Competition and Distortion: A Theory of Information Bias on the Peer-to-Peer Lending Market

Online Appendix

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A Proofs in Section 4

We first establish the following lemma, which will be useful for the subsequent analysis.

Proof of Lemma 1. This result is supported by the following claims.

Claim. In equilibrium the high tech type platform will truthfully disclose the signal in $t = 2$.

Proof. Given the investor's decision in $t = 2$. At the beginning of $t = 2$, for any $\omega_2 \in \{l, h\}$, a high tech platform j's payoff from truthfully disclosing the signal is B. The payoff from non-truthful disclosure is $B - C$. Thus, truthful disclosure is optimal in $t = 2$. $\square$

Claim. In equilibrium the mediocre type platform will truthfully disclose the signal in $t = 2$.

Proof. Given the investor's decision in $t = 2$. At the beginning of $t = 2$, if $\omega_2 = l$, a mediocre platform j's expected payoff from truthfully disclosing the signal is $\psi B + (1 - \psi)(B - C)$ where $\psi \equiv \psi(l|l) = \dfrac{qp}{1 + 2qp - q - p} > \dfrac{1}{2}$. The expected payoff from non-truthful disclosure is $(1 - \psi)B + \psi(B - C)$. It is easy to check it is true that $\psi B + (1 - \psi)(B - C) > (1 - \psi)B + \psi(B - C)$. If $\omega_2 = h$, platform j's expected payoff from truthfully disclosing the signal

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is $(1-\hat{\psi})B + \hat{\psi}(B-C)$ where $\hat{\psi} \equiv \hat{\psi}(l|h) = \dfrac{(1-q)p}{q+p-2qp} < \dfrac{1}{2}$. The expected payoff from non-truthful disclosure is $\hat{\psi}B + (1-\hat{\psi})(B-C)$. It is easy to check $(1-\hat{\psi})B + \hat{\psi}(B-C) > \hat{\psi}B + (1-\hat{\psi})(B-C)$. Therefore, for any signal $\omega_2 \in \{l, h\}$, the mediocre type will truthfully disclose it. $\square$

Claim. In equilibrium, the investor will always purchase in $t = 2$.

Proof. In $t = 2$, if $x_2 = l$, then the investor's expected utility from investing is $\hat{\mu}(l|l)v(R_l, l) + \hat{\mu}(h|l)v(R_l, h)$, where $\hat{\mu}(l|l) = \text{Prob}\{s_2 = l|x_2 = l\}$. By Assumption 2, we have $v(R_l, h) > v_0$, then the investor will purchase in $t = 2$ if $x_2 = l$. If $x_2 = h$, then the investor's expected utility from investing is $\hat{\mu}(h|h)v(R_h, h) + \hat{\mu}(s = l|h)v(R_h, l)$, where $\hat{\mu}(h|h) = \text{Prob}\{s_2 = h|x_2 = h\}$. Similarly, we have $v(R_h, h) > v(R_l, h) > v_0$, then the investor will purchase in $t = 2$ if $x_2 = h$. $\square$

■

Proof of Lemma 2. Given $\sigma_{1,1}(x = l|\omega_1 = l, \theta_T) = 1$ and let $\sigma(x|\omega_1) \equiv \sigma_{1,1}(x|\omega_1, \theta_M)$, then[1]

$$\mu(l, \emptyset) = \frac{\lambda_1 p}{\lambda_1 p + (1-\lambda_1)\Big[[pq + (1-p)(1-q)]\sigma(l|\hat{l}) + [p(1-q) + (1-p)q]\sigma(l|\hat{h})\Big]}$$

and

$$\mu(h, \emptyset) = \frac{\lambda_1(1-p)}{\lambda_1(1-p) + (1-\lambda_1)\Big[[pq + (1-p)(1-q)]\sigma(h|\hat{l}) + [p(1-q) + (1-p)q]\sigma(h|\hat{h})\Big]}$$

Therefore,

$$\bar{\mu}(l, \emptyset) = \frac{\lambda_1 p}{\lambda_1 p + (1-\lambda_1)[pq + (1-p)(1-q)]},$$

and

$$\bar{\mu}(h, \emptyset) = \frac{\lambda_1(1-p)}{\lambda_1(1-p) + (1-\lambda_1)[p(1-q) + (1-p)q]}.$$

Furthermore, we can check, if $1 > p > 1/2$, then $\bar{\mu}(l, \emptyset) > \lambda_1$ and $\bar{\mu}(h, \emptyset) < \lambda_1$. ■

[1] In the appendix, to simplify notation, we let $\sigma(x|\hat{s}) \equiv \sigma(x|\omega_1 = s)$ for $s \in \{l, h\}$, when we drop ω_1.

