Online Appendix to

Show Me the Money: The Economic Impact of Membership-based Free Shipping Programs on e-Tailers

In Part A, we solve for the e-tailer's optimal pricing decisions under different pricing schemes. Proofs of Lemmas and Propositions are provided in Part B. In Part C, we present three model extensions to generate more insights into the economic value of MFS. An additional analysis on asymmetric misfit costs is available in Part D.

A. Analysis with Different Pricing Schemes

A.1 Analysis with PS Pricing

Under PS pricing, the e-tailer charges price p_P and face the following pricing problem:

$$\max_{p_P} \pi_P = p_P \cdot Q_P \left[\beta f_h + (1-\beta) f_l \right], \tag{A1}$$

where $Q_P = v - p - s$. Throughout this Appendix we use a general form for demand and do not specify the strategy-specific pricing terms in it. The solution to (A1) can be derived using the first-order condition. Specifically, by setting $\partial\pi/\partial p_P = \left(v - s - 2p_P\right)\overline{f} = 0$, where $\overline{f} \equiv \beta \cdot f_h + (1-\beta) f$, we can solve for the optimal price $p_P^* = \left(v - s\right)/2$. Substituting p_P^* into (A1) we can derive the optimal profit $\pi_P^* = (v-s)^2\, \overline{f} / 4$.

A.2 Analysis with CFS Pricing

Similarly, we can write the e-tailer's problem under CFS pricing as:

$$\max_{p_C} \pi_C = p_C \cdot Q_P \alpha \overline{f} + \left(p_C - s + \mu\right) \cdot Q_C \left(1-\alpha\right) \overline{f}, \tag{A2}$$

where $Q_C = v - p - k$. Using the first-order condition, we can solve for the optimal price:

$$p_C^* = \frac{1}{2}\left(s + v - \left(\mu + k\right)\left(1-\alpha\right) - 2s\alpha\right). \tag{A3}$$

Substituting (A3) into (A2), we can derive the optimal profit under CFS pricing:

$$\pi_C^* = \frac{\overline{f}}{4}\left\{\left(2s-k-\mu\right)^2\alpha^2 - 2\left[k^2-\left(s+v\right)k+\left(\mu-s\right)\left(\mu-2s\right)+v\mu\right]\alpha+\left(v+\mu-k-s\right)^2\right\}. \quad \text{(A4)}$$

A.3 Analysis with MFS Pricing

In our main text, we are interested in three configurations as the outcome of segmentation: 1) *MPCC*, 2) *MPMC*, and 3) *MMCC*. The e-tailer's choice of *m* dictates which configuration the outcome would be. When the value of *m* is large and satisfies $\max(f_l s,\ f_h k) < m < f_h s$, *MPCC* configuration would emerge. When $m < \max(f_l s,\ f_h k)$, two cases arise: *MPMC* configuration, if $f_l s < f_h k$; *MMCC* configuration, if $f_l s > f_h k$.

1) *MPCC Configuration*

The e-tailer's problem in *MPCC* configuration can be formulated as:

$$\max_{p_{M1},\, m_{M1}} \pi_{M1} = Q_M\alpha\beta\left[\left(p_{M1}-s\right)\cdot f_h + m_{M1}\right]+\left(p_{M1}-s+\mu\right)\cdot Q_C\left(1-\alpha\right)\overline{f} + p_{M1}\cdot Q_P\alpha\left(1-\beta\right)f_l, \quad \text{(A5)}$$

subject to

$$\max(f_l s,\ f_h k) < m_{M1} < f_h s. \quad \text{(A6)}$$

Solving (A5) using the first-order condition, we have the optimal pricing terms given as:

$$p_{M1}^* = \frac{1}{2}\frac{\left(v+s-\mu-k\right)\left(1-\alpha\right)\beta f_h + \left[s+v-\left(\mu+k\right)\left(1-\alpha\right)-2s\alpha\right]\left(1-\beta\right)f_l}{\left(1-\alpha\right)\beta f_h+\left(1-\beta\right)f_l}, \quad \text{(A7)}$$

$$m_{M1}^* = \frac{f_h}{2}\frac{\left(\mu+k\right)\left(1-\alpha\right)\beta f_h + \left[\left(\mu+k\right)\left(1-\alpha\right)+2s\alpha\right]\left(1-\beta\right)f_l}{\left(1-\alpha\right)\beta f_h+\left(1-\beta\right)f_l}. \quad \text{(A8)}$$

Plugging (A7)-(A8) into (A5), we can derive the optimal profit given as:

$$\pi_{M1}^* = \frac{\overline{f}\left[c_1\left(1-\alpha\right)\beta f_h + c_2\left(1-\beta\right)f_l\right]}{4\left[\left(1-\alpha\right)\beta f_h+\left(1-\beta\right)f_l\right]}, \text{ where}$$

$$c_1 = \left(1-\alpha\right)\mu^2 + 2\left(v-s\right)\left(1-\alpha\right)\mu + \left[\left(v-s\right)^2 - 2\left(1-\alpha\right)\left(v+\mu-s\right)k+\left(1-\alpha\right)k^2\right],$$

$$c_2 = (1-2\alpha)^2 s^2 - 2\left(v+(1-\alpha)(1-2\alpha)\mu\right)s + k^2(1-\alpha)^2 - 2(1-\alpha)\left(v-s-2s\alpha+(1+\alpha)\mu\right)k+\left(v+(1-\alpha)\mu\right)^2.$$

Plugging (A8) into (A6) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A7)-(A8) as long as market parameters fall into the region R_{M1}, where

$$R_{M1} = \left\{ (s,\mu,k,\alpha,\beta,f_l,f_h) \middle| (d_1-d_2)\mu + 2d_2 s > (d_1+d_2)k, \frac{2s(f_l d_1 - f_h d_2)}{f_h(d_1-d_2)} < \mu + k < 2s \right\},$$

$$d_1 = (1-\alpha)\beta f_h + (1-\beta) f_l,\ d_2 = \alpha(1-\beta) f_l\,.$$

2) ***MPMC Configuration***

The e-tailer's problem in *MPMC* configuration can be written as:

$$\max_{p_{M2},\, m_{M2}} \pi_{M2} = Q_M \beta\left[\left(p_{M2}-s\right) f_h + m_{M2}\right] + \left(p_{M2}-s+\mu\right) Q_C \left(1-\alpha\right)\left(1-\beta\right) f_l + p_{M2} Q_P \alpha\left(1-\beta\right) f_l, \qquad \text{(A9)}$$

subject to

$$\max(f_l s,\ f_h k) < m_{M1} < f_h s. \qquad \text{(A10)}$$

Solving (A9) we obtain the optimal pricing terms given as:

$$p^*_{M2} = \frac{1}{2}\left[s + v - \left(\mu + k\right)\left(1-\alpha\right) - 2s\alpha\right], \qquad \text{(A11)}$$

$$m^*_{M2} = \frac{f_h}{2}\left[\left(\mu + k\right)\left(1-\alpha\right) + 2s\alpha\right]. \qquad \text{(A12)}$$

Plugging (A11)-(A12) into (A9), we can derive the optimal profit given as:

$$\pi^*_{M2} = \frac{\left(v-s\right)^2 \beta f_h + c_3(1-\beta) f_l}{4}, \text{ where}$$

$$c_3 = (1-2\alpha)^2 s^2 - 2[v + (1-\alpha)(1-2\alpha)\mu]s + (1-\alpha)^2 k^2 - 2(1-\alpha)[v - s - 2s\alpha + (1+\alpha)\mu]k + [v + (1-\alpha)\mu]^2.$$

