end{aligned}$$

By taking expectations on both sides of the equations above, the optimal expected profits of the seller and the retailer are as follows.

$$\begin{aligned}\Pi_r^{AG-RS} = E[\pi_r] =& \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta\sigma^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2},\\ :=& \Pi_r^{AG} + I_r^{AG}.\\ \Pi_s^{AG-RS} = E[\pi_s] =& \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} := \Pi_s^{AG} + \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}.\end{aligned}$$

Note that $I_r^{AG} > 0 \iff Af^2+Bf+C > 0$ where $A = -(1-\gamma^2)\gamma^2 < 0$, $B = \gamma^4 - 5\gamma^2 + 4$, and $C = (1-\gamma)(\gamma+2)^2$. This inequality yields $f_1 < f < f_2$, as $A < 0$, where f_1 and f_2 are two roots for $Af^2+Bf+C = 0$. Thus, $I_r^{AG} > 0$ always holds because $f_1 < 0 < f < 1 < f_2$. □

Technical Insights of Proposition 1

Under both policies, the seller leverages the demand signal to extract surplus associated with predictable demand. This is captured by the second term of the seller's profit function Π_s^{ζ} for $\zeta \in \{AG-S, AG-RS\}$ as presented in Table 1 and formalized in Lemma 1. When the signal is withheld from the retailer, it restricts the retailer's flexibility, thereby precluding gains that would otherwise arise—reflected in the second term of Π_r^{AG-R}. Likewise, the same logic applies when no policy protects the seller from the retailer's information monopoly. Notably, the informed party's pricing is also bounded, as uncertainty about the uninformed party's reaction hinders full exploitation of the information advantage. For this reason, restoring mutual flexibility yields superior performance relative to the scenarios with guaranteed signal privacy, thereby enhancing efficiency for both parties, as demonstrated by I_r^{AG} in Table 1.

Proof of Proposition 1

Note that Π_j^{AG-R}, Π_j^{AG-S}, and Π_j^{AG-RS} denote the expected profits of firm $j \in \{s, r\}$ under Case R, Policy S, and Policy RS, respectively, as derived in Lemma 1.

(i) For the seller, note that,

$$\Pi_s^{AG-S} > \Pi_s^{AG-R} \iff \Pi_s^{AG-S} - \Pi_s^{AG-R} = \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)} > 0.$$

This inequality always holds because $f, \gamma, \beta \in (0,1)$ and $\sigma > 0$.

For the retailer, note that,

$$\Pi_r^{AG-S} < \Pi_r^{AG-R} \iff \Pi_r^{AG-S} - \Pi_r^{AG-R} = -\frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} < 0.$$

This inequality always holds because $\gamma, \beta \in (0,1)$ and $\sigma > 0$.

(ii) Similarly, for the seller, note that, under AG,

$$\Pi_s^{AG-RS} > \Pi_s^{AG-R} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-R} = \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} > 0.$$

This inequality always holds, because $f, \gamma, \beta \in (0,1)$ and $\sigma > 0$.

For the retailer, note that, under AG,

$$\Pi_r^{AG-RS} > \Pi_r^{AG-R} \iff \Pi_r^{AG-RS} - \Pi_r^{AG-R} = \frac{\beta(1-\gamma)(\gamma+2)\sigma^2(Af^2+Bf+C)}{4(\gamma+1)(\gamma^2(f+1)-4)^2} > 0.$$

Note that $A = -(\gamma^3 + 2\gamma^2) < 0$, $B = 8 + 4\gamma - 2\gamma^3$, and $C = 8\gamma + 2\gamma^2 - \gamma^3$. This inequality yields $f_1 < f < f_2$, as A<0, where f_1 and f_2 are two roots of $Af^2 + Bf + C = 0$. Therefore, $\Pi_r^{AG-RS} > \Pi_r^{AG-R}$ always holds because $f_1 < 0 < f < 1 < f_2$ for $f, \gamma, \beta \in (0,1)$, and $\sigma > 0$. □

Technical Insights of Proposition 2

As established in Proposition 1 and detailed in its proof, the information advantage becomes more pronounced as the competing counterpart's information set is also unconstrained. The information rent embedded in the second term of Π_s^{AG-S} is therefore limited, relative to that in Π_s^{AG-RS}. A higher informational value amplifies the corresponding informational payoff, such that $\frac{\partial \Pi_s^{AG-S}}{\partial \beta} < \frac{\partial \Pi_s^{AG-RS}}{\partial \beta}$. The same rationale applies to the retailer's gains, implying that Policy RS is also the most beneficial to the retailer, even more so than in Case R, thereby aligning both parties' best outcomes.

Proof of Proposition 2

Note that Π_j^{AG-S}, and Π_j^{AG-RS} denote the expected profits of firm $j \in \{s, r\}$ under Policies S and RS, respectively, as derived in Lemma 1.

Similar to the proof of Proposition 1, note that, for the seller,

$$\Pi_s^{AG-RS} > \Pi_s^{AG-S} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-S} = \frac{\beta(1-\gamma)\gamma(1-f)\sigma^2(\gamma+\gamma f+2)(8+\gamma(2-\gamma-\gamma f))}{4(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} > 0.$$

This inequality always holds because $8+\gamma(2-\gamma-\gamma f)>0$ and $f,\gamma,\beta\in(0,1)$, and $\sigma>0$.

Similarly, note that, for the retailer,

$$\Pi_r^{AG-RS}>\Pi_r^{AG-S} \iff \Pi_r^{AG-RS}-\Pi_r^{AG-S}=I_r^{AG}>0.$$

This inequality always holds for all $f,\gamma,\beta\in(0,1)$, because $I_r^{AG}>0$ from the Proof of Lemma 1. □

Proof of Lemma 2

Under ***WH-R***, the retailer chooses their optimal prices to maximize his profit as

$$\begin{aligned}\max_{p_s,p_r}\pi_r&=p_rE[q_r|Y]+(p_s-w_s)E[q_s|Y]\\&=\frac{p_r}{1+\gamma}\left(a+\beta Y-\frac{1}{1-\gamma}p_r+\frac{\gamma}{1-\gamma}p_s\right)+\frac{p_s-w_s}{1+\gamma}\left(a+\beta Y-\frac{1}{1-\gamma}p_s+\frac{\gamma}{1-\gamma}p_r\right).\end{aligned}\tag{S4}$$

This retailer's objective function is concave in p_r and p_s and is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives his best response as $p_r=\frac{a+\beta Y}{2}$ and $p_s=\frac{a+\beta Y+w_s}{2}$. Then, given the seller's expectation on his best response (i.e., $E[p_r]=\frac{a}{2}$ and $E[p_s]=\frac{a+w_s}{2}$), the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\begin{aligned}\max_{w_s}\pi_s=w_sE[q_s]&=\frac{w_s}{1+\gamma}\left(a-\frac{1}{1-\gamma}E[p_s]+\frac{\gamma}{1-\gamma}E[p_r]\right)\\&=\frac{w_s}{1+\gamma}\left(a-\frac{1}{1-\gamma}\left(\frac{a+w_s}{2}\right)+\frac{\gamma}{1-\gamma}\left(\frac{a}{2}\right)\right).\end{aligned}\tag{S5}$$

This seller's objective function is concave in w_s and is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives the seller's optimal wholesale price as $w_s=\frac{a(1-\gamma)}{2}$.

Substituting w_s into $p_s=\frac{a+\beta Y+w_s}{2}$ gives $p_s=\frac{a(3-\gamma)}{4}+\frac{\beta Y}{2}$. Then, substituting w_s, p_s, and p_r into Equation (S4) and (S5) gives

$$\pi_r=\tfrac{a^2(3\gamma+5)}{16(\gamma+1)}+\tfrac{a\beta(\gamma+3)Y}{4(\gamma+1)}+\tfrac{\beta^2Y^2}{2(\gamma+1)},\ \text{and}\ \pi_s=\tfrac{a^2(1-\gamma)}{8(\gamma+1)}.$$

By taking expectations on both sides of the equations above, the optimal expected profits of the seller and the retailer are as follows.

$$\Pi_r^{WH-R}=E[\pi_r]=\tfrac{a^2(3\gamma+5)}{16(\gamma+1)}+\tfrac{\beta\sigma^2}{2(\gamma+1)}:=\Pi_r^{WH}+\tfrac{\beta\sigma^2}{2(\gamma+1)},\ \text{and}\ \Pi_s^{WH-R}=E[\pi_s]=\tfrac{a^2(1-\gamma)}{8(\gamma+1)}:=\Pi_s^{WH}.$$

Similarly, the proofs for the remaining two policies, WH-S and WH-RS, can be derived straightforwardly and are therefore omitted. □

Technical Insights of Propositions 3 and 4 (1)

As established in Propositions 1 and 2, the way the information advantage manifests remains unchanged under WH. That is, the informed party can more effectively leverage its pricing decision, and recognizing the counterpart's information set further reinforces this effect. However, the value of data exclusivity becomes more pronounced under a linear wholesale pricing contract characterized by typical double marginalization, which reflects the inefficiency that arises when both the supplier and the retailer apply their own markups. Furthermore, compared with the seller, the retailer is relatively free from direct competition in his retail pricing stage (i.e., $\frac{\partial p_r^{\zeta}}{\partial \gamma} = 0$ for $\forall \zeta \in \{WH-R, WH-S, WH-RS\}$), where both products are priced by the retailer. For instance, *ceteris paribus*, as the retailer becomes more exclusively informed, his benefit from setting markups on the seller's wholesale price can be greater and the relative competitive advantage derived from his joint retail pricing across both products increases accordingly, i.e., $\frac{\partial^2 p_s^{WH-R}}{\partial Y \partial \gamma} = 0 > \frac{\partial^2 p_s^{WH-RS}}{\partial Y \partial \gamma}$. Anticipating such behavior, maintaining data exclusivity enables the seller to achieve her best possible performance.

Proof of Proposition 3

Note that Π_j^{WH-R}, Π_j^{WH-S}, and Π_j^{WH-RS} denote the expected profits of firm $j \in \{s, r\}$ under Case R, Policy S, and Policy RS, respectively, as derived in Lemma 2. Similar to the proof of Proposition 1, we can easily show that $\Pi_s^{WH-S} > \Pi_s^{WH-R}$ and $\Pi_s^{WH-RS} > \Pi_s^{WH-R}$ for the seller, and, for retailer, $\Pi_r^{WH-S} < \Pi_r^{WH-R}$, and $\Pi_r^{WH-RS} < \Pi_r^{WH-R}$ for $\gamma, \beta \in (0,1)$, and $\sigma > 0$ as follows:

For the seller, note that

$$\Pi_s^{WH-S} > \Pi_s^{WH-R} \iff \Pi_s^{WH-S} - \Pi_s^{WH-R} = \frac{\beta(1-\gamma)\sigma^2}{\gamma+1} > 0,$$
$$\Pi_s^{WH-RS} > \Pi_s^{WH-R} \iff \Pi_s^{WH-RS} - \Pi_s^{WH-R} = \frac{\beta(1-\gamma)\sigma^2}{8(\gamma+1)} > 0.$$

For the retailer, note that

$$\Pi_r^{WH-S} < \Pi_r^{WH-R} \iff \Pi_r^{WH-S} - \Pi_r^{WH-R} = -\frac{\beta\sigma^2}{2(\gamma+1)} < 0,$$
$$\Pi_r^{WH-RS} < \Pi_r^{WH-R} \iff \Pi_r^{WH-RS} - \Pi_r^{WH-R} = -\frac{3\beta(1-\gamma)\sigma^2}{16(\gamma+1)} < 0.$$

These inequalities always hold because $\gamma, \beta \in (0,1)$, and $\sigma > 0$. □

Technical Insights of Propositions 3 and 4 (2)

Unlike AG, given the retailer's downstream pricing and the prioritized emphasis on his retail brand capitalization, though present, the mutual benefit from information advantage is attenuated under WH, i.e., $\frac{\partial \Pi_s^{WH-R}}{\partial \beta} < \frac{\partial \Pi_s^{WH-RS}}{\partial \beta} < \frac{\partial \Pi_s^{WH-S}}{\partial \beta}$. As explained, information gain exists, but it increases with data exclusivity in light of double marginalization for both parties, i.e., $\frac{\partial \Pi_r^{WH-R}}{\partial \beta} > \frac{\partial \Pi_r^{WH-RS}}{\partial \beta} > \frac{\partial \Pi_r^{WH-S}}{\partial \beta}$. Propositions 3 and 4 show that policy enforcement under WH is therefore more crucial than under AG, where incentives are markedly misaligned.

Proof of Proposition 4

Note that Π_j^{WH-S} and Π_j^{WH-RS} denote the expected profits of firm $j \in \{s, r\}$ under Policies S and RS, respectively, as derived in Lemma 2. Similar to the proof of Proposition 3, $\Pi_s^{WH-RS} < \Pi_s^{WH-S}$ and $\Pi_r^{WH-RS} > \Pi_r^{WH-S}$ can be derived straightforwardly, and the details are thus omitted. □

Proof of Corollary 1

Using the expected profits, Π_s^{AG-R}, Π_s^{AG-S}, and Π_s^{AG-RS} from Lemma 1, together with Π_s^{WH-R} from Lemma 2, we derive the pairwise profit differences (denoted by $\Delta\Pi$) between AG and the unregulated WH benchmark. Specifically, under the unregulated setting of both AG and WH:

$$\Delta\Pi_s^{AG-R} = \Pi_s^{WH-R} - \Pi_s^{AG-R} = \frac{a^2(1-\gamma)}{8(\gamma+1)}\left(\frac{8(\gamma+2)^2(f-1)}{(\gamma^2(f+1)-4)^2}+1\right)$$

The right-hand side is negative at $f = 0$, and strictly increasing in f. It follows that there exists a threshold $\overline{f}_s \in (0, 1]$ such that the right-hand side is negative, i.e., $\Delta\Pi_s^{AG-R} < 0$, whenever $f < \overline{f}_s$.

Similarly, comparing the regulated AG settings with the unregulated WH benchmark, we obtain

$$\Delta\Pi_s^{AG-S} = \Pi_s^{WH-R} - \Pi_s^{AG-S} = \Delta\Pi_s^{AG-R} - \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)},$$
$$\Delta\Pi_s^{AG-RS} = \Pi_s^{WH-R} - \Pi_s^{AG-RS} = \Delta\Pi_s^{AG-R} - \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.$$

This establishes Corollary 1 since $\Delta\Pi_s^{AG-RS} - \Delta\Pi_s^{AG-S} = \frac{\beta(1-\gamma)\gamma(1-f)\sigma^2(\gamma+\gamma f+2)(\gamma(\gamma+\gamma f-2)-8)}{4(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} < 0$ for all admissible parameter values of f, γ, β, and σ. Using the expected profit Π_r^{AG-R} from Lemma 1 and Π_r^{WH-R} from Lemma 2, we derive the retailer's profit difference between *AG-R* and *WH-R*:

$$\begin{aligned}\Delta\Pi_r^{AG-R} &= \Pi_r^{WH-R} - \Pi_r^{AG-R}\\ &= \frac{a^2}{16(1+\gamma)}\left(3\gamma + \frac{16\left(\gamma^3+\gamma^4(-f)(f+1)+\gamma^2(f(f+5)+3)-4(f+1)\right)}{(\gamma^2(f+1)-4)^2}+5\right)+\frac{\beta\sigma^2}{4}.\end{aligned}$$

The first term on the right-hand side is positive at $f = 0$, and the second term is always positive. Moreover, the entire expression is strictly decreasing in f. It follows that there exists a threshold $\overline{f}_r \in (0, 1]$ such that the retailer prefers *WH-R* whenever $f < \overline{f}_r$. Hence, letting $f_{rs} := \min\left[\overline{f}_r, \overline{f}_s\right]$, it follows that for $f < f_{rs}$, the retailer switches from *AG-R* to *WH-R*, and this shift reduces the seller's payoff. The remaining comparison cases can be established analogously. □

Technical Insights of Proposition 5

As shown in Propositions 3 and 4, the retailer's downstream pricing for both products can neutralize the competitive pressure on his own brand, as indicated by $\frac{\partial p_r^{WH-S}}{\partial\gamma} = 0 > \frac{\partial p_r^{AG-S}}{\partial\gamma}$. This strategic decoupling suggests that, under intense market competition that suppresses the retailer's payoffs from both products under the agency contract, he may be incentivized to steer the seller toward the wholesale arrangement. The retailer's inclination toward the wholesale contract may, however, harm the seller when the quality of exclusive data is insufficient to offset the adverse effect of double marginalization under wholesale pricing. The seller's profit sensitivity to data accuracy is higher under wholesale pricing than under the agency arrangement, i.e., $\frac{\partial \Pi_s^{WH-S}}{\partial \beta} > \frac{\partial \Pi_s^{AG-S}}{\partial \beta} > 0$. Moreover, an unfavorable commission fee reinforces this tendency by tightening the retailer's portion.

Proof of Proposition 5

We first derive the retailer's preference for WH under Policy S, followed by the seller's preference.

Under Policy S, the retailer prefers WH if and only if $\Pi_r^{AG-S} < \Pi_r^{WH-S}$, which can be further reduced to:

$$F_1(\gamma, f) := (\gamma+2)^2(\gamma(\gamma(3\gamma-7)+8)+4) + ((3\gamma-11)\gamma^2+16)\,\gamma^2 f^2 + 2(\gamma-2)(\gamma+2)\,(3(\gamma-1)\gamma^2+8)\,f > 0.$$

Observe that $F_1(\gamma, f)$ is monotonically increasing in γ, but is monotonically decreasing in f.

$$\frac{\partial F_1(\gamma,f)}{\partial \gamma} = \gamma\left(15\gamma^3(f+1)^2 - 4\gamma^2(f+1)(11f-5) - 24\gamma(3f+1) + 16f(2f+5) + 16\right) + 48 > 0.$$

$$\frac{\partial F_1(\gamma,f)}{\partial f} = -24\gamma^3 + 6\gamma^5(f+1) - 2\gamma^4(11f+3) + 8\gamma^2(4f+5) - 64 < 0.$$

Note that, for $\gamma = 0$, $F_1(0, f) = 16 - 64f > 0$, if and only if $f < \frac{1}{4}$. Under this condition, $F_1(0, f) > 0$ also holds for all γ, because it is monotonically increasing in γ. This leads to $\Pi_r^{AG-S} < \Pi_r^{WH-S}$. In addition, for $\gamma = 1$, $F_1(1, f) = 8(3-f)^2 \ > \ 0$, which always holds for $0 < f < 1$. This means that, when $f > \frac{1}{4}$, there exists a threshold γ_{r1} such that $F_1(\gamma_{r1}, f) = 0$, because $F_1(0, f) < 0$, and $F_1(\gamma, f)$ is monotonically increasing in γ. Thus, $F_1(\gamma, f) > 0$ when $\gamma > \gamma_{r1}$. Altogether, $F_1(\gamma, f) > 0$, if and only if $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$. Therefore, under this condition, $\Pi_r^{AG-S} < \Pi_r^{WH-S}$ holds.

Next, the seller prefers WH if and only if $\Pi_s^{AG-S} < \Pi_s^{WH-S}$, which can be further reduced to:

$$F_2(\gamma, f) + G_1(\gamma, f)\beta\sigma^2 > 0,$$

where $F_2(\gamma, f) := \frac{a^2(1-\gamma)}{8(1+\gamma)}\left(\frac{8(\gamma+2)^2(f-1)}{(\gamma^2(f+1)-4)^2} + 1\right)$ and $G_1(\gamma, f) := \frac{1-\gamma}{1+\gamma}\left(\frac{2(f-1)(\gamma^2(f+1)-2)}{(\gamma^2(f+1)-4)^2} + 1\right)$.

Note that $G_1(\gamma, f) > 0$ always holds for all $f, \gamma \in (0, 1)$ and that $F_2(\gamma, f) > 0$ if and only if $f > f_{s1} := \frac{4(\gamma+2)\sqrt{\gamma\left(\gamma^3-\gamma+4\right)+4} - 16 - 16\gamma - \gamma^4}{\gamma^4}$.

(i) When $f < f_{s1}$, $\Pi_s^{AG-S} < \Pi_s^{WH-S} \iff \beta > \frac{-F_2(\gamma,f)}{G_1(\gamma,f)\sigma^2} := \beta_{s1}$. In other words, when $f < f_{s1}$, $\Pi_s^{AG-S} > \Pi_s^{WH-S} \iff \beta < \beta_{s1}$. Thus, the switch to WH hurts the seller under this condition.

(ii) when $f > f_{s1}$, $\Pi_s^{AG-S} < \Pi_s^{WH-S}$ always holds since $F_2(\gamma, f) > 0$ and $G_1(\gamma, f) > 0$ for $f > f_{s1}$. This implies that the switch to WH benefits the seller under this condition. □

Technical Insights of Proposition 6

The insight articulated in Propositions 1 – 5 remains robust. However, under RS, the quality of non-exclusive data cannot counteract the adverse effects of double marginalization, nor can it improve upon the optimal outcome achieved under AG-RS—unless the profit under AG is substantially eroded by the retailer's fee extraction.

Proof of Proposition 6

We first derive the retailer's preference for WH under Policy RS, followed by the seller's preference.

Under Policy RS, the retailer prefers WH if and only if $\Pi_r^{AG-RS} < \Pi_r^{WH-RS}$, which can be further reduced to $F_1(\gamma, f) > 0$, where $F_1(\gamma, f)$ is the function defined in the proof of Proposition 5, ensuring that the condition remains consistent with Proposition 5. Specifically, $\Pi_r^{AG-RS} < \Pi_r^{WH-RS}$ holds if and only if $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$.

Next, the seller prefers WH if and only if $\Pi_s^{AG-RS} < \Pi_s^{WH-RS}$ which can be reduced to $F_2(\gamma, f) > 0$, where $F_2(\gamma, f)$ is defined in the proof of Proposition 5. Recall that $F_2(\gamma, f) > 0$ holds if and only if $f > f_{s1} = \frac{4(\gamma+2)\sqrt{\gamma\left(\gamma^3-\gamma+4\right)+4}-16-16\gamma-\gamma^4}{\gamma^4}$. Thus, $\Pi_s^{AG-RS} < \Pi_s^{WH-RS}$ holds if and only if $f > f_{s1}$. □

Proof of Corollary 2

The proof is straightforward: the seller benefits from the retailer's switch to WH when $f > f_{s1}$ under both Policies S and RS, as shown in Propositions 5 and 6. However, when $f < f_{s1}$, the retailer's switch to WH benefits the seller under Policy S if $\beta > \beta_{s1}$, whereas the switch always harms the seller under Policy RS. □

Technical Insights of Proposition 7

The central technical insight derived from the previous propositions remains valid.