Lemma 1. *At the end of the first period, the investor will hold to her investment in platform 1 if and only if she believes that platform 1 is more likely to be a high tech type, that is,* $\mu_1(x_1, a_1) \geq \lambda_2$.

Proof.

At the end of the first period, the investor's expected payoff from investing in platform j in the second period is

$$\begin{aligned} \mathrm{E}_{s_2}[v(R(x_{j,2}, s_2)|\mu_j(A)] = p\left[\mu_j(A)v(R_l, l) + (1 - \mu_j(A))\left(qv(R_l, l) + (1-q)v(R_h, l)\right)\right] \\ + (1-p)\left[\mu_j(A)v(R_h, h) + (1 - \mu_j(A))\left(qv(R_h, h) + (1-q)v(R_l, h)\right)\right] \end{aligned}$$

Then, by Assumption 2, the above increases in $\mu_j(A)$. At the end of the first period, the investor's belief of platform 1 being a high tech type is her posterior about platform 1's type after observing the risk assessment and uncertainty-resolution outcome in the first period, that is, $\mu_1(A) = \mu_1(x_1, a_1)$; the investor's belief of platform 2 being a high tech type is just her prior about platform 2's type, that is, $\mu_1(A) = \lambda$. Therefore, by monotonicity of investor's payoff of investing, she will hold to her investment in platform 1 if and only if $\mu_1(x_1, a_1) \geq \lambda_2$.

■

Proof of Proposition 1. The proof of the monopoly platform's equilibrium strategy in the second period is the same as the proof of Lemma 1. Following backward induction, we go back to the beginning of $t = 1$. For any $\omega_1 \in \{l, h\}$, a high tech platform's expected payoff from truthful disclosure in $t = 1$ is $B + \rho\beta B + (1-\rho)d(s, \emptyset)\beta B$. The expected payoff from non-truthful disclosure is $B - C + (1-\rho)d(s', \emptyset)\beta B$, where $s \neq s'$. Since the investor always purchases from the monopoly platform in equilibrium, i.e., $d(l, \emptyset) = d(h, \emptyset) = 1$, then the high tech type will truthfully disclose the signal in $t = 1$. For the same reason, the mediocre platform will truthfully report in $t = 1$. ■

Proof of Proposition 2. The proof of the platform's equilibrium strategy in the second period is the same as the proof of Lemma 1. Following backward induction, we go back to the beginning of $t = 1$. For any $\omega_1 \in \{l, h\}$, a high tech platform's expected payoff from truthful disclosure in $t = 1$ is $B + \rho\beta B + (1-\rho)d(s,s)\beta B$. The expected payoff from non-truthful disclosure is $B - C + (1-\rho)d(s,s')\beta B$. Since the investor prefers high tech platform, then we have $d(s,s) = 1 > d(s,s') = 0$. Thus, the high tech type will truthfully disclose the signal in $t = 1$. For the same reason, the mediocre platform will truthfully report in $t = 1$. ■

Proof of main results. The proofs of Propositions 3, 4, and Corollary 1 are composed of the following claims.

Claim. Given $\bar{\mu}(h, \emptyset) < \lambda_2 < \lambda_1 < \bar{\mu}(l, \emptyset)$, there is an equilibrium, such that

1. for a mediocre type platform ($\theta = \theta_M$) which observes $\omega_1 = l$, we have $\sigma(x = l|\omega_1 = l) = 1$;
2. for a mediocre type platform ($\theta = \theta_M$) which observes $\omega_1 = h$, we have $\sigma(x = l|\omega_1 = h) \in [0, 1)$.
3. for the investor, we have $d(x = l, s = l) = d(x = h, s = h) = 1$, $d(x = l, s = h) = d(x = h, s = l) = 0$, $d(x = l, \emptyset) = 1$ and

$$d(x = h, \emptyset) = 1 - \frac{(1 - 2\hat{\psi})(C + \rho\beta Q)}{(1-\rho)\beta Q}.$$