Plugging (A12) into (A10) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A11)-(A12) as long as market parameters fall into the region R_{M2}, where

$$R_{M2} = \left\{ (s,\mu,k,\alpha,f_h,f_l) \middle| (1+\alpha)k - (1-\alpha)\mu > 2\alpha s,\ \mu + k > \frac{2s(f_l - \alpha f_h)}{(1-\alpha) f_h} \right\}.$$

3) ***MMCC Configuration***

The e-tailer's problem in *MMCC* configuration can be expressed as:

$$\max_{p_{M3},\, m_{M3}} \pi_{M3} = \alpha\left\{(p_{M3}-s)\left[Q_M^H \beta f_h + Q_M^L (1-\beta) f_l\right] + m_{M3}\left[Q_M^H \beta + Q_M^L (1-\beta)\right]\right\} + (1-\alpha)(p_{M3}-s+\mu)Q_C \overline{f}, \tag{A13}$$

subject to

$$f_h k < m_{M3} < f_l s. \tag{A14}$$

Solving (A13) we have the optimal pricing terms given as:

$$p_{M3}^* = \frac{1}{2}\left\{v + s - \frac{(k+\mu)(1-\alpha)\left[(1-\beta) f_h + \beta f_l\right]\overline{f}}{\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l}\right\}, \tag{A15}$$

$$m_{M3}^* = \frac{1}{2}\frac{(k+\mu)(1-\alpha) f_h f_l \overline{f}}{\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l}. \tag{A16}$$

Plugging (A15)-(A16) into (A13), we derive the optimal profit:

$$\pi_{M3}^* = \frac{\overline{f}}{4}\left\{(1-\alpha)^2 k^2 - 2(1-\alpha)\left[v-s+(1+\alpha)\mu\right]k + \left[v-s+(1-\alpha)\mu\right]^2 + c_4\right\}, \text{ where}$$

$$c_4 = \frac{\alpha(1-\alpha)^2 (k+\mu)^2 f_h f_l}{\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l}.$$

Plugging (A16) into (A14) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A15)-(A16) as long as market parameters fall into the region R_{M3}, where

$$R_{M3} = \left\{(s,\mu,k,\alpha,\beta, f_h, f_l) \,\middle|\, \mu > \left(\frac{2d_3}{f_l} - 1\right)k,\ \mu + k < \frac{2d_3 s}{f_h}\right\},$$

$$d_3 = \frac{(1-\beta)\beta(f_h - f_l)^2 + (1-\alpha) f_h f_l}{(1-\alpha)\overline{f}}.$$

B. Proofs

B.1 Proof of Lemma 1

Setting $p_C^* > p_P^*$ and rearranging terms, we can show that $p_C^* > p_P^*$, *if* $\mu + k < 2s$. Similarly, we can characterize the conditions for $p_C^* = p_P^*$. Since we are interested in the case where the e-tailer charges a positive price, we need to identify the intersection $\{p_C^* < p_P^*\} \cap \{p_C^* > 0\}$ over the parameter space. Combining all three conditions, we have:

$$
\begin{aligned}
&(a)\ \ p_C^* > p_P^*, \quad if\ \ \mu + k < 2s\ and\ \ 0 < \alpha < 1;\\
&(b)\ \ p_C^* = p_P^*, \quad if\ \ \mu + k = 2s\ and\ \ 0 < \alpha < 1;\\
&(c)\ \ p_C^* < p_P^*, \quad if\ \begin{cases} 2s < \mu + k < v + s\ and\ \ 0 < \alpha < 1, \\ v + s < \mu + k\ and\ \ \dfrac{\mu + k - (v+s)}{\mu + k - 2s} < \alpha < 1. \end{cases}
\end{aligned}
$$

B.2 Proof of Lemma 2

Setting $\pi_C^* > \pi_P^*$ and rearranging terms, we can characterize conditions under which CFS pricing is more profitable than PS pricing when market configuration falls in the region R_C, where

$$
R_C = \left\{ (\mu, k, s, \alpha) \left| \begin{array}{ll} \mu \geq s, & 0 < \alpha < 1 \\ \mu < s, & \begin{cases} k < \mu, & \begin{Bmatrix} v \geq \hat{v}_C,\ 0 < \alpha < 1 \\ v < \hat{v}_C,\ 0 < \alpha < \bar{\alpha} \end{Bmatrix} \\ k > \mu + 2(v - s), & 0 < \alpha < \bar{\alpha}_C \end{cases} \end{array} \right. \right\},
$$

where $\hat{v}_C = \dfrac{2(k-s)^2}{\mu - k} + 2k - s$ and $\bar{\alpha} = \dfrac{(k-\mu)\left[k - \mu - 2(v-s)\right]}{(k - 2s + \mu)^2}$.

B.3 Proof of Proposition 1

Proof of Proposition 1 is straightforward. Similar to the procedure used in C.1, we can characterize the conditions under which the price under MFS pricing is strictly higher than its CFS counterpart. Reducing intersection $\{p_{M1}^* > p_C^*\} \cap \{R_{M1}\}$, we can show that the inequality $p_{M1}^* > p_C^*$ holds under all conditions. Note that the second set of the intersection is the feasible condition for the optimal solution set (p_{M1}^*, m_{M1}^*) in *MPCC* configuration.

B.4 Proof of Proposition 2(a)

We can follow the procedure used in C.3 to characterize the conditions under which *MPCC* pricing outperforms CFS pricing with respect to profit. Reducing intersection $\{\pi_{M1}^* > \pi_C^*\} \cap \{R_{M1}\}$, we can show that the inequality $\pi_{M1}^* > \pi_C^*$ holds under all feasible conditions for the optimal solution set $(p_{M1}^*,\ m_{M1}^*)$.

B.5 Proof of Proposition 2(b)

We can characterize the conditions under which *MPMC* pricing outperforms CFS pricing by reducing intersection $\{\pi_{M2}^* > \pi_C^*\} \cap \{R_{M2}\}$. Again, the second set of the intersection is to ensure that the optimal solution set $(p_{M2}^*,\ m_{M2}^*)$ is feasible. We can show that the inequality $\pi_{M2}^* > \pi_C^*$ holds under all feasible conditions for the optimal solution set $(p_{M2}^*,\ m_{M2}^*)$.

B.6 Proof of Proposition 2(c)

Similarly, we can characterize the conditions under which *MMCC* pricing outperforms CFS pricing by reducing intersection $\{\pi_{M3}^* < \pi_C^*\} \cap \{R_{M3}\}$. Again, the second set of the intersection is to ensure that the optimal solution set $(p_{M3}^*,\ m_{M3}^*)$ is feasible. We can show that the inequality $\pi_{M3}^* > \pi_C^*$ holds under all feasible conditions for the optimal solution set $(p_{M3}^*,\ m_{M3}^*)$.

B.7 Proof of Proposition 3

The best choice of shipping strategy comes mainly from the feasible region of each configuration. If market parameters only fall in the feasible region of one configuration, then the best choice of shipping strategy should be that configuration. Therefore, we only need to consider region which is not in the feasible region of any configuration or which is in the feasible region of multiple configurations.