Proof of Proposition 7

(i) The seller prefers WH-S to AG-RS if and only if $\Pi_s^{AG-RS} < \Pi_s^{WH-S}$, which can be reduced to:

$$F_2(\gamma, f) + G_3(\gamma, f)\beta\sigma^2 > 0,$$

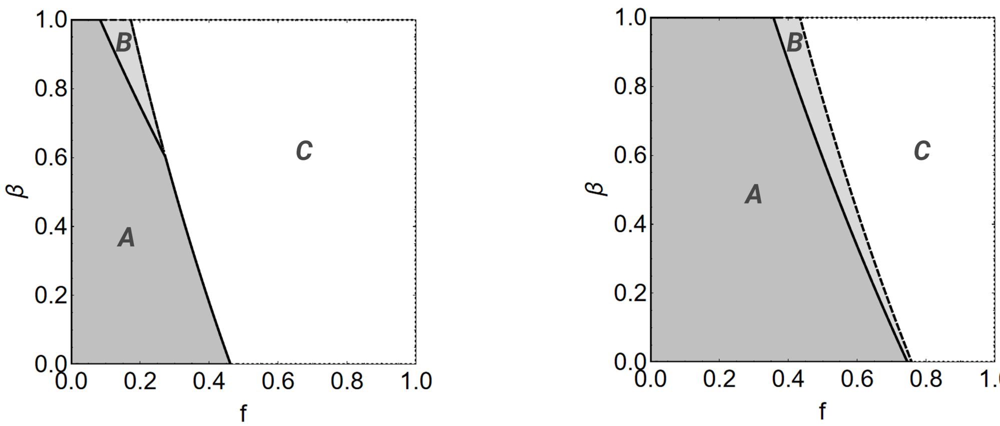
where $F_2(\gamma, f)$ is defined in Proposition 5, and $F_2(\gamma, f) > 0$ holds if and only if $f > f_{s1}$, which also leads to $G_3(\gamma, f) := \frac{1-\gamma}{1+\gamma}\left(\frac{(\gamma+2)^2(f-1)}{(\gamma^2(f+1)-4)^2} + 1\right) > 0$ Thus, when $f > f_{s1}$, $\Pi_s^{AG-RS} < \Pi_s^{WH-S}$ holds. When $f < f_{s1}$, $\Pi_s^{AG-RS} < \Pi_s^{WH-S}$ holds if and only if $\beta > \frac{-F_2(\gamma,f)}{G_3(\gamma,f)\sigma^2} := \beta_{s3}$.

(ii) The retailer prefers WH-S to AG-RS if and only if $\Pi_r^{AG-RS} < \Pi_r^{WH-S}$, which can be reduced to:

$$F_1(\gamma, f) - G_4(\gamma, f)\beta\sigma^2 > 0,$$

where $G_4(\gamma, f) := \frac{(\gamma-1)(-(\gamma+2)^2+(\gamma+1)\gamma^2 f^2+(\gamma-2)(\gamma+1)(\gamma+2)f)}{(\gamma+1)(\gamma^2(f+1)-4)^2} > 0$. From Proposition 5, if $\gamma < \gamma_{r1}$ and $f > \frac{1}{4}$, then $F_1(\gamma, f) < 0$, which leads to $F_1(\gamma, f) - G_4(\gamma, f)\beta\sigma^2 < 0$. So, the retailer has no incentive to switch to WH-S. Thus, only when $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$, the retailer's preference for WH-S can align with the seller's. Under this condition, $F_1(\gamma, f) - G_4(\gamma, f)\beta\sigma^2 > 0 \iff \beta < \frac{F_1(\gamma,f)}{G_4(\gamma,f)\sigma^2} := \beta_{r2}$.

We derive the condition under which both prefer WH. The seller prefers WH when $f > f_{s1}$ and the retailer does so if and only if $\beta < \beta_{r2}$ and either $f < \frac{1}{4}$ or $\gamma > \gamma_{r1}$. We verify that $f_{s1} > \frac{1}{4}$. Thus, when $f > f_{s1}$, the retailer's preference for WH aligns with the seller's if and only if $\gamma > \gamma_{r1}$ and $\beta < \beta_{r2}$. When $f < f_{s1}$, the seller prefers WH if and only if $\beta > \beta_{s3}$. Therefore, the retailer's preference for WH aligns with the seller's if and only if $\beta_{r3} < \beta < \beta_{r2}$ and either $f < \frac{1}{4}$ or $\gamma > \gamma_{r1}$ holds. Figure 5 highlights the region within the feasible space where both firms prefer WH over AG. □



(a) $\gamma = 0.25$ (b) $\gamma = 0.55$

Notes. Region A is the area where the retailer prefers WH while the seller prefers AG. In B, both the retailer and the seller prefer WH. In C, the retailer does not prefer WH. We use $a = 1$, and $\sigma = 0.5$.

Figure 5 Seller and retailer preferences: AG-RS vs. WH-S

Proof of Corollary 3

Note that $\Delta\Pi_s^{S/RS}$ is given by

$$\Delta\Pi_s^{S/RS} = \Pi_s^{WH-S} - \Pi_s^{AG-RS} = \frac{a^2(1-\gamma)}{8(1+\gamma)}\left(\frac{8(\gamma+2)^2(f-1)}{(\gamma^2(f+1)-4)^2}+1\right) + \frac{\beta\sigma^2}{1+\gamma}\left(\frac{(\gamma-1)(\gamma+2)^2(1-f)}{(\gamma^2(f+1)-4)^2}+(1-\gamma)\right).$$

Taking the partial derivative of $\Delta\Pi_s^{S/RS}$ with respect to β and $\frac{\partial\Delta\Pi_s^{S/RS}}{\partial\beta} > 0$ yields

$$\frac{\partial\Delta\Pi_s^{S/RS}}{\partial\beta} = \frac{\sigma^2}{1+\gamma}\left(\frac{(\gamma-1)(\gamma+2)^2(1-f)}{(\gamma^2(f+1)-4)^2}+1-\gamma\right) > 0 \iff Af^2 + Bf + C > 0,$$

where $A = (1-\gamma)\gamma^4 > 0$, $B = (1-\gamma)(\gamma+2)\left(2(\gamma-2)\gamma^2+\gamma+2\right)$, and $C = (3-\gamma)\left(\gamma^2+\gamma-2\right)^2$. This inequality yields $f < f_1$ or $f > f_2$, as $A > 0$, where f_1 and f_2 are two roots for $Af^2 + Bf + C = 0$. However, because $f_1 < f_2 < 0$, $\frac{\partial\Delta\Pi_s^{S/RS}}{\partial\beta} > 0$ always holds.

Similarly, note that $\Delta\Pi_r^{S/RS}$ is given by

$$\begin{aligned}\Delta\Pi_r^{S/RS} &= \Pi_r^{WH-S} - \Pi_r^{AG-RS}\\ &= \frac{a^2}{16(1+\gamma)}\left(3\gamma+5+\frac{16(\gamma^3+\gamma^4(-f)(f+1)+\gamma^2(f(f+5)+3)-4(f+1))}{(\gamma^2(f+1)-4)^2}\right) - \frac{\beta\sigma^2\left(\gamma^4 f(f+1)-\gamma^3-\gamma^2(f(f+5)+3)+4(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.\end{aligned}$$

Taking the partial derivative of $\Delta\Pi_r^{S/RS}$ with respect to β and $\frac{\partial\Delta\Pi_r^{S/RS}}{\partial\beta}<0$ yields

$$\frac{\partial\Delta\Pi_r^{S/RS}}{\partial\beta}=-\frac{\sigma^2\left(\gamma^4 f(f+1)-\gamma^3-\gamma^2(f(f+5)+3)+4(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}<0 \iff Af^2+Bf+C>0,$$

where $A=(\gamma^2-1)\gamma^2<0$, $B=\gamma^4-5\gamma^2+4$, and $C=-(\gamma-1)(\gamma+2)^2$. This inequality yields $f_3<f<f_4$, as $A<0$, where f_3 and f_4 are two roots for $Af^2+Bf+C=0$. Thus, $\frac{\partial\Delta\Pi_r^{S/RS}}{\partial\beta}<0$ always holds, because $f_3<0<f<1<f_4$. □

Technical Insights of Proposition 8

The core insight holds in reverse: while firm-level consumer data facilitates consumer surplus extraction, especially as all firms have unconstrained access (i.e., $\frac{\partial\Pi_c^\zeta}{\partial\beta}\leq 0$ for $\zeta\in\{AG-RS, WH-RS\}$), competition can offset this through downward pricing pressure. However, vertical structures, particularly linear wholesale contracts, remain inefficient due to double marginalization, lowering stakeholders' welfare including that of consumers. Unlike in agency models, where direct competition among brands is preserved, wholesale arrangements incentivize the retailer to weaken brand competition to promote his own offerings, thereby exacerbating inefficiencies and further harming consumer welfare, e.g., $\Pi_c^{AG-RS}>\Pi_c^{WH-RS}$. Therefore, Π_c^{WH-RS} reflects the lowest consumer surplus, driven by the compounded effects of double marginalization (WH) and intensified extraction by both informed firms (RS).

Proof of Proposition 8

Our demand functions are derived from the quadratic consumption utility of a representative consumer, expressed as $\sum_j\left(A(q_j)-\frac{q_j^2}{2}-p_jq_j\right)-\gamma\sum_{i\neq j}q_iq_j$. By solving utility maximization problems for the representative consumer, our linear demand functions are derived, which is extensively used in the literature (Singh and Vives 1984, Zhang and Zhang 2020, Ha et al. 2022). In specific, solving the FOCs of π_c with respect to q_s and q_r gives

$$q_s=\frac{1}{1+\gamma}\left(A-\frac{1}{1-\gamma}p_s+\frac{\gamma}{1-\gamma}p_r\right),\ \text{ and } q_r=\frac{1}{1+\gamma}\left(A-\frac{1}{1-\gamma}p_r+\frac{\gamma}{1-\gamma}p_s\right).$$

These are the same as the demand functions in Equation (1). As pointed out in Zhang and Zhang (2020), this type of formulation aligns with a formulation in which a continuum of identical consumers whose utility functions are separable and linear in the good.

(i) In AG-R, substituting the expressions for p_r and p_s from Lemma 1 into Equation (6) yields:

$$\begin{aligned}\pi_c=&\frac{a^2(\gamma(2(\gamma+4)+(\gamma+1)f(\gamma(f-2)-4))+8)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}\\&+\frac{a\theta(\gamma(\gamma f+f-2)-4)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)}+Y\left(\frac{a\beta(\gamma-1)(\gamma(\gamma f+f-1)-2)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)}+\frac{\beta(\gamma-1)\theta}{2(\gamma+1)}\right)+\frac{\theta^2}{\gamma+1}-\frac{\beta^2(\gamma-1)Y^2}{8(\gamma+1)}.\end{aligned}$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{AG-R} := E[\pi_c] = \frac{a^2(\gamma(2(\gamma+4)+(\gamma+1)f(\gamma(f-2)-4))+8)}{\left(\gamma^2(f+1)-4\right)^2 2(1+\gamma)} + \frac{2\sigma^2}{2(1+\gamma)} - \frac{3\beta(1-\gamma)\sigma^2}{8(\gamma+1)}.$$

In AG-S, substituting the expressions for p_r and p_s from Lemma 1 into Equation (6) yields:

$$\begin{aligned}\pi_c = & \frac{a^2(2\gamma(\gamma+4)+\gamma(\gamma+1)f(\gamma(f-2)-4)+8)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} \\ & + \frac{a\theta(\gamma(\gamma f+f-2)-4)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)} + Y\left(\frac{\beta(\gamma-1)\theta}{2(\gamma+1)} - \frac{a\beta\left(\gamma^2+\gamma-2\right)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)}\right) + \frac{\theta^2}{\gamma+1} - \frac{\beta^2(\gamma-1)Y^2}{8(\gamma+1)}.\end{aligned}$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{AG-S} := E[\pi_c] = \frac{a^2(\gamma(2(\gamma+4)+(\gamma+1)f(\gamma(f-2)-4))+8)}{\left(\gamma^2(f+1)-4\right)^2 2(1+\gamma)} + \frac{2\sigma^2}{2(1+\gamma)} - \frac{3\beta(1-\gamma)\sigma^2}{8(\gamma+1)}.$$

Clearly, $\Pi_c^{AG-R} = \Pi_c^{AG-S}$ always holds.

In AG-RS, substituting the expressions for p_r and p_s from Lemma 1 into Equation (6) yields:

$$\begin{aligned}\pi_c = & \frac{a^2(\gamma(2(\gamma+4)+(\gamma+1)f(\gamma(f-2)-4))+8)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{a\theta(\gamma(\gamma f+f-2)-4)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)} + \frac{\theta^2}{\gamma+1} + \frac{\beta^2(\gamma-1)Y^2\left(\gamma\left(2\gamma(\gamma+3)-\gamma f^2+2\left(\gamma^2+\gamma-2\right)f\right)-8\right)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} \\ & + Y\left(-\frac{a\beta(\gamma-1)(\gamma(\gamma(f(\gamma+\gamma f+f+2)-2)-8)-8)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} - \frac{\beta(\gamma-1)\theta(\gamma(f+2)+4)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)}\right).\end{aligned}$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{AG-RS} := E[\pi_c] = \frac{a^2(\gamma(2(\gamma+4)+(\gamma+1)f(\gamma(f-2)-4))+8)}{\left(\gamma^2(f+1)-4\right)^2 2(1+\gamma)} - \frac{\beta(1-\gamma)\sigma^2\left(24-2\gamma^3(f+1)^2-\gamma^2(f(f+6)+2)+4\gamma(f+4)\right)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.$$

Note that $\Pi_c^{AG-S} > \Pi_c^{AG-RS}$ can be reduced to $\frac{(\gamma-1)(\gamma+2)(\gamma+\gamma f+2)\left(3\gamma^2(f+1)+2\gamma(f-2)-12\right)}{8(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} > 0$. This inequality always holds for all $f, \gamma \in (0,1)$.

Therefore, $\Pi_c^{AG-S} = \Pi_c^{AG-R} > \Pi_c^{AG-RS}$ holds.

(ii) In WH-R, substituting the expressions for p_r and p_s from Lemma 2 into Equation (6) gives

$$\pi_c = \frac{a^2(3\gamma+5)}{32(\gamma+1)} + \frac{a(\gamma+3)\theta}{4(\gamma+1)} + Y\left(-\frac{a\beta(\gamma+3)}{8(\gamma+1)} - \frac{\beta\theta}{\gamma+1}\right) + \frac{\theta^2}{\gamma+1} + \frac{\beta^2 Y^2}{4\gamma+4}.$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{WH-R} := E[\pi_c] = \frac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)} - \frac{3\beta\sigma^2}{4\gamma+4}.$$

In WH-S, substituting the expressions for p_r and p_s from Lemma 2 into Equation (6) gives

$$\pi_c = \frac{a^2(3\gamma+5)}{32(\gamma+1)} + \frac{a(\gamma+3)\theta}{4(\gamma+1)} + \frac{\theta^2}{\gamma+1}.$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{WH-S} := E[\pi_c] = \frac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)}.$$

In WH-RS, substituting the expressions for p_r and p_s from Lemma 2 into Equation (6) gives

$$\pi_c = \frac{a^2(3\gamma+5)}{32(\gamma+1)} + \frac{a(\gamma+3)\theta}{4(\gamma+1)} + Y\left(\frac{\beta(\gamma-5)\theta}{4(\gamma+1)} - \frac{a\beta(\gamma+7)}{16(\gamma+1)}\right) + \frac{\theta^2}{\gamma+1} + \frac{\beta^2(13-5\gamma)Y^2}{32(\gamma+1)}.$$

Taking the expectation on both sides of the equation above, the expected consumer surplus is

$$\Pi_c^{WH-RS} := E[\pi_c] = \frac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)} - \frac{3\beta(9-\gamma)\sigma^2}{32(\gamma+1)}.$$

Note that $\Pi_c^{WH-S} > \Pi_c^{WH-R}$ can be reduced to $\frac{3}{4\gamma+4} > 0$, which always holds. Similarly, $\Pi_c^{WH-R} > \Pi_c^{WH-RS}$ reduces to $\frac{3(1-\gamma)}{32(\gamma+1)} > 0$, which also holds.

Altogether, $\Pi_c^{WH-S} > \Pi_c^{WH-R} > \Pi_c^{WH-RS}$. □

Appendix B: Analysis for Retailer Market Entry

In this analysis, we examine the retailer's market entry decision prior to the pricing game. For the benchmark (Case R), Policy S, and Policy RS under both AG and WH, we derive the equilibrium outcomes including optimal prices and entry decisions. Subsequently, we assess whether our main results continue to hold qualitatively.

This extension uncovers additional insights about the impact of data policies on the retailer's market entry. Under AG, compared to the benchmark (i.e., AG-R), Policies S and RS discourage the retailer from entering the market. Specifically, in Case R, the retailer enters the market when the commission rate f is low, competition intensity γ is not high,[1] or the data accuracy β is high (i.e., $f \le \tilde{f}_1$, $\gamma \le \tilde{\gamma}_1$, or $\beta > \tilde{\beta}_1$). Intuitively, lower commission revenue and low competition reduce the opportunity cost of entry, motivating the retailer to introduce a private-label product. In addition, when data accuracy is high, the retailer's exclusive access to superior demand information enhances its competitive position against the seller. This informational advantage, as also outlined in Lemma 1 and Proposition 1, can outweigh the effects of competition and commission margins. Under Policy S, however, the retailer's entry decision becomes independent of β, as the policy eliminates the informational benefit that would otherwise favor entry. Likewise, Policy RS neutralizes the effect of information asymmetry by granting both firms equal access to market data, leading to the same equilibrium outcome.

In contrast, under WH, neither Policy S nor RS meaningfully curtails the retailer's incentive to enter the market. Upon entry, the retailer gains an additional revenue stream by directly sourcing and selling its own product. This advantage is amplified under WH, as the retailer's control over downstream prices for both products mitigates competition, as discussed in Proposition 3. As a result, the retailer secures a higher margin than it would by reselling the seller's product at the wholesale price—even if the retail price of its own product is lower. Consequently, market entry remains an attractive strategy. Interestingly, this observation of his aggressive encroachment under WH aligns with regulatory concerns about antitrust issues in markets where Amazon exercises full pricing control through wholesale contracts with its sellers (Sager 2024). Overall, our extended model reveals that while Policies S and RS benefit sellers in competitive markets upon retailer entry, they can also offer additional protection to sellers, especially under AG.

[1] This finding aligns with observations from the Amazon marketplace, where Amazon has actively promoted private-label and exclusive brands, particularly in electronics—a sector known for lower commission rates (around 8%) (Numerator 2019, Amazon 2023). This lower commission structure likely incentivizes Amazon to prioritize its own-brand products in these categories. Notable examples include *Aplenty* and *Happy Belly* in the food category and *Rivet* in the furniture category, which operate in less saturated, less competitive markets (Forbes 2022).

Proofs: We exclude the analysis for the retailer's entry scenario, as the equilibrium outcomes in the subgame are identical to those derived in the main analyses. We use p_j^{NE} and π_j^{NE} to denote the price and profit of firm $j \in \{s, r\}$ under the scenario where the retailer does not enter the market.

AG-R: Under AG in the benchmark case, without the retailer's entry, the seller chooses her optimal price p_s^{NE} to maximize her profit π_s^{NE} as follows:

$$\max_{p_s^{NE}} \pi_s^{NE} = \quad (1-f)p_s^{NE}E[q_s^{NE}] = \quad (1-f)p_s^{NE}(a - p_s^{NE}). \tag{S6}$$

This seller's objective function is concave in p_s^{NE} and is sufficient to solve its first-order conditions (FOCs) to derive the best response. Thus, solving the FOCs gives her optimal price $p_s^{NE} = \frac{a}{2}$. Substituting the optimal retail price into Equation (S6), the expected profit of the seller can be derived as $\Pi_s^{NE} := E[\pi_s^{NE}] = \frac{a^2(1-f)}{4}$. Accordingly, substituting the optimal price into the retailer's profit $\pi_r^{NE} = fp_s^{NE}(a - p_s^{NE})$ and taking the expectation of the profit over the distribution of Y, the expected profit of the retailer can be derived as $\Pi_r^{NE} := E[\pi_r^{NE}] = \frac{a^2 f}{4}$.

Note that, under AG in Policy R with his entry, the equilibrium results remain the same as those derived in Lemma 1 (i.e., AG-R in the main analysis).

Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{AG-R} > \Pi_r^{NE} = \frac{a^2 f}{4},$$

where Π_r^{AG-R} is presented in Lemma 1. The inequality holds if and only if

$$\tilde{F}_1(f,\gamma) + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} > 0,$$

where $\tilde{F}_1(f,\gamma) := \frac{a^2(1-\gamma)\left((\gamma+2)^2+(\gamma+1)\gamma^2\left(-f^2\right)-(\gamma-2)(\gamma+1)(\gamma+2)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{a^2 f}{4}$. Based on Cardano's approach (Nelson 2008), we can easily verify that $\tilde{F}_1(f,\gamma)$ has three real roots. However, Galois's theory (Artin and Milgram 1998) shows that without a rational root, the roots cannot be expressed by an algebraic expression.

Alternatively, note that $\frac{\partial \tilde{F}_1(f,\gamma)}{\partial f} < 0$, for any γ, $\gamma \in (0,1)$ and $\frac{\partial \tilde{F}_1(f,\gamma)}{\partial \gamma} < 0$, for any f, $f \in [0,1]$. In other words, $\tilde{F}_1(f,\gamma)$ is a strictly decreasing function of f and γ, respectively. In addition, note that the maximum (minimum) value of $F_1(f,\gamma)$ is positive (negative) in all possible parameter ranges of f and γ. Altogether, there exist such values of f and γ (e.g., $\tilde{f}_1 \in [0,1]$ and $\gamma_1 \in (0,1)$) such that $\tilde{F}_1(f,\gamma) = 0$). Meanwhile, solving $\tilde{F}_1(f,\gamma) + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} = 0$ gives $\beta = \frac{a^2(\gamma+1)}{(1-\gamma)\sigma^2}\left(\frac{4\left(\gamma^3+\gamma^4(-f)(f+1)+\gamma^2(f(f+5)+3)-4(f+1)\right)}{(\gamma+1)\left((1-\gamma)\sigma^2\gamma^2(f+1)-4\right)^2} + \frac{f}{(1-\gamma)\sigma^2}\right) := \tilde{\beta}_1$. Therefore, the retailer enters the market (i.e., $\pi_r^{AG-R} > \pi_r^{NE}$) if and only if the competition is not intense, the commission fee is low, or the information is highly accurate (i.e., $f \leq \tilde{f}_1$, $\gamma \leq \tilde{\gamma}_1$, or $\beta > \tilde{\beta}_1$).

