Supplementary Online Appendix for

The Attention Economy: Measuring the Value of Free Goods on the Internet

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Equilibrium

We study a competitive equilibrium in which households take prices as given and markets clear. The model is static, and we study a sequence of static equilibria to compare the model to the data.

The solution to the problem in the equation (2) in the paper determines the demand functions for the time share of Internet, television and composite good consumption. Since the marginal utility of zero Internet time is bounded above (due to the term $\kappa_1 > 0$), the solution to the individual's maximization problem could be at a corner where $T_1 = 0$. Note that the price of the composite good (i.e., numeraire) is normalized to one: P =1. For any given real wage rate W, there exists a threshold quality level $\tilde{Q}(W)$ such that the equilibrium time spent on Internet will be zero if $Q \leq \tilde{Q}(W)$, i.e., quality of Internet is low and hence the relative implicit price for Internet $1/Q$ is high, where $\tilde{Q}(W)$ solves for the following equation:

$$\frac{1}{\Gamma}\cdot\frac{1}{(1-\alpha_0)}+\left[\frac{1-\theta_1}{\theta_1}\cdot\frac{1}{\tilde{Q}}\right]^{\sigma_1}<\frac{1-F/W+\kappa_2}{\kappa_1} \tag{S1}$$

where $$\Gamma=\left[\frac{1-\theta_1}{\theta_1}\frac{1}{\tilde{Q}}\right]^{\sigma_1[\sigma_0-1]}\left[\frac{\theta_1}{1-\alpha_0}\right]^{\sigma_0}\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right)^{-\alpha_0\sigma_0}\left[\frac{\theta_0}{1-\theta_0}\right]^{\sigma_0}\left[\tilde{Q}\right]^{\sigma_0}. \tag{S2}$$

The demand for Internet and television time share is written, respectively, as:

$$T_1=T_1(P,Q,W,F)=\begin{cases}0, & if\, Q<\tilde{Q}(W);\\ \dfrac{\Gamma\left[1+\kappa_2-\dfrac{F}{W}-\dfrac{\kappa_1}{Q}\right]}{\left[\dfrac{1}{1-\alpha_0}+\dfrac{1}{Q}\cdot\Gamma+\Gamma\cdot\left[\dfrac{1-\theta_1}{\theta_1}\cdot\dfrac{1}{Q}\right]^{\sigma_1}\right]}-\kappa_1, & if\, Q\geq\tilde{Q}(W),\end{cases} \tag{S3}$$

$$T_2=T_2(P,Q,W,F)=\begin{cases}(\text{see equation S5}) & if\, Q<\tilde{Q}(W);\\ \left[\dfrac{1-\theta_1}{\theta_1}\dfrac{1}{Q}\right]^{\sigma_1}\cdot\dfrac{\Gamma\left[1+\kappa_2-\dfrac{F}{W}-\dfrac{\kappa_1}{Q}\right]}{\left[\dfrac{1}{1-\alpha_0}+\dfrac{1}{Q}\cdot\Gamma+\Gamma\cdot\left[\dfrac{1-\theta_1}{\theta_1}\cdot\dfrac{1}{Q}\right]^{\sigma_1}\right]}-\kappa_2 & if\, Q\geq\tilde{Q}(W).\end{cases} \tag{S4}$$

We can rewrite the corner solution of $T_1=0$ as:

$$(1-\alpha_0)\left[1-T_2-\frac{F}{W}\right]=\left[\frac{\theta_0}{1-\theta_0}\frac{1-\theta_1}{1-\alpha_0}\right]^{-\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right]^{\alpha_0(\sigma_0-1)}\cdot\left[\theta_1\kappa_1^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_0-\sigma_1}{1-\sigma_1}}\cdot(T_2+\kappa_2)^{\frac{\sigma_0}{\sigma_1}}. \tag{S5}$$