We first explore the situation of overlapping feasible region. From the expression of feasible regions of three configurations, we find that *MPCC* and *MPMC* would never overlap, and *MPCC* and *MMCC* would overlap in most cases. To find out which strategy we should use in the overlapping feasible region of *MPCC* and *MMCC*, we need to compare their profit. Simplifying the condition $\pi^*_{M3} < \pi^*_{M1}$, we obtain the following condition:

$$r_1(\mu+k)^2 - 4r_2 s(\mu+k) + 4r_2 s^2 < 0.$$

This is the condition of region R_{M13}:

$$R_{M13} = \left\{(s,\mu,k,\alpha,\beta,f_h,f_l) \middle| r_1(\mu+k)^2 - 4r_2 s(\mu+k) + 4r_2 s^2 < 0\right\}, \text{ where}$$

$$r_1 = (1-\alpha)\, f_h \left(f_l^{\,2} - \beta^2 \left(f_h - f_l \right)^2 \right),\ r_2 = (1-\beta)\beta\left(f_h - f_l \right)^2 + (1-\alpha)\, f_h f_l\,.$$

Thus, the region where *MMCC* is the best choice should be $R'_{M3} = R_{M3} - R_{M13}$ and the region where *MPCC* is the best choice should be $R'_{M1} = R_{M1} - R'_{M3}$.

We then turn to the region between *MPCC* and *MPMC*. In this region, the best membership fees of *MPCC* and *MPMC* are binding solutions, and the solutions are both $m^* = f_h s$. This means that the best choice of this region is the binding solution between *MPCC* and *MPMC*. This region is characterized as:

$$R_{M12} = \left\{(s,\mu,k,\alpha,\beta,f_h,f_l) \middle| (d_1+d_2)k - (d_1-d_2)\mu > 2d_2 s, (1+\alpha)k - (1-\alpha)\mu < 2\alpha s, kf_h > sf_l\right\}.$$

Finally, we need to consider other regions where the optimal strategy is not determined. Comparing the profits of PS and CFS, we can find that PS is better when μ is small. Thus, the region where μ is large should have the best strategy as CFS, and the region where μ is small should have the best strategy as PS. Specifically, the region where CFS is the best choice is:

$$R'_C = \left\{(v,s,\mu,k,\alpha) \middle| 2s < \mu + k < \frac{v+(1-2\alpha)s}{1-\alpha}\right\}.$$

The e-tailer should resort to the PS option otherwise. With all analysis above, we can determine the best shipping strategy under every market parameter.

B.8 Proof of Corollary 1

Corollary 1 can be directly derived from Proposition 3.

C. Extensions of Basic Model

In this Appendix, we relax various assumptions used in the basic model to evaluate the robustness of our results and uncover more insights. We also examine the strategic role of MFS pricing in a competitive setting.

C.1 Demand Boosting of MFS

So far, our model considers a setting where order frequency is deterministic and independent of consumer choice of the shipping policy. In reality, since the MFS subscription fee is charged upfront as a lump-sum payment, it becomes a sunk cost for the members once they subscribe to the program. Consumers may end up buying more because the effective price for the extra demand is low (Kasarda and Lindsay 2011). In this section, we extend our basic model to incorporate this boosted demand resulting from customers treating the membership fee as a sunk cost. Now suppose that consumers make the same decision in the first stage, but those who choose to become members will place ψ extra orders (in total) over the membership period in the second stage. Parameter ψ describes the magnitude of the increment in demand boosted by the free-shipping incentive of the program. As the value of ψ goes up, the MFS program entices more additional purchases. We formally present our findings concerning the *demand boosting effect* of MFS in the following remark.

Proposition A1(a). *Compared to the benchmark case of CFS, the impact of including the MFS option on the e-tailer's shipping menu increases the unit price under all three MFS configurations, after taking the demand boosting effect into account.*

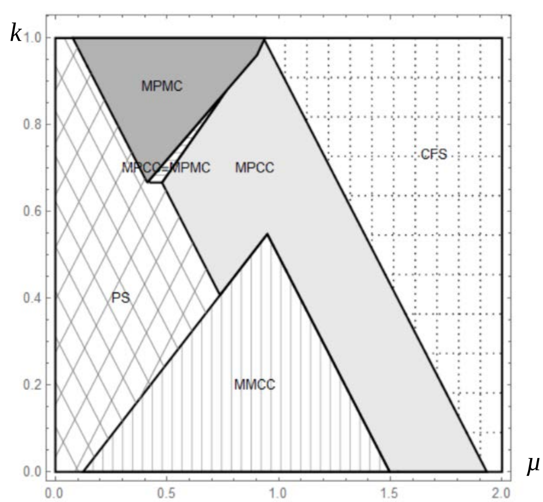
Proposition A1(b). *In the presence of the demand boosting effect, the e-tailer would raise the product price but lower the membership fee when product value is large enough, as compared to the MFS configurations when the effect is absent.*

Proposition A1(a) suggests that the results from our basic analyses still hold even after we consider the membership fee as a sunk cost. That is, the e-tailer still increases the price after introducing the MFS program under all three configurations. This is a small deviation from our basic analyses, where the e-tailer only keeps the price unchanged after introducing the MFS program under *MPMC* configuration.

Proposition A1(b) unveils more interesting implications. The demand boosting effect has asymmetric impacts on the two pricing decisions (i.e., price and membership fee) the e-tailer can leverage. The intuition developed in Proposition 1 carries over that the price surge puts downward pressure on the size of consumer base. To combat the loss in demand, the e-tailer at optimum would lower the *effective price* for program members by reducing the membership fee. Moreover, it is also intuitive that the incentive to adopt MFS pricing is decreasing in shipping cost as the e-tailer would need to subsidize an increased amount of orders due to the demand boosting effect.

We also illustrate the e-tailer's best shipping strategy with the demand boosting effect in Figure A1. The overall pattern is the same as the equilibrium discussed earlier. The most salient difference is that the *MMCC* configuration now dominates in the whole feasible regions that overlap the feasible regions of *MPCC* and even encroaches on the region dominated by CFS. The reason is obvious. The *MMCC* configuration has more consumers (two segments, all high-disutility consumers) in the membership program, therefore, enjoying a greater demand boosting effect.

Figure A1. Best Choice of Shipping Strategy for the E-tailer (with Demand Boosting Effect) [1]



[1] In Figure A1, the parameters used for plotting are $\alpha = 0.3$, $\beta = 0.3$, $f_h = 1.5$, $f_l = 1$, $s = 1$, $v = 2$, $\psi = 0.2$.

C.2 MFS Programs under E-tailer Competition

Our analyses so far have generated useful insights for the monopolistic e-tailer. What remains unclear is the strategic role of MFS in the presence of e-tailer competition. A real-world observation is that only some e-tailers have embraced the practice of MFS. Furthermore, there is no clear understanding of the desired customer segmentation in a competitive environment. In this extension, we first consider a duopolistic setting where the e-tailers can choose between CFS and MFS (*MPCC*). We then consider a case where e-tailers can only choose MFS but are flexible in achieving any segmentation configurations.

Suppose that the market is served by two e-tailers who offer horizontally differentiated products. Following the classic Hotelling model (d'Aspremont et al. 1979; Hotelling 1929), we assume that the two competing e-tailers are located at the two edges of a consumer preference line, whose length is normalized to 1. Consumers are uniformly distributed along the line segment, and the distance between a particular consumer and an e-tailer captures the degree of the misfit between the consumer's ideal product and the one offered by that e-tailer. For example, a consumer at distance x away from e-tailer 1 incurs misfit costs tx toward the product offered by e-tailer 1 and $t(1 - x)$ toward e-tailer 2, respectively. Note that t is the cost per unit distance attributed to misfit.