AG-S: Under AG in Policy S, without his entry, the seller chooses her optimal price to maximize her profits π_s^{NE} as

$$\max_{p_s^{NE}} \pi_s^{NE} = \quad (1-f)p_s^{NE}E[q_s^{NE}|Y] = \quad (1-f)p_s^{NE}(a+\beta Y - p_s^{NE}). \tag{S7}$$

Similarly, we derive her optimal price as $p_s^{NE} = \frac{a+\beta Y}{2}$. Substituting the optimal retail price into Equation (S7), the expected profit of the seller can be derived as $\Pi_s^{NE} := E[\pi_s^{NE}] := \frac{a^2(1-f)}{4} + \frac{(1-f)\beta\sigma^2}{4}$. Accordingly, substituting the optimal retail price into the retailer's profit $\pi_r^{NE} = fp_s^{NE}(a - p_s^{NE})$ and taking the expectation of the profit over the distribution of Y, the expected profit of the retailer can be derived as $\Pi_r^{NE} = E[\pi_r^{NE}] := \frac{a^2 f}{4}$.

Note that, under AG in Policy S with his entry, the equilibrium results remain the same as those derived in Lemma 1 (i.e., for AG-S in the main analysis).

Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{AG-S} > \Pi_r^{NE} = \frac{a^2 f}{4},$$

where Π_r^{AG-S} is presented in Lemma 1. The inequality holds if and only if

$$\tilde{F}_1(f,\gamma) > 0.$$

In the proof for AG-R, we showed that $\tilde{F}_1(f,\gamma) > 0$ holds if and only if the competition is not intense or the commission fee is low (i.e., $f \le \tilde{f}_1$ or $\gamma \le \tilde{\gamma}_1$). Thus, the condition for his market entry becomes stricter, compared to AG-R.

AG-RS: Under AG in Policy RS, without his entry, the seller chooses her optimal price to maximize her profits π_s^{NE} as

$$\max_{p_s^{NE}} \pi_s^{NE} = \quad (1-f)p_s^{NE}E[q_s^{NE}|Y] = \quad (1-f)p_s^{NE}(a+Y\beta - p_s^{NE}). \tag{S8}$$

Similarly, the seller's optimal price can be derived as $p_s^{NE} = \frac{a}{2} + \frac{\beta Y}{2}$. Substituting the optimal retail price into Equation (S8), the profit of the seller, given a signal Y, can be derived as $\pi_s^{NE} = \frac{(1-f)(a+\beta Y)^2}{4}$. Accordingly, the profit of the retailer can be derived as $\pi_r^{NE} = \frac{f(a+\beta Y)^2}{4}$. Then, taking expectations on both sides of the equations above (i.e., $E[\pi_s^{NE}]$ and $E[\pi_r^{NE}]$) gives their optimal expected profits as follows.

$$\Pi_s^{NE} := E[\pi_s^{NE}] = \frac{a^2(1-f)}{4} + \frac{\beta(1-f)\sigma^2}{4}, \quad \Pi_r^{NE} := E[\pi_r^{NE}] = \frac{a^2 f}{4} + \frac{\beta f \sigma^2}{4}.$$

Note that, with his entry, the equilibrium results remain the same as those derived in Lemma 1 (i.e., AG-RS in the main analysis).

Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{AG-RS} > \Pi_r^{NE} = \frac{a^2 f}{4} + \frac{\beta f \sigma^2}{4},$$

where Π_r^{AG-RS} is presented in Lemma 1. The inequality holds if and only if

$$\tilde{F}_1(f, \gamma) > 0.$$

Again, we already showed that $\tilde{F}_1(f,\gamma) > 0$ holds if and only if $f \leq \tilde{f}_1$ or $\gamma \leq \tilde{\gamma}_1$. Thus, the condition for his market entry becomes stricter, as compared to the benchmark case AG-R.

Note that Propositions 1 and 2 are not contingent on specific conditions. Therefore, it is evident that these propositions remain valid under the entry conditions.

WH-R: Under WH in the benchmark case, without his market entry, the retailer chooses the optimal price p_s^{NE} to maximize his profit:

$$\max_{p_s^{NE}} \pi_r^{NE} = \quad (p_s^{NE} - w_s^{NE}) E[q_s^{NE}|Y] = \quad (p_s^{NE} - w_s^{NE})(a + Y\beta - p_s^{NE}). \tag{S9}$$

This retailer's objective function is concave in p_s^{NE}, and it is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives his best response as $p_s^{NE} = \frac{a+\beta Y + w_s^{NE}}{2}$, where w_s^{NE} is the wholesale price of product s. Then, given p_s^{NE}, the seller chooses the optimal wholesale price w_s^{NE} to maximize her profit as

$$\max_{w_s^{NE}} \pi_s^{NE} = \quad w_s^{NE} E[q_s^{NE}] = \quad w_s^{NE}\left(a - E[p_s^{NE}]\right) = \quad w_s^{NE}\left(a - \frac{a + w_s^{NE}}{2}\right). \tag{S10}$$

This seller's objective function is concave in w_s^{NE}, and it is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives the seller's optimal wholesale price as $w_s^{NE} = \frac{a}{2}$. Substituting w_s^{NE} in p_s^{NE} gives $p_s^{NE} = \frac{3a}{4} + \frac{\beta Y}{2}$. Substituting w_s^{NE} and p_s^{NE} into Equation (S9) gives $\Pi_r^{NE} := E[\pi_r^{NE}] = \frac{a^2}{16} + \frac{\beta\sigma^2}{4}$. Substituting w_s^{NE} and $E[p_s^{NE}]$ into Equation (S10) gives $E[\pi_s^{NE}] = \frac{a^2}{8}$.

With his entry, the equilibrium results remain the same as those derived in Lemma 2 (i.e., WH-R in the main analysis).

Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{WH-R} > \Pi_r^{NE} \iff \frac{a^2(\gamma+2)}{8(\gamma+1)} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} > 0,$$

where Π_r^{WH-R} is presented in Lemma 2. The inequality always holds because $\frac{a^2(\gamma+2)}{8(\gamma+1)} > 0$ and $\frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} > 0$. This implies that the retailer always enters the market.

WH-S: Under WH in Policy S, without his entry, the retailer chooses the optimal price to maximize his profit:

$$\max_{p_s^{NE}} \pi_r^{NE} = \quad (p_s^{NE} - E[w_s^{NE}]) E[q_s^{NE}] = \quad (p_s^{NE} - E[w_s^{NE}])(a - p_s^{NE}). \tag{S11}$$

Similarly, his best response can be derived as $p_s^{NE} = \frac{a+E[w_s^{NE}]}{2}$. Then, given p_s^{NE}, the seller chooses the optimal wholesale price w_s^{NE} to maximize her profit as

$$\max_{w_s^{NE}} \pi_s^{NE} = \quad w_s^{NE} E[q_s^{NE}|Y] = \quad w_s^{NE} \left(a + \beta Y - \frac{a + w_s^{NE}}{2} \right). \tag{S12}$$

Similarly, the seller's optimal wholesale price can be derived as $w_s^{NE} = \frac{a}{2} + \beta Y$. Substituting $E[w_s^{NE}] = \frac{a}{2}$ in $p_s^{NE} = \frac{a+E[w_s^{NE}]}{2}$ gives $p_s^{NE} = \frac{3a}{4}$. Substituting w_s^{NE} and p_s^{NE} into Equations (S11) and (S12) gives $\Pi_r^{NE} = E[\pi_r^{NE}] = \frac{a^2}{16}$ and $E[\pi_s^{NE}] = \frac{a^2}{8} + \beta\sigma^2$, respectively. Note that with his entry, the equilibrium results remain the same as those derived in Lemma 2 (i.e., WH-S in the main analysis).

Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{WH-S} > \Pi_r^{NE} = \frac{a^2}{16},$$

where Π_r^{WH-S} is presented in Lemma 2. The inequality always holds because $\frac{a^2(\gamma+2)}{8(\gamma+1)} > 0$. This implies that the retailer always enters the market.

WH-RS: Under WH in Policy RS, without his entry, the retailer chooses the optimal price to maximize his profit as

$$\max_{p_s^{NE}} \pi_r^{NE} = \quad (p_s^{NE} - w_s^{NE}) E[q_s^{NE}|Y] = \quad (p_s^{NE} - w_s^{NE}) \left(a + \beta Y - p_s^{NE} \right). \tag{S13}$$

Similarly, his best response can be derived as $p_s^{NE} = \frac{a+\beta Y + w_s^{NE}}{2}$. Then, given his best response p_s^{NE}, the seller chooses the optimal wholesale price w_s^{NE} to maximize her profit as

$$\max_{w_s^{NE}} \pi_s^{NE} = \quad w_s^{NE} E[q_s^{NE}|Y] = \quad w_s^{NE} \left(a + \beta Y - \frac{a + \beta Y + w_s^{NE}}{2} \right). \tag{S14}$$

Note that the seller's optimal wholesale price can be derived as $w_s^{NE} = \frac{a+\beta Y}{2}$. Substituting w_s^{NE} in $p_s^{NE} = \frac{a+\beta Y + w_s^{NE}}{2}$ gives $p_s^{NE} = \frac{3(a+\beta Y)}{4}$. Substituting w_s^{NE} and p_s^{NE} into Equations (S13) and (S14) gives $\pi_r^{NE} = \frac{(a+\beta Y)^2}{16}$ and $\pi_s^{NE} = \frac{(a+\beta Y)^2}{8}$, respectively. Taking the expectation on both sides of the equations above gives their optimal expected profits as $E[\pi_s^{NE}] := \Pi_s^{NE} = \frac{a^2}{8} + \frac{\beta\sigma^2}{8}$ and $E[\pi_r^{NE}] := \Pi_r^{NE} = \frac{a^2}{16} + \frac{\beta\sigma^2}{16}$, respectively. Note that with his entry, the equilibrium results remain the same as those derived in Lemma 2 (i.e., WH-RS in the main analysis).

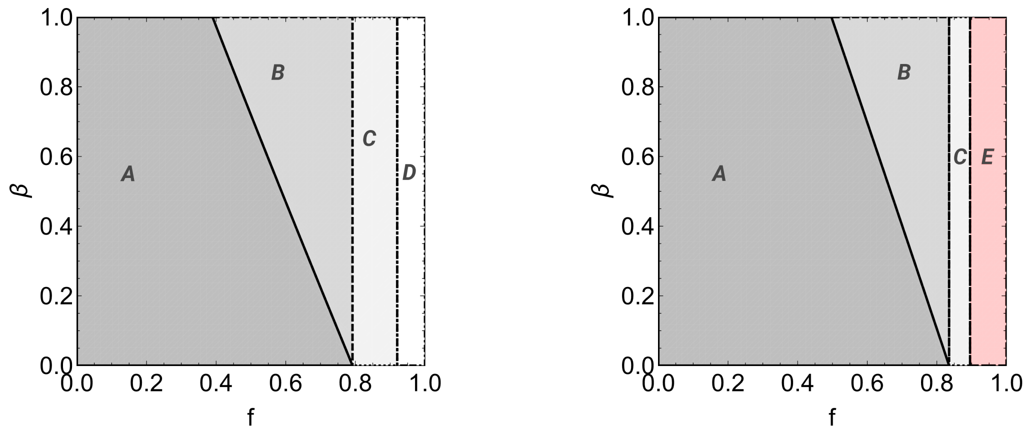
Next, we derive his decision on market entry. The retailer enters the market if and only if

$$\Pi_r^{WH-RS} = \frac{a^2(3\gamma+5)}{16(\gamma+1)} + \frac{\beta(3\gamma+5)\sigma^2}{16(\gamma+1)} > \Pi_r^{NE} = \frac{a^2}{16} + \frac{\beta\sigma^2}{16}.$$

where Π_r^{WH-RS} is presented in Lemma 2. The inequality always holds because $\frac{a^2(\gamma+2)}{8(\gamma+1)} > 0$ and $\frac{\beta(\gamma+2)\sigma^2}{8(\gamma+1)}$. This implies that the retailer always enters the market.[2]

[2] Our observation of the retailer's aggressive encroachment under WH aligns with strong regulatory concerns about antitrust issues in markets where Amazon exercises full price control under wholesale contracts with its sellers (Sager 2024).

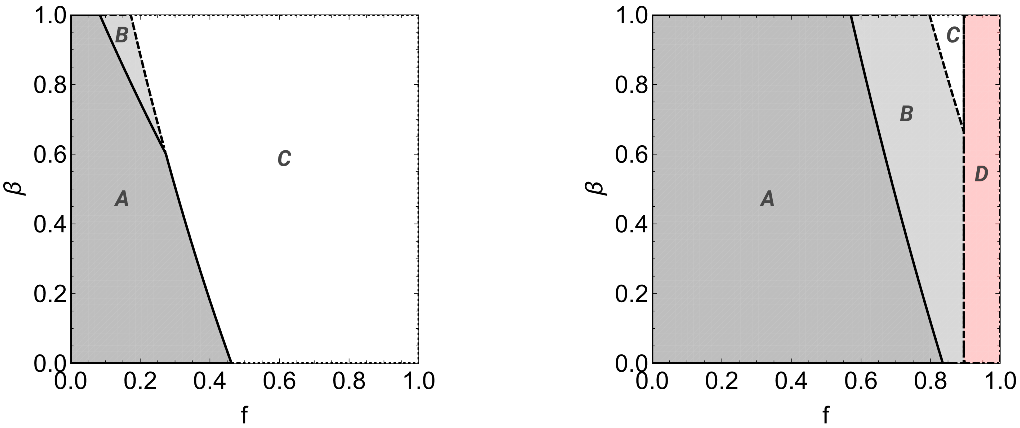
Note that Propositions 3, 4, and 8 are not contingent on specific conditions. Therefore, it is evident that these propositions remain valid. In addition, proofs of Propositions 5 – 7 are omitted because they are analogous to those for the main analysis. Figure 6 illustrates Propositions 5 and 6, and Figure 7 illustrates Proposition 7 under the entry conditions.



(a) $\gamma = 0.6$ (b) $\gamma = 0.7$

Notes. Region A (C) is the area where the retailer is incentivized to switch to WH and the switch decreases (increases) the seller's profit under Policies S and RS. In B, the retailer is incentivized to switch to WH and the switch decreases the seller's profit under Policy RS but increases the seller's profit under Policy S. In D, the retailer is not incentivized to switch to WH. In E, the retailer does not enter the market. We use $a = 1$ and $\sigma = 0.5$.

Figure 6 **Retailer's market entry: Alignment of Profitability from the Retailer's Practice in Response to Data Regulations**



(a) $\gamma = 0.25$

(b) $\gamma = 0.7$

Notes. Region A is the area where the retailer is incentivized to switch to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch to WH. In D, the retailer does not enter the market. We use $a = 1$ and $\sigma = 0.5$.

Figure 7 Retailer's market entry: Seller and retailer preferences for contract modes (AG-RS vs. WH-S)

Appendix C: Analysis for Retailer Demand Advantage

In this analysis, we consider the asymmetric base demand, captured by $\delta \in (-\frac{a}{2}, \frac{a}{2})$, between the retailer and the seller. For Case R, Policy S, and Policy RS under AG and WH, we derive the equilibrium results and assess whether our main results hold qualitatively.

Overall, we confirm that our main results remain qualitatively the same even when the retailer benefits from demand advantage (i.e., $\delta > 0$). As expected, his incentive to mitigate policy-induced losses by switching from AG to WH becomes stronger as the retailer can more effectively capitalize on the demand advantage with full control over both products (e.g., $\frac{\partial p_r^\zeta}{\partial \gamma} = 0$ for $\forall \zeta \in \{WH-S, WH-RS\}$). At the same time, a stronger demand advantage can also benefit the seller with the retailer's switching, because under AG, the seller's demand disadvantage can be exacerbated with a high commission fee (e.g., $\frac{\partial \Pi_s^{AG-RS}}{\partial \delta} < \frac{\partial \Pi_s^{WH-RS}}{\partial \delta}$ if $f > \frac{\gamma^4 - 4\gamma^3 - 4\sqrt{-\gamma^7 + 2\gamma^6 + 12\gamma^5 - 5\gamma^4 - 40\gamma^3 + 32\gamma + 16} + 16\gamma + 16}{16\gamma^2 - \gamma^4}$). This dynamic highlights the complex interplay between market advantages and strategic pricing decisions under WH. This extension reinforces regulatory concerns about the dual-role retailer, as the retailer can leverage various marketing tools to redirect demand toward his own products, further exacerbating competitive imbalances.

Proofs: In AG-R, both firms choose their optimal prices to maximize their profits as

$$\begin{aligned} \max_{p_r} \pi_r &= p_r E[q_r|Y] + f p_s E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a + \delta + \beta Y - \tfrac{1}{1-\gamma} p_r + \tfrac{\gamma}{1-\gamma} p_s\right) + \tfrac{f p_s}{1+\gamma}\left(a - \delta + \beta Y - \tfrac{1}{1-\gamma} p_s + \tfrac{\gamma}{1-\gamma} p_r\right), \\ \max_{p_s} \pi_s &= (1-f) p_s E[q_s] = \tfrac{(1-f)p_s}{1+\gamma}\left(a - \delta - \tfrac{1}{1-\gamma} p_s + \tfrac{\gamma}{1-\gamma} E[p_r]\right). \end{aligned} \tag{S15}$$

The retailer's (seller's) objective function is concave in p_r (p_s) and is sufficient to solve its FOCs to derive the best response. Solving their best responses as a system of equations gives the optimal retail prices $p_r = \frac{(1-\gamma)(a(\gamma+\gamma f+2) - \delta(\gamma+\gamma f-2))}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)Y}{2}$ and $p_s = \frac{(1-\gamma)(a(\gamma+2)-(2-\gamma)\delta)}{4-\gamma^2(f+1)}$. Substituting the optimal retail prices p_r and p_s into Equation (S15), given a signal Y, the profits of the retailer and the seller can be derived as

$$\begin{aligned} \pi_r = &\ \frac{a^2(\gamma-1)\left(-(\gamma+2)^2 + (\gamma+1)\gamma^2 f^2 + \left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{2a(1-\gamma)\delta\left(-\gamma^2+\gamma^2 f^2 - \left(\gamma^2+4\right)f + 4\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta^2(1-\gamma)Y^2}{4(\gamma+1)} \\ &- \frac{(\gamma-1)\delta^2\left((\gamma-2)^2 + (\gamma-1)\gamma^2 f^2 + \left(\gamma^3-\gamma^2-4\gamma+4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + Y\left(\frac{a\beta(\gamma-1)(\gamma+2(\gamma+1)f+2)}{(\gamma+1)(\gamma^2(f+1)-4)} + \frac{\beta(1-\gamma)\delta(\gamma+2f-2)}{(\gamma+1)(\gamma^2(f+1)-4)}\right), \\ \pi_s = &\ \frac{(\gamma-1)(f-1)(a(\gamma+2)-(2-\gamma)\delta)^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}. \end{aligned}$$

Taking expectations on both sides of the equations above gives the optimal expected profit of the seller and the retailer as follows.

$$\begin{aligned} \Pi_r^{AG-R} := E[\pi_r] = &\ \frac{a^2(\gamma-1)\left(-(\gamma+2)^2 + (\gamma+1)\gamma^2 f^2 + \left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{2a(1-\gamma)\delta\left(-\gamma^2+\gamma^2 f^2 - \left(\gamma^2+4\right)f + 4\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} \\ &+ \frac{(1-\gamma)\delta^2\left((\gamma-2)^2 + (\gamma-1)\gamma^2 f^2 + \left(\gamma^3-\gamma^2-4\gamma+4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)}, \\ \Pi_s^{AG-R} := E[\pi_s] = &\ \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} - \frac{2a(2-\gamma)(1-\gamma)(\gamma+2)\delta(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{(2-\gamma)^2(\gamma-1)\delta^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}. \end{aligned}$$

In AG-S, the retailer and the seller choose their optimal prices to maximize their profits as

$$\begin{aligned}
\max_{p_r} \pi_r &= p_r E[q_r] + fE[p_s]E[q_s] \\
&= \tfrac{p_r}{1+\gamma}\left(a+\delta-\tfrac{1}{1-\gamma}p_r+\tfrac{\gamma}{1-\gamma}E[p_s]\right)+\tfrac{fE[p_s]}{1+\gamma}\left(a-\delta-\tfrac{1}{1-\gamma}E[p_s]+\tfrac{\gamma}{1-\gamma}p_r\right), \qquad \text{(S16)}\\
\max_{E[p_s]} \pi_s &= (1-f)p_s E[q_s|Y] = \tfrac{(1-f)p_s}{1+\gamma}\left(a-\delta+\beta Y-\tfrac{1}{1-\gamma}p_s+\tfrac{\gamma}{1-\gamma}p_r\right).
\end{aligned}$$

Similarly, by solving their best responses as a system of equations (i.e., $p_r = \frac{(a+\delta)(1-\gamma)}{2}+\frac{\gamma(f+1)}{2}E[p_s]$ and $p_s = \frac{a(1-\gamma)}{2}+\frac{(-\delta+\gamma(\delta+p_r-\beta Y)+\beta Y)}{2}$), the optimal retail prices can be derived as follows:

$$p_s = \tfrac{(1-\gamma)(a(\gamma+2)+(\gamma-2)\delta)}{4-\gamma^2(f+1)}+\tfrac{\beta(1-\gamma)Y}{2}, \text{ and } p_r = \tfrac{(1-\gamma)(a(\gamma+\gamma f+2)-\delta(\gamma+\gamma f-2))}{4-\gamma^2(f+1)}.$$

Substituting the optimal retail prices into Equation (S16), their expected profits can be derived.

$$\begin{aligned}
\Pi_r^{AG-S} := E[\pi_r] = {} & \tfrac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}+\tfrac{2a(1-\gamma)\delta\left(-\gamma^2+\gamma^2f^2-\left(\gamma^2+4\right)f+4\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}\\
& -\tfrac{(\gamma-1)\delta^2\left((\gamma-2)^2+(\gamma-1)\gamma^2f^2+\left(\gamma^3-\gamma^2-4\gamma+4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2},\\
\Pi_s^{AG-S} := E[\pi_s] = {} & \tfrac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}-\tfrac{2a(2-\gamma)(1-\gamma)(\gamma+2)\delta(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}+\tfrac{(\gamma-2)^2(1-\gamma)\delta^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}+\tfrac{(1-\gamma)(1-f)\beta\sigma^2}{4}.
\end{aligned}$$

In AG-RS, the retailer and the seller choose their optimal prices to maximize their profits as

$$\begin{aligned}
\max_{p_r} \pi_r &= p_r E[q_r|Y] + f p_s E[q_s|Y] \\
&= \tfrac{p_r}{1+\gamma}\left(a+\delta+\beta Y-\tfrac{1}{1-\gamma}p_r+\tfrac{\gamma}{1-\gamma}p_s\right)+\tfrac{fp_s}{1+\gamma}\left(a-\delta+\beta Y-\tfrac{1}{1-\gamma}p_s+\tfrac{\gamma}{1-\gamma}p_r\right), \qquad \text{(S17)}\\
\max_{p_s} \pi_s &= (1-f)p_s E[q_s|Y] = \tfrac{(1-f)p_s}{1+\gamma}\left(a-\delta+\beta Y-\tfrac{1}{1-\gamma}p_s+\tfrac{\gamma}{1-\gamma}p_r\right).
\end{aligned}$$

Similarly, their optimal retail prices can be derived as $p_r = \frac{(1-\gamma)(a(\gamma+\gamma f+2)-\delta(\gamma+\gamma f-2))}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)Y(\gamma+\gamma f+2)}{4-\gamma^2(f+1)}$ and $p_s = \frac{(1-\gamma)(a(\gamma+2)-(2-\gamma)\delta)}{4-\gamma^2(f+1)}+\frac{\beta(1-\gamma)(\gamma+2)Y}{4-\gamma^2(f+1)}$. Then, substituting the optimal retail prices into Equation (S17), given a signal Y, their expected profits can be derived as

$$\begin{aligned}
\Pi_r^{AG-RS} := E[\pi_r] = {} & \tfrac{a^2(\gamma-1)\left(-(\gamma+2)^2+(\gamma+1)\gamma^2f^2+\left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}+\tfrac{2a(1-\gamma)\delta\left(-\gamma^2+\gamma^2f^2-\left(\gamma^2+4\right)f+4\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}\\
& +\tfrac{(1-\gamma)\delta^2\left((\gamma-2)^2+(\gamma-1)\gamma^2f^2+\left(\gamma^3-\gamma^2-4\gamma+4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}+\tfrac{\beta(\gamma-1)\sigma^2\left(-(\gamma+2)^2+(\gamma+1)\gamma^2f^2+\left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2},\\
\Pi_s^{AG-RS} := E[\pi_s] = {} & \tfrac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}-\tfrac{2a(2-\gamma)(1-\gamma)(\gamma+2)\delta(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}+\tfrac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}+\tfrac{(2-\gamma)^2(\gamma-1)\delta^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}.
\end{aligned}$$

Note that, for the seller, $\Pi_s^{AG-S} > \Pi_s^{AG-R} \iff \Pi_s^{AG-S} - \Pi_s^{AG-R} = \frac{(1-\gamma)(1-f)\beta\sigma^2}{4} > 0$ and that, for the retailer, $\Pi_r^{AG-S} < \Pi_r^{AG-R} \iff \Pi_r^{AG-S} - \Pi_r^{AG-R} = -\frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} < 0$. These inequalities always hold for all $\gamma, f \in (0,1)$. Hence, Proposition 1(i) remains to hold, for any given $\delta \in (-\frac{a}{2}, \frac{a}{2})$.