We can write composite good consumption C and leisure L as:

$$C = C(P,Q,W,F) = \begin{cases} \left(\dfrac{\alpha_0}{1-\alpha_0} \cdot \dfrac{W}{P} \right) \cdot L & if\, Q < \tilde{Q}(W); \\ \dfrac{\alpha_0}{1-\alpha_0} \cdot \dfrac{W}{P} \cdot \left\{ \dfrac{1 + \kappa_2 - \dfrac{F}{W} - \dfrac{\kappa_1}{Q}}{\left[\dfrac{1}{1-\alpha_0} + \dfrac{1}{Q} \cdot \Gamma + \Gamma \cdot \left[\dfrac{1-\theta_1}{\theta_1} \cdot \dfrac{1}{Q} \right]^{\sigma_1} \right]} \right\} & if\, Q \geq \tilde{Q}(W), \end{cases} \tag{S6}$$

$$L = L(P,Q,W,F) = \begin{cases} (1-\alpha_0)\left[1 - T_2 - \dfrac{F}{W} \right], & if\, Q < \tilde{Q}(W); \\ \dfrac{1 + \kappa_2 - \dfrac{F}{W} - \dfrac{\kappa_1}{Q}}{\left[\dfrac{1}{1-\alpha_0} + \dfrac{1}{Q} \cdot \Gamma + \Gamma \cdot \left[\dfrac{1-\theta_1}{\theta_1} \cdot \dfrac{1}{Q} \right]^{\sigma_1} \right]}, & if\, Q \geq \tilde{Q}(W). \end{cases} \tag{S7}$$

Note that the case of the corner solution of $T_1 = 0$ will be used in the counterfactual experiment of making Internet consumption unavailable to household so as to calculate the welfare gain from Internet.

Analytical proof

This section considers the time model and analyzes the solution to the household's problem.

$$\begin{aligned} L = & \left\{ \theta_0 \left[\theta_1 (T_1 + \kappa_1)^{1-\frac{1}{\sigma_1}} + (1-\theta_1)(T_2 + \kappa_2)^{1-\frac{1}{\sigma_1}} \right]^{\left(\frac{\sigma_1}{\sigma_1 - 1}\right)\cdot\left(1-\frac{1}{\sigma_0}\right)} + (1-\theta_0)\left[C^{\alpha_0} \cdot L^{1-\alpha_0} \right]^{\left(1-\frac{1}{\sigma_0}\right)} \right\}^{\frac{\sigma_0}{\sigma_0 - 1}} \\ & + \lambda \left\{ W \left[1 - \frac{T_1}{Q} - T_2 - L \right] - P \cdot C - F \right\} + \mu_1 T_1 + \mu_2 T_2 + \mu_3 C + \mu_4 L \end{aligned}$$

$$(T_1, T_2): \quad \frac{\theta_1}{1-\theta_1} \cdot \left(\frac{T_1 + \kappa_1}{T_2 + \kappa_2} \right)^{-\frac{1}{\sigma_1}} = \frac{\lambda \cdot \dfrac{W}{Q} - \mu_1}{\lambda \cdot W - \mu_2}$$

$$(C,L): \quad \frac{\alpha_0}{1-\alpha_0}\cdot\frac{L}{C}=\frac{\lambda\cdot P-\mu_3}{\lambda\cdot W-\mu_4}$$

$$(T_1,C)\quad : \quad \frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\left(\frac{\sigma_1}{\sigma_1-1}\right)\cdot\left(\frac{\sigma_0-1}{\sigma_0}\right)-1}\cdot\theta_1[T_1+\kappa_1]^{-\frac{1}{\sigma_1}}}{\alpha_0\cdot\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot\cdot\left(\frac{1}{C}\right)}=\frac{\lambda\cdot\frac{W}{Q}-\mu_1}{\lambda\cdot P-\mu_3}$$

$$(T_1,L): \frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\left(\frac{\sigma_1}{\sigma_1-1}\right)\cdot\left(\frac{\sigma_0-1}{\sigma_0}\right)-1}\cdot\theta_1[T_1+\kappa_1]^{-\frac{1}{\sigma_1}}}{\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot(1-\alpha_0)\cdot\left(\frac{1}{L}\right)}=\frac{\lambda\cdot\frac{W}{Q}-\mu_1}{\lambda\cdot W-\mu_4}$$

$$(T_2,C)\quad : \quad \frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\left(\frac{\sigma_1}{\sigma_1-1}\right)\cdot\left(\frac{\sigma_0-1}{\sigma_0}\right)-1}\cdot(1-\theta_1)(T_2+\kappa_2)^{-\frac{1}{\sigma_1}}}{\alpha_0\cdot\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot\cdot\left(\frac{1}{C}\right)}=\frac{\lambda\cdot W-\mu_2}{\lambda\cdot P-\mu_3}$$

$$(T_2,L): \frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}\cdot\frac{\sigma_0-1}{\sigma_0}-1}\cdot(1-\theta_1)(T_2+\kappa_2)^{-\frac{1}{\sigma_1}}}{\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot(1-\alpha_0)\cdot\left(\frac{1}{L}\right)}=\frac{\lambda\cdot W-\mu_2}{\lambda\cdot W-\mu_4}$$