We assume that the two e-tailers are completely symmetric in all aspects (i.e., v, s, μ) and are facing the same population of consumers. We consider a baseline case where both e-tailers offer a PS/CFS menu. This way, we can clearly identify the profit change after the MFS program is introduced by one or both e-tailers. We only consider the *MPCC* configuration as a representative of adding the MFS option because if *MPCC* strictly dominates CFS in the equilibrium, the other configurations are also preferable.

Proposition A2. *In a duopolistic setting where the two competing e-tailers are symmetric in their shipping cost, operating the MFS program is always the equilibrium strategy for both e-tailers.*

The intuition is similar to that under the monopolistic setting. The additional decision variable provides more opportunities for e-tailers to price-discriminate and to obtain more profits. This finding partially explains why more and more e-tailers have adopted their own loyalty programs that offer unlimited free shipping (Thau 2017).

Next, suppose two e-tailers can only operate the MFS program, and they choose to implement *MPCC* or *MMCC* configuration. We are able to compare the profit of each strategy combination and find the equilibrium. Because the optimal price and membership fee are too complicated when both e-tailers adopt *MMCC*, we implement a simulation with the various value of parameters and find the equilibrium respectively. We summary the result from these analyses in Proposition A3, and the details of simulation can be found in Appendix C.4.3.

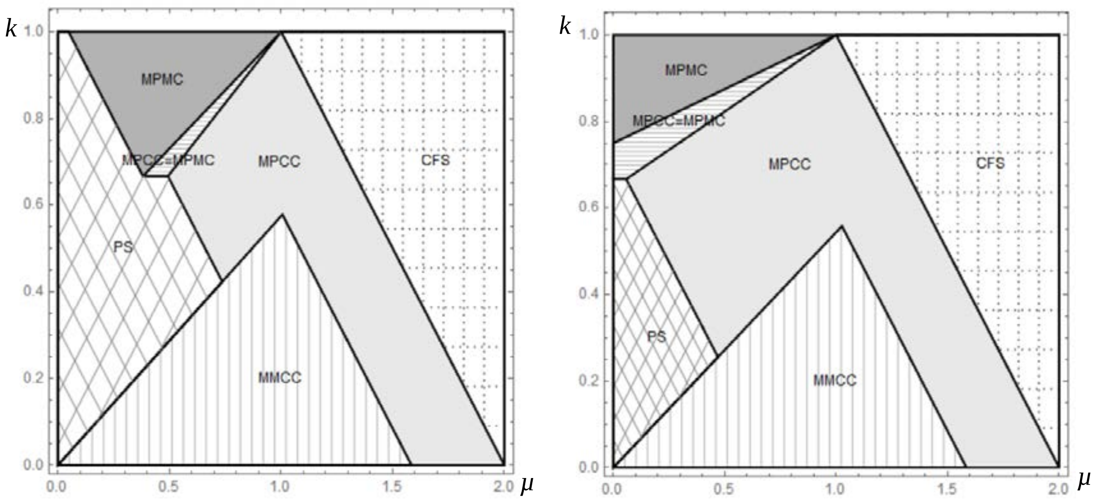
Proposition A3. *In a duopolistic setting where the two competing e-tailers are symmetric in their shipping cost, operating MPCC is always an equilibrium configuration for both e-tailers, whereas operating MMCC is sometimes an equilibrium configuration for both e-tailers but always worse than the situation when both are operating MPCC.*

Proposition A3 provides an implication that the high membership fee is more attractive to both e-tailers in a competitive environment. The intuition is that: competition prevents e-tailers from setting a high price, thus losing the market power of monopoly. This finding explains why most e-tailers choose to gradually raise up the membership fee instead of attracting all consumers to join the MFS program.

C.3 Sensitivity Analysis of CFS Market Parameters

In the basic model, we assume the fraction of low-disutility shoppers (i.e., α), disutility (i.e., k), and extra margin (i.e., μ) the e-tailer makes through the sales of the auxiliary product to be exogenously given. However, these parameters are likely to be determined by different levels of the CFS threshold. With a large CFS threshold, consumers bear a higher cost to acquire the auxiliary product, leading to a higher disutility and a smaller fraction of low-disutility shoppers. In this section, we analyze the impact when the parameter α varies while the other parameters stay unchanged. This allows us to understand the impact of CFS threshold from the proxy of α. Note that we still assume the order frequency is fixed and independent of price and CFS threshold, given that our research scope excludes stockpiling behavior, for example, in the context of online grocery shopping. With an endogenous order frequency, practitioners should set a lower price and membership fee as the optimal strategy under all configurations. But the general pattern of strategy equilibrium carries over even with an endogenous order frequency.

Figure A2. Impact of α Changes on the Equilibrium of Basic Model [2]



(a) $\alpha = 0.3$ (b) $\alpha = 0.6$

We find that a lower CFS threshold leads to a larger α, which enhances the advantage of MFS programs to encroach on the dominant region of PS (Figure A2). Specific to configurations, *MPCC* is more desirable at a larger parameter region, and *MMCC* is optimal at a smaller parameter region. Under *MMCC* configuration, more members with the free-shipping perk need more subsidies from CFS shoppers who are now smaller in size, resulting in an inferior performance compared to *MPCC*.

It is not difficult to show that the results from our main model are robust, and the insights carry over here except for some extreme cases of k and μ. Specifically, MFS always dominates in its feasible regions. Moreover, an endogenous CFS threshold can even lead to more advantages of MFS because the additional decision variable of α enlarges the optimal parameter region of MFS, decreases the optimal parameter region of PS, and has no impact on the optimal parameter region of CFS because it only depends on incentive compatibility constraint of *MPCC*. [3]

We further present more details of the sensitivity analysis on CFS market parameters under the duopoly model. Our results of Remarks A2 and A3 carry over here. When only comparing CFS and *MPCC* in the

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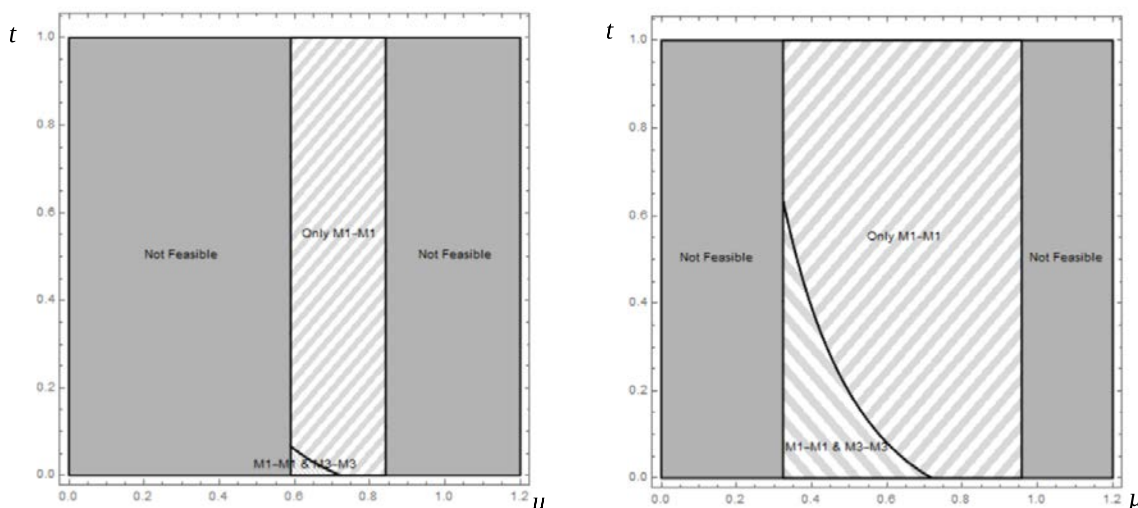
[2] In Figure A2, the parameters used for plotting are $\beta = 0.3$, $f_h = 1.5$, $f_l = 1$, $s = 1$, $v = 2$.