Similarly, $\Pi_s^{AG-RS} > \Pi_s^{AG-R} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-R} = \frac{\beta(\gamma-1)(\gamma+2)^2(f-1)\sigma^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} > 0$, for the seller. This inequality is satisfied for all $\gamma, f \in (0,1)$.

Note that, for the retailer, $\Pi_r^{AG-RS} > \Pi_r^{AG-R} \iff \Pi_r^{AG-RS} - \Pi_r^{AG-R} > 0 \iff Af^2+Bf+C > 0$, where $A = -(\gamma^3+2\gamma^2) < 0$, $B = -2\gamma^3+4\gamma+8$, and $C = -\gamma^3+2\gamma^2+8\gamma$. This inequality holds if

$f_1 < f < f_2$, where f_1 and f_2 are the solutions of $Af^2 + Bf + C = 0$. In addition, we can easily verify that $f_1 < 0 < f < 1 < f_2$, which implies that $\Pi_r^{AG-RS} > \Pi_r^{AG-R}$ always holds. Together, Proposition 1(ii) remains to hold, for any given $\delta \in (-\frac{a}{2}, \frac{a}{2})$.

In addition, we can easily show that Proposition 2 remains to hold, for any given $\delta \in (-\frac{a}{2}, \frac{a}{2})$ in a similar manner so we omit the details.

In WH-R, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned} \max_{p_s, p_r} \pi_r &= p_r E[q_r|Y] + (p_s - w_s)E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a + \delta + \beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s - w_s}{1+\gamma}\left(a - \delta + \beta Y - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right). \end{aligned} \tag{S18}$$

The retailer's objective function is concave in p_r and p_s and is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives his best response as $p_r = \frac{a+\beta Y}{2} + \frac{(1-\gamma)\delta}{2\gamma+2}$ and $p_s = \frac{a+\beta Y + w_s}{2} - \frac{(1-\gamma)\delta}{2\gamma+2}$. Then, given his expected best response, $E[p_r]$ and $E[p_s]$, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s] = \tfrac{w_s}{1+\gamma}\left(a - \delta - \tfrac{1}{1-\gamma}\left(\tfrac{a+\beta Y + w_s}{2} - \tfrac{(1-\gamma)\delta}{2(\gamma+1)}\right) + \tfrac{\gamma}{1-\gamma}\left(\tfrac{a}{2} + \tfrac{(1-\gamma)\delta}{2(\gamma+1)}\right)\right). \tag{S19}$$

The seller's objective function is also concave in w_s and is sufficient to solve its FOCs to derive the best response. Thus, solving the FOCs gives her optimal wholesale price as $w_s = \frac{(a-\delta)(1-\gamma)}{2}$. Substituting w_s into p_s and p_r gives $p_s = \frac{a(3-\gamma)}{4} + \left(\frac{\gamma+1}{4} - \frac{1}{\gamma+1}\right)\delta + \frac{\beta Y}{2}$ and $p_r = \frac{a}{2} + \left(\frac{1}{2} + \frac{1}{\gamma+1}\right)\delta + \frac{\beta Y}{2}$. By substituting w_s, p_s, and p_r into Equations (S18) and (S19), given a signal Y, their expected profits can be derived as:

$$\begin{aligned} \Pi_r^{WH-R} &:= E[\pi_r] = \tfrac{a^2(3\gamma+5)}{16(\gamma+1)} - \tfrac{3a(\gamma-1)\delta}{8(\gamma+1)} + \tfrac{(\gamma-1)(3\gamma-5)\delta^2}{16(\gamma+1)^2} + \tfrac{\beta\sigma^2}{2(\gamma+1)}, \\ \Pi_s^{WH-R} &:= E[\pi_s] = \tfrac{a^2(1-\gamma)}{8(\gamma+1)} - \tfrac{\delta(a-a\gamma)}{4\gamma+4} + \tfrac{(1-\gamma)\delta^2}{8\gamma+8}. \end{aligned}$$

In WH-S, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned} \max_{p_s, p_r} \pi_r &= p_r E[q_r] + (p_s - E[w_s])E[q_s] \\ &= \tfrac{p_r}{1+\gamma}\left(a + \delta - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s - E[w_s]}{1+\gamma}\left(a - \delta - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right). \end{aligned} \tag{S20}$$

Similarly, his best response can be derived as $p_r = \frac{a}{2} + \frac{(1-\gamma)\delta}{2\gamma+2}$ and $p_s = \frac{a+E[w_s]}{2} - \frac{(1-\gamma)\delta}{2\gamma+2}$. Then, given his best response, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s|Y] = \tfrac{w_s}{1+\gamma}\left(a + \beta Y - \delta - \tfrac{1}{1-\gamma}\left(\tfrac{a+w_s}{2} - \tfrac{(1-\gamma)\delta}{2(\gamma+1)}\right) + \tfrac{\gamma}{1-\gamma}\left(\tfrac{a}{2} + \tfrac{(1-\gamma)\delta}{2\gamma+2}\right)\right) \tag{S21}$$

Solving the FOCs gives the seller's optimal wholesale price as $w_s = \frac{(1-\gamma)(a+2\beta Y-\delta)}{2}$. Substituting w_s into $p_s = \frac{a+E[w_s]}{2} - \frac{(1-\gamma)\delta}{2\gamma+2}$ gives $p_s = \frac{a(3-\gamma)}{4} + \left(\frac{\gamma+1}{4} - \frac{1}{\gamma+1}\right)\delta$. Substituting w_s, p_s, and p_r into Equations (S20) and (S21) gives

$$\begin{aligned} \Pi_r^{WH-S} &:= E[\pi_r] = \tfrac{(3\gamma+5)a^2}{16(\gamma+1)} + \tfrac{3a(1-\gamma)\delta}{8(\gamma+1)} + \tfrac{(1-\gamma)(5-3\gamma)\delta^2}{16(\gamma+1)^2}, \\ \Pi_s^{WH-S} &:= E[\pi_s] = \tfrac{(1-\gamma)a^2}{8(\gamma+1)} - \tfrac{\delta a(1-\gamma)}{4(\gamma+1)} + \tfrac{(1-\gamma)\delta^2}{8(\gamma+1)} + \tfrac{\beta(1-\gamma)\sigma^2}{\gamma+1}. \end{aligned}$$

In WH-RS, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned}\max_{p_s,p_r} \pi_r &= p_r E[q_r|Y] + (p_s - w_s)E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a+\delta+\beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s-w_s}{1+\gamma}\left(a-\delta+\beta Y - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S22}$$

Similarly, his best response can be derived as $p_r = \frac{a+\beta Y}{2} + \frac{(1-\gamma)\delta}{2\gamma+2}$ and $p_s = \frac{a+\beta Y+w_s}{2} - \frac{(1-\gamma)\delta}{2\gamma+2}$. Then, given p_r and p_s, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s|Y] = \tfrac{w_s}{1+\gamma}\left(a-\delta+\beta Y - \tfrac{1}{1-\gamma}\left(\tfrac{a+\beta Y+w_s}{2} - \tfrac{(1-\gamma)\delta}{2\gamma+2}\right) + \tfrac{\gamma}{1-\gamma}\left(\tfrac{a+\beta Y}{2} + \tfrac{(1-\gamma)\delta}{2\gamma+2}\right)\right). \tag{S23}$$

The seller's optimal wholesale price can be derived as $w_s = \frac{(a-\delta+\beta Y)(1-\gamma)}{2}$. Substituting w_s into p_s gives $p_s = \frac{a(3-\gamma)}{4} + \left(\frac{\gamma+1}{4} - \frac{1}{\gamma+1}\right)\delta + \frac{\beta(3-\gamma)Y}{4}$. Substituting w_s, p_s, and p_r into Equations (S22) and (S23) gives

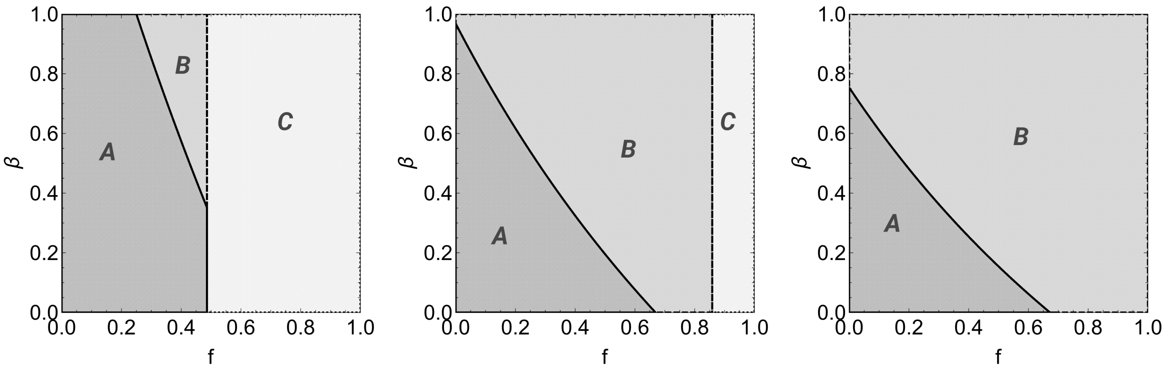
$$\begin{aligned}\Pi_r^{WH-RS} := E[\pi_r] = &\quad \tfrac{(3\gamma+5)\left(a^2+\beta\sigma^2\right)}{16(\gamma+1)} + \tfrac{3a(1-\gamma)\delta}{8(\gamma+1)} + \tfrac{(\gamma-1)(3\gamma-5)\delta^2}{16(\gamma+1)^2}, \\ \Pi_s^{WH-RS} := E[\pi_s] = &\quad \tfrac{(1-\gamma)\left(a^2+\beta\sigma^2\right)}{8(\gamma+1)} - \tfrac{\delta(a-a\gamma)}{4\gamma+4} + \tfrac{(1-\gamma)\delta^2}{8\gamma+8}.\end{aligned}$$

For the seller, note that $\Pi_s^{WH-S} > \Pi_s^{WH-R} \iff \Pi_s^{WH-S} - \Pi_s^{WH-R} = \frac{\beta(1-\gamma)\sigma^2}{\gamma+1} > 0$ and that $\Pi_s^{WH-RS} > \Pi_s^{WH-R} \iff \Pi_s^{WH-RS} - \Pi_s^{WH-R} = \frac{\beta(1-\gamma)\sigma^2}{8(\gamma+1)} > 0$. These inequalities always hold for all $\gamma \in (0,1)$. Thus, both Policies S and RS benefit the seller.

However, for the retailer, note that $\Pi_r^{WH-S} < \Pi_r^{WH-R} \iff \Pi_r^{WH-S} - \Pi_r^{WH-R} = -\frac{\beta\sigma^2}{2(\gamma+1)} < 0$ and that $\Pi_r^{WH-RS} < \Pi_r^{WH-R} \iff \Pi_r^{WH-RS} - \Pi_r^{WH-R} = -\frac{3\beta(1-\gamma)\sigma^2}{16(\gamma+1)} < 0$. These inequalities always hold for all $\gamma \in (0,1)$. Thus, both Policies S and RS harm the retailer. Hence, Proposition 3 remains to hold, for any given $\delta \in (-\frac{a}{2}, \frac{a}{2})$.

In a similar way, we can easily verify that Proposition 4 remains to hold, for any given $\delta \in (-\frac{a}{2}, \frac{a}{2})$. So, the details are excluded here.

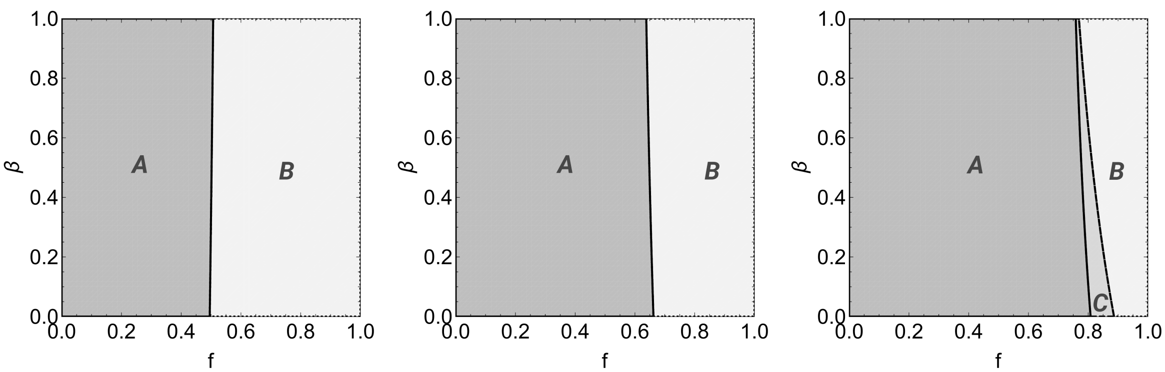
Due to the tractability issue, we are unable to analytically derive the retailer's strategy to counter policy-induced losses and its impact on the seller's profit. Instead, we conduct numerical analyses to confirm the robustness of our findings in Propositions 5 through 7, as illustrated in Figures 8 to 10.



(a) $\delta = -0.2$ (b) $\delta = 0.2$ (c) $\delta = 0.3$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a = 1$, $\gamma = 0.3$, and $\sigma = 0.5$.

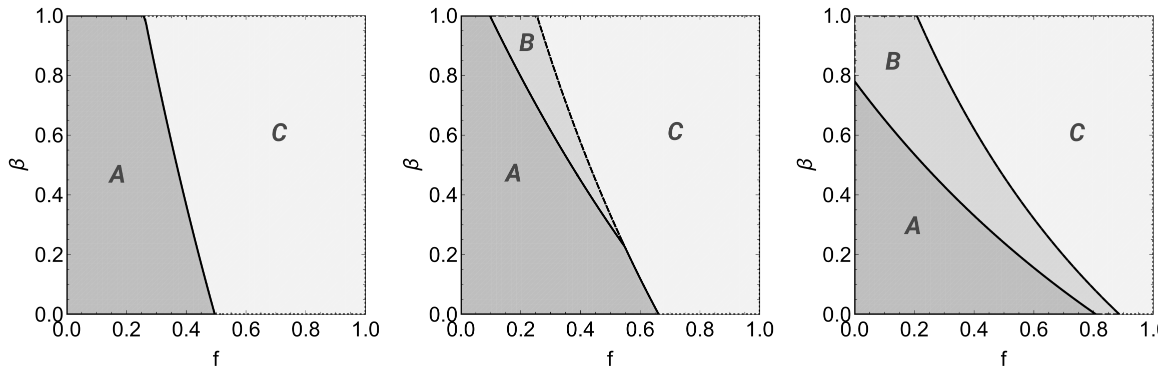
Figure 8 **Demand Advantage: Seller and retailer preferences for contract modes under Policy S**



(a) $\delta = -0.2$ (b) $\delta = 0.2$ (c) $\delta = 0.3$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a = 1$, $\gamma = 0.3$, and $\sigma = 0.5$.

Figure 9 **Demand Advantage: Seller and retailer preferences for contract modes under Policy RS**

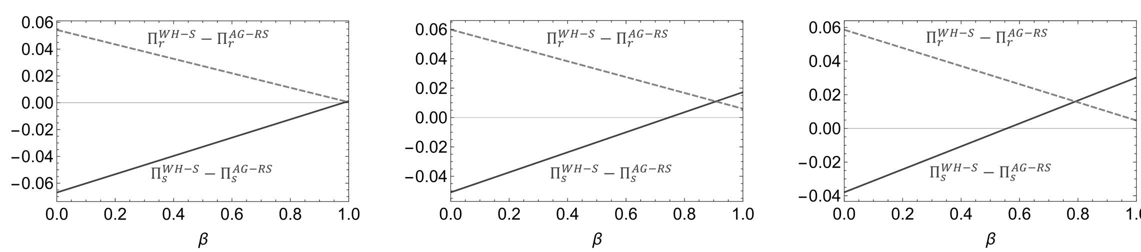


(a) $\delta = -0.2$ (b) $\delta = 0.2$ (c) $\delta = 0.3$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a = 1$, $\gamma = 0.3$, and $\sigma = 0.5$.

Figure 10 Demand Advantage: Seller and retailer preferences for contract modes (AG-RS vs. WH-S)

Overall, we observe similar trends in the preferences of the seller and the retailer regarding contract modes, but with thresholds shifted by δ. Specifically, as δ increases, the retailer's incentive to switch to WH becomes stronger. Interestingly, when the retailer has a greater demand advantage, the retailer's switch to WH results in higher profits for the seller. This creates a scenario where the seller's data advantage becomes more pronounced, as illustrated in Figure 11.



(a) $\delta = -0.2$ (b) $\delta = 0.2$ (c) $\delta = 0.3$

Notes. We use $a = 1$, $f = 0.35$, $\gamma = 0.45$, and $\sigma = 0.5$.