Interior solution

We focus on the interior solution: $T_1>0$. The solution (T_1,T_2,C,L) are characterized by the following three first order conditions.

$$T_2+\kappa_2=\left[\frac{1-\theta_1}{\theta_1}\cdot\frac{1}{Q}\right]^{\sigma_1}\cdot[T_1+\kappa_1] \qquad \text{(E1)}$$

$$C=\left(\frac{\alpha_0}{1-\alpha_0}\cdot\frac{W}{P}\right)\cdot L \qquad \text{(E2)}$$

$$\frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}\cdot\frac{\sigma_0-1}{\sigma_0}-1}\cdot\theta_1[T_1+\kappa_1]^{-\frac{1}{\sigma_1}}}{\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot(1-\alpha_0)\cdot\left(\frac{1}{L}\right)}=\frac{1}{Q} \quad \text{(E3)}$$

The last condition used in solving for the solution is the budget constraint.

For the utility function, terms inside the first bracket $\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}$ is simplified to:

$$[T_1+\kappa_1]^{\frac{\sigma_1-1}{\sigma_1}}[\theta_1+(1-\theta_1)\Psi] \quad where \quad \Psi\equiv\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1-1}. \quad \text{(E4)}$$

Using equation (E1), (E2), and (E4), we can simplify equation (E3) as:

$$\frac{\theta_0}{1-\theta_0}\cdot\frac{\theta_1[T_1+\kappa_1]^{-\frac{1}{\sigma_0}}[\theta_1+(1-\theta_1)\Psi]^{\left(\frac{\sigma_1}{\sigma_1-1}\right)\cdot\left(\frac{\sigma_0-1}{\sigma_0}\right)-1}}{(1-\alpha_0)\cdot L^{-\frac{1}{\sigma_0}}\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right)^{\alpha_0\left(\frac{\sigma_0-1}{\sigma_0}\right)}}=\frac{1}{Q}. \quad \text{(E5)}$$

Rearranging terms of equation (E5), we have:

$$\theta_1[T_1+\kappa_1]^{-\frac{1}{\sigma_0}}[\theta_1+(1-\theta_1)\Psi]^{\left(\frac{\sigma_1}{\sigma_1-1}\right)\cdot\left(\frac{\sigma_0-1}{\sigma_0}\right)-1}=\frac{1-\theta_0}{\theta_0}\cdot\frac{1}{Q}.(1-\alpha_0)\cdot L^{-\frac{1}{\sigma_0}}\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right)^{\alpha_0\left(\frac{\sigma_0-1}{\sigma_0}\right)}$$

Which is simplified to:

$$T_1+\kappa_1=\left\{[\theta_1+(1-\theta_1)\Psi]^{\frac{\sigma_0-\sigma_1}{\sigma_1-1}}\left[\frac{\theta_1}{1-\alpha_0}\right]^{\sigma_0}\left[\frac{\theta_0}{1-\theta_0}\right]^{\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right]^{\alpha_0(1-\sigma_0)}Q^{\sigma_0}\right\}\cdot L \quad \text{(E6)}$$

Finally, we have budget constraint simplified to:

$$P\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\cdot L\right)+F=W\left[1+\kappa_2-\frac{T_1}{Q}-\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}[T_1+\kappa_1]-L\right]. \quad \text{(E7)}$$

$$L=(1-\alpha_0)\left[1+\kappa_2-\frac{F}{W}-\frac{T_1}{Q}-\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}\cdot(T_1+\kappa_1)\right]$$

Rearranging to above two equations (E6) and (E7), we have:

$$L = \frac{(1-\alpha_0)\cdot\left[1+\kappa_2 - \frac{F}{W} + \frac{\kappa_1}{Q}\right]}{1+(1-\alpha_0)\cdot\frac{\Gamma}{Q} + (1-\alpha_0)\cdot\Gamma\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}}.$$

$$where \quad \Gamma \equiv \left[\theta_1 + (1-\theta_1)\Psi\right]^{\frac{\sigma_0-\sigma_1}{\sigma_1-1}}\left[\frac{\theta_1}{1-\alpha_0}\right]^{\sigma_0}\left[\frac{\theta_0}{1-\theta_0}\right]^{\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P}\right]^{\alpha_0(1-\sigma_0)} Q^{\sigma_0}$$