[3] Substituting (11) into the second inequality of (9), we show that the border of *MPCC* and CFS is exactly $\mu + k < 2s$.

duopolistic setting, *MPCC* is always the dominant strategy for both e-tailers, no matter what parameter values are. As shown in Figure A3, when comparing *MPCC* with *MMCC* in the duopolistic setting, the equilibrium always exists, except for extreme values of μ. However, *M*3-*M*3 can only become equilibrium with small *t* and relatively small μ. *M*1-*M*1 is always the better equilibrium, as postulated by Proposition A3. Besides, we also conduct a sensitivity analysis on the fraction of high-disutility shoppers (i.e., α). With a smaller α, the region with feasible parameters shrinks, and the region with two equilibria also shrinks, leading to a larger advantage of *MPCC* configuration. That is to say, *MPCC* is always favored by both e-tailers. The parameter α only influences the size of that advantage.

Figure A3. Impact of α changes on the equilibrium of duopoly model [4]



(a) α = 0.3 (b) α = 0.6

C.4 Proofs

C.4.1 Proof of Proposition A1

To model the demand boosting effect of MFS, we consider that at the purchase stage, all members will place ψ extra orders in total over the subscription period. Parameter ψ governs the magnitude of the *demand*

4 In Figure A3, the parameters used for plotting are β = 0.3, f_h = 1.5, f_l = 1, s = 1, k = 0.1.

boosting effect of the MFS program. As the value of ψ goes up, we say that the MFS program entices more extra orders from its members. We conduct analysis on three different MFS configurations.

1) ***MPCC Configuration***

Under *the MPCC* configuration, the e-tailer's problem can be formulated as:

$$\max_{p_{B1},\, m_{B1}} \pi_{B1} = (p_{B1}-s)Q_M\alpha\beta(f_h+\psi) + m_{B1}Q_M\alpha\beta + (p_{B1}-s+\mu)Q_C(1-\alpha)\overline{f} + p_{B1}Q_P\alpha(1-\beta)f_l. \quad \text{(A17)}$$

subject to

$$\max(f_l s,\ f_h k) < m_{B1} < f_h s. \quad \text{(A18)}$$

We use subscript *B* to denote the quantities in this case. Solving (A18), we derive the optimal pricing terms given as:

$$p^*_{B1} = \frac{2(v+s-\mu-k)(1-\alpha)\beta f_h^2 + 2\left[s+v-(\mu+k)(1-\alpha)-2s\alpha\right](1-\beta)f_h f_l - \alpha\beta\left[s\psi^2+(s-v)f_h\psi\right]}{4f_h\left[(1-\alpha)\beta f_h+(1-\beta)f_l\right]-\alpha\beta\psi^2}, \quad \text{(A19)}$$

$$m^*_{B1} = \frac{2(\mu+k)(1-\alpha)\beta f_h^3 + \left[(\mu+k)(1-\alpha)+2s\alpha\right]\left[2(1-\beta)f_h f_l+\psi\overline{f}\right]f_h - \left[(v-s)(\alpha\beta\psi+\overline{f})+2\alpha\beta s f_h\right]\psi f_h}{4f_h\left[(1-\alpha)\beta f_h+(1-\beta)f_l\right]-\alpha\beta\psi^2}. \quad \text{(A20)}$$

Plugging (A19)-(A20) into (A17), we can derive the optimal profit given as:

$$\pi^*_{B1} = \frac{f_h\overline{f}\left[c_5(1-\alpha)\beta f_h + c_6(1-\beta)f_l\right] - c_7\alpha\beta(v-s)f_h\psi - c_8\alpha\beta\psi^2}{4f_h\left[(1-\alpha)\beta f_h+(1-\beta)f_l\right]-\alpha\beta\psi^2}, \text{ where}$$

$$c_5 = (1-\alpha)\mu^2 + 2(v-s)(1-\alpha)\mu + \left[(v-s)^2 - 2(1-\alpha)(v+\mu-s)k + (1-\alpha)k^2\right],$$

$$c_6 = \left[(1-\alpha)(\mu-k)+(v-s)\right]^2 + 4\alpha(1-\alpha)\left[(\mu+k-s)s-\mu k\right],$$

$$c_7 = (1-\alpha)(k+\mu+s-v)\overline{f} + \alpha(1-\beta)(3s-v)f_l,$$

$$c_8 = (1-\alpha)(v-s-k)\mu\overline{f} + \alpha(1-\beta)(v-2s)f_l s.$$

Plugging (A20) into (A18) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A19)-(A20) as long as market parameters fall into the region R_{B1}, where

$$R_{B1}=\left\{(\psi,v,s,\mu,k,\alpha,\beta,f_l,f_h)\left|\begin{array}{l}2f_h\left[d_4\mu+d_5s-(d_4+d_5)k\right]+\left\{\left[(1-\alpha)(k+\mu-2s)-(v-3s)\right]\overline{f}-2\alpha\beta sf_h\right\}\psi\\ \quad -\alpha\beta\psi^2(v-s-k)>0,\\ \mu+k>\dfrac{4s(\overline{f}-\alpha f_h)f_hf_l+\alpha\beta\psi^2\left[(v-s)f_h-sf_l\right]+\left[(v-s-2s\alpha)\overline{f}+2\alpha\beta sf_h\right]f_h\psi}{d_4f_h(2f_h+\psi)},\\ \mu+k<2s+\dfrac{\alpha\beta\psi^2(v-2s)+\left[(v-3s)\overline{f}+2\alpha\beta sf_h\right]\psi}{d_4(2f_h+\psi)}\end{array}\right.\right\},$$

where $d_4=(1-\alpha)\overline{f}$, $d_5=2\alpha(1-\beta)f_l$.

2) ***MPMC Configuration***

Under *the MPMC* configuration, the e-tailer's problem can be formulated as:

$$\max_{p_{B2},\,m_{B2}}\pi_{B2}=(p_{B2}-s)Q_M\beta(f_h+\psi)+m_{B2}Q_M\beta+(p_{B2}-s+\mu)Q_C(1-\alpha)(1-\beta)f_l+p_{B2}Q_P\alpha(1-\beta)f_l. \quad \text{(A21)}$$

subject to

$$f_ls<m_{B2}<f_hk. \quad \text{(A22)}$$

Solving (A21), we derive the optimal pricing terms given as:

$$p^*_{B2}=\frac{2\left[s+v-(\mu+k)(1-\alpha)-2s\alpha\right](1-\beta)f_hf_l+\left[(v-s)f_h-s\psi\right]\beta\psi}{4(1-\beta)f_hf_l-\beta\psi^2}, \quad \text{(A23)}$$

$$m^*_{B2}=\frac{(1-\beta)\left[(1-\alpha)(k+\mu)+2\alpha s\right](2f_h+\psi)f_hf_l-(v-s)(\overline{f}+\beta\psi)f_h\psi}{4(1-\beta)f_hf_l-\beta\psi^2}. \quad \text{(A24)}$$