Figure 11 Demand advantage: Impact of Data Accuracy on Profits Between AG-RS and WH-S

Finally, we verify whether Proposition 8 remains valid in this extension. In AG-R, substituting $p_r = \frac{(\gamma-1)(a(\gamma+\gamma f+2)-\delta(\gamma+\gamma f-2))}{\gamma^2(f+1)-4} - \frac{\beta(\gamma-1)Y}{2}$ and $p_s = \frac{(\gamma-1)(a(\gamma+2)+(\gamma-2)\delta)}{\gamma^2(f+1)-4}$ into

$$\pi_c = \left((a+\delta+\theta)q_r - \frac{q_r^2}{2} - p_r q_r\right) + \left((a-\delta+\theta)q_s - \frac{q_s^2}{2} - p_s q_s\right) - \gamma q_r q_s,$$

where $q_r = a-\delta+\theta-\frac{1}{1-\gamma}p_r+\frac{\gamma}{1-\gamma}p_s$ and $q_s = a+\delta+\theta-\frac{1}{1-\gamma}p_s+\frac{\gamma}{1-\gamma}p_r$, gives

$$\begin{aligned}\pi_c = &\tfrac{a^2(2\gamma(\gamma+4)+\gamma(\gamma+1)f(\gamma(f-2)-4)+8)}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}+\tfrac{a\theta(\gamma(\gamma f+f-2)-4)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)}+\tfrac{\delta}{\gamma+1}\left(\tfrac{a\gamma^2 f(2(\gamma-1)\gamma+(2\gamma^2+\gamma-1)f-6)}{(\gamma^2(f+1)-4)^2}+\tfrac{(\gamma-1)\gamma f\theta}{(\gamma^2(f+1)-4)}\right)\\ &+\tfrac{\delta^2\left(4\gamma^5 f(f+1)+\gamma^4(f(2-3f)+8)-2\gamma^3(f(f+12)+9)+\gamma^2(f(f+6)-22)+4\gamma(f+12)+8\right)}{2(\gamma+1)^2\left(\gamma^2(f+1)-4\right)^2}-\tfrac{\beta^2(\gamma-1)Y^2}{8(\gamma+1)}+\tfrac{\theta^2}{\gamma+1}\\ &+\tfrac{Y}{\gamma+1}\left(\tfrac{a\beta(\gamma-1)(\gamma(\gamma f+f-1)-2)}{2(\gamma^2(f+1)-4)}+\tfrac{\beta(\gamma-1)\theta}{2}+\tfrac{\beta\delta(\gamma(-2\gamma^2+\gamma-(\gamma+2)\gamma f+f+5)+2)}{2(\gamma^2(f+1)-4)}\right).\end{aligned}$$

Taking expectations on both sides of the equation gives

$$\begin{aligned}\Pi_c^{AG-R} := E[\pi_c] = &\tfrac{a\gamma^2\delta f\left(2(\gamma-1)\gamma+\left(2\gamma^2+\gamma-1\right)f-6\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}+\tfrac{a^2(2\gamma(\gamma+4)+\gamma(\gamma+1)f(\gamma(f-2)-4)+8)+2\sigma^2\left(\gamma^2(f+1)-4\right)^2}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}\\ &+\tfrac{3\beta(\gamma-1)\sigma^2}{8(\gamma+1)}+\tfrac{\delta^2\left(4\gamma^5 f(f+1)+\gamma^4(f(2-3f)+8)-2\gamma^3(f(f+12)+9)+\gamma^2(f(f+6)-22)+4\gamma(f+12)+8\right)}{2(\gamma+1)^2\left(\gamma^2(f+1)-4\right)^2}.\end{aligned}$$

Similarly, in AG-S, substituting $p_r = \frac{(\gamma-1)(a(\gamma+\gamma f+2)-\delta(\gamma+\gamma f-2))}{\gamma^2(f+1)-4}$ and $p_s = \frac{(\gamma-1)(a(\gamma+2)+(\gamma-2)\delta)}{\gamma^2(f+1)-4}-\frac{\beta(\gamma-1)Y}{2}$ into

$$\pi_c = \left((a+\delta+\theta)q_r-\frac{q_r^2}{2}-p_r q_r\right)+\left((a-\delta+\theta)q_s-\frac{q_s^2}{2}-p_s q_s\right)-\gamma q_r q_s,$$

where $q_r = a-\delta+\theta-\frac{1}{1-\gamma}p_r+\frac{\gamma}{1-\gamma}p_s$ and $q_s = a+\delta+\theta-\frac{1}{1-\gamma}p_s+\frac{\gamma}{1-\gamma}p_r$, and taking expectations on both sides of the equation gives

$$\begin{aligned}\Pi_c^{AG-S} := E[\pi_c] = &\tfrac{a\gamma^2\delta f\left(2(\gamma-1)\gamma+\left(2\gamma^2+\gamma-1\right)f-6\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}+\tfrac{a^2(2\gamma(\gamma+4)+\gamma(\gamma+1)f(\gamma(f-2)-4)+8)+2\sigma^2\left(\gamma^2(f+1)-4\right)^2}{2(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}\\ &+\tfrac{3\beta(\gamma-1)\sigma^2}{8(\gamma+1)}+\tfrac{\delta^2\left(4\gamma^5 f(f+1)+\gamma^4(f(2-3f)+8)-2\gamma^3(f(f+12)+9)+\gamma^2(f(f+6)-22)+4\gamma(f+12)+8\right)}{2(\gamma+1)^2\left(\gamma^2(f+1)-4\right)^2}=\Pi_c^{AG-R}.\end{aligned}$$

In a similar way, in AG-RS, we can easily derive Π_c^{AG-RS} and observe that

$$\Pi_c^{AG-RS}-\Pi_c^{AG-R} = -\frac{\beta(\gamma-1)(\gamma+2)\sigma^2(\gamma+\gamma f+2)\left(3\gamma^2(f+1)+2\gamma(f-2)-12\right)}{8(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.$$

Note that $\Pi_c^{AG-RS}-\Pi_c^{AG-R}<0$, because $-\frac{\beta(\gamma-1)(\gamma+2)\sigma^2(\gamma+\gamma f+2)\left(3\gamma^2(f+1)+2\gamma(f-2)-12\right)}{8(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}<0$ holds for any given $f,\gamma\in(0,1)$.

Therefore, Proposition 8(i) holds with $\Pi_c^{AG-S}=\Pi_c^{AG-R}$ verified above.

In WH-R, similarly, we can easily derive Π_c^{WH-R}, Π_c^{WH-S} and Π_c^{WH-RS} as follows:

$$\begin{aligned}\Pi_c^{WH-R} &:= E[\pi_c] = \tfrac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)}+\tfrac{a(5\gamma+3)\delta}{16(\gamma+1)}-\tfrac{3\beta\sigma^2}{4\gamma+4}+\tfrac{(\gamma(40-13\gamma)+5)\delta^2}{32(\gamma+1)^2},\\ \Pi_c^{WH-S} &:= E[\pi_c] = \tfrac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)}+\tfrac{a(5\gamma+3)\delta}{16(\gamma+1)}+\tfrac{(\gamma(40-13\gamma)+5)\delta^2}{32(\gamma+1)^2},\\ \Pi_c^{WH-RS} &:= E[\pi_c] = \tfrac{a^2(3\gamma+5)+32\sigma^2}{32(\gamma+1)}+\tfrac{a(5\gamma+3)\delta}{16(\gamma+1)}+\tfrac{3\beta(\gamma-9)\sigma^2}{32(\gamma+1)}+\tfrac{(\gamma(40-13\gamma)+5)\delta^2}{32(\gamma+1)^2}.\end{aligned}$$

Note that $\Pi_c^{WH-R}-\Pi_c^{WH-S} = -\frac{3\beta\sigma^2}{4\gamma+4}<0$, and that $\Pi_c^{WH-RS}-\Pi_c^{WH-R} = -\frac{3\beta(1-\gamma)\sigma^2}{32(\gamma+1)}<0$. The inequalities always hold. Therefore, $\Pi_c^{WH-RS}<\Pi_c^{WH-R}<\Pi_c^{WH-S}$ holds and Proposition 8(ii) remains to hold in this extension. $\square$

Appendix D: Analysis for Competition with Multiple Sellers

In this analysis, we relax the assumption that there is only one seller in the market. As explained in the proof of Proposition 8, our demand functions are derived from the quadratic consumption utility of a representative consumer, expressed as

$$\sum_j \left(A\,q_j - \frac{q_j^2}{2} - p_j q_j\right) - \gamma \sum_{i \neq j} q_i q_j.$$

By solving the FOCs of the consumer utility above with respect to the retail prices, we can generalize the demand functions as follows

$$q_j = \frac{1}{1+n\gamma}\left(A - \frac{1+(n-1)\gamma}{1-\gamma} p_j + \frac{\gamma}{1-\gamma}\sum_{j\neq k} p_k\right), \tag{S24}$$

where n is the number of sellers in the market, i.e., $n \geq 2$. For notational simplicity, we use $j \in \{1,...,n\}$ to denote the seller j and r to denote the retailer. Note that the number of products is $n+1$.

We confirm that our main results remain qualitatively the same in this extension. This outcome is expected because our baseline model represents a specific case of the generalized model shown in this section, where the retailer's pricing decisions play a more pronounced role. However, unlike the main model— where consumer welfare remains unaffected under Policy S compared to the benchmark under AG (Proposition 8(i))—this extension reveals a decrease in consumer welfare under Policy S (i.e., $\Pi_c^{AG-S} < \Pi_c^{AG-R}$). This difference arises because, in the main model, market data is utilized by a single firm (either the seller or the retailer) to extract consumer surplus under Policy S or benchmark (Case R). In contrast, this extension involves multiple firms (sellers) utilizing the data under Policy S, whereas only the retailer does so under Case R (e.g., $\frac{\partial \Pi_c^{AG-S} - \Pi_c^{AG-R}}{\partial \beta} = 0$ for $n = 1$ and $\frac{\partial \Pi_c^{AG-S} - \Pi_c^{AG-R}}{\partial \beta} < 0$ for $n \geq 2$). Additionally, this extended model offers further insights relevant to policy design. As the number of sellers increases and competition intensifies, the retailer's incentive to switch to WH strengthens. By gaining full control over the retail prices of all products, the retailer can more effectively mitigate the negative impact of heightened competition. However, this strategic shift can place greater strain on the sellers, as the retailer's pricing power under WH amplifies the pressures on them. Consequently, policy intervention—particularly under RS—becomes even more critical to ensure a balanced and equitable market structure.

Proofs: In AG-R, the retailer and the sellers ($j \in \{1,...,n\}$) choose their optimal prices to maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r = p_r E[q_r|Y] + f\textstyle\sum_j p_j E[q_j|Y] = \frac{p_r}{1+(n)\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}\sum_j p_j\right)& \\ + \textstyle\sum_j \frac{f p_j}{1+(n)\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_j + \frac{\gamma}{1-\gamma}\sum_{j\neq k} p_k + \frac{\gamma}{1-\gamma}p_r\right),&\end{aligned} \tag{S25}$$

$$\max_{p_j} \pi_j = (1-f)p_j E[q_j] = \frac{(1-f)p_j}{1+(n)\gamma}\left(a - \frac{1+(n-1)\gamma}{1-\gamma}p_j + \frac{\gamma}{1-\gamma}\sum_{j\neq k} p_k + \frac{\gamma}{1-\gamma}E[p_r]\right).$$

The retailer's (seller's) objective function is concave in p_r (p_j) and is sufficient to solve its FOCs to derive the best response. Thus, solving their best responses gives the optimal retail prices $p_j = \frac{a(1-\gamma)(\gamma(2n-1)+2)}{\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4}$ for $j \in \{1,...,n\}$ and $p_r = \frac{(1-\gamma)(a)(\gamma((f+2)n-1)+2)}{\gamma(2-\gamma+n(\gamma(f-2n+5)-6)+6)-4} + \frac{\beta(1-\gamma)Y}{2\gamma(n-1)+2}$. Substituting p_j and p_r into Equation (S25), given a signal Y, and taking expectations on both sides of the equations, the expected profits of the retailer and the seller j, $j \in \{1,...,n\}$, can be derived as

$$\begin{aligned}\Pi_r^{AG-R} := E[\pi_r] = & \frac{(1-\gamma)\left((\gamma(n-1)-1)(\gamma-2\gamma n-2)^2\right)a^2}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma(n-1)+1)(\gamma n+1)} \\ & + \frac{(\gamma-1)a^2\left(\gamma^2 f^2 n^2(\gamma n+1) - fn(\gamma n+1)(\gamma(2n-3)+2)(\gamma(2n-1)+2)\right)}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2},\end{aligned}$$

$$\Pi_j^{AG-R} := E[\pi_j] = \frac{a^2(\gamma-1)(f-1)n(\gamma(n-1)+1)(\gamma-2\gamma n-2)^2}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2}.$$

In AG-S, the retailer and the sellers choose their optimal prices to maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r = p_r E[q_r] + f\textstyle\sum_j p_j E[q_j] = & \frac{p_r}{1+(n)\gamma}\left(a - \frac{1+(n-1)\gamma}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}\textstyle\sum_j E[p_j]\right) \\ & + \textstyle\sum_j \frac{fE[p_j]}{1+(n)\gamma}\left(a - \frac{1+(n-1)\gamma}{1-\gamma}E[p_j] + \frac{\gamma}{1-\gamma}\textstyle\sum_{j\neq k} E[p_k] + \frac{\gamma}{1-\gamma}p_r\right), \end{aligned} \tag{S26}$$
$$\max_{p_j} \pi_j = (1-f)p_j E[q_j|Y] = \frac{(1-f)p_j}{1+(n)\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_j + \frac{\gamma}{1-\gamma}\textstyle\sum_{j\neq k} p_k + \frac{\gamma}{1-\gamma}p_r\right).$$

Similarly, solving their best responses gives $p_j = \frac{a(1-\gamma)(\gamma(2n-1)+2)}{\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4} + \frac{Y(\beta-\beta\gamma)}{\gamma(n-1)+2}$ for $j \in \{1,...,n\}$ and $p_r = \frac{(1-\gamma)(a)(\gamma((f+2)n-1)+2)}{\gamma(2-\gamma+n(\gamma(f-2n+5)-6)+6)-4}$. Substituting p_j and p_r into Equation (S26), given a signal Y, and taking expectations on both sides of equations, the expected profits of the retailer and the seller j, $j \in \{1,...,n\}$, can be derived as

$$\Pi_r^{AG-S} := E[\pi_r] = \frac{(1-\gamma)(\gamma(n-1)+1)(\gamma-2\gamma n-2)^2 a^2}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2} + \frac{(\gamma-1)a^2\left(\gamma^2 f^2 n^2(\gamma n+1) - fn(\gamma n+1)(\gamma(2n-3)+2)(\gamma(2n-1)+2)\right)}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2},$$

$$\Pi_s^{AG-S} := E[\pi_j] = \frac{a^2(\gamma-1)(f-1)n(\gamma(n-1)+1)(\gamma-2\gamma n-2)^2}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2} + \frac{\beta(\gamma-1)(f-1)n\sigma^2(\gamma(n-1)+1)}{(\gamma(n-1)+2)^2(\gamma n+1)}.$$

In AG-RS, the retailer and the sellers choose their optimal prices to maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r \quad & = p_r E[q_r|Y] + f\textstyle\sum_j p_j E[q_j|Y] = \frac{p_r}{1+(n)\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}\textstyle\sum_j p_j\right) \\ & + \textstyle\sum_j \frac{fp_j}{1+\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_j + \frac{\gamma}{1-\gamma}\textstyle\sum_{j\neq k} p_k + \frac{\gamma}{1-\gamma}p_r\right), \end{aligned} \tag{S27}$$
$$\max_{p_j} \pi_j = (1-f)p_j E[q_j|Y] = \frac{(1-f)p_j}{1+(n)\gamma}\left(a + \beta Y - \frac{1+(n-1)\gamma}{1-\gamma}p_j + \frac{\gamma}{1-\gamma}\textstyle\sum_{j\neq k} p_k + \frac{\gamma}{1-\gamma}p_r\right).$$

Solving their best responses gives the optimal prices $p_j = \frac{(1-\gamma)(\gamma(2n-1)+2)(a+\beta Y)}{2(\gamma-3)\gamma+\gamma n(6-\gamma(f-2n+5))+4}$ for $j \in \{1,...,n\}$ and $p_r = \frac{(\gamma-1)(a+\beta Y)(\gamma((f+2)n-1)+2)}{\gamma(-2\gamma+n(\gamma(f-2n+5)-6)+6)-4}$. Substituting the optimal prices p_j and p_r into Equation (S27), given a signal Y, and taking the expectation on both sides of equations, the expected profits of the retailer and the seller j, $j \in \{1, 2, \ldots, n\}$ can be derived as

$$\Pi_r^{AG-RS} := E[\pi_r] = \frac{(1-\gamma)(\gamma(n-1)-1)(\gamma-2\gamma n-2)^2(a^2+\beta\sigma^2)}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2} + \frac{(\gamma-1)(a^2+\beta\sigma^2)(\gamma^2 f^2 n^2(\gamma n+1) - fn(\gamma n+1)(\gamma(2n-3)+2)(\gamma(2n-1)+2))}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2},$$

$$\Pi_s^{AG-RS} := E[\pi_j] = \frac{(a^2+\beta\sigma^2)(1-\gamma)(1-f)n(\gamma(n-1)+1)(\gamma-2\gamma n-2)^2}{(\gamma n+1)(\gamma(2(\gamma-3)+n(6-\gamma(f-2n+5)))+4)^2}.$$

Note that, for the seller, $\Pi_s^{AG-S} > \Pi_s^{AG-R} \iff \Pi_s^{AG-S} - \Pi_s^{AG-R} = \frac{\beta(1-\gamma)(1-f)n\sigma^2(\gamma(n-1)+1)}{(\gamma(n-1)+2)^2(\gamma n+1)} > 0$ and that, for the retailer, $\Pi_r^{AG-S} < \Pi_r^{AG-R} \iff \Pi_r^{AG-S} - \Pi_r^{AG-R} = -\frac{\beta(1-\gamma)\sigma^2}{4(\gamma(n-1)+1)(\gamma n+1)} < 0$. These inequalities also hold for all $\gamma, f \in (0,1)$ and $n \geq 2$. Thus, Proposition 1(i) remains to hold.

Also note that $\Pi_s^{AG-RS} > \Pi_s^{AG-R} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-R} = \frac{\beta(1-\gamma)(1-f)n\sigma^2(\gamma(n-1)+1)(\gamma-2\gamma n-2)^2}{(\gamma n+1)(2(\gamma-3)\gamma+\gamma n(6-\gamma(f-2n+5))+4)^2} > 0$ for the seller. This inequality holds for all $\gamma, f \in (0,1)$ and $n \geq 2$. In addition, note that, for the retailer,

$$\Pi_r^{AG-RS} > \Pi_r^{AG-R} \iff \Pi_r^{AG-RS} - \Pi_r^{AG-R} > 0 \iff Af^2 + Bf + C > 0,$$

where $A = \gamma^2 n(\gamma - 2\gamma n - 2) < 0$, $B = 2\gamma\left(8\gamma + \gamma^2(n(2n(2n-5)+7)-2) + 4\gamma n(3n-5) + 12n - 10\right) + 8$ and $C = \gamma(\gamma(2n-1)+2)(\gamma(3n-4)+4)$. This inequality holds if $f_1 < f < f_2$, where f_1 and f_2 are two roots of $Af^2 + Bf + C = 0$. In addition, we can easily verify that $f_1 < 0 < f < 1 < f_2$, which implies that $\Pi_r^{AG-RS} > \Pi_r^{AG-R}$ always holds. Thus, Proposition 1(ii) remains to hold.

In addition, we can easily show that Proposition 2 remains to hold, for any given $n \geq 2$ in a similar manner, so we omit the details.

In WH-R, similarly, we can easily derive the expected profit of the seller j, $j = \{1, 2, \ldots, n\}$ and the retailer as follows.

Under WH-R, note that

$$\begin{aligned}
\Pi_r^{WH-R} := E[\pi_r] &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{4(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{\beta(n+1)\sigma^2}{4(\gamma n+1)},\\
\Pi_s^{WH-R} := E[\pi_j] &= \frac{a^2(1-\gamma)(\gamma(n-1)+1)}{2(\gamma(n-1)+2)^2(\gamma n+1)}.
\end{aligned}$$

Under WH-S, note that

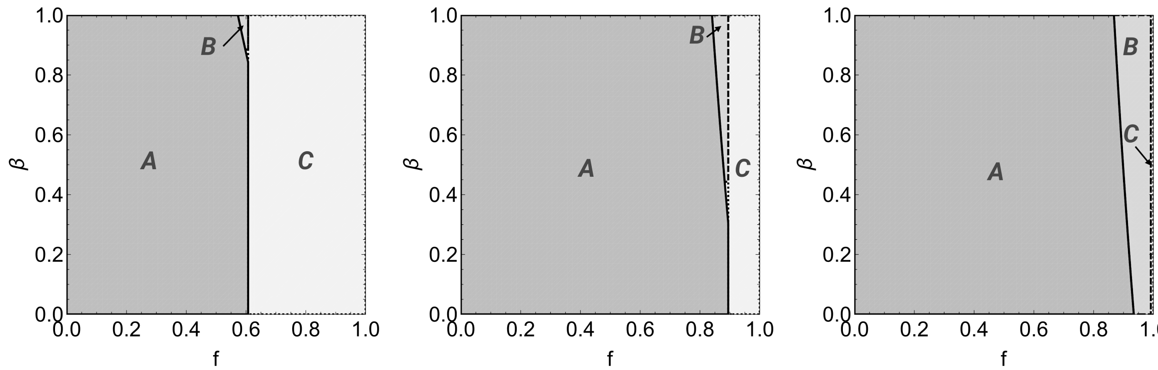
$$\begin{aligned}
\Pi_r^{WH-S} := E[\pi_r] &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{4(\gamma(n-1)+2)^2(\gamma n+1)},\\
\Pi_s^{WH-S} := E[\pi_j] &= \frac{a^2(1-\gamma)(\gamma(n-1)+1)}{2(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{2\beta(1-\gamma)\sigma^2}{(\gamma(n-1)+2)(\gamma n+1)}.
\end{aligned}$$

Under WH-RS, note that

$$\begin{aligned}
\Pi_r^{WH-RS} := E[\pi_r] &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{4(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{\beta\sigma^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{4(\gamma(n-1)+2)^2(\gamma n+1)},\\
\Pi_s^{WH-RS} := E[\pi_j] &= \frac{a^2(1-\gamma)(\gamma(n-1)+1)}{2(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{\beta(1-\gamma)\sigma^2(\gamma(n-1)+1)}{2(\gamma(n-1)+2)^2(\gamma n+1)}.
\end{aligned}$$

Similar to the proof of Propositions 3 and 4, simple algebraic manipulation yields our comparative results. For example, for the retailer, $\Pi_r^{WH-S} < \Pi_r^{WH-R} \iff \Pi_r^{WH-S} - \Pi_r^{WH-R} = -\frac{\beta(n+1)\sigma^2}{4(\gamma n+1)} < 0$ and that, for the seller, $\Pi_s^{WH-RS} > \Pi_s^{WH-R} \iff \Pi_s^{WH-RS} - \Pi_s^{WH-R} = \frac{\beta(1-\gamma)\sigma^2(\gamma(n-1)+1)}{2(\gamma(n-1)+2)^2(\gamma n+1)} > 0$. These inequalities hold for all $\gamma \in (0,1)$ and $n \geq 2$. Moreover, it is straightforward to verify that Propositions 3 and 4 remain to hold for any given $n \geq 2$. For brevity, we omit the detailed proofs.

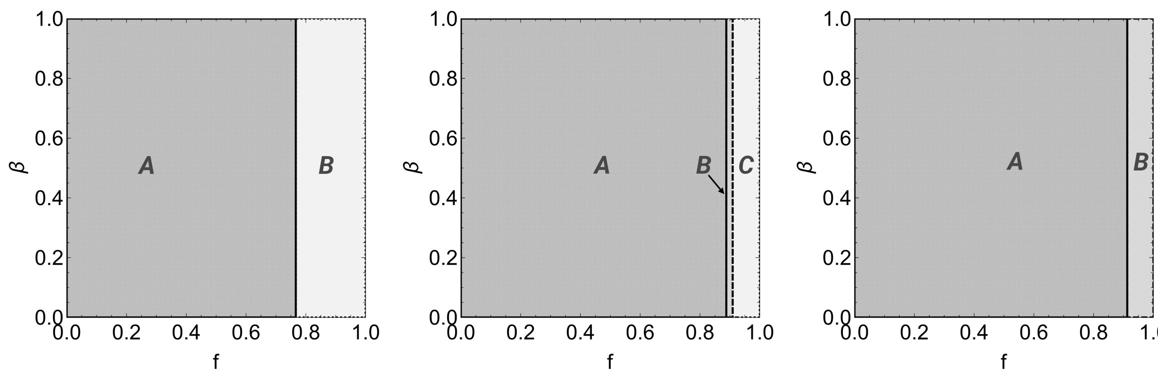
Due to the tractability issue, we are unable to analytically derive the retailer's strategy to counter policy-induced losses and its impact on the seller's profit. Instead, we conduct numerical analyses to confirm the robustness of our findings in Propositions 5 through 7, as illustrated in Figures 12 to 14.