Hence, we have (T_1, T_2, C, L) as follows.

$$C = \left(\frac{\alpha_0}{1-\alpha_0}\cdot\frac{W}{P}\right)\cdot\left\{\frac{(1-\alpha_0)\cdot\left[1+\kappa_2 - \frac{F}{W} + \frac{\kappa_1}{Q}\right]}{1+(1-\alpha_0)\frac{\Gamma}{Q} + (1-\alpha_0)\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}\Gamma}\right\}$$

$$T_1 = \frac{(1-\alpha_0)\cdot\left[1+\kappa_2 - \frac{F}{W} + \frac{\kappa_1}{Q}\right]}{\frac{1}{\Gamma} + (1-\alpha_0)\frac{1}{Q} + (1-\alpha_0)\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}} - \kappa_1 = \frac{\Gamma(1-\alpha_0)\cdot\left[1+\kappa_2 - \frac{F}{W} + \frac{\kappa_1}{Q}\right]}{1+(1-\alpha_0)\frac{\Gamma}{Q} + (1-\alpha_0)\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}\Gamma} - \kappa_1$$

$$T_2 = \left[\frac{1-\theta_1}{\theta_1}\cdot\frac{1}{Q}\right]^{\sigma_1}\cdot\left\{\frac{\Gamma(1-\alpha_0)\cdot\left[1+\kappa_2 - \frac{F}{W} + \frac{\kappa_1}{Q}\right]}{1+(1-\alpha_0)\frac{\Gamma}{Q} + (1-\alpha_0)\left[\frac{1-\theta_1}{\theta_1}\frac{1}{Q}\right]^{\sigma_1}\Gamma}\right\} - \kappa_2$$

Corner solution $(T_1 = 0)$

We discuss the case of Q too low such that $T_1 = 0$. In this case, the solution (T_1, T_2, C, L) is characterized by the following first order conditions.

$$C = \left(\frac{\alpha_0}{1-\alpha_0}\cdot\frac{W}{P}\right)\cdot L \qquad \text{(E8)}$$

$$\frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1\kappa_1^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}\cdot\frac{\sigma_0-1}{\sigma_0}-1}\cdot(1-\theta_1)(T_2+\kappa_2)^{-\frac{1}{\sigma_1}}}{\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot(1-\alpha_0)\cdot\left(\frac{1}{L}\right)}=1 \qquad \text{(E9)}$$

Combining equations (E8) and (E9), and arranging terms, we derive L as a function of T_2:

$$L=\left[\frac{\theta_0}{1-\theta_0}\frac{1-\theta_1}{1-\alpha_0}\right]^{-\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right]^{\alpha_0(\sigma_0-1)}\cdot\left[\theta_1\kappa_1^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_0-\sigma_1}{1-\sigma_1}}\cdot(T_2+\kappa_2)^{\frac{\sigma_0}{\sigma_1}}.$$

Finally, the budget constraint is given by:

$$P\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right)L+F=W\left[1-T_2-L\right]$$

which is simplified to:

$$L=(1-\alpha_0)\cdot\left[1-T_2-\frac{F}{W}\right]. \qquad \text{(E10)}$$

Therefore, the solution is characterized by the following one equation with one unknown T_2:

$$(1-\alpha_0)\left[1-T_2-\frac{F}{W}\right]=\left[\frac{\theta_0}{1-\theta_0}\frac{1-\theta_1}{1-\alpha_0}\right]^{-\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right]^{\alpha_0(\sigma_0-1)}\cdot\left[\theta_1\kappa_1^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)(T_2+\kappa_2)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_0-\sigma_1}{1-\sigma_1}}\cdot(T_2+\kappa_2)^{\frac{\sigma_0}{\sigma_1}}.$$

Corner solution (T_2)

We discuss the case where $T_2=0$. In this case, the solution (T_1,T_2,C,L) is characterized by the following first order conditions.