Proof. To prove the first part, we need the following condition to be true:

$$\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M),$$

where $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M)$ is

$$\psi B + (1-\psi)(B - C) + \rho\psi\beta[qB + (1-q)(B-C)] + (1-\rho)d(l, \emptyset)\beta[qB + (1-q)(B-C)]$$

here $\psi = \psi(l|\hat{l}) = \mathrm{Prob}\{s_1 = l|\omega_1 = l\} = \dfrac{qp}{1 + 2qp - q - p} > \dfrac{1}{2}$. In the meantime,

$\mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$ is

$$(1-\psi)B+\psi(B-C)+\rho(1-\psi)\beta[qB+(1-q)(B-C)]+(1-\rho)d(h,\emptyset)\beta[qB+(1-q)(B-C)]$$

We can check when $1 > q > p > 1/2$, $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$ holds for $d(l, \emptyset) = 1$ and any $d(h, \emptyset) \in [0, 1]$.

To prove the second part, we need to find $\sigma(x = l|\omega_1 = h) \in [0, 1)$, such that, the following condition is true: $\mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M) = \mathrm{E}U(x = h|\omega_1 = l, \theta_M)$. Here, $\mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M)$ is

$$\hat{\psi}B+\left(1-\hat{\psi}\right)(B-C)+\rho\hat{\psi}\beta[qB+(1-q)(B-C)]+(1-\rho)d(l,\emptyset)\beta[qB+(1-q)(B-C)],$$

where $\hat{\psi} = \hat{\psi}(l|\hat{h}) = \text{Prob}\{s_1 = l|\omega_1 = h\} = \dfrac{(1-q)p}{q+p-2qp} < \dfrac{1}{2}$. In the meantime, $\mathrm{E}U_{1,1}(x = h|\omega_1 = h, \theta_M)$ is

$$(1-\hat{\psi})B+\hat{\psi}(B-C)+\rho(1-\hat{\psi})\beta[qB+(1-q)(B-C)]+(1-\rho)d(h,\emptyset)\beta[qB+(1-q)(B-C)].$$

Therefore, $\mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M) = \mathrm{E}U(x = h|\omega_1 = h, \theta_M)$ induces

$$d(h, \emptyset) = 1 - \frac{(1-2\hat{\psi})(C+\rho\beta Q)}{(1-\rho)\beta Q}.$$

Here, we let $Q = qB + (1-q)(B-C) = B - (1-q)C$. To guarantee $d(h, \emptyset) \in (0, 1)$, we need

$$\rho \leq \rho^* \equiv \frac{\beta Q - (1-2\hat{\psi})C}{2\beta Q(1-\hat{\psi})}.$$

We can check $\rho^* < 1$.

For the investor, $d(x = l, s = l) = d(x = h, s = h) = 1$, $d(x = l, s = h) = d(x = h, s = l) = 0$ are determined by the order of the posterior and Lemma 1. At the information set $(x = h, \emptyset)$, recall that $\sigma(h|\hat{l}) = 0$, then the investor's posterior on platform 1 being high tech is

$$\mu(h, \emptyset) = \frac{\lambda_1(1-p)}{\lambda_1(1-p) + (1-\lambda_1)[p(1-q) + (1-p)q]\sigma(h|\hat{h})}.$$

Since $d(h, \emptyset) \in (0,1)$, then mixing is optimal for the investor, thus we have $\mu(\theta_T|h, \emptyset) = \lambda_2$, which induces

$$\sigma(h|\hat{h}) = \frac{\lambda_1(1-\lambda_2)(1-p)}{\lambda_2(1-\lambda_1)[p(1-q)+(1-p)q]}.$$

At the information set $(x = l, \emptyset)$, recall that $\sigma(l|\hat{l}) = 1$, then the investor's posterior on platform 1 being high tech is

$$\mu(l, \emptyset) = \frac{\lambda_1\lambda_2 p}{\lambda_1 p + \lambda_2 - \lambda_1}.$$

Given $\lambda_1 > \lambda_2$, we have $\dfrac{\lambda_1\lambda_2 p}{\lambda_1 p + \lambda_2 - \lambda_1} > \lambda_2$, thus $d(l, \emptyset) = 1$.