(a) $n=2$ (b) $n=5$ (c) $n=6$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a=1$, $\gamma=0.4$, and $\sigma=0.5$.

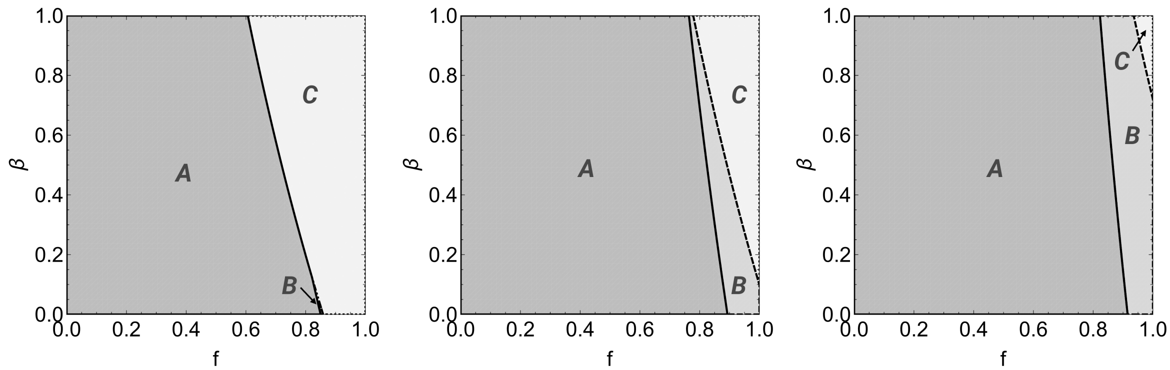
Figure 12 Multiple Sellers: Seller and retailer preferences for contract modes under Policy S



(a) $n=2$ (b) $n=3$ (c) $n=6$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a=1$, $\gamma=0.4$, and $\sigma=0.5$.

Figure 13 Multiple Sellers: Seller and retailer preferences for contract modes under Policy RS

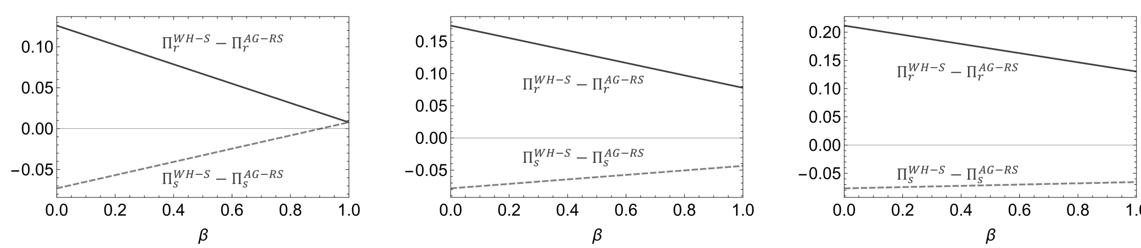


(a) $n=2$ (b) $n=3$ (c) $n=4$

Notes. Region A is the area where the retailer is incentivized to switch from AG to WH and the switch decreases the seller's profit. In B, the retailer is incentivized to switch from AG to WH and the switch increases the seller's profit. In C, the retailer is not incentivized to switch from AG to WH. We use $a=1$, $\gamma=0.35$, and $\sigma=0.5$.

Figure 14 Multiple Sellers: Seller and retailer preferences for contract modes (AG-RS vs. WH-S)

Overall, we observe similar trends in the preferences of the seller and the retailer regarding contract modes, but with thresholds shifted by n. Specifically, as n increases, the retailer's incentive to switch to WH becomes stronger. This creates a scenario where the seller's data advantage becomes less pronounced, as illustrated in Figure 15.



(a) $n=2$ (b) $n=3$ (c) $n=4$

Notes. We use $a=1$, $f=0.45$, $\gamma=0.55$, and $\sigma=0.85$.

Figure 15 Multiple Sellers: Impact of Data Accuracy on Profits Between AG-RS and WH-S

Finally, we verify whether Proposition 8 remains valid in this extension. Similar to the proof of Proposition 8, we can easily derive the expected consumer surplus under AG as follows:

$$\Pi_c^{AG-R} = \frac{a^2(Af^2+Bf+C)}{2(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)^2} + \frac{(n+1)\sigma^2}{2\gamma n+2} + \frac{3\beta(\gamma-1)\sigma^2}{8(\gamma(n-1)+1)(\gamma n+1)},$$

$$\Pi_c^{AG-S} = \frac{a^2(Af^2+Bf+C)}{2(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)^2} + \frac{(n+1)\sigma^2}{2\gamma n+2} + \frac{\beta(\gamma-1)n\sigma^2(2\gamma(n-1)+3)}{2(\gamma(n-1)+2)^2(\gamma n+1)},$$

$$\begin{aligned}\Pi_c^{AG-RS} = {} & \frac{a^2(Af^2+Bf+C)}{2(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)^2} + \frac{(n+1)\sigma^2}{2\gamma n+2} + \frac{\beta(\gamma-1)\left(-\gamma+n(\gamma+\gamma f+2)+2\gamma n^2+2\right)\sigma^2}{(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)} \\ & + \frac{\beta(\gamma-1)\sigma^2\left(-(\gamma-2)^2(\gamma-1)+\gamma^2n^3\left(\gamma\left(f^2-4\right)+4\right)\right)}{2(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)^2} + \frac{\beta(\gamma-1)\sigma^2\left((1-\gamma)\gamma n^2\left(\gamma f^2+4\gamma f+8\right)+\left(\gamma^2-3\gamma+2\right)n(\gamma(2f+3)+2)\right)}{2(\gamma n+1)\left(\gamma^2\left(-(f+5)n+2n^2+2\right)+6\gamma(n-1)+4\right)^2},\end{aligned}$$

where $A=\gamma^2n^2(\gamma n+1)>0$, $B=-2\gamma n\left(\gamma^3 n\left(2n^2-3n+1\right)+\gamma^2\left(6n^2-6n+1\right)+\gamma(6n-3)+2\right)$ and $C=(n+1)\left(\gamma^2\left(2n^2-3n+1\right)+\gamma(4n-3)+2\right)^2$.

We can easily show that Proposition 8 holds except for the case where $\Pi_c^{AG-S}<\Pi_c^{AG-R}$ in this extension. The deviation from the main result arises because, unlike in the main model where market data is utilized by a single firm (either the seller or the retailer) to extract consumer surplus under Policy S or the benchmark (i.e., AG-S or AG-R), this extension involves multiple firms (sellers) leveraging the data under Policy S. In contrast, under the benchmark (i.e., $AG-R$), the retailer only utilizes the data.

Now, we present a proof of this difference. Note that

$$\Pi_c^{AG-S}<\Pi_c^{AG-R} \iff \Pi_c^{AG-S}-\Pi_c^{AG-R} = -\frac{\beta(1-\gamma)\sigma^2}{8(\gamma n+1)}\left(\frac{4n(2\gamma(n-1)+3)}{(\gamma(n-1)+2)^2}-\frac{3}{\gamma(n-1)+1}\right)<0.$$

This inequality always holds for all $\gamma, f\in(0,1)$ and $n\geq 2$. Therefore, $\Pi_c^{AG-S}<\Pi_c^{AG-R}$.
In WH-R, similarly, we can easily derive Π_c^{WH-R}, Π_c^{WH-S} and Π_c^{WH-RS} as follows:

$$\begin{aligned}\Pi_c^{WH-R} &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{8(\gamma(n-1)+2)^2(\gamma n+1)} - \frac{3\beta(n+1)\sigma^2}{8\gamma n+8} + \frac{(n+1)\sigma^2}{2\gamma n+2}, \\ \Pi_c^{WH-S} &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{8(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{(n+1)\sigma^2}{2\gamma n+2}, \\ \Pi_c^{WH-RS} &= \frac{a^2\left(\gamma^2\left(n\left(n^2+n-3\right)+1\right)+\gamma(n(2n+5)-4)+n+4\right)}{8(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{\beta\sigma^2\left(-3(\gamma-2)^2-3\gamma^2n^3+\gamma(5\gamma-14)n^2+(\gamma(\gamma+5)-15)n\right)}{8(\gamma(n-1)+2)^2(\gamma n+1)} + \frac{(n+1)\sigma^2}{2\gamma n+2}.\end{aligned}$$

Observe that $\Pi_c^{WH-S}-\Pi_c^{WH-R}=\frac{3\beta(n+1)\sigma^2}{8\gamma n+8}>0$. Therefore, $\Pi_c^{WH-S}>\Pi_c^{WH-R}$ for all $\gamma\in(0,1)$ and $n\geq 2$. Also observe that $\Pi_c^{WH-RS}-\Pi_c^{WH-R}=\frac{\beta(\gamma-1)n\sigma^2(2\gamma(n-1)+3)}{8(\gamma(n-1)+2)^2(\gamma n+1)}<0$. Therefore, $\Pi_c^{WH-RS}<\Pi_c^{WH-R}$ for all $\gamma\in(0,1)$ and $n\geq 2$. Hence, Proposition 8(ii) holds. □

Appendix E: Analysis for Regulatory Challenges: Endogenizing Fees under AG

In this extension, we examine the robustness of our results when the commission fee under AG is endogenized and allowed to differ across Case R, Policy S, and Policy RS. We confirm that our results remain qualitatively unchanged even when the commission rate is determined endogenously.

To proceed, we introduce an upper bound $\overline{f}$ on the commission rate. This bound reflects platform-level frictions observed in practice, where commission schedules are typically set at the product–category level, strictly regulated, and largely influenced by external market or policy considerations rather than case-by-case optimization (Ha et al. 2022, Hao and Tan 2019, Liu et al. 2021). Hence, it is reasonable to treat $\overline{f}$ as given. In addition to this platform-level bound, the commission rate must also satisfy the seller's participation constraint, which we analyze in detail later.

Under AG-R, from Lemma 1, the expected profits of the retailer and the seller are

$$\Pi_r^{AG-R} = E[\pi_r] = \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} := \Pi_r^{AG} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)},$$
$$\Pi_s^{AG-R} = E[\pi_s] = \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} := \Pi_s^{AG}.$$

Taking the first derivative of Π_r^{AG-R} with respect to f, we obtain $\frac{\partial \Pi_r^{AG-R}}{\partial f} = \frac{a^2\left(16-2\gamma^5-\gamma^6(f+1)+\gamma^4(5f+3)-4\gamma^2(f+4)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^3}$. It is straightforward to verify that $\frac{\partial \Pi_r^{AG-R}}{\partial f} > 0$ for all $\gamma, f \in (0,1)$. Hence, the retailer's profit Π_r^{AG-R} is monotonically increasing in f, and the retailer has an incentive to raise f to the highest feasible level unless constrained by the seller's participation and the category level bound $\overline{f}$.

Suppose that the seller has an outside option with expected profit $\Pi_s^O > 0$. The retailer then selects f subject to the seller's participation constraint, i.e., $\Pi_s^{AG-R} \geq \Pi_s^O$. Differentiating the seller's profit gives $\frac{\partial \Pi_s^{AG-R}}{\partial f} = -\frac{a^2(1-\gamma)(\gamma+2)^2\left(4-\gamma^2(3-f)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^3}$, which is strictly negative for all $\gamma, f \in (0,1)$. Hence, the seller's profit is strictly decreasing in f, and there exists a unique commission rate f^R such that $\Pi_s^{AG-R}(f^R) = \Pi_s^O$. Consequently, the retailer's optimal commission rate f^{R*} is determined by both the platform-imposed bound and the participation constraint, and is given by $f^{R*} = min[\overline{f}, f^R]$.

Similarly, under AG-S, the expected profits of the retailer and the seller are

$$\Pi_r^{AG-S} = E[\pi_r] = \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} := \Pi_r^{AG},$$
$$\Pi_s^{AG-S} = E[\pi_s] = \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)} := \Pi_s^{AG} + \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)}.$$

Taking the first derivative of Π_r^{AG-S} with respect to f, we obtain $\frac{\partial \Pi_r^{AG-S}}{\partial f} = \frac{a^2\left(16-2\gamma^5-\gamma^6(f+1)+\gamma^4(5f+3)-4\gamma^2(f+4)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^3} = \frac{\partial \Pi_r^{AG-R}}{\partial f}$, which implies $\frac{\partial \Pi_r^{AG-S}}{\partial f} > 0$ for all $\gamma, f \in (0,1)$. Thus, the retailer's profit Π_r^{AG-S} is monotonically increasing in f, and the retailer has an incentive to raise f to the highest feasible level unless constrained by the seller's participation and the category level bound $\overline{f}$.

The retailer then selects f subject to the seller's participation constraint, i.e., $\Pi_s^{AG-S} \geq \Pi_0^S$, where Π_0^S is an outside option under Policy S. Differentiating the seller's profit gives $\frac{\partial \Pi_s^{AG-S}}{\partial f} = \frac{(\gamma-1)}{4(1+\gamma)}\left(\frac{4a^2(\gamma+2)^2\left(\gamma^2(f-3)+4\right)}{\left(\gamma^2(f+1)-4\right)^3} - \beta\sigma^2\right)$, which is strictly negative for any $\gamma, f \in (0,1)$. Thus, the seller's profit decreases monotonically in f, and there exists a unique commission rate f^S such that $\Pi_s^{AG-S}|_{f=f^S} = \Pi_0^S$. As a result, the retailer's optimal commission rate f^{S*} is determined by both the platform-imposed bound and the participation constraint, and is given by $f^{S*} = min[\overline{f}, f^S]$.

Under AG-RS, the expected profits of the retailer and the seller are provided as:

$$\begin{aligned}\Pi_r^{AG-RS} = E[\pi_r] &= \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta\sigma^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}, \\ &:= \Pi_r^{AG} + I_r^{AG}.\end{aligned}$$

$$\Pi_s^{AG-RS} = E[\pi_s] = \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} := \Pi_s^{AG} + \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}.$$

Taking the first derivative of Π_r^{AG-RS} with respect to f, we obtain $\frac{\partial \Pi_r^{AG-RS}}{\partial f} = \frac{\left(2-\gamma^2-\gamma\right)\left(\gamma\left(\gamma\left(\gamma^2+\gamma+(\gamma-2)(\gamma+1)f-2\right)+4\right)+8\right)\left(a^2+\beta\sigma^2\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^3}$. $\frac{\partial \Pi_r^{AG-RS}}{\partial f}$ is monotonically increasing function in f for any given $\gamma, f \in (0,1)$. Thus, similar to previous two cases, case R and Policy S, the retailer has an incentive to raise f to the highest feasible level.

The retailer then selects f subject to the seller's constraint, i.e., $\Pi_s^{AG-RS} \geq \Pi_0^{RS}$, where Π_0^{RS} is the expected profit from the seller's outside option. Differentiating the seller's profit gives $\frac{\partial \Pi_s^{AG-RS}}{\partial f} = -\frac{(1-\gamma)(\gamma+2)^2\left(\gamma^2(f-3)+4\right)\left(a^2+\beta\sigma^2\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^3}$, which is strictly negative for any $\gamma, f \in (0,1)$. Thus, the seller's profit decreases monotonically in f, and there exists a commission rate f^{RS} such that $\Pi_s^{AG-RS}|_{f=f^{RS}} = \Pi_0^{RS}$. As a result, the retailer's optimal commission rate f^{RS*} is determined by both the platform-imposed bound and the participation constraint, and is given by $f^{RS*} = min[\overline{f}, f^{RS}]$.

Note that $f^R < f^S < f^{RS}$ since $\Pi_s^{RS} = \Pi_s^R + \frac{\beta(1-\gamma)(\gamma+2)^2(1-f)\sigma^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} > \Pi_s^S = \Pi_s^R + \frac{\beta(1-\gamma)(1-f)\sigma^2}{4(\gamma+1)} > \Pi_s^R$. It follows that, for any $f \in (0,1)$, $\Pi_s^{RS} > \Pi_s^S > \Pi_s^R$, and hence $\Pi_s^{RS}|_{f=f^{RS*}} \geq \Pi_s^S|_{f=f^{S*}} \geq \Pi_s^R|_{f=f^{R*}}$ because $\Pi_s^{RS} > \Pi_s^S > \Pi_s^R(f^R)$ holds for all $f \in (0,1)$, and

(i) $\Pi_s^{RS}|_{f=f^{RS*}} = \Pi_s^S|_{f=f^{S*}} = \Pi_s^R|_{f=f^{R*}}$ if $f^{RS} \leq \overline{f}$;

(ii) $\Pi_s^{RS}|_{f=f^{RS*}} > \Pi_s^S|_{f=f^{S*}} = \Pi_s^R|_{f=f^{R*}}$ if $f^S \leq \overline{f} < f^{RS}$;

(iii) $\Pi_s^{RS}|_{f=f^{RS*}} > \Pi_s^S|_{f=f^{S*}} > \Pi_s^R|_{f=f^{R*}}$ if $\overline{f} < f^S$

holds. Thus, the seller's profit under AG-RS is always at least as large as under AG-S and AG-R, which aligns with the results in Propositions 1 and 2.

Intuitively, when the retailer's market commission fee f is high under AG, the retailer already internalizes most of the channel surplus. Thus, moving to WH, which forces the retailer to concede margin to the seller under the wholesale arrangement, diminishes the retailer's payoff and hence cannot be profitable. To avoid lengthy algebra, we state a general dominance argument and prove only one representative case. Specifically, we consider the *worst* AG case for the retailer (AG-S) against the *best* WH case (WH-R).

Let $\Pi_r^{AG\text{-}S}(f)$ and Π_r^{WH-R} denote the retailer's profits under these two contract modes. We define

$$\tilde{f} := \inf \left\{ f \in (0,1) : \Pi_r^{AG-S}(f) \geq \Pi_r^{WH-R} \right\}.$$

By continuity and $\Pi_r^{AG-S}(f=1) > \Pi_r^{WH-R}$, such a threshold $\tilde{f}$ exists and is strictly less than one. Thus, if $f^{S*} \geq \tilde{f}$, $\Pi_r^{AG-S}(f^{S*}) \geq \Pi_r^{WH-R}$ holds and therefore the retailer has no incentive to switch from AG to WH. Since AG-S is the least favorable AG case and WH-R the most favorable case under WH, this dominance result immediately extends to all other AG-WH comparisons. $\square$

Appendix F: Analysis for Regulatory Challenges: Evasion through Observing Seller Data

Consider the retailer's circumvention strategy under WH, denoted as O, in which the retailer utilizes the seller-side signal Y despite the intended restriction imposed under the regulation WH-S. Given the wholesale price w_s, the retailer chooses p_r and p_s to maximize

$$\begin{aligned}\max_{p_s,p_r}\ \pi_r &= p_r E[q_r|Y] + (p_s - w_s)E[q_s|Y] \\ &= \frac{p_r}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_r + \frac{\gamma}{1-\gamma}p_s\right) + \frac{p_s-w_s}{1+\gamma}\left(a+\beta Y - \frac{1}{1-\gamma}p_s + \frac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S28}$$

Then, anticipating that the retailer will circumvent the policy and exploit the signal, the seller uses the shared signal to set the wholesale price w_s, maximizing her profit as follows:

$$\begin{aligned}\max_{w_s}\ \pi_s = w_s E[q_s|Y] &= \frac{w_s}{1+\gamma}\left(a - \frac{1}{1-\gamma}E[p_s|Y] + \frac{\gamma}{1-\gamma}E[p_r|Y]\right) \\ &= \frac{w_s}{1+\gamma}\left(a - \frac{1}{1-\gamma}\left(\frac{a+\beta Y + w_s}{2}\right) + \frac{\gamma}{1-\gamma}\left(\frac{a+\beta Y}{2}\right)\right).\end{aligned} \tag{S29}$$

Substituting the derived optimal prices into Equation (S29) and taking expectations over Y yields the seller's expected profit Π_s^{WH-O}. Comparing this with the seller's expected profits under WH-R and WH-S in Lemma 2 establishes Result 1. Furthermore, we conduct additional comparative analyses by comparing Π_s^{WH-O} with the seller's profits in Lemma 1, which yield that $\Pi_s^{WH-O} - \Pi_s^{AG-R} < 0$, $\Pi_s^{WH-O} - \Pi_s^{AG-S} < 0$, and $\Pi_s^{WH-O} - \Pi_s^{AG-RS} < 0$ when the fee is sufficiently small.

Similarly, substituting the derived prices into Equation (S28) and taking expectations yields the retailer's expected profit Π_r^{WH-O}. Comparing this and the retailer's expected profits under WH-R/WH-S with those under AG-S establishes Result 2. For instance, we derive the difference:

$$\begin{aligned}\Delta\Pi_r^O := \Pi_r^{WH-O} - \Pi_r^{AG-S} &= \frac{a^2(3\gamma+5)}{16(\gamma+1)} \\ &+ \frac{16a^2\left(\gamma^3+\gamma^4(-f)(f+1)+\gamma^2(f(f+5)+3)-4(f+1)\right)}{16(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta(3\gamma+5)\sigma^2}{16(\gamma+1)}, \\ \Delta\Pi_r^R := \Pi_r^{WH-R} - \Pi_r^{AG-S} &= \frac{a^2(3\gamma+5)}{16(\gamma+1)} \\ &+ \frac{16a^2\left(\gamma^3+\gamma^4(-f)(f+1)+\gamma^2(f(f+5)+3)-4(f+1)\right)}{16(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{8\beta\sigma^2}{16(\gamma+1)}.\end{aligned}$$

Comparing the two payoff gains above shows that $\Delta\Pi_r^R > \Delta\Pi_r^O$, since $\Delta\Pi_r^R - \Delta\Pi_r^O > 0$ if and only if $\frac{3\beta(1-\gamma)\sigma^2}{16(\gamma+1)} > 0$, which holds for all $\gamma, \beta \in (0,1)$ and $\sigma^2 > 0$. Moreover, when the fee is sufficiently small, our additional comparative analyses between Π_r^{WH-O} and the retailer's profits in Lemma 1 yield $\Pi_r^{WH-O} - \Pi_r^{AG-R} > 0$, $\Pi_r^{WH-O} - \Pi_r^{AG-S} > 0$, and $\Pi_r^{WH-O} - \Pi_r^{AG-RS} > 0$, highlighting the retailer's incentive for evasion through access to seller data, which consistently harms the seller and, together with our other results, underscores the pressing need for stronger regulatory restrictions on WH.