$$C=\left(\frac{\alpha_0}{1-\alpha_0}\cdot\frac{W}{P}\right)\cdot L \qquad \text{(E11)}$$

$$\frac{\theta_0}{1-\theta_0}\cdot\frac{\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)\kappa_2^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}\cdot\frac{\sigma_0-1}{\sigma_0}-1}\cdot\theta_1(T_1+\kappa_1)^{-\frac{1}{\sigma_1}}}{\left[C^{\alpha_0}\cdot L^{1-\alpha_0}\right]^{\frac{\sigma_0-1}{\sigma_0}}\cdot(1-\alpha_0)\cdot\left(\frac{1}{L}\right)}=\frac{1}{Q} \quad \text{(E12)}$$

Combining equations (E11) and (E12), and arranging terms, we derive L as a function of T_1:

$$L=\left[\frac{Q\theta_0}{1-\theta_0}\frac{\theta_1}{1-\alpha_0}\right]^{-\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right]^{\alpha_0(\sigma_0-1)}\cdot\left[\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}+(1-\theta_1)\kappa_2^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_0-\sigma_1}{1-\sigma_1}}\cdot(T_1+\kappa_1)^{\frac{\sigma_0}{\sigma_1}}.$$

Finally, the budget constraint is given by:

$$P\left(\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right)L+F=W\left[1-\frac{T_1}{Q}-L\right]$$

which is simplified to:

$$L=(1-\alpha_0)\cdot\left[1-\frac{T_1}{Q}-\frac{F}{W}\right]. \quad \text{(E13)}$$

Therefore, the solution is characterized by the following one equation with one unknown T_1:

$$(1-\alpha_0)\left[1-\frac{T_1}{Q}-\frac{F}{W}\right]=\left[\frac{Q\theta_0}{1-\theta_0}\frac{\theta_1}{1-\alpha_0}\right]^{-\sigma_0}\left[\frac{\alpha_0}{1-\alpha_0}\frac{W}{P_0}\right]^{\alpha_0(\sigma_0-1)}\cdot\left[(1-\theta_1)\kappa_2^{\frac{\sigma_1-1}{\sigma_1}}+\theta_1(T_1+\kappa_1)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_0-\sigma_1}{1-\sigma_1}}\cdot(T_1+\kappa_1)^{\frac{\sigma_0}{\sigma_1}}.$$

Data Materials

In Table S1, we present an overview of changes in Internet penetration rate, subscription price index and expenditure share of Internet subscription price, from the National Income and Product Accounts (NIPA) and World Bank. The data shows strong growth in the Internet penetration rate and share of Internet expenditure as illustrated in Figure S1. Internet expenditure share in the population is on average, 0.4% of total expenditure. The Internet expenditure share of an average Internet user, the adjusted expenditure share is about 0.6%.[1] Internet expenditure share of a user grew quickly before 2000, stayed relatively stable in early 2000s, and increased again in the following five years.

Table S1. Internet time share, money expenditure share and elasticity of substitution

Year	Expenditure share	Internet adoption	Adj. Expenditure share (F/W)	Hours spent on Internet	Hours spent on Television
1998	0.0009	0.301	0.0031	1.423	16.46
1999	0.0016	0.359	0.0045	1.898	16.88
2000	0.0024	0.431	0.0056	2.372	17.22
2001	0.0026	0.492	0.0052	2.688	17.41
2002	0.0029	0.589	0.0050	3.004	17.56
2003	0.0034	0.619	0.0055	3.162	18.06
2004	0.0036	0.650	0.0055	3.320	18.55
2005	0.0034	0.682	0.0049	3.478	18.06
2006	0.0036	0.692	0.0052	3.637	18.06
2007	0.0043	0.752	0.0058	4.111	18.34
2008	0.0050	0.742	0.0068	4.269	19.39
2009	0.0057	0.712	0.0080	4.585	19.74
2010	0.0062	0.742	0.0083	4.649	18.90
2011	0.0067	0.778	0.0085	4.743	19.60

Note: this table presents Internet penetration rate, subscription price index and expenditure share of Internet subscription price, from the National Income and Product Accounts (NIPA) and World Bank. Expenditure share is calculated from NIPA Table 2.4.5., Internet access divided by personal income. Adjusted expenditure share represents Internet expenditure share in Internet household.

