

Online Supplement to “Spillover Effect of Consumer Awareness on
Third Parties’ Selling Strategies and Retailers’ Platform
Openness”

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In this online supplement, we consider the case in which the retailer carries products 1 and 2. To be consistent with how the valuation of product 1 is modeled for the retailer and the third-party seller, we consider the valuation of product 2 sold by the retailer to be $4t$, and the valuation of product 2 sold by the third-party seller to be $4kt$, where k is uniformly distributed over $[0, 1]$.¹ In addition, we consider a unit cost c associated with selling product 2 for the retailer, and we assume that it is more efficient for the third-party seller than the retailer to provide the product. The latter assumption, technically, requires $c \geq 2t$, because the surplus created from providing one unit to consumers by the retailer is $4t - c$, and the expected surplus created by the third party is $2t$. Everything else remains the same as in the paper.

We use backward induction to solve the game. We first derive the equilibrium outcome for each price-competition subgame. We then examine the third-party seller’s incentive to join the retailer’s platform under a given commission rate. Finally, we derive the equilibrium of the super game on

¹Recall that we assume “ t as the expected profit of product 2 for the third-party seller from each additional consumer who becomes aware of its product 2.” With the micro foundation that the valuation of product 2 sold by the third-party seller is uniformly distributed over $[0, 4t]$, without competition from the retailer, the third-party seller optimally charges price $2t$ and the expected profit is t , which reduces to the assumption imposed in the paper.

the retailer's openness decision.

1 Price Competition

When the retailer sells products 1 and 2 directly, for an open platform and a given commission rate θ , the third party also has four selling choices—selling no products, product 2, product 1, and products 1 and 2 on the retailer's platform. The competition in product 1 remains the same as in the case that the retailer only carries product 1.

We denote p_{ij} , $i \in \{s, r\}$ and $j \in \{1, 2\}$, as seller i 's price of product j . As with product 1, consumers aware of both sellers' offerings of product 2 buy from the retailer only when $4t - p_{r2} > 4kt - p_{s2}$. In other words, consumers with $k \leq 1 - \frac{p_{r2} - p_{s2}}{4t}$ buy from the retailer and the rest buy from the third party. We can formulate the retailer's and the third party's profit functions in different scenarios as follows.

Scenario I: Selling Separately

$$\begin{aligned}\pi_r &= p_{r1} [\alpha (1 - (p_{r1} - p_{s1})) + (1 - \alpha)] + (p_{r2} - c) [\alpha (1 - \frac{p_{r2} - p_{s2}}{4t}) + (1 - \alpha)] \\ \pi_s &= \alpha p_{s1} (p_{r1} - p_{s1}) + \alpha p_{s2} (\frac{p_{r2} - p_{s2}}{4t})\end{aligned}\tag{1}$$

Scenario II: Selling Product 2 through the Retailer

$$\begin{aligned}\pi_r &= p_{r1} [\mu (1 - (p_{r1} - p_{s1})) + (1 - \mu)] + (p_{r2} - c) (1 - \frac{p_{r2} - p_{s2}}{4t}) + \theta p_{s2} (\frac{p_{r2} - p_{s2}}{4t}) \\ \pi_s &= \mu p_{s1} (p_{r1} - p_{s1}) + (1 - \theta) p_{s2} (\frac{p_{r2} - p_{s2}}{4t})\end{aligned}\tag{2}$$

Scenario III: Selling Product 1 through the Retailer

$$\begin{aligned}\pi_r &= p_{r1} (1 - (p_{r1} - p_{s1})) + (p_{r2} - c) [\mu (1 - \frac{p_{r2} - p_{s2}}{4t}) + (1 - \mu)] + \theta p_{s1} (p_{r1} - p_{s1}) \\ \pi_s &= (1 - \theta) p_{s1} (p_{r1} - p_{s1}) + \mu p_{s2} (\frac{p_{r2} - p_{s2}}{4t})\end{aligned}\tag{3}$$