If $\rho > \rho^*$, it is easy to check, given $d(l, \emptyset) = 1$ and $d(h, \emptyset) = 0$, we have

$$\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M),$$

and

$$\mathrm{E}U(x = h|\omega_1 = h, \theta_M) \geq \mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M).$$

Thus, we have an equilibrium with $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$. The proof of results for the case of $\bar{\mu}(\theta_T|h, \emptyset) < \lambda_1 < \lambda_2 < \bar{\mu}(\theta_T|l, \emptyset)$ is similar to the above process. □

Claim. If $\bar{\mu}(l, \emptyset) < \lambda_2 < \bar{\mu}(h, h)$, there is an equilibrium, such that

1. $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$;
2. $d(h, h) = d(l, l) = 1$ and $d(h, l) = d(l, h) = d(h, \emptyset) = d(l, \emptyset) = 0$.

Proof. Given $\bar{\mu}(l, \emptyset) < \lambda_2 < \bar{\mu}(h, h)$, then, by Lemma 1, we know that the investor would always switch to platform 2 in $t = 2$ if the uncertainty is not resolved, i.e., $d(h, \emptyset) = d(l, \emptyset) = 0$, and stay on platform 1 if uncertainty is resolved and matches the assessment, i.e., $d(h, h) = d(l, l) = 1$. Therefore $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M)$ is $\psi B + (1-\psi)(B-C) + \rho\psi\beta[qB + (1-q)(B-C)]$, and $\mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$is $(1-\psi)B + \psi(B-C) + \rho(1-\psi)\beta[qB + (1-q)(B-C)]$. Since $\psi > 1/2$, then $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$. Similarly, we can prove, given $\hat{\psi} < \frac{1}{2}$, it is true that $\mathrm{E}U(x = h|\omega_1 = h, \theta_M) \geq \mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M)$. Thus, we have $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$. □

Claim. Given $\lambda_2 < \bar{\mu}(h, \emptyset)$, there is an equilibrium, such that

1. $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$;

2. $d(h, \emptyset) = d(l, \emptyset) = d(h, h) = d(l, l) = 1$ and $d(h, l) = d(l, h) = 0$.

Proof. Given $\lambda_2 < \bar{\mu}(h, \emptyset)$, then, by Lemma 1, we know that the investor would always stay on platform 1 as long as the uncertainty is not resolved or is resolved to match the assessment, i.e., $d(h, \emptyset) = d(l, \emptyset) = d(h, h) = d(l, l) = 1$ and $d(h, l) = d(l, h) = 0$. Therefore $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M)$ is

$$\psi B + (1 - \psi)(B - C) + \rho\psi\beta[qB + (1 - q)(B - C)] + (1 - \rho)\beta[qB + (1 - q)(B - C)]$$

and $\mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$ is

$$(1 - \psi)B + \psi(B - C) + \rho(1 - \psi)\beta[qB + (1 - q)(B - C)] + (1 - \rho)\beta[qB + (1 - q)(B - C)]$$

Since $\psi > 1/2$, then $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$. Similarly, we can prove, given $\hat{\psi} < \frac{1}{2}$, it is true that $\mathrm{E}U(x = h|\omega_1 = h, \theta_M) \geq \mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M)$. Thus, we have $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$. □

Claim. When $\lambda_2 > \bar{\mu}(h, h)$, there is an equilibrium, such that

1. $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$;

2. $d(h, h) = d(l, l) = d(h, l) = d(l, h) = d(h, \emptyset) = d(l, \emptyset) = 0$.

Proof. Given $\lambda_2 > \bar{\mu}(h, h)$, then, by Lemma 1, we know that the investor would always switch to platform 2, i.e., $d(h, h) = d(l, l) = d(h, l) = d(l, h) = d(h, \emptyset) = d(l, \emptyset) = 0$. Therefore $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M)$ is $\psi B - (1 - \psi)C$ and $\mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$ is $(1 - \psi)B - \psi C$. Since $\psi > 1/2$, then $\mathrm{E}U_{1,1}(x = l|\omega_1 = l, \theta_M) \geq \mathrm{E}U_{1,1}(x = h|\omega_1 = l, \theta_M)$. Similarly, we can prove, given $\hat{\psi} < \frac{1}{2}$, it is true that $\mathrm{E}U_{1,1}(x = h|\omega_1 = h, \theta_M) \geq \mathrm{E}U_{1,1}(x = l|\omega_1 = h, \theta_M)$. Thus, we have $\sigma(l|\hat{l}) = 1$ and $\sigma(l|\hat{h}) = 0$. □

Claim. The high tech type platform will truthfully disclose the observed signal in $t = 1$.