Table S2. Balanced Panel Data from Forrester Research: Hours spent on Internet and Demographics

Balanced panel: 2007-2011	Year	# Obs.	Mean	Median	S.D.	Min	Max
Internet Service Features and Hours Online							
Hours spent on Internet for leisure	2007	241	3.84	0.5	7.07	0	35.5
	2008	235	3.9	0.5	6.9	0	32
	2009	237	4.3	0.5	7.22	0	32
	2010	236	5.01	2.5	7.63	0	32
	2011	236	5.32	2.5	7.76	0	32
Hours spent on Internet for work	2007	236	2.07	0	5.7	0	35.5
	2008	239	2.76	0	5.79	0	32

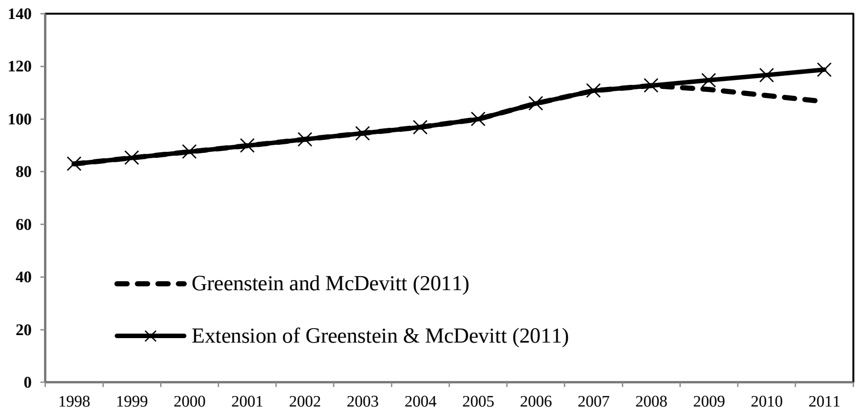
[1] We divided the Internet expenditure share by the Internet penetration rate to calculate the adjusted Internet expenditure share per user.

	2009	239	3.49	0	7.7	0	32
	2010	238	3.13	0	7.04	0	32
	2011	236	3.76	0	7.43	0	32
Individual Demographics							
Age	2007	243	51.02	47	16.72	18	90
Gender (1: Male, 2: Female)	2007	243	1.52	2	0.5	1	2
Income	2007	243	50,051	43,749	42,283	3,750	325,000
Financial asset	2007	194	356,765	37,500	2,528,209	12,500	25,000,000
Internet experience (years)	2007	164	3.44	3.5	1.73	0.5	6.5
Education	2007	243	3.25	3	1.84	1	8
Marital Status (Married)	2007	243	0.48	0	0.5	0	1
Region	2007	243	4.53	4	2.32	1	9
Kids (Has kids under 18)	2007	243	1.29	1	0.46	1	2

To account for overall speed improvement of Internet over recent years, we introduce quality-adjusted Internet price index, an extension of Internet quality index estimated by Greenstein and McDevitt (2011).[2] The annual growth rate of Internet quality index by Greenstein and McDevitt is about 1.7%, depicting a slight decreasing trend since 2008. This is partly due to the limitations of the data that only covers until 2009. We extend Greenstein and McDevitt (2011) index to preserve the least positive growth rate on 2007, around 1.7% up to 2011. This index is illustrated as a real line in Figure S1. It shows about 2.8% of annual increase and we adopted this modest index in the quantitative analysis for the Internet quality index.

Figure S1. Internet Quality Index

2 Internet subscription price index from NIPA indicates about 1.4% annual drop during 1998-2006 and shows about 0.9% increase in 2007-2011. Greenstein et al. (2009,2011) note drawbacks of this data by not accounting for the quality changes, e.g., speed improvement, increasing adoption of broadband. We extend the Internet quality index estimated by Greenstein and McDevitt (2011) to preserve the least positive growth rate on 2007, around 1.7% upto 2011. This index shows about 2.8% of annual increase.



Note: this figure presents the Internet speed index estimated by Greenstein and McDevitt (2011) and our extension of this index index upto 2011. This index shows about 2.8% of annual increase.

Methods

We use our model to analyze quantitatively the sources of changes in the welfare gain from Internet. In order to compute this, we have to calibrate six preference parameters: the elasticity of substitution parameters, σ_1, σ_0, the weight on the utility from the time spent on Internet and the bundle of Internet television together, θ_1, θ_0, and the parameters κ_1, κ_2, that determine the utility level when the hours spent on Internet and television are zero. Altogether, these parameters specify the utility from Internet, television and other goods. We find those sources to be primarily changes in observed hours spent on Internet and television, income, adoption, and quality improvement of Internet.