Scenario IV: Selling Both Products through the Retailer

$$\begin{aligned}\pi_r &= p_{r1} (1 - (p_{r1} - p_{s1})) + (p_{r2} - c) (1 - \frac{p_{r2} - p_{s2}}{4t}) + \theta p_{s1} (p_{r1} - p_{s1}) + \theta p_{s2} (\frac{p_{r2} - p_{s2}}{4t}) \\ \pi_s &= (1 - \theta) p_{s1} (p_{r1} - p_{s1}) + (1 - \theta) p_{s2} (\frac{p_{r2} - p_{s2}}{4t})\end{aligned}\tag{4}$$

Both firms maximize their profits by choosing prices. Based on the best response to each seller, we can derive the equilibrium outcome in the above four scenarios as summarized in the following lemma.

Lemma 1. *If the retailer carries products 1 and 2, the equilibrium outcome is as follows.*

(a) *When the third party sells its products separately, the equilibrium profits are*

$$\begin{aligned} \pi_r^* &= \begin{cases} \frac{2-\alpha}{2} + (4t-c) \left(\frac{2-\alpha}{2}\right) & \text{if } \alpha \leq \frac{2}{3} \\ \frac{4}{9\alpha} + (4t-c) \left(\frac{2-\alpha}{2}\right) & \text{otherwise} \end{cases} \\ \pi_s^* &= \begin{cases} \frac{\alpha}{4} + \alpha t & \text{if } \alpha \leq \frac{2}{3} \\ \frac{1}{9\alpha} + \alpha t & \text{otherwise} \end{cases} \end{aligned} \quad (5)$$

(b) *When the third party sells product 2 on the retailer's platform, the equilibrium profits are*

$$\begin{aligned} \pi_r^* &= \begin{cases} \frac{2-\mu}{2} + \frac{4t-c}{2} + \theta t & \text{if } \mu \leq \frac{2}{3} \\ \frac{4}{9\mu} + \frac{4t-c}{2} + \theta t & \text{otherwise} \end{cases} \\ \pi_s^* &= \begin{cases} \frac{\mu}{4} + (1-\theta)t & \text{if } \mu \leq \frac{2}{3} \\ \frac{1}{9\mu} + (1-\theta)t & \text{otherwise} \end{cases} \end{aligned} \quad (6)$$

(c) *When the third party sells product 1 on the retailer's platform, the equilibrium profits are*

$$\begin{cases} \pi_r^* = \frac{4-\theta}{(3-\theta)^2} + (4t-c) \left(\frac{2-\mu}{2}\right) \\ \pi_s^* = \frac{1-\theta}{(3-\theta)^2} + \mu t \end{cases} \quad (7)$$

(d) *When the third party sells products 1 and 2 on the retailer's platform, the equilibrium profits are*

$$\begin{cases} \pi_r^* = \frac{4-\theta}{(3-\theta)^2} + \frac{4t-c}{2} + \theta t \\ \pi_s^* = \frac{1-\theta}{(3-\theta)^2} + (1-\theta)t \end{cases} \quad (8)$$

Proof. (a) The third party's optimal prices are characterized by the first-order conditions on π_s in

Equation (1):

$$\begin{aligned}\frac{d\pi_s}{dp_{s1}} &= \alpha (p_{r1} - p_{s1}) - \alpha p_{s1} = 0 \\ \frac{d\pi_s}{dp_{s2}} &= \alpha \left(\frac{p_{r2} - p_{s2}}{4t} \right) - \frac{\alpha p_{s2}}{4t} = 0\end{aligned}$$

Therefore, we conclude $p_{s1}^*(p_{r1}) = \frac{p_{r1}}{2}$ and $p_{s2}^*(p_{r2}) = \frac{p_{r2}}{2}$.