To summarize, we show that the regulatory restriction under WH effectively reduces the retailer's incentive for evasion that shifts the seller toward WH, and this result continues to hold even when the retailer strategically accesses seller data under WH-S. Altogether, stronger regulation on WH can further mitigate such evasion toward WH, thereby reinforcing seller profits. □

Appendix G: Analysis for Regulatory Challenges: Retailer Informational Advantage

In practice, even when data are shared, sellers and retailers differ in how effectively they can exploit the information, leading to heterogeneous insights. Moreover, pricing mode can affect the scope of data available to the seller. Under the wholesale regime, for instance, the seller effectively operates as a vendor to Amazon, while Amazon directly manages end-customer interactions. As a result, the proprietary data shared with the seller may be more limited. To capture these nuances, we extend our model to allow the seller to observe a private signal X_m–derived from the shared data under Policies S and RS– conditional on the pricing mode $m \in \{A, W\}$, where $m = A$ denotes agency (AG) and $m = W$ wholesale (WH). By contrast, the retailer always observes a private signal Y under the benchmark case R and Policy RS, regardless of the pricing arrangement.

Next, we establish the relationships between signals, which are central to the extended model. Recall that θ has mean zero and variance σ^2. Following the standard linear information structure, the signals are given by

$$X_A = \theta + \epsilon_A, \quad X_W = \theta + \epsilon_W, \quad Y = \theta + \epsilon_Y,$$

where ϵ_A, ϵ_W, and ϵ_Y are independent error terms, each independent of θ, with zero means and variances σ_A^2, σ_W^2, and σ_Y^2, respectively. Consequently, the variances of the signals are

$$Var(X_A) = \sigma^2 + \sigma_A^2, \quad Var(X_W) = \sigma^2 + \sigma_W^2, \quad Var(Y) = \sigma^2 + \sigma_Y^2.$$

Using the properties of the linear information structure (e.g., Li and Zhang 2008, Shang et al. 2016, Ha et al. 2017), the conditional expectations of θ are

$$E[\theta \mid X_A] = \frac{t_A\sigma^2}{1 + t_A\sigma^2} X_A, \quad E[\theta \mid X_W] = \frac{t_W\sigma^2}{1 + t_W\sigma^2} X_W, \quad E[\theta \mid Y] = \frac{t\sigma^2}{1 + t\sigma^2} Y,$$

where

$$t_A = Var[X_A \mid \theta]^{-1} = \frac{1}{\sigma_A^2}, \quad t_W = Var[X_W \mid \theta]^{-1} = \frac{1}{\sigma_W^2}, \quad t = Var[Y \mid \theta]^{-1} = \frac{1}{\sigma_Y^2}.$$

We let

$$\alpha_A = \frac{t_A\sigma^2}{1 + t_A\sigma^2}, \quad \alpha_W = \frac{t_W\sigma^2}{1 + t_W\sigma^2}, \quad \beta = \frac{t\sigma^2}{1 + t\sigma^2},$$

and assume $\alpha_m < \beta$ for $m \in \{A, W\}$ to reflect the seller's comparatively weaker capability.

Note that due to the independence between θ and ϵ_m for $m \in \{A, W, Y\}$, we have

$$E[X_m \mid Y] = E[\theta + \epsilon_m \mid Y] = E[\theta \mid Y] + E[\epsilon_m \mid Y] = \beta Y,$$

$$E[Y \mid X_m] = E[\theta + \epsilon_Y \mid X_m] = E[\theta \mid X_m] + E[\epsilon_Y \mid X_m] = \alpha_m X_m.$$

Similarly, we have

$$E[YX_m] = E\big[E[YX_m \mid \theta]\big] = E\big[E[Y \mid \theta]\, E[X_m \mid \theta]\big] = E[\theta \cdot \theta] = \sigma^2.$$

We will use these properties when deriving the expected profits of the two firms and consumer welfare.

Technical Insights of Propositions 1 through 4 under Retailer Informational Advantage

The two-signal extension verifies that our main propositions are robust when the seller's informational capability is explicitly weaker than the retailer's. Under AG, Policy S reallocates informational rents from the retailer to the seller, while Policy RS improves both firms' profits, consistent with the baseline analysis. Under WH, both policies benefit the seller but reduce the retailer's payoff, with Policy RS relatively less harmful to the retailer than Policy S. Overall, heterogeneous signal precision alters the magnitude of profit changes but preserves the directional effects, highlighting the persistence of asymmetries across contract modes.

Proofs of Propositions 1 through 4 under Retailer Informational Advantage

In AG-R, both firms choose their optimal prices to maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r &= p_r E[q_r|Y] + f p_s E[q_s|Y] = \tfrac{p_r}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{f p_s}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right),\\ \max_{p_s} \pi_s &= (1-f)p_s E[q_s] = \tfrac{(1-f)p_s}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}E[p_r]\right).\end{aligned}$$

Similar to the proofs for case AG-R in Lemma 1, solving the FOCs gives the optimal retailer prices $p_r = \frac{(1-\gamma)a(\gamma+\gamma f+2)}{4-\gamma^2(f+1)} + \frac{\beta(1-\gamma)Y}{2}$ and $p_s = \frac{(1-\gamma)(a(\gamma+2))}{4-\gamma^2(f+1)}$. By substituting the optimal prices, given a signal Y, the profits of the retailer and the seller can be derived as

$$\begin{aligned}\pi_r = &\ \frac{a^2(\gamma-1)\left(-(\gamma+2)^2+(\gamma+1)\gamma^2 f^2+\left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta^2(1-\gamma)Y^2}{4(\gamma+1)} + Y\left(\frac{a\beta(\gamma-1)(\gamma+2(\gamma+1)f+2)}{(\gamma+1)(\gamma^2(f+1)-4)}\right),\\ \pi_s = &\ \frac{(\gamma-1)(f-1)(a(\gamma+2))^2}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2}.\end{aligned}$$

Taking expectations on both sides of the equations, their optimal expected profits can be obtained

$$\begin{aligned}\Pi_r^{AG-R} := E[\pi_r] = &\ \frac{a^2(\gamma-1)\left(-(\gamma+2)^2+(\gamma+1)\gamma^2 f^2+\left(\gamma^3+\gamma^2-4\gamma-4\right)f\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)},\\ \Pi_s^{AG-R} := E[\pi_s] = &\ \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2}.\end{aligned}$$

In AG-S, the retailer and the seller choose their optimal prices to maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r &= p_r E[q_r] + f E[p_s]E[q_s] = \tfrac{p_r}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}E[p_s]\right) + \tfrac{f E[p_s]}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}E[p_s] + \tfrac{\gamma}{1-\gamma}p_r\right),\\ \max_{E[p_s]} \pi_s &= (1-f)p_s E[q_s|X_A] = \tfrac{(1-f)p_s}{1+\gamma}\left(a + \alpha_A X_A - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right).\end{aligned}$$

Similarly, by solving their best responses as a system of equations (i.e., $p_r = \frac{(a)(1-\gamma)}{2} + \frac{\gamma(f+1)}{2}E[p_s]$ and $p_s = \frac{a(1-\gamma)}{2} + \frac{(+\gamma(p_r-\alpha_A X_A)+\alpha_A X_A)}{2}$), the optimal retail prices can be derived as follows:

$$p_s = \frac{(1-\gamma)a(\gamma+2)}{4-\gamma^2(f+1)} + \frac{\alpha_A(1-\gamma)X_A}{2}, \text{ and } p_r = \frac{(1-\gamma)a(\gamma+\gamma f+2)}{4-\gamma^2(f+1)}.$$

Substituting the optimal retail prices, their expected profits can be derived

$$\begin{aligned}\Pi_r^{AG-S} := E[\pi_r] = &\ \frac{a^2\left(4(f+1)-\gamma^2(f(f+5)+3)-\gamma^3+\gamma^4 f(f+1)\right)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2},\\ \Pi_s^{AG-S} := E[\pi_s] = &\ \frac{a^2(1-\gamma)(\gamma+2)^2(1-f)}{(\gamma+1)\left(4-\gamma^2(f+1)\right)^2} + \frac{(1-\gamma)(1-f)\alpha_A\sigma^2}{4}.\end{aligned}$$

In AG-RS, both firms utilize their private signals (Y for the retailer, and X_A for the seller) to determine the optimal prices that maximize their profits as

$$\begin{aligned}\max_{p_r} \pi_r &= p_r E[q_r|Y] + f E[p_s|Y] E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}E[p_s|Y]\right) + \tfrac{fE[p_s|Y]}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}E[p_s|Y] + \tfrac{\gamma}{1-\gamma}p_r\right), \\ \max_{p_s} \pi_s &= (1-f)p_s E[q_s|X_A] = \tfrac{(1-f)p_s}{1+\gamma}\left(a+\alpha_A X_A - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}E[p_r|X_A]\right).\end{aligned} \tag{S30}$$

Following the standard procedure for establishing equilibrium (e.g., Vives 1984, Li 2002), we specify each firm's set of beliefs, derive rational strategies accordingly, and demonstrate consistency between strategies and beliefs. Specifically, we conjecture a linear strategy for each firm as follows:

$$E[p_r|X_A] = A_r + B_r E[Y|X_A] = A_r + B_r(\alpha_A X_A),$$
$$E[p_s|Y] = A_s + B_s E[X_A|Y] = A_s + B_s(\beta Y),$$

where A_r, A_s, B_r, and B_s are constant values to be determined. Using the linear strategy and the FOCs, we derive $p_r = \frac{a(1-\gamma)+A_s\gamma(f+1)}{2} + \frac{\beta Y(\gamma(B_s f+B_s-1)+1)}{2}$ and $p_s = \frac{a(1-\gamma)+A_r\gamma}{2} + \frac{X_A\alpha_A((B_r-1)\gamma+1)}{2}$. To determine A_r, A_s, B_r, and B_s, we solve the following system of equations

$$A_r = \tfrac{a(1-\gamma)+A_s\gamma(f+1)}{2}, B_r = \tfrac{\beta(\gamma(B_s f+B_s-1)+1)}{2}, A_s = \tfrac{a(1-\gamma)+A_r\gamma}{2}, \text{ and } B_s = \tfrac{\alpha_A((B_r-1)\gamma+1)}{2},$$

which gives

$$A_r = \tfrac{a(\gamma-1)(\gamma+\gamma f+2)}{\gamma^2(f+1)-4}, B_r = \tfrac{\beta(\gamma-1)(\gamma(f+1)\alpha_A+2)}{\beta\gamma^2(f+1)\alpha_A-4}, A_s = \tfrac{a\left(\gamma^2+\gamma-2\right)}{\gamma^2(f+1)-4}, \text{ and } B_s = \tfrac{(\gamma-1)\alpha_A(\beta\gamma+2)}{\beta\gamma^2(f+1)\alpha_A-4}.$$

Substituting $p_r = A_r + B_r(Y)$ and $p_s = A_s + B_s(\beta Y)$ into the retailer's objective function in Equation (S30) provides

$$\begin{aligned}\Pi_r^{AG-RS} := E[\pi_r] = &\tfrac{a^2\left(-\gamma^3+\gamma^4 f(f+1)-\gamma^2(f(f+5)+3)+4(f+1)\right)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \tfrac{\beta(\gamma-1)\sigma^2(-4\alpha_A(\gamma+f(\beta\gamma+\gamma+2))-4)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2} \\ &+ \tfrac{\beta(\gamma-1)\sigma^2\left(\alpha_A^2\left(\beta^2\gamma^3 f(f+1)+\gamma^2\left(\beta^2 f+2\beta f(f+1)-(f+1)^2\right)+4\beta\gamma f+4f\right)\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}.\end{aligned}$$

Similarly, $p_r = A_r + B_r(\alpha_A X_A)$ and $p_s = A_s + B_s(X_A)$ into the seller's objective function in Equation (S30) provides

$$\Pi_s^{AG-RS} := E[\pi_s] = \tfrac{a^2(\gamma-1)(\gamma+2)^2(f-1)}{(\gamma+1)\left(\gamma^2(f+1)-4\right)^2} + \tfrac{(\gamma-1)(f-1)\sigma^2\alpha_A(\beta\gamma+2)^2}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}.$$

The proofs of Propositions 1 and 2 for the retailer's and the seller's profits are analogous to those provided in the main analysis. We first examine the retailer's profits in different cases. Specifically, $\Pi_r^{AG-R} > \Pi_r^{AG-S} \iff \Pi_r^{AG-S} - \Pi_r^{AG-R} = -\frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} < 0$, which always holds because $\gamma \in (0,1)$.

Similarly, $\Pi_r^{AG-R} < \Pi_r^{AG-RS} \iff \Pi_r^{AG-RS} - \Pi_r^{AG-R} > 0 \iff \mathcal{F} > 0$, where

$$\mathcal{F} = \tfrac{\beta(\gamma-1)\sigma^2}{4(1+\gamma)}\left(\tfrac{4\left(\alpha_A^2\left(\beta^2\gamma^3 f(f+1)+\gamma^2\left(\beta^2 f+2\beta f(f+1)-(f+1)^2\right)+4\beta\gamma f+4f\right)-4\alpha_A(\gamma+f(\beta\gamma+\gamma+2))-4\right)}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}+1\right).$$

Note that $\mathcal{F}$ is an increasing function of α_A because $\frac{\partial \mathcal{F}}{\partial \alpha_A} > 0 \iff \frac{4\beta(1-\gamma)\sigma^2(\beta\gamma+2)\left(\alpha_A\left(-\left(\gamma^2(f+1)((\beta-1)f-1)\right)-2\beta\gamma f-4f\right)+2(\gamma+(\gamma+2)f)\right)}{(\gamma+1)\left(4-\beta\gamma^2(f+1)\alpha_A\right)^3} < 0$. This inequality holds for any given $\beta,\ \gamma \in (0,1)$. In addition, we can easily prove that $\mathcal{F}|_{\alpha_A=0} = 0$. Thus, $\mathcal{F} > 0$ holds for any given $\alpha_A \in (0,\beta)$, and hence $\Pi_r^{AG-R} < \Pi_r^{AG-RS}$.

Lastly, $\Pi_r^{AG-S} < \Pi_r^{AG-RS} \iff \Pi_r^{AG-RS} - \Pi_r^{AG-S} > 0 \iff \tilde{\mathcal{F}} > 0$, where

$$\tilde{\mathcal{F}} = \frac{\beta(\gamma-1)\sigma^2\left(\alpha_A^2\left(\beta^2\gamma^3 f(f+1)+\gamma^2\left(\beta^2 f+2\beta f(f+1)-(f+1)^2\right)+4\beta\gamma f+4f\right)-4\alpha_A(\gamma+f(\beta\gamma+\gamma+2))-4\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}.$$

Note that $\tilde{\mathcal{F}}$ is an increasing function of α_A because $\frac{\partial \tilde{\mathcal{F}}}{\partial \alpha_A} > 0 \iff \frac{4\beta(\gamma-1)\sigma^2(\beta\gamma+2)\left(\alpha_A\left(-\left(\gamma^2(f+1)((\beta-1)f-1)\right)-2\beta\gamma f-4f\right)+2(\gamma+(\gamma+2)f)\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^3} > 0$. This inequality holds for any given $\beta,\ \gamma \in (0,1)$. In addition, we can easily prove that $\tilde{\mathcal{F}}|_{\alpha_A=0} = \frac{\beta(1-\gamma)\sigma^2}{4(\gamma+1)} > 0$ for any given $\gamma \in (0,1)$. Thus, $\tilde{\mathcal{F}} > 0$ holds for any given $\alpha_A \in (0,\beta)$, and hence $\Pi_r^{AG-S} < \Pi_r^{AG-RS}$.

For the seller, note that $\Pi_s^{AG-R} < \Pi_s^{AG-S} \iff \Pi_s^{AG-S} - \Pi_s^{AG-R} = \frac{(1-\gamma)(1-f)\sigma^2\alpha_A}{4(\gamma+1)} > 0$, and $\Pi_s^{AG-R} < \Pi_s^{AG-RS} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-R} = \frac{(1-\gamma)(1-f)\sigma^2\alpha_A(\beta\gamma+2)^2}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2} > 0$. These inequalities hold because $f, \gamma \in (0,1)$ and $\alpha_A \in (0,\beta)$.

Lastly, note that

$$\Pi_s^{AG-S} < \Pi_s^{AG-RS} \iff \Pi_s^{AG-RS} - \Pi_s^{AG-S} > 0 \iff \mathcal{G} > 0,$$

where $\mathcal{G} = \frac{(1-\gamma)(1-f)\sigma^2\alpha_A}{4(1+\gamma)}\left(\frac{4(\beta\gamma+2)^2}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2} - 1\right)$. This is an increasing function of α_A, because $\frac{\partial \mathcal{G}}{\partial \alpha_A} > 0$ always holds for all $\gamma, f \in (0,1)$ and $\alpha_A \in (0,\beta)$ and $\mathcal{G}|_{\alpha_A=0} = 0$. Thus, $\mathcal{G} > 0$ holds for any given $\alpha_A \in (0,\beta)$, and hence $\Pi_s^{AG-S} < \Pi_s^{AG-RS}$.

In WH-R, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned}\max_{p_s,p_r} \pi_r &= p_r E[q_r|Y] + (p_s - E[w_s|Y])E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s - E[w_s|Y]}{1+\gamma}\left(a+\beta Y - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S31}$$

The retailer's objective function is concave in p_r and p_s and is sufficient to solve its FOCs to derive the best response. Thus, his best response can be derived as $p_r = \frac{a+\beta Y}{2}$ and $p_s = \frac{a+\beta Y+E[w_s|Y]}{2}$.

Then, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s] = \tfrac{w_s}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}E[p_r] + \tfrac{\gamma}{1-\gamma}E[p_s]\right). \tag{S32}$$

The seller's objective function is also concave in w_s and is sufficient to solve its FOCs to derive the best response. Thus, her optimal wholesale price can be derived as $w_s = \frac{(a)(1-\gamma)}{2}$. Substituting w_s into p_s and p_r gives $p_s = \frac{a(3-\gamma)}{4} + \frac{\beta Y}{2}$ and $p_r = \frac{a}{2} + \frac{\beta Y}{2}$. By substituting w_s, p_s, and p_r into Equations (S31) and (S32), given a signal Y, their expected profits can be derived as:

$$\Pi_r^{WH-R} := E[\pi_r] = \tfrac{a^2(3\gamma+5)}{16(\gamma+1)} + \tfrac{\beta\sigma^2}{2(\gamma+1)},\ \text{ and } \Pi_s^{WH-R} := E[\pi_s] = \tfrac{a^2(1-\gamma)}{8(\gamma+1)}.$$

In WH-S, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned}\max_{p_s,p_r} \pi_r &= p_r E[q_r] + (p_s - E[w_s])E[q_s] \\ &= \tfrac{p_r}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s - E[w_s]}{1+\gamma}\left(a - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S33}$$

Similarly, his best response can be derived as $p_r = \frac{a}{2}$ and $p_s = \frac{a+E[w_s]}{2}$. Then, given his best response, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s|X_W] = \tfrac{w_s}{1+\gamma}\left(a + \alpha_W X_W - \tfrac{1}{1-\gamma}\left(\tfrac{a+w_s}{2}\right) + \tfrac{\gamma}{1-\gamma}\left(\tfrac{a}{2}\right)\right). \tag{S34}$$

Solving the FOCs gives the seller's optimal wholesale price as $w_s = \frac{(1-\gamma)(a+2\alpha_W X_W)}{2}$. Substituting w_s into $p_s = \frac{a+E[w_s]}{2}$ gives $p_s = \frac{a(3-\gamma)}{4}$. Substituting w_s, p_s, and p_r into Equations (S33) and (S34) gives

$$\Pi_r^{WH-S} := E[\pi_r] = \tfrac{(3\gamma+5)a^2}{16(\gamma+1)}, \text{ and } \Pi_s^{WH-S} := E[\pi_s] = \tfrac{(1-\gamma)a^2}{8(\gamma+1)} + \tfrac{\alpha_W(1-\gamma)\sigma^2}{\gamma+1}.$$

In WH-RS, the retailer chooses the optimal prices to maximize his profit as

$$\begin{aligned}\max_{p_s,p_r} \pi_r &= p_r E[q_r|Y] + (p_s - E[w_s|Y])E[q_s|Y] \\ &= \tfrac{p_r}{1+\gamma}\left(a + \beta Y - \tfrac{1}{1-\gamma}p_r + \tfrac{\gamma}{1-\gamma}p_s\right) + \tfrac{p_s - E[w_s|Y]}{1+\gamma}\left(a + \beta Y - \tfrac{1}{1-\gamma}p_s + \tfrac{\gamma}{1-\gamma}p_r\right).\end{aligned} \tag{S35}$$

Similarly, by solving the FOCs, his best response can be derived as $p_r = \frac{a+\beta Y}{2}$ and $p_s = \frac{a+\beta Y+E[w_s|Y]}{2}$. Then, the seller chooses the optimal wholesale price w_s to maximize her profit as

$$\max_{w_s} \pi_s = w_s E[q_s|X_W] = \tfrac{w_s}{1+\gamma}\left(a + \alpha_W X_W - \tfrac{1}{1-\gamma}E[p_r|X_W] + \tfrac{\gamma}{1-\gamma}E[p_s|X_W]\right). \tag{S36}$$

Similar to AG-RS, with a linear conjecture, the seller uses $E[p_r|X_W] = A_r + B_r E[Y|X_W] = A_r + B_r(\alpha_W X_W)$ and $E[p_s|X_W] = A_s + B_s E[Y|X_W] + C_s E[w_s|X_W] = A_s + B_s(\alpha_W X_W) + C_s w_s$, where A_r, B_r, A_s, B_s, and C_s are constant values to be determined. The retailer also uses a linear conjecture such as $E[w_s|Y] = A_w + B_w E[X_W|Y] = A_w + B_w(\beta Y)$, where A_w and B_w are constant values to be determined. Solving the FOCs gives

$$p_r = \tfrac{a+\beta Y}{2}, \quad p_s = \tfrac{(a+A_w+\beta(B_w+1)Y)}{2}, \text{ and } w_s = \tfrac{a(-\gamma)+a+A_r\gamma-A_s}{2C_s} - \tfrac{X_W\alpha_W(-B_r\gamma+B_s+\gamma-1)}{2C_s},$$

which leads to $A_r = \frac{a}{2}, B_r = \frac{\beta}{2}, A_s = \frac{a}{2}, B_s = \frac{\beta}{2}, C_s = \frac{1}{2}, A_w = \frac{a(1-\gamma)}{2}$, and $B_w = \frac{(\beta-2)(\gamma-1)}{2}\alpha_W$. Then, we have

$$w_s = \tfrac{a(1-\gamma)}{2} + \tfrac{\alpha_W(2-\beta)(1-\gamma)}{2}X_W, \quad p_s = \tfrac{a(3-\gamma)}{4} + \tfrac{\beta Y(\alpha_W(2-\beta)(1-\gamma)+2)}{4}, \text{ and } p_r = \tfrac{a+\beta Y}{2}.$$

Substituting $p_r = A_r + B_r(Y)$, $p_s = A_s + B_s(Y) + C_s E[w_s|Y]$, and $E[w_s|Y] = A_w + B_w(\beta Y)$ into Equation (S35) provides

$$\Pi_r^{WH-RS} := E[\pi_r] = \tfrac{a^2(3\gamma+5)}{16(\gamma+1)} + \tfrac{\beta\sigma^2(8-(2-\beta)(1-\gamma)\alpha_W((\beta-2)\alpha_W+4))}{16(\gamma+1)}.$$

Similarly, substituting $E[p_r|X_W] = A_r + B_r(\alpha_W X_W)$, $E[p_s|X_W] = A_s + B_s(\alpha_W X_W) + C_s w_s$, and $w_s = A_w + B_w(X_W)$ into Equation (S36) provides

$$\Pi_s^{WH-RS} := E[\pi_s] = \frac{a^2(1-\gamma)}{8(\gamma+1)} + \frac{(2-\beta)^2(1-\gamma)\sigma^2\alpha_W}{8(\gamma+1)}.$$

The proofs of Propositions 3 and 4 for the retailer's and the seller's profits are analogous to those provided in the main analysis. We first examine the retailer's profits in different cases. Specifically, $\Pi_r^{WH-R} > \Pi_r^{WH-S} \iff \Pi_r^{WH-R} - \Pi_r^{WH-S} = \frac{\beta\sigma^2}{2(\gamma+1)} > 0$. The inequality holds because $\beta, \gamma \in (0,1)$.