The quantitative analysis is two parts. The goal is to calibrate the set of parameters $(\sigma_1, \theta_0, \theta_1, \kappa_1, \kappa_2)$ that fit annual data from 1998 to 2011, based on the estimated value of σ_0.[3] First, we start with initial range of σ_0 from the regression and calibrate initial set of parameters. Then

[3] We estimated σ_0 based on the *Consumer Technographics* in 2007 from Forrester Research that includes detailed information about Internet household, e.g., average years of Internet experience, household income level, wealth, education, employment and characteristics of Internet services.

we update the parameter value of σ_0, by plugging in the set of initial parameters $(\sigma_1, \theta_0, \theta_1, \kappa_1, \kappa_2)$ and iterate this process until the estimated value of σ_0 converge to the calibrated value.

The calibration is based on the following steps. The predicted time spent on Internet at year t, h_1^{*t}, is computed by plugging in the corresponding quality, price and income level, (Q^t, P^t, W^t), into the demand functions. The preference parameters can be determined by minimizing the sum of the squared differences between the actual time spent on Internet observed in the data $h_1^{Data,t}$ during the sample period from 1998 to 2011 and the model-predicted time $h_1^{*t} = T_1^{*t}(\sigma_0, \sigma_1, \theta_{0,}\theta_1, \kappa_1, \kappa_2; Q^t, P^t, W^t)/Q^t$.[4] We calibrate the parameters by solving the following minimization problem:

$$\min_{\sigma_0,\sigma_1,\theta_{0,}\theta_1,\kappa_1,\kappa_2} \sum_{t=1998}^{2011} \left[h_1^{Data,t} - \frac{T_1^{*t}(\sigma_0, \sigma_1, \theta_{0,}\theta_1, \kappa_1, \kappa_2; Q^t, P^t, W^t)}{Q^t} \right]^2 \qquad \text{(S8)}$$

Parameter Calibration

From the optimality condition in equation (3), we derive equation (S9) by taking natural log each side.[5] In equation (S9), we estimate the implied value of elasticity of substitution parameter σ_0 between Internet (Television) and all other goods.

[4] In our analysis, Internet hours (at home), increasing rapidly over time during our sample period, are of primary importance, while TV hours, which do not increase much during our sample period, are of the secondary importance. Therefore, we do not include the distance between the observed and model-predicted TV hours in the objective function of our estimation. Alternatively, we can include the distance between the observed and model-predicted TV hours, with a weight less than one, in the objective function.

[5] The Internet expenditure share, F/W is about 0.0006 in the data, nearly zero. The other variables in the right-hand side of equation (3) is reduced to the notation A in equation (S9).

$$\ln\left(\frac{1+\kappa_2-T_1-\left[\frac{1-\theta_1}{\theta_1}\right]^{\sigma_1}\cdot(T_1+\kappa_1)}{T_1+\kappa_1}\right)=\ln(A)+\alpha_0(\sigma_0-1)\ln(W)+\sigma_0\ln\left(\frac{1-\theta_0}{\theta_0}\right) \qquad \text{(S9)}$$

We start estimation by assuming the unknown value of parameters κ_1,κ_2 to be zero and $\theta_1=0.5$. In this condition, the left side of equation (S9) simplifies to $\ln\left(\frac{1-T_1-T_1}{T_1}\right)$. Table S3 presents the result from this regression.

Model 1 is the basic regression only with log income and constant. Model 2 is regressing work-purpose Internet hours, instead of leisure-purpose Internet hours, as a dependent variable, different from all other models. Model 3-5 are extension of Model 1, by adding demographic variables (Model 3), adding work-purpose Internet hours and years of Internet access (Model 4), and finally adding financial assets (Model 5).

In table S3, positive sign of income regression coefficient in Models 1, 3, 4 and 5 in the Internet group indicates that people with higher income spend less time on the Internet for leisure at home. This is consistent with their higher opportunity cost of time. In contrast, the income coefficient of Model 2 is negative in any case. This result confirms that although people with high income use less Internet for leisure at home, they spend more time on Internet for work.

Table S3: Estimation with Internet Subscription at home

	Model 1	Model2	Model3	Model4	Model5
logIncome	0.046***	-0.264***	0.087***	0.159***	0.176***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Internet working hours				-0.021***	-0.022***
				(0.000)	(0.000)
Financial Assets					-1.96e-08***
					(0.004)
Control variables	No	No	Yes	Yes	Yes

Constant	2.586***	6.060***	2.101***	2.023***	1.816***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Implied σ_0	1.12	N/A	1.24	1.43	1.48
Prob > F	0.000	0.000	0.000	0.000	0.000
Adjusted R^2	0.001	0.013	0.031	0.087	0.089
N	34,252	22,174	34,252	33,313	25,706

Note: Regressions are based on the *Consumer Technographics* in 2007. P-value in parentheses. ***p<0.01, **p<0.05, *p<0.1.