The retailer's optimal prices are characterized by the first-order derivatives of π_r in Equation (1):

$$\begin{aligned}\frac{d\pi_r}{dp_{r1}} &= (1 - \alpha) + \alpha [1 - (p_{r1} - p_{s1})] - \alpha p_{r1} = 1 - \frac{3\alpha}{2} p_{r1} \\ \frac{d\pi_r}{dp_{r2}} &= (1 - \alpha) + \alpha \left[1 - \left(\frac{p_{r2} - p_{s2}}{4t} \right) \right] - \frac{\alpha(p_{r2} - c)}{4t} = 1 + \frac{\alpha c}{4t} - \frac{3\alpha}{8t} p_{r2}\end{aligned}$$

where the second equality comes from substituting $p_{s_j}^*(p_{r_j})$. Notice that $\frac{d\pi_r}{dp_{r2}} \geq 0$ for any $p_{r2} \leq 4t$, and thus $p_{r2}^* = 4t$. When $\alpha \leq \frac{2}{3}$, $\frac{d\pi_r}{dp_{r1}} \geq 0$ for any $p_{r1} \in [0, 1]$, and thus $p_{r1}^* = 1$. When $\alpha > \frac{2}{3}$, $p_{r1}^* = \frac{2}{3\alpha}$ by letting $\frac{d\pi_r}{dp_{r1}} = 0$. Therefore, $p_{r2}^* = 4t$ and $p_{s2}^* = 2t$. If $\alpha > \frac{2}{3}$, $p_{r1}^* = \frac{2}{3\alpha}$ and $p_{s1}^* = \frac{1}{3\alpha}$; Otherwise, $p_{r1}^* = 1$ and $p_{s1}^* = \frac{1}{2}$. Substituting these optimal prices into Equation (1), we derive the equilibrium profits as in Equation (5).

(b) The third party's optimal prices are characterized by the first-order conditions on π_s in Equation (2):

$$\begin{aligned}\frac{d\pi_s}{dp_{s1}} &= \mu (p_{r1} - p_{s1}) - \mu p_{s1} = 0 \\ \frac{d\pi_s}{dp_{s2}} &= (1 - \theta) \left(\frac{p_{r2} - p_{s2}}{4t} \right) - \frac{(1 - \theta)p_{s2}}{4t} = 0\end{aligned}$$

Therefore, we conclude $p_{s1}^*(p_{r1}) = \frac{p_{r1}}{2}$ and $p_{s2}^*(p_{r2}) = \frac{p_{r2}}{2}$.

The retailer's optimal prices are characterized by the first-order derivatives of π_r in Equation (2),

$$\begin{aligned}\frac{d\pi_r}{dp_{r1}} &= (1 - \mu) + \mu [1 - (p_{r1} - p_{s1})] - \mu p_{r1} = 1 - \frac{3\mu}{2} p_{r1} \\ \frac{d\pi_r}{dp_{r2}} &= \left(1 - \frac{p_{r2} - p_{s2}}{4t} \right) - \frac{(p_{r2} - c)}{4t} + \frac{\theta p_{s2}}{4t} = 1 + \frac{c}{4t} - \frac{3 - \theta}{8t} p_{r2}\end{aligned}$$

where the second equality comes from substituting $p_{s_j}^*(p_{r_j})$. Notice that $\frac{d\pi_r}{dp_{r2}} \geq 0$ for any $p_{r2} \leq 4t$, and thus $p_{r2}^* = 4t$. When $\mu \leq \frac{2}{3}$, $\frac{d\pi_r}{dp_{r1}} \geq 0$ for any $p_{r1} \in [0, 1]$, and thus $p_{r1}^* = 1$. When $\mu > \frac{2}{3}$, $p_{r1}^* = \frac{2}{3\mu}$ by letting $\frac{d\pi_r}{dp_{r1}} = 0$. Therefore, $p_{r2}^* = 4t$ and $p_{s2}^* = 2t$. If $\mu > \frac{2}{3}$, $p_{r1}^* = \frac{2}{3\mu}$ and $p_{s1}^* = \frac{1}{3\mu}$; Otherwise, $p_{r1}^* = 1$ and $p_{s1}^* = \frac{1}{2}$. Substituting these optimal prices into Equation (2), we derive the equilibrium profits as in Equation (6).