Plugging (A23)-(A24) into (A21), we can derive the optimal profit given as:

$$\pi^*_{B2}=\frac{(1-\beta)f_hf_l\left[(v-s)^2\beta f_h+c_9(1-\beta)f_l\right]+(1-\beta)\beta f_l\psi c_{10}}{4(1-\beta)f_hf_l-\beta\psi^2}, \text{ where}$$

$$c_9=(1-2\alpha)^2s^2-2[v+(1-\alpha)(1-2\alpha)\mu]s+(1-\alpha)^2k^2-2(1-\alpha)[v-s-2s\alpha+(1+\alpha)\mu]k+[v+(1-\alpha)\mu]^2,$$

$$c_{10}=\left[(1-\alpha)(k+s-v)\mu+\alpha s(2s-v)\right]\psi-\left[(1-\alpha)(k+\mu)-v+s+2\alpha s\right](v-s)f_h.$$

Plugging (A24) into (A22) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A23)-(A24) as long as market parameters fall into the region R_{B2}, where

$$R_{B2}=\left\{(\psi,v,s,\mu,k,\alpha,\beta,f_l,f_h)\left|\begin{array}{c} d_6k-d_7\mu>d_8 \\ \mu+k>\dfrac{d_9 f_l-d_8 f_h}{d_7 f_h}\end{array}\right.\right\}, \text{where}$$

$$d_6=\left[2(1+\alpha)f_h-(1-\alpha)\psi\right](1-\beta)f_l-\beta\psi^2,\ d_7=(1-\alpha)(1-\beta)(2f_h+\psi)f_l,$$

$$d_8=2\alpha(1-\beta)sf_l(2f_h+\psi)-(v-s)(\overline{f}+\beta\psi)\psi,\ d_9=4s(1-\beta)f_hf_l-s\beta\psi^2.$$

3) *MMCC Configuration*

Under *the MMCC* configuration, the e-tailer's problem can be formulated as:

$$\begin{aligned}\max_{p_{B3},\,m_{B3}}\pi_{B3}=\alpha(p_{B3}-s)\left[Q_M^H\beta(f_h+\psi)+Q_M^L(1-\beta)(f_l+\psi)\right]+\alpha m_{B3}\left[Q_M^H\beta+Q_M^L(1-\beta)\right]\\ +(1-\alpha)(p_{B3}-s+\mu)Q_C\overline{f}.\end{aligned}\tag{A25}$$

subject to

$$f_hk<m_{B3}<f_ls.\tag{A26}$$

Solving (A25), we derive the optimal pricing terms given as:

$$p_{B3}^*=\frac{2c_{11}f_hf_l(v+s)-2(1-\alpha)(k+\mu)c_{12}f_hf_l\overline{f}+\alpha f_hf_lc_{12}(v-s)\psi-\alpha c_{12}^2s\psi^2}{4c_{11}f_hf_l-\alpha c_{12}^2\psi^2},\tag{A27}$$

$$m_{B3}^*=\frac{2(1-\alpha)(k+\mu)f_h^2f_l^2\overline{f}+\left[(1-\alpha)(k+\mu)+s-v\right]c_{12}f_hf_l\overline{f}\psi-\alpha c_{12}(v-s)f_hf_l\psi^2}{4c_{11}f_hf_l-\alpha c_{12}^2\psi^2},\tag{A28}$$

where

$$c_{11}=\beta(1-\beta)(f_h-f_l)^2+(1-\alpha)f_hf_l,$$

$$c_{12}=f_h+f_l-\overline{f}.$$

Plugging (A27)-(A28) into (A25), we can derive the optimal profit given as:

$$\pi_{B3}^*=\frac{c_6\psi^2-\alpha f_hf_l(v-s)c_7\psi+c_8f_hf_l\overline{f}}{4c_4f_hf_l-\alpha c_5^2\psi^2}, \text{where}$$

$$c_{13}=\alpha(1-\alpha)(k+s-v)\mu c_{12}^2\overline{f},$$

$$c_{14}=\left[(1-\alpha)(k+\mu)-(v-s)\right]c_{11}+\alpha(1-\alpha)(k+\mu)f_hf_l,$$

$$c_{15}=\left\{\left[(v-s)-(1-\alpha)(k-\mu)\right]^2-4\alpha(1-\alpha)\mu k\right\}c_{11}+\alpha\left(1-\alpha\right)^2\left(k+\mu\right)^2 f_h f_l.$$

Plugging (A28) into (A26) and rearranging terms, we can characterize the *feasible* conditions for the solution set (A27)-(A28) as long as market parameters fall into the region R_{B3}, where

$$R_{B3}=\left\{(\psi,v,s,\mu,k,\alpha,\beta,f_l,f_h)\left|\begin{array}{c}\mu+k<\dfrac{4sc_{11}f_hf_l+\left(v-s\right)c_{12}f_h\overline{f}\psi+\alpha c_{12}\left[(v-s)f_h-c_{12}s\right]\psi^2}{\left(1-\alpha\right)\left(2f_hf_l+c_{12}\psi\right)f_h\overline{f}}\\ d_9k+d_{10}\mu>d_{11}\end{array}\right.\right\},$$

where

$$d_9=2f_hf_l\left[\left(1-\alpha\right)f_l\overline{f}-2c_{11}\right]+\left(1-\alpha\right)c_{12}f_l\overline{f}\psi+\alpha c_{12}^2\psi^2,$$

$$d_{10}=\left(1-\alpha\right)\left(2f_hf_l+c_{12}\psi\right)f_l\overline{f},$$

$$d_{11}=\left(v-s\right)\left(\overline{f}+\alpha\psi\right)c_{12}f_l\psi.$$

To compare the MFS pricing schemes before and after the demand boosting effect, we have more requirements on the product value v. Under different configurations, we find that when v is large enough (i.e. $v>\hat{v}$), the e-tailer would raise the price and lower the membership as long as the demand boosting effect exists.

Under *MPCC* configuration:

(1) To make $p_{B1}^*>p_{M1}^*$, $\hat{v}=s+\max\left\{\mu+k,\left(1-\alpha\right)\left(\mu+k\right)+2s\alpha\right\}$;

(2) To make $m_{B1}^*<m_{M1}^*$, $\hat{v}=s+\max\left\{\dfrac{\mu+k}{2},\left(1-\alpha\right)\left(\mu+k\right)+2s\alpha\right\}$.

Under *MPMC* configuration:

(3) To make $p_{B2}^*>p_{M2}^*$, $\hat{v}=s+\left(1-\alpha\right)\left(\mu+k\right)+2s\alpha$;

(4) To make $m_{B2}^*<m_{M2}^*$, $\hat{v}=s+\left[\left(1-\alpha\right)\left(\mu+k\right)+2s\alpha\right]\cdot\max\left\{\dfrac{1}{2},\dfrac{\left(1-\beta\right)f_l}{\overline{f}}\right\}$.

Under *MMCC* configuration:

(5) To make $p_{B3}^{*} > p_{M3}^{*}$, $\hat{v} = s + \frac{(1-\alpha)(\mu+k)c_{12}\overline{f}}{c_{11}}$;

(6) To make $m_{B3}^{*} < m_{M3}^{*}$, $\hat{v} = s + (1-\alpha)(\mu+k)\cdot \max\left\{1, \frac{c_{12}\overline{f}}{2c_{11}}\right\}$.

C.4.2 Proof of Proposition A2

In this proof, we first specify the two-competing e-tailers' problem in four cases. Then, we solve for each e-tailer's optimal prices and, if applicable, membership fees.