The estimation results shows the range of elasticity of substitution parameter σ_0 from 1.12 to 1.48 based on the household with Internet subscription in Table S3. We update our regression coefficient by plugging in the calibrated values of $(\sigma_1, \theta_1, \kappa_1, \kappa_2)$ in equation (S8). Note that in equation (S8), we need to plug in the value of $(\sigma_1, \theta_1, \kappa_1, \kappa_2)$ in the left-hand side for the regression. We calibrated the set of parameter values, $(\sigma_1, \theta_1, \kappa_1, \kappa_2)$ using the estimated value of σ_0 from the regression.

We iterate this process until the estimation results of σ_0 and the value of calibrated parameters converge. Under the updated set of parameters $(\sigma_1, \theta_0, \theta_1, \kappa_1, \kappa_2)$ in Table S4, the implied value of σ_0 ranges from 1.03 to 1.26.[6] Table S4 summarizes the results of the updated calibration. We perform our analysis based on the value of σ_0, 1.26 which gives us a consistent result based on both estimation and calibration.

Table S4: Robustness check of Estimation with Calibrated Parameters: With Internet Subscription at home

	Model 1	Model2	Model3	Model4	Model5
logIncome	0.009	-0.153***	0.035***	0.083***	0.096***
	(0.193)	(0.000)	(0.000)	(0.000)	(0.000)
Internet working hours				-0.013***	-0.014***
				(0.000)	(0.000)
Financial Assets					-1.58e-08***

[6] While the initial range of σ_0 is from 1.03 to 1.26, we dropped the case of parameter value under 1.1 that shows statistically insignificant results from the regression.

					(0.002)
Control variables	No	No	Yes	Yes	Yes
Constant	1.908***	3.763***	1.565***	1.446***	1.280***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Implied σ_0	1.03	N/A	1.09	1.22	1.26
Prob > F	0.193	0.000	0.000	0.000	0.000
Adjusted R_sq	0.000	0.011	0.020	0.057	0.060
N	32,171	19,799	32,171	31,308	24,128

Note: P-value in parentheses. ***p<0.01, **p<0.05, *p<0.1.

In general, the findings are consistent with findings in Table S3. The results show that while people with high income spend less hours on the leisure-purpose Internet, they spend more hours on the Internet for work. In addition, people with high level of financial assets actually spend more hours on the Internet for leisure. This pair of results is consistent with the opportunity cost of time being higher for people with work-based income, but not necessarily unearned income.

Table S5: Calibration of parameters

	σ_0	σ_1	θ_0	θ_1	κ_1	κ_2	R^2
Initial Parameters	1.12	1.315	0.448	0.236	0.048	0.258	0.990
	1.24	1.408	0.470	0.257	0.045	0.244	0.988
	1.43	1.595	0.535	0.264	0.038	0.259	0.983
	1.48	1.649	0.548	0.267	0.037	0.268	0.994
Updated Parameters	1.25	1.410	0.476	0.256	0.045	0.244	0.987
	1.26	1.419	0.486	0.253	0.044	0.247	0.989
	1.28	1.438	0.492	0.255	0.044	0.246	0.988
	1.30	1.445	0.499	0.255	0.044	0.250	0.990

Table S5 summarizes the calibration results. We obtained a value for σ_1 of 1.419, when the value for σ_0 is equal to 1.26. As we predicted, the elasticity of substitution between Internet and television, σ_1, is much higher than that between Internet and other goods, σ_0. This implies that the Internet and Television are closer substitutes than Internet and all other goods. The value of weight parameter θ_1 is 0.25, and θ_0 is 0.49. The weight parameters compare relative importance of Internet bundle with respect to the Television bundle and both bundles with respect to the other goods. The constant parameters κ_1, κ_2 are estimated as low as 0.044 and 0.25. The value of κ_1, κ_2 implies that the measured surplus from Internet and Television could be over-estimated without considering these parameters in the model. The calculated R^2 fitting the hours spent on Internet is over 0.98.