(c) The third party's optimal prices are characterized by the first-order conditions on π_s in

Equation (3):

$$\begin{aligned}\frac{d\pi_s}{dp_{s1}} &= (1 - \theta)(p_{r1} - p_{s1}) - (1 - \theta)p_{s1} = 0 \\ \frac{d\pi_s}{dp_{s2}} &= \mu \left(\frac{p_{r2} - p_{s2}}{4t} \right) - \frac{\mu p_{s2}}{4t} = 0\end{aligned}$$

Therefore, we conclude $p_{s1}^*(p_{r1}) = \frac{p_{r1}}{2}$ and $p_{s2}^*(p_{r2}) = \frac{p_{r2}}{2}$.

The retailer's optimal prices are characterized by the first-order derivatives of π_r in Equation (3):

$$\begin{aligned}\frac{d\pi_r}{dp_{r1}} &= [1 - (p_{r1} - p_{s1})] - p_{r1} + \theta p_{s1} = 1 - \frac{3-\theta}{2}p_{r1} \\ \frac{d\pi_r}{dp_{r2}} &= (1 - \mu) + \mu \left(1 - \frac{p_{r2} - p_{s2}}{4t} \right) - \frac{\mu(p_{r2} - c)}{4t} = 1 + \frac{\mu c}{4t} - \frac{3\mu}{8t}p_{r2}\end{aligned}$$

where the second equality comes from substituting $p_{s_j}^*(p_{r_j})$. Solving the above system of equations, we can derive the equilibrium prices. Substituting these optimal prices into Equations (3), we derive the equilibrium profits as in Equation (7).

(d) The third party's optimal prices are characterized by the first-order conditions on π_s in Equation (4):

$$\begin{aligned}\frac{d\pi_s}{dp_{s1}} &= (1 - \theta)(p_{r1} - p_{s1}) - (1 - \theta)p_{s1} = 0 \\ \frac{d\pi_s}{dp_{s2}} &= (1 - \theta) \left(\frac{p_{r2} - p_{s2}}{4t} \right) - \frac{(1-\theta)p_{s2}}{4t} = 0\end{aligned}$$

Therefore, we conclude $p_{s1}^*(p_{r1}) = \frac{p_{r1}}{2}$ and $p_{s2}^*(p_{r2}) = \frac{p_{r2}}{2}$.

The retailer's optimal prices are characterized by the first-order derivatives of π_r in Equation (4):

$$\begin{aligned}\frac{d\pi_r}{dp_{r1}} &= [1 - (p_{r1} - p_{s1})] - p_{r1} + \theta p_{s1} = 1 - \frac{3-\theta}{2}p_{r1} \\ \frac{d\pi_r}{dp_{r2}} &= \left(1 - \frac{p_{r2} - p_{s2}}{4t} \right) - \frac{(p_{r2} - c)}{4t} + \frac{\theta p_{r2}}{4t} = 1 + \frac{c}{4t} - \frac{3-\theta}{8t}p_{r2}\end{aligned}$$

where the second equality comes from substituting $p_{s_j}^*(p_{r_j})$. Solving the above system of equations, we can derive the equilibrium prices. Substituting these optimal prices into Equations (4), we derive the equilibrium profits as in Equation (8). \square

2 Third Party's Selling Choice

By comparing the equilibrium payoffs under these four selling choices, we can derive the conditions under which the third party chooses, in equilibrium, to sell no products, product 1 only, product 2 only, or products 1 and 2 on the retailer's platform, as summarized in the following proposition. Because the third party's equilibrium profits in Lemma 1 are similar to those when the retailer only

sells product 1 (presented in Lemmas 1–4 in the paper), the third party’s selling choices remain the same as those presented in Proposition 1 in the paper.