(a) ***Under Case CC*** (both e-tailer 1 e-tailer 2 use CFS):

$$\max_{p_1^{CC}} \pi_1^{CC} = \left[\alpha p_1^{CC} Q_{1P}^{CC} + (1-\alpha)\left(p_1^{CC} - s + \mu\right) Q_{1C}^{CC}\right]\overline{f}, \tag{A29}$$

$$\max_{p_2^{CC}} \pi_2^{CC} = \left[\alpha p_2^{CC} Q_{2P}^{CC} + (1-\alpha)\left(p_2^{CC} - s + \mu\right) Q_{2C}^{CC}\right]\overline{f}. \tag{A30}$$

where

$$Q_{1C}^{CC} = Q_{1P}^{CC} = \frac{t - p_1^{CC} + p_2^{CC}}{2t}, Q_{2C}^{CC} = Q_{2P}^{CC} = \frac{t - p_2^{CC} + p_1^{CC}}{2t}.$$

(b) ***Under Case MM*** (both e-tailer 1 and e-tailer 2 use MFS):

$$\max_{p_1^{MM}, m_1^{MM}} \pi_1^{MM} = Q_{1M}^{MM} \alpha\beta\left[\left(p_1^{MM} - s\right) f_h + m_1^{MM}\right] + \left(p_1^{MM} - s + \mu\right) Q_{1C}^{MM} (1-\alpha)\overline{f} + p_1^{MM} Q_{1P}^{MM} \alpha(1-\beta) f_l, \tag{A31}$$

$$\max_{p_2^{MM}, m_2^{MM}} \pi_2^{MM} = Q_{2M}^{MM} \alpha\beta\left[\left(p_2^{MM} - s\right) f_h + m_2^{MM}\right] + \left(p_2^{MM} - s + \mu\right) Q_{2C}^{MM} (1-\alpha)\overline{f} + p_2^{MM} Q_{2P}^{MM} \alpha(1-\beta) f_l, \tag{A32}$$

subject to

$$\max\{f_h k, f_l s\} < m_1^{MM} \le f_h s,\ \max\{f_h k, f_l s\} < m_2^{MM} \le f_h s, \tag{A33}$$

where

$$Q_{1M}^{MM} = \frac{1}{2t}\left(t - p_1^{MM} + p_2^{MM} - \frac{m_1^{MM} - m_2^{MM}}{f_h}\right),\ Q_{1C}^{MM} = Q_{1P}^{MM} = \frac{t - p_1^{MM} + p_2^{MM}}{2t},$$

$$Q_{2M}^{MM} = \frac{1}{2t}\left(t - p_2^{MM} + p_1^{MM} + \frac{m_1^{MM} - m_2^{MM}}{f_h}\right),\ Q_{2C}^{MM} = Q_{2P}^{MM} = \frac{t - p_2^{MM} + p_1^{MM}}{2t}.$$

(c) ***Under Case MC*** (e-tailer 1 uses MFS whereas e-tailer 2 uses CFS):

$$\max_{p_1^{MC}, m_1^{MC}} \pi_1^{MC} = Q_{1M}^{MC} \alpha\beta \left[\left(p_1^{MC} - s \right) f_h + m_1^{MC} \right] + \left(p_1^{MC} - s + \mu \right) Q_{1C}^{MC} \left(1 - \alpha \right) \overline{f} + p_1^{MC} Q_{1P}^{MC} \alpha \left(1 - \beta \right) f_l, \quad \text{(A34)}$$

$$\max_{p_2^{MC}} \pi_2^{MC} = p_2^{MC} Q_{2M}^{MC} \alpha\beta f_h + \left(p_2^{MC} - s + \mu \right) Q_{2C}^{MC} \left(1 - \alpha \right) \overline{f} + p_2^{MC} Q_{2P}^{MC} \alpha \left(1 - \beta \right) f_l, \quad \text{(A35)}$$

subject to

$$\max\left\{ f_h k, f_l s \right\} < m_1^{MC} < f_h s, \quad \text{(A36)}$$

where

$$Q_{1M}^{MC} = \frac{1}{2t}\left(t - p_1^{MC} + p_2^{MC} + s - \frac{m_1^{MC}}{f_h} \right),\ Q_{1C}^{MC} = Q_{1P}^{MC} = \frac{t - p_1^{MC} + p_2^{MC}}{2t},$$

$$Q_{2M}^{MC} = \frac{1}{2t}\left(t - p_2^{MC} + p_1^{MC} - s + \frac{m_1^{MC}}{f_h} \right),\ Q_{2C}^{MC} = Q_{2P}^{MC} = \frac{t - p_2^{MC} + p_1^{MC}}{2t}.$$

Given the well-defined problems above, we can solve for each e-tailer's pricing terms given as:

(a) ***Under Case CC*** (both e-tailer 1 e-tailer 2 use CFS):

$$p_1^{CC*} = p_2^{CC*} = \left(1 - \alpha \right)\left(s - \mu \right) + t.$$

(b) ***Under Case MM*** (both e-tailer 1 and e-tailer 2 use MFS):

$$p_1^{MM*} = p_2^{MM*} = \frac{\left(1 - \alpha \right) \beta f_h \left(s - \mu + t \right) + \left(1 - \beta \right) f_l \left[\left(1 - \alpha \right)\left(s - \mu \right) + t \right]}{\left(1 - \alpha \right) \beta f_h + \left(1 - \beta \right) f_l},$$

$$m_1^{MM*} = m_2^{MM*} = \frac{\left(1 - \alpha \right) \beta f_h^2 \mu + \left(1 - \beta \right) f_h f_l \left[\left(1 - \alpha \right) \mu + \alpha s \right]}{\left(1 - \alpha \right) \beta f_h + \left(1 - \beta \right) f_l}.$$

(c) ***Under Case MC*** (e-tailer 1 uses MFS, whereas e-tailer 2 uses CFS):

$$p_1^{MC*} = \frac{\left[\left(2 - \alpha \right)(s - \mu) + 2t \right]\left(1 - \alpha \right) \beta f_h + 2\left[(1 - \alpha)(s - \mu) + t \right]\left(1 - \beta \right) f_l}{2\left[\left(1 - \alpha \right) \beta f_h + \left(1 - \beta \right) f_l \right]},$$

$$m_1^{MC*} = \frac{f_h}{2} \frac{\left(1 - \alpha \right) \beta f_h \left(\mu + s \right) + \left(1 - \beta \right) f_l \left[\left(\mu + s \right)\left(1 - \alpha \right) + 2s\alpha \right]}{\left(1 - \alpha \right) \beta f_h + \left(1 - \beta \right) f_l},$$

$$p_2^{MC*} = \left(1 - \alpha \right)\left(s - \mu \right) + t.$$

$$R_D = \left\{ (s,\mu,k,\alpha,\beta,f_l,f_h) \left| \begin{array}{l} k < \dfrac{(1-\alpha)\overline{f}\mu + \alpha(1-\beta) f_l s}{(1-\alpha)\beta f_h + (1-\beta) f_l},\ \mu < s, \\ \alpha f_h \geq \overline{f} \text{ or } \mu > \dfrac{\left(\overline{f} - \alpha f_h\right) f_l s}{(1-\alpha)\overline{f} f_h} \end{array} \right. \right\}.$$

It is obvious to find that Case *MM* is the equilibrium strategy for both e-tailer 1 and e-tailer 2.