Similarly, $\Pi_r^{WH-R} > \Pi_r^{WH-RS} \iff \Pi_r^{WH-RS} - \Pi_r^{WH-R} < 0$. In turn,

$$\Pi_r^{WH-RS} - \Pi_r^{WH-R} < 0 \iff \mathcal{F} = -\frac{(2-\beta)\beta(1-\gamma)\sigma^2\alpha_W((\beta-2)\alpha_W+4)}{16(\gamma+1)} < 0.$$

The inequality always holds because $\beta, \gamma \in (0,1)$. Thus $\Pi_r^{WH-R} > \Pi_r^{WH-RS}$ holds.

Lastly, $\Pi_r^{WH-S} < \Pi_r^{WH-RS} \iff \Pi_r^{WH-RS} - \Pi_r^{WH-S} > 0$. In turn,

$$\Pi_r^{WH-RS} - \Pi_r^{WH-S} > 0 \iff \frac{\beta\sigma^2(8-(2-\beta)(1-\gamma)\alpha_W(4-(2-\beta)\alpha_W))}{16(\gamma+1)} > 0.$$

The inequality holds because $\beta, \gamma \in (0,1)$.

For the seller, note that $\Pi_s^{WH-R} < \Pi_s^{WH-S} \iff \Pi_s^{WH-S} - \Pi_s^{WH-R} = \frac{(1-\gamma)\sigma^2\alpha_W}{2(\gamma+1)} > 0$. The inequality holds because $\gamma \in (0,1)$. In addition, note that $\Pi_s^{WH-R} < \Pi_s^{WH-RS} \iff \Pi_s^{WH-RS} - \Pi_s^{WH-R} = \frac{(2-\beta)^2(1-\gamma)\sigma^2\alpha_W}{8(\gamma+1)} > 0$. The inequality holds because $\gamma \in (0,1)$ and $\alpha_W \in (0,\beta)$. Lastly, note that $\Pi_s^{WH-S} > \Pi_s^{WH-RS} \iff \Pi_s^{WH-S} - \Pi_s^{WH-RS} = \frac{(4+4\beta-\beta^2)(1-\gamma)\sigma^2\alpha_W}{8(\gamma+1)} > 0$. The inequality holds because $\beta, \gamma \in (0,1)$. Overall, Propositions 1 – 4 continue to hold in this extended model. □

Technical Insights of Proposition 5 under Retailer Informational Advantage

Proposition 5 confirms that the retailer's switching incentive under Policy S is robust to the two-signal setting, driven by competition intensity and commission fees. However, the seller's outcome depends on relative signal precision: weak wholesale signals exacerbate losses, while sufficiently accurate signals allow her to benefit, particularly at higher commission rates. Thus, the qualitative insights remain valid, though refined by thresholds on information quality.

Proof of Proposition 5 under Retailer Informational Advantage

We first derive the retailer's preference for WH under Policy S, followed by the seller's preference.

Under Policy S, the retailer prefers WH if and only if $\Pi_r^{AG-S} < \Pi_r^{WH-S}$, which can be reduced to:

$$F_1(\gamma, f) := (\gamma+2)^2(\gamma(\gamma(3\gamma-7)+8)+4) + ((3\gamma-11)\gamma^2+16)\gamma^2 f^2 + 2(\gamma-2)(\gamma+2)(3(\gamma-1)\gamma^2+8)f > 0,$$

where the function $F_1(\gamma, f)$ is identical to the one used in the proof of Proposition 5. Accordingly, the condition under which the retailer prefers WH under Policy S coincides with that stated in Proposition 5. Specifically, $F_1(\gamma, f) > 0$ if and only if $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$. Under this condition, it follows that $\Pi_r^{AG-S} < \Pi_r^{WH-S}$ under the two-signal setting.

Next, the seller prefers WH if and only if $\Pi_s^{AG-S} < \Pi_s^{WH-S}$, which can be further reduced to:

$$F_2(\gamma, f) - \overline{G}_1(\gamma, f)\alpha_A\sigma^2 + \tilde{G}_1(\gamma, f)\alpha_W\sigma^2 > 0,$$

where the function $F_2(\gamma, f)$ is provided in the proof of Proposition 5 in Section 6.1, and $\overline{G}_1(\gamma, f) := \frac{(1-\gamma)(1-f)}{4(\gamma+1)}$ and $\tilde{G}_1(\gamma, f) := \frac{(1-\gamma)}{2(\gamma+1)}$.

Recall from the proof of Proposition 5 that $F_2(\gamma, f) > 0 \iff f > f_{s1}$. Note that $\tilde{G}_1(\gamma, f) > 0$ always holds for any $\gamma, f \in (0,1)$. Thus,

(i) When $f < f_{s1}$, $\Pi_s^{AG-S} < \Pi_s^{WH-S} \iff \alpha_W > \frac{-F_2(\gamma,f)+\overline{G}_1(\gamma,f)\alpha_A\sigma^2}{\tilde{G}_1(\gamma,f)\sigma^2} := \alpha_{W1}$. That is, when $f < f_{s1}$, $\Pi_s^{AG-S} > \Pi_s^{WH-S} \iff \alpha_W < \alpha_{W1}$. Thus, the switch to WH hurts the seller under this condition.

(ii) When $f > f_{s1}$, $\Pi_s^{AG-S} < \Pi_s^{WH-S}$ holds when $\alpha_W > \frac{(1-f)\alpha_A}{2}$ because $\tilde{G}_1(\gamma, f) > \overline{G}_1(\gamma, f)$. This implies that under the two-signal setting, Proposition 5 remains to hold qualitatively if the signal accuracy under WH is not too low, i.e., $\alpha_W > \frac{(1-f)\alpha_A}{2}$. Obviously, when $\alpha_W = \alpha_A$, $\alpha_W > \frac{(1-f)\alpha_A}{2}$ holds and thus Proposition 5 holds. □

Technical Insights of Proposition 6 under Retailer Informational Advantage

Proposition 6 demonstrates that under Policy RS the retailer's switching incentive is preserved, but its interaction with signal precision alters the seller's outcome. When the seller's signal under AG is weak or the wholesale signal under WH is poor, the retailer gains more from switching, while the seller is more likely to be harmed, especially at low commission rates. Conversely, stronger wholesale signals can mitigate these losses and align incentives, showing that information accuracy shapes the profitability of contract switching.

Proof of Proposition 6 under Retailer Informational Advantage

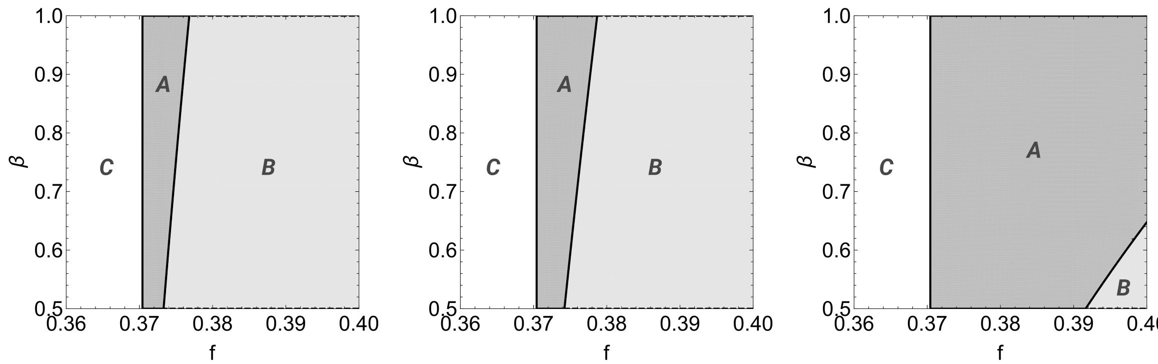
We first derive the retailer's preference for WH under Policy RS, followed by the seller's preference.

Under Policy RS, the retailer prefers WH if and only if

$$F_1(\gamma, f) - \overline{H}_1(\alpha_A, \beta)\sigma^2 + \tilde{H}_1(\alpha_W, \beta)\sigma^2 > 0,$$

where $\overline{H}_1(\alpha_A, \beta) := \frac{\beta(\gamma-1)\left(\alpha_A^2\left(\beta^2\gamma^3 f(f+1)+\gamma^2\left(\beta^2 f+2\beta f(f+1)-(f+1)^2\right)+4\beta\gamma f+4f\right)-4\alpha_A(\gamma+f(\beta\gamma+\gamma+2))-4\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2} > 0$ and $\tilde{H}_1(\alpha_W\beta) := \frac{\beta(8-(2-\beta)(1-\gamma)\alpha_W(4-(2-\beta)\alpha_W))}{16(\gamma+1)} > 0$.

Recall from the proof of Proposition 5 that $F_1(\gamma, f) > 0 \iff \gamma > \gamma_{r1}$ or $f < \frac{1}{4}$. Note that $\frac{\partial\overline{H}_1(\alpha_A,\beta))}{\partial\alpha_A} = \frac{4\beta(\gamma-1)(\beta\gamma+2)\left(\alpha_A\left(-\left(\gamma^2(f+1)((\beta-1)f-1)\right)-2\beta\gamma f-4f\right)+2(\gamma+(\gamma+2)f)\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^3} > 0$ and $\frac{\partial\tilde{H}_1(\alpha_W,\beta))}{\partial\alpha_W} = -\frac{(\beta-2)\beta(\gamma-1)((\beta-2)\alpha_W+2)}{8(\gamma+1)} < 0$ for any $\beta, \gamma \in (0,1)$. Thus, when $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$, the retailer prefers WH if α_A is not too large. In other words, when $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$, Proposition 6 remains to hold if $\alpha_A < \overline{\alpha}_A$, where $\overline{\alpha}_A$ is a value of α_A such that $F_1(\gamma, f) - \overline{H}_1(\alpha_A, \beta)|_{\alpha_A=\overline{\alpha}_A}\sigma^2 + \tilde{H}_1(\alpha_W, \beta)\sigma^2 = 0$.



(a) $\alpha_A = 0.2,\ \alpha_W = 0.2$ (b) $\alpha_A = 0.2,\ \alpha_W = 0.5$ (c) $\alpha_A = 0.5,\ \alpha_W = 0.2$

Notes. Region A is the area where the retailer is not incentivized to switch from AG to WH. In B, the retailer is incentivized to switch from AG to WH. Region C denotes the region where $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$ holds. We use $a = 1$, $\gamma = 0.15$, and $\sigma = 0.1$.

Figure 16 Information Advantage: Retailer preferences for WH under Policy RS

When $\gamma \leq \gamma_{r1}$ and $f \geq \frac{1}{4}$, we numerically illustrate the retailer's preference for a contract mode, as provided in Figure 16.

As shown in Figure 16, the retailer's incentive to switch from AG-RS to WH-RS becomes stronger when the seller's signal accuracy under AG (i.e., α_A) is low or when the seller's signal accuracy under WH (i.e., α_W) is low. By contrast, when the seller's signal accuracy is close to the retailer's signal accuracy (e.g., Figure 16(c)), the retailer's incentive to switch from AG to WH becomes weaker, a consistent finding from Proposition 6. This is because under AG, data sharing through RS allows the retailer to benefit jointly with the seller. Under WH, however, if the seller gains higher signal accuracy through data sharing, the seller can use this improved information when setting the wholesale price, thereby strengthening its bargaining position and reducing the retailer's payoff.

Next, we examine the seller's preference on pricing modes. The seller prefers WH if and only if $\Pi_s^{AG-RS} < \Pi_s^{WH-RS}$ which can be reduced to

$$F_2(\gamma, f) + H_2(\alpha_W, \alpha_A, \beta)\sigma^2 > 0,$$

where the function $F_2(\gamma, f) > 0$ is provided in the proof of Proposition 5 and $H_2(\alpha_W, \alpha_A, \beta) := \frac{(1-\gamma)}{8(1+\gamma)}\left((2-\beta)^2\alpha_W - \frac{(1-f)\alpha_A(\beta\gamma+2)^2}{(\beta\gamma^2(f+1)\alpha_A-4)^2}\right)$. Recall from the proof of Proposition 5 that $F_2(\gamma, f) > 0 \iff f > f_{s1} := \frac{4(\gamma+2)\sqrt{\gamma\left(\gamma^3-\gamma+4\right)+4}-16-16\gamma-\gamma^4}{\gamma^4}$.

Note that $\frac{\partial H_2(\alpha_W,\alpha_A,\beta))}{\partial \alpha_W} = \frac{(\beta-2)^2(1-\gamma)}{8(\gamma+1)} > 0$ and $\frac{\partial H_2(\alpha_W,\alpha_A,\beta))}{\partial \alpha_A} = -\frac{(1-\gamma)(1-f)(\beta\gamma+2)^2\left(\beta\gamma^2(f+1)\alpha_A+4\right)}{(\gamma+1)\left(4-\beta\gamma^2(f+1)\alpha_A\right)^3} < 0$ because $\beta, \gamma \in (0,1)$. This implies that the seller is hurt less (more) by the retailer's switch to WH when the seller's information disadvantage under AG (under WH), relative to the retailer's, becomes greater. Also note that $H_2(\alpha_W,\alpha_A,\beta))|_{\alpha_W=0}<0$ and $H_2(\alpha_W,\alpha_A,\beta))|_{\alpha_A=0}>0$. Thus, the retailer's switch is more likely to reduce the seller's profit if $f<f_{s1}$, which is consistent with the result in Proposition 6. Specifically, given α_A, $F_2(\gamma,f)+H_2(\alpha_W,\alpha_A,\beta)\sigma^2>0$ holds if $\alpha_W>\frac{8(1-f)\alpha_A(\beta\gamma+2)^2}{(\beta-2)^2\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}$, where $\frac{8(1-f)\alpha_A(\beta\gamma+2)^2}{(\beta-2)^2\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}\in(0,\beta)$ for $\gamma,\beta\in(0,1)$ and $\alpha_A\in(0,\beta)$. The result is intuitive because the seller's profit increases with a higher signal accuracy under WH.

In the special case of $\alpha_A=\alpha_W$, the condition reduces to

$$\frac{\sqrt{8(1-f)}(\beta\gamma+2)}{2-\beta}>\beta\gamma^2(f+1)\alpha_A-4.$$

This inequality always holds because $\frac{\sqrt{8(1-f)}(\beta\gamma+2)}{2-\beta}>0$ and $\beta\gamma^2(f+1)\alpha_A-4<0$ for $\beta,f,\gamma,\alpha_A\in(0,1)$. Thus, the switch leads to a reduction in the seller's profit if $f<f_{s1}$, similar to the condition provided in Proposition 6. ◻

Technical Insights of Proposition 7 under Retailer Informational Advantage

The main technical insight derived from the previous propositions remains valid.

Proof of Proposition 7 under Retailer Informational Advantage

(i) The seller prefers WH-S to AG-RS if and only if $\Pi_s^{AG-RS}<\Pi_s^{WH-S}$, which can be reduced to:

$$F_2(\gamma,f)+\overline{G}_3(\alpha_A,\alpha_W,\beta)\sigma^2>0,$$

where $F_2(\gamma,f)>0 \iff f>f_{s1}$ is derived in Proposition 5 in Section 6.1 and $\overline{G}_3(\alpha_A,\alpha_W,\beta):=\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{(f-1)\alpha_A(\beta\gamma+2)^2}{(\beta\gamma^2(f+1)\alpha_A-4)^2}+\frac{\alpha_W}{2}\right)$. Moreover, note that $\overline{G}_3(\cdot)>0 \iff \alpha_W>\frac{2(1-f)\alpha_A(\beta\gamma+2)^2}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}$ and $\frac{\partial\overline{G}_3(\cdot)}{\partial\beta}=-\frac{4(1-\gamma)\gamma(1-f)\alpha_A(\beta\gamma+2)(\gamma(f+1)\alpha_A+2)}{(\gamma+1)\left(4-\beta\gamma^2(f+1)\alpha_A\right)^3}<0$. Thus,

(a) If $f>f_{s1}$ and $\alpha_W>\frac{2(1-f)\alpha_A(\beta\gamma+2)^2}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}$, $F_2(\cdot)>0$ and $\overline{G}_3(\cdot)>0$. Thus, the seller prefers WH.

(b) If $f\le f_{s1}$ and $\alpha_W>\frac{2(1-f)\alpha_A(\beta\gamma+2)^2}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}$, then $F_2(\cdot)\le 0$ and $\overline{G}_3(\cdot)>0$. Thus, the seller prefers WH if and only if $\beta<\overline{\beta}_{s3}$, where $\overline{\beta}_{s3}$ is a value of β such that $F_2(\cdot)+\overline{G}_3(\alpha_A,\alpha_W,\beta)|_{\beta=\overline{\beta}_{s3}}\sigma^2=0$.

(c) Similarly, if $f>f_{s1}$ and $\alpha_W\le\frac{2(1-f)\alpha_A(\beta\gamma+2)^2}{\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2}$, then $F_2(\cdot)>0$ and $\overline{G}_3(\cdot)<0$. Thus, the seller prefers WH if and only if $\beta<\overline{\beta}_{s3}$.

(d) If $f\le f_{s1}$ and $\alpha_W\le\frac{2(1-f)\alpha_A(\beta\gamma+2)^2}{(\beta\gamma^2(f+1)\alpha_A-4)^2}$, then $F_2(\cdot)\le 0$ and $\overline{G}_3(\cdot)\le 0$. Thus, the seller prefers AG.

As compared to the main result, a key distinction is that a higher accuracy of the retailer's signal (i.e., β) does not necessarily enhance the seller's payoff. In fact, a higher β benefits the seller only under AG, because the retailer's improved information leads to more efficient joint use of data, which indirectly increases the seller's payoff. Under WH, by contrast, the retailer internalizes the gain from improved accuracy when setting the retail price, leaving little or no benefit for the seller. Consequently, as the retailer's informational advantage increases (e.g., $\beta>\overline{\beta}_{s3}$), the seller prefers to remain under AG. By contrast, when the seller's signal improves under WH (i.e., a higher α_W), the seller is better off under WH, consistent with Proposition 7(i).

(ii) The retailer prefers WH-S to AG-RS if and only if $\Pi_r^{AG-RS} < \Pi_r^{WH-S}$, which can be reduced to:

$$F_1(\gamma, f) - \overline{G}_4(f, \gamma, \beta)\sigma^2 > 0,$$

where the function $F_1(\gamma, f)$ is defined in Proposition 5. Note that $\overline{G}_4(f,\gamma,\beta) := \frac{\beta(\gamma-1)\left(\alpha_A^2\left(\beta^2\gamma^3 f(f+1)+\gamma^2\left(\beta^2 f+2\beta f(f+1)-(f+1)^2\right)+4\beta\gamma f+4f\right)-4\alpha_A(\gamma+f(\beta\gamma+\gamma+2))-4\right)}{(\gamma+1)\left(\beta\gamma^2(f+1)\alpha_A-4\right)^2} > 0$. In addition, if $\gamma < \gamma_{r1}$ and $f > \frac{1}{4}$, then $F_1(\gamma, f) < 0$, which leads to $F_1(\gamma, f) - \overline{G}_4(f, \gamma, \beta)\sigma^2 < 0$. Under this condition, the retailer has no incentive to switch to WH-S. Thus, only when $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$, the retailer's preference for WH-S can align with the seller's.

Note that $\frac{\partial \overline{G}_4(f,\gamma,\beta)}{\partial \beta}|_{\beta=0} = \frac{(1-\gamma)\left(\left(\alpha_A^2\left(\gamma^2(f+1)^2-4f\right)\right)+4\alpha_A(\gamma+(\gamma+2)f)+4\right)}{16(\gamma+1)} > 0$ holds, and that $\frac{\partial^2 \overline{G_4}(\gamma,f,\beta)}{\partial \beta^2} > 0 \iff \beta\gamma^4(f+1)^3\alpha_A^2 + 2\beta\gamma^3(f+1)^2\alpha_A - 4\gamma^2(f+1)\alpha_A\left(\beta f\alpha_A + 2(\beta-1)f - 2\right) + 8\gamma\left(-3\beta f\alpha_A + 2f + 2\right) + 32f\left(1-\alpha_A\right) > 0$. The inequality holds because $8\gamma\left(-3\beta f\alpha_A + 2f + 2\right) > 0$ and $-4\gamma^2(f+1)\alpha_A\left(\beta f\alpha_A + 2(\beta-1)f - 2\right) > 0$ holds. Thus, $\frac{\partial \overline{G}_4(\cdot)}{\partial \beta}$ is an increasing function of β.

Therefore, under the condition $\gamma > \gamma_{r1}$ or $f < \frac{1}{4}$, $F_1(\gamma, f) - \overline{G}_4(f, \gamma, \beta)\sigma^2 > 0 \iff \beta < \overline{\beta}_{r2}$, where $\overline{\beta}_{r2}$ is a value of β such that $F_1(\gamma, f) - \overline{G}_4(f, \gamma, \beta)|_{\beta=\overline{\beta}_{r2}}\sigma^2 = 0$. Thus, the retailer prefers WH when β is not too high and the competition level γ is not low, a result consistent with Proposition 7(ii). $\square$