Proposition 1. *When the retailer sells products 1 and 2 by itself, for a given θ , the third party sells on the retailer’s platform only in the following cases:*

(a) *When $\mu \leq \frac{4}{9}$, if $\theta \leq K(\frac{\mu}{4})$, the third party sells products 1 and 2; otherwise, (a.1) when $\beta < \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta_{s2}^*]$, the third party sells product 2; (a.2) when $\beta \geq \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta_{21}^*]$, the third party sells product 2, and if $\theta \in (\theta_{21}^*, \theta_{s1}^*]$, the third party sells product 1;*

(b) *When $\mu > \frac{4}{9}$, (b.1) when $\beta < \bar{\beta}(\alpha)$, if $\theta \leq \theta_{s2}^*$, the third party sells product 2; (b.2) when $\beta \geq \bar{\beta}(\alpha)$, if $\theta \leq \theta_{21}^*$, the third party sells product 2, and if $\theta \in (\theta_{21}^*, \theta_{s1}^*]$, the third party sells product 1, where $\bar{\beta}(\alpha)$, $K(\cdot)$, θ_{s2}^* , θ_{s1}^* , and θ_{21}^* are defined in Equations (14), (15), (20), (23), and (25), respectively, in the paper.*

Proof. The proof is the same as that of Proposition 1 in the paper. □

As in Proposition 1 in the paper, Proposition 1 describes the third party’s optimal selling strategies: how the equilibrium choice varies with the initial awareness level, the extent of the spillover effect, and the commission rate. When the expected mixed awareness is low, if the spillover effect relative to the initial awareness is salient, the third party sells products 1 and 2, product 2, product 1, and no products, as the commission rate varies from low to high; if the relative spillover effect is mild, the third party sells products 1 and 2, product 2, and no products, as the commission rate varies from low to high. When the expected mixed awareness is high, if the relative spillover effect is salient, the third party sells product 2, product 1, and no products, as the commission rate varies from low to high; if the relative spillover effect is mild, the third party sells product 2 and no products, as the commission rate varies from low to high.

3 Retailer’s Openness Decision

The retailer compares the optimal profits when opening and not opening its platform to choose whether to open its platform. As in the main paper, first, we show that the retailer has no incentive to induce the third party to sell both products on its platform, as summarized in the following lemma.

Lemma 2. *When the retailer sells products 1 and 2 by itself, the third party selling products 1 and 2 on the retailer's platform cannot arise as an equilibrium in the super game.*

Proof. According to Proposition 1, selling products 1 and 2 can be the equilibrium choice for the third party only when $\mu \leq \frac{4}{9}$ and $\theta \in [0, K(\frac{\mu}{4})]$. However, when $\mu \leq \frac{4}{9}$, by Equation (26) in the main paper and $\frac{\partial \pi_{r3}^*}{\partial \theta} > 0$, for any $\theta \in [0, K(\frac{\mu}{4})]$, we have

$$\pi_{r3}^*(\theta) \leq \pi_{r3}^*(K(\frac{\mu}{4})) < \pi_{r3}^*(\theta_{21}^*) < \pi_{r2}^*(\theta_{21}^*)$$

where the last inequality is because $\pi_{r3}^*(\theta) < \pi_{r2}^*(\theta)$ for $\theta \in [0, 1]$ by Lemma 1. Therefore, when $\mu \leq \frac{4}{9}$, the retailer prefers to induce the third party to sell product 2 on its platform by setting $\theta \in (K(\frac{\mu}{4}), \theta_{21}^*)$ rather than to induce the third party to sell both products. \square

Next, we examine the conditions under which the retailer can benefit if the third party only sells product 2 or product 1 on its platform.

Lemma 3. *When the retailer sells products 1 and 2 by itself, (a) if the third party only sells product 2 on the retailer's platform, the retailer can benefit if and only if $\theta \in (\theta_{r2}^*, 1]$, where θ_{r2}^* is defined as*

$$\theta_{r2}^* = \begin{cases} \frac{(1-\alpha)\beta + (4t-c)(1-\alpha)}{2t} & \text{if } \mu \leq \frac{2}{3} \\ \frac{1}{t} \left(1 - \frac{\alpha}{2} - \frac{4}{9\mu} + \frac{(4t-c)(1-\alpha)}{2} \right) & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \leq \frac{2}{3} \\ \frac{1}{t} \left(\frac{4}{9\alpha} - \frac{4}{9\mu} + \frac{(4t-c)(1-\alpha)}{2} \right) & \text{otherwise} \end{cases} \quad (9)$$