Proof. Following backward induction, we go back to the beginning of $t = 1$. For any $\omega_1 \in \{l, h\}$, a high tech platform's expected payoff from truthful disclosure in $t = 1$ is $B + \rho\beta B + (1-\rho)d(l, \emptyset)\beta B$. The expected payoff from non-truthful disclosure is $B - C + (1-\rho)d(h, \emptyset)\beta B$. Therefore, the high tech platform will truthfully disclose signals if

$$\frac{C}{\beta B} + \rho > (1-\rho)[d(h, \emptyset) - d(l, \emptyset)].$$

Since $d(h, \emptyset) \leq d(l, \emptyset)$, then the high tech type will truthfully disclose the signal in $t = 1$. □

■

B Proofs in Section 5

Proof of Lemma 3 and Corollary 2. First, following the same logic as in the baseline model, investor α ex ante prefers high tech type, and thus, at the end of the first period, would choose to hold to her investment in platform 1 if $\mu(x_1, a_1) \geq \lambda$, and withdraw her investment and consider investing in platform 2 otherwise. Thus, if the investor invests in platform 1 at the second period, $\mathrm{E}_s[v(R(\omega_{1,2}), s; \alpha)|\mu_1(x_1, a_1)] \geq \mathrm{E}_s[v(R(\omega_{2,2}), s; \alpha)|\lambda]$ must hold. As a result, the expected payoff of the investor investing in the P2P market in the second period is

$$\begin{aligned}\mathrm{E}(V_2|\text{investing}) = d(x_1, a_1)\mathrm{E}_s[v(R(\omega_{1,2}), s; \alpha)|\mu_1(x_1, a_1)] + \\ (1 - d(x_1, a_1))\mathrm{E}_s[v(R(\omega_{2,2}), s; \alpha)|\lambda] \geq \mathrm{E}_s[v(R(\omega_{2,2}), s; \alpha)|\lambda]. \quad (1)\end{aligned}$$

Then, the expected payoff of the investor at the beginning of the second period is $\mathrm{E}(V_2) = \max\{\mathrm{E}(V_2|\text{investing}), v_0\}$.

At the beginning of the first period, the expected payoff of the investor is

$$E(V_1|\text{investing}) = \mathrm{E}_s[v(R(x_{1,1}), s; \alpha)|\lambda] + \beta\mathrm{E}(V_2).$$

Note that the platform 1 and platform 2 are the same ex-ante (i.e., $\lambda_1 = \lambda_2$), and thus

$$\mathrm{E}_s[v(R(\omega_{1,1}), s; \alpha)|\lambda] = \mathrm{E}_s[v(R(\omega_{2,2}), s; \alpha)|\lambda]. \tag{2}$$

Then $\mathrm{E}_s[v(R(x_{1,1}), s; \alpha)|\lambda] = \mathrm{E}_s[v(R(\omega_{1,1}), s; \alpha)|\lambda]$, if platform 1 truthfully discloses the signal in the first period; otherwise, $\mathrm{E}_s[v(R(x_{1,1}), s; \alpha)|\lambda] \leq \mathrm{E}_s[v(R(\omega_{1,1}), s; \alpha)|\lambda]$ because of Assumption 1 and Assumption 4 part (i). By Assumption 1, the platform more likely reports the true state when it truthfully discloses the signal than non-truthfully discloses the signal. Then, by Assumption 4 part (i), an investor gains a higher payoff when the probability that the platform she invests in reports the true state is higher. Therefore, by Equations (1) and (2), we have

$$\mathrm{E}_s[v(R(x_{1,1}), s; \alpha)|\lambda] \leq \mathrm{E}_s[v(R(\omega_{2,2}), s; \alpha)|\lambda] \leq E(V_2|\text{investing}) \leq E(V_2).$$

As a result,

$$E(V_1) \leq (1+\beta)E(V_2). \tag{3}$$

The demands of the P2P lending market in the two periods are, respectively,

$$\begin{aligned} D_1 =& \{\alpha \in [0, \alpha] : E(V_1) \geq (1+\beta)v_0\}; \\ D_2 =& D_1 \cap \{\alpha \in [0, \alpha] : E(V_2) \geq v_0\}. \end{aligned}$$

Thus, to show $D_1 = D_2$, is equivalent to show $D_1 \subseteq \{\alpha \in [0, \alpha] : E(V_2) \geq v_0\}$, which is established by Condition (3).