(b) *If the third party only sells product 1 on the retailer's platform, the retailer can benefit if and only if $\alpha \in (\frac{1}{2}, 1)$ and $\theta \in (\theta_{r1}^*, 1]$, where θ_{r1}^* is defined as*

$$\theta_{r1}^* = \begin{cases} \frac{5-3\alpha+3(1-\alpha)\beta(4t-c)-\sqrt{5-2\alpha+2(1-\alpha)\beta(4t-c)}}{2-\alpha+(1-\alpha)\beta(4t-c)} & \text{if } \alpha \leq \frac{2}{3} \\ \frac{24-9\alpha+27\alpha(1-\alpha)\beta(4t-c)-3\sqrt{16\alpha+9\alpha^2+18\alpha^2(1-\alpha)\beta(4t-c)}}{8+9\alpha(1-\alpha)\beta(4t-c)} & \text{otherwise} \end{cases} \quad (10)$$

Proof. (a) The retailer's equilibrium profit difference between Scenario II ("Selling Product 2 through

the Retailer”) and Scenario I (“Selling Separately”) is

$$\pi_{r2}^* - \pi_{r0}^* = \begin{cases} \theta t - \left[\left(\frac{2-\alpha}{2} - \frac{2-\mu}{2} \right) + (4t-c) \left(\frac{1-\alpha}{2} \right) \right] & \text{if } \mu \leq \frac{2}{3} \\ \theta t - \left[\left(\frac{2-\alpha}{2} - \frac{4}{9\mu} \right) + (4t-c) \left(\frac{1-\alpha}{2} \right) \right] & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \leq \frac{2}{3} \\ \theta t - \left[\left(\frac{4}{9\alpha} - \frac{4}{9\mu} \right) + (4t-c) \left(\frac{1-\alpha}{2} \right) \right] & \text{if } \alpha > \frac{2}{3} \end{cases} \quad (11)$$

Because the above profit difference increases in θ , we have $\pi_{r0}^* \leq \pi_{r2}^*$ if and only if $\theta_{r2}^* \leq \theta$, where θ_{r2}^* is the solution to $\pi_{r2}^* - \pi_{r0}^* = 0$ and can be derived as in Equation (9).

(b) The retailer’s equilibrium profit difference between Scenario III (“Selling Product 1 through the Retailer”) and Scenario I is

$$\pi_{r1}^* - \pi_{r0}^* = \begin{cases} \frac{4-\theta}{(3-\theta)^2} - \frac{2-\alpha}{2} - (4t-c) \left(\frac{\mu-\alpha}{2} \right) & \text{if } \alpha \leq \frac{2}{3} \\ \frac{4-\theta}{(3-\theta)^2} - \frac{4}{9\alpha} - (4t-c) \left(\frac{\mu-\alpha}{2} \right) & \text{otherwise} \end{cases} \quad (12)$$

The above profit difference increases in θ . Notice that when $\alpha \leq \frac{1}{2}$, $\pi_{r1}^* \leq \pi_{r0}^*$ even with the highest commission rate 1. Therefore, in this case, the retailer has no incentive to let the third party sell product 1 only. When $\alpha \in (\frac{1}{2}, 1]$, we have $\pi_{r0}^* \leq \pi_{r1}^*$ if and only if $\theta_{r1}^* \leq \theta$, where θ_{r1}^* is the solution to $\pi_{r1}^* - \pi_{r0}^* = 0$ and can be derived as in Equation (10). \square

Next, we present the retailer’s equilibrium decision on whether to open its platform and the third party’s equilibrium selling strategies when the retailer’s platform is open.

Proposition 2. *In equilibrium, the retailer opens its platform in the following cases:*

- (a.1) when $\beta < \bar{\beta}(\alpha)$ and $\theta_{r2}^* < \theta_{s2}^*$;
- (a.2) when $\beta \geq \bar{\beta}(\alpha)$, $\alpha \leq \frac{1}{2}$, and $\theta_{r2}^* < \theta_{21}^*$;
- (a.3) when $\beta \geq \bar{\beta}(\alpha)$, $\alpha > \frac{1}{2}$, $\theta_{r2}^* < \theta_{21}^*$, and $\pi_{r2}^*(\theta_{21}^*) > \pi_{r1}^*(\min\{\theta_{s1}^*, 1\})$;
- (b) when $\beta \geq \bar{\beta}(\alpha)$, $\alpha > \frac{1}{2}$, $\theta_{r1}^* < \theta_{s1}^*$, and $\pi_{r2}^*(\theta_{21}^*) < \pi_{r1}^*(\min\{\theta_{s1}^*, 1\})$.

In cases (a.1)–(a.3), the third party sells product 2 on the platform, and in case (b) the third party sells product 1. Thresholds θ_{r2}^ and θ_{r1}^* are defined in Equations (9) and (10), respectively, and $\bar{\beta}(\alpha)$, θ_{s2}^* , θ_{s1}^* , and θ_{21}^* are defined in Equations (14), (20), (23), and (25), respectively, in the paper.*

Proof. The proof is the same as that of Proposition 2 in the paper. \square

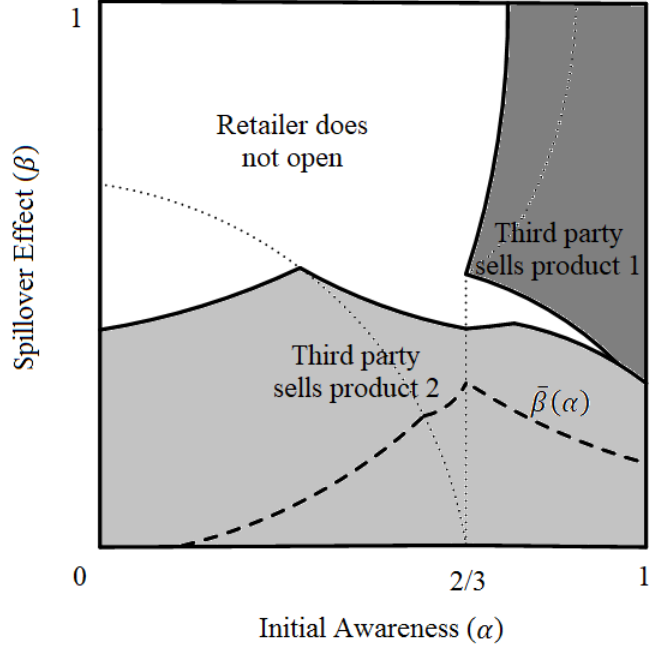


Figure 1: Equilibrium Outcome in the Super Game Considering Cost ($c = 3t$ and $t = 1$)

Similar to Proposition 2 in the paper, Proposition 2 shows that the retailer’s equilibrium openness decision depends on the extent of the spillover effect and the third party’s initial awareness level. The equilibrium outcome is illustrated in Figure 1 (in which the solid lines define three possible outcomes). As seen in the figure, although there are some distortions in the boundaries that segment different cases, the overall pattern and thus the insights remain the same: When the spillover effect is mild (i.e., in the gray area), the retailer opens its platform and the third party sells product 2 on it; When the spillover effect is salient, but the third party’s initial awareness level is low (i.e., in the white area), the retailer does not open its platform; When the spillover effect is salient and the third party’s initial awareness level is high (i.e., in the dark area), the retailer opens its platform and the third party sells product 1 on it.

Altogether, we illustrate that even when the retailer carries both products, given an exogenous commission rate, the third party’s optimal selling strategies (presented in Proposition 1) remain qualitatively the same as in the case that the retailer only carries product 1 (presented in Proposition 1 in the paper). Further, the retailer’s equilibrium openness decision (presented in Proposition 2) also remains qualitatively the same as in the case that the retailer only carries product 1 (presented in Proposition 2 in the paper). Therefore, the insights delivered in the paper regarding the effect

of spillover on a third party's optimal selling strategies and on the retailer's platform openness are robust to some extent.