Next, we show $m(D_1) > 0$. Write $\hat{\alpha} \equiv max_\alpha \min_{(x,s)} v(R(x), s; \alpha)$ for the investor who has the highest utility under the worst pair of realizations (x, s). By Assumption 4 part (ii), $\min_{(x,s)} v(R(x), s; \hat{\alpha}) > v_0$. By continuity of investors' payoff function on α, there exists a neighborhood of $\mathcal{N}(\hat{\alpha})$ with $m(\mathcal{N}(\hat{\alpha})) > 0$, such that, for any $\alpha \in \mathcal{N}(\hat{\alpha})$, we have $\min_{(x,s)} v(R(x), s; \hat{\alpha}) > v_0$. Because $\min_{(x,s)} v(R(x), s; \hat{\alpha}) \leq \mathrm{E}_s[v(R(x_{1,1}), s; \alpha)|\lambda] \leq$ E(V_2), we have $E(V_1) \geq (1+\beta)\min_{(x,s)} v(R(x), s; \hat{\alpha}) \geq (1+\beta)v_0$ for any $\alpha \in \mathcal{N}(\hat{\alpha})$. As a result, $\mathcal{N}(\hat{\alpha}) \subseteq D_1$ and, in turn, $m(D_1) > 0$. ■

Proof of Proposition 5. First, we prove, in any n-TDE, $d_1(x_1^*, x_2^*; \emptyset) \neq 0$ or $d_2(x_1^*, x_2^*; \emptyset) = 0$. Suppose there is an n-TDE such that $d_j^*(x_j^*, x_{-j}^*; \emptyset) = 0$ where $j \in \{1, 2\}$. Then the following condition must be satisfied

$$\mathrm{E}U_j(x_j^* = l, x_{-j}^*, d^* | \omega_1 = \hat{h}, \theta_M) \geq \mathrm{E}U_j(x_j = h, x_{-j}^*, d^* | \omega_1 = \hat{h}, \theta_M),$$

which turns to $\rho[\hat{\psi} d_j(l, x_{-j}^*; l)(B+Z) \geq (1-\rho)[\hat{\psi} d_j(h, x_{-j}^*; \emptyset)(B-C) + (1-\hat{\psi}) d_j(h, x_{-j}^*; \emptyset)(B + Z)] + \rho(1-\hat{\psi}) d_1(h, x_{-j}^*; h)(B+Z)$. It cannot be true because $\hat{\psi} < 1/2$ and $d_j(l, x_{-j}^*; l) = d_j(h, x_{-j}^*; h)$.

Second, the investor's expected payoff from l, denoted as

$$V(l) = \mu_j(l) v(R_l, l) + (1 - \mu_j(l)) \left[p v(R_l, l) + (1-p) v(R_l, h) \right]$$

should be larger than the one from h, denoted as

$$V(h) = \mu_j(h) v(R_h, h) + (1 - \mu_j(h)) \left[p(1-q) v(R_h, l) + (1-p) q v(R_h, h) \right].$$

We can check $V(h) < V(l)$ if it is true that $q v(R_l, l) + (1-q) v(R_l, h) \geq q v(R_h, h) + (1 - q) v(R_h, l)$. Additionally, for the platform $j \in \{1, 2\}$, the following condition must be true:

$$\mathrm{E}U_j(x_j^* = l, x_{-j}^* = l, d^* | \omega_1 = \hat{h}, \theta_M) \geq \mathrm{E}U_1(x_j = h, x_{-j}^* = l, d^* | \omega_1 = \hat{h}, \theta_M), \quad (4)$$

where $d^* = (d_1^*(l, l; \emptyset) = d_2^*(l, l; \emptyset) = 1/2)$ is the equilibrium strategy of the investor. Condition (4) turns to be

$$\frac{1-\rho}{2}[\hat{\psi}(B+Z) + (1-\hat{\psi})(B-C)] + \frac{\rho}{2}\hat{\psi}(B+Z) \geq \rho(1-\hat{\psi}) d_j(h, x_2 = l; h)(B+Z)$$

which is equivalent to

$$\rho < \frac{\hat{\psi}(B+Z) + (1-\hat{\psi})(B-C)}{2(1-\hat{\psi})(B+Z) + (1-\hat{\psi})(B-C)}.$$

where $\hat{\psi} \equiv \psi(l|\hat{h}) = \text{Prob}(s = l|\omega_j = \hat{h}) = \dfrac{(1-q)p}{q+p-2qp}$. ■