	MPCC	CFS
MPCC	$\pi_1^{MM*} = \pi_2^{MM*} \frac{1}{2}\overline{f}t$	$\pi_1^{MC*} = \frac{1}{2}\overline{f}t + \frac{\alpha(1-\alpha)^2 \beta f_h \overline{f}(\mu-s)^2}{8t\left[(1-\alpha)\beta f_h + (1-\beta) f_l\right]}$, $\pi_2^{MC*} = \frac{1}{2}\overline{f}t - \frac{\alpha(1-\alpha)^2 \beta f_h \overline{f}(\mu-s)^2}{4\left[(1-\alpha)\beta f_h + (1-\beta) f_l\right]}$
CFS	$\pi_1^{CM*} = \frac{1}{2}\overline{f}t - \frac{\alpha(1-\alpha)^2 \beta f_h \overline{f}(\mu-s)^2}{4\left[(1-\alpha)\beta f_h + (1-\beta) f_l\right]}$, $\pi_2^{CM*} = \frac{1}{2}\overline{f}t + \frac{\alpha(1-\alpha)^2 \beta f_h \overline{f}(\mu-s)^2}{8t\left[(1-\alpha)\beta f_h + (1-\beta) f_l\right]}$	$\pi_1^{CC*} = \pi_2^{CC*} = \frac{1}{2}\overline{f}t$

C.4.3 Proof of Proposition A3

In this proof, we first specify the two-competing e-tailers' problem in four cases. Then, we solve for each e-tailer's optimal prices and, if applicable, membership fees.

(a) ***Under Case M*****1-*****M*****1** (both e-tailer 1 e-tailer 2 use *MPCC*):

$$\max_{p_1^{11}, m_1^{11}} \pi_1^{11} = Q_{1M}^{11}\alpha\beta\left[\left(p_1^{11} - s\right) f_h + m_1^{11}\right] + \left(p_1^{11} - s + \mu\right) Q_{1C}^{11}(1-\alpha)\overline{f} + p_1^{11} Q_{1P}^{11}\alpha(1-\beta) f_l, \tag{A37}$$

$$\max_{p_2^{11}, m_2^{11}} \pi_2^{11} = Q_{2M}^{11}\alpha\beta\left[\left(p_2^{11} - s\right) f_h + m_2^{11}\right] + \left(p_2^{11} - s + \mu\right) Q_{2C}^{11}(1-\alpha)\overline{f} + p_2^{11} Q_{2P}^{11}\alpha(1-\beta) f_l, \tag{A38}$$

subject to

$$\max\left\{f_h k, f_l s\right\} < m_1^{11} < f_h s,\ \ \max\left\{f_h k, f_l s\right\} < m_2^{11} < f_h s, \tag{A39}$$

where

$$Q_{1M}^{11} = \frac{1}{2t}\left(t - p_1^{11} + p_2^{11} - \frac{m_1^{11} - m_2^{11}}{f_h}\right),\ Q_{1C}^{11} = Q_{1P}^{11} = \frac{t - p_1^{11} + p_2^{11}}{2t},$$

$$Q_{2M}^{11} = \frac{1}{2t}\left(t - p_2^{11} + p_1^{11} + \frac{m_1^{11} - m_2^{11}}{f_h}\right),\ Q_{2C}^{11} = Q_{2P}^{11} = \frac{t - p_2^{11} + p_1^{11}}{2t}.$$

(b) ***Under Case M*3-*M*3** (both e-tailer 1 and e-tailer 2 use *MMCC*):

$$\max_{p_1^{33},m_1^{33}} \pi_1^{33} = Q_{1MH}^{33}\alpha\beta\left[\left(p_1^{33}-s\right)f_h + m_1^{33}\right] + Q_{1ML}^{33}\alpha\left(1-\beta\right)\left[\left(p_1^{33}-s\right)f_l + m_1^{33}\right] + \left(p_1^{33}-s+\mu\right)Q_{1C}^{33}\left(1-\alpha\right)\overline{f}, \tag{A40}$$

$$\max_{p_2^{33},m_2^{33}} \pi_2^{33} = Q_{2MH}^{33}\alpha\beta\left[\left(p_2^{33}-s\right)f_h + m_2^{33}\right] + Q_{2ML}^{33}\alpha\beta\left[\left(p_2^{33}-s\right)f_l + m_2^{33}\right] + \left(p_2^{33}-s+\mu\right)Q_{2C}^{33}\left(1-\alpha\right)\overline{f}, \tag{A41}$$

subject to

$$f_h k < m_1^{33} < f_l s,\ f_h k < m_2^{33} < f_l s, \tag{A42}$$

where

$$Q_{1MH}^{33} = \frac{1}{2t}\left(t - p_1^{33} + p_2^{33} - \frac{m_1^{33}-m_2^{33}}{f_h}\right),\ Q_{1ML}^{33} = \frac{1}{2t}\left(t - p_1^{33} + p_2^{33} - \frac{m_1^{33}-m_2^{33}}{f_l}\right),\ Q_{1C}^{33} = \frac{t-p_1^{33}+p_2^{33}}{2t},$$

$$Q_{2MH}^{33} = \frac{1}{2t}\left(t - p_2^{33} + p_1^{33} + \frac{m_1^{33}-m_2^{33}}{f_h}\right),\ Q_{2ML}^{33} = \frac{1}{2t}\left(t - p_2^{33} + p_1^{33} + \frac{m_1^{33}-m_2^{33}}{f_l}\right),\ Q_{2C}^{33} = \frac{t-p_2^{33}+p_1^{33}}{2t}.$$

(c) ***Under Case M*1-*M*3** (e-tailer 1 uses MPCC, whereas e-tailer 2 uses *MMCC*):

$$\max_{p_1^{13},m_1^{13}} \pi_1^{13} = Q_{1M}^{13}\alpha\beta\left[\left(p_1^{13}-s\right)f_h + m_1^{13}\right] + \left(p_1^{13}-s+\mu\right)Q_{1C}^{13}\left(1-\alpha\right)\overline{f} + p_1^{13}Q_{1P}^{13}\alpha\left(1-\beta\right)f_l, \tag{A43}$$

$$\max_{p_2^{13},m_2^{13}} \pi_2^{13} = Q_{2M}^{13}\alpha\beta\left[\left(p_2^{13}-s\right)f_h + m_2^{13}\right] + Q_{2P}^{13}\alpha\left(1-\beta\right)\left[\left(p_2^{13}-s\right)f_l + m_2^{13}\right] + \left(p_2^{13}-s+\mu\right)Q_{2C}^{13}\left(1-\alpha\right)\overline{f}, \tag{A44}$$

subject to

$$\max\left\{f_h k, f_l s\right\} < m_1^{13} < f_h s,\ f_h k < m_2^{13} < f_l s, \tag{A45}$$

where

$$Q_{1M}^{13} = \frac{1}{2t}\left(t - p_1^{13} + p_2^{13} - \frac{m_1^{13}-m_2^{13}}{f_h}\right),\ Q_{1P}^{13} = \frac{1}{2t}\left(t - p_1^{13} + p_2^{13} - s + \frac{m_2^{13}}{f_l}\right),\ Q_{1C}^{13} = \frac{t-p_1^{13}+p_2^{13}}{2t},$$

$$Q_{2M}^{13} = \frac{1}{2t}\left(t - p_2^{13} + p_1^{13} + \frac{m_1^{13}-m_2^{13}}{f_h}\right),\ Q_{2P}^{13} = \frac{1}{2t}\left(t - p_2^{13} + p_1^{13} + s - \frac{m_2^{13}}{f_l}\right),\ Q_{2C}^{13} = \frac{t-p_2^{13}+p_1^{13}}{2t}.$$

Given the well-defined problems above, we can solve for each e-tailer's pricing terms given as:

(a) ***Under Case M*1-*M*1**(both e-tailer 1 e-tailer 2 use *MPCC*):

$$p_1^{11*} = p_2^{11*} = \frac{(1-\alpha)\beta f_h (s-\mu+t) + (1-\beta) f_l \left[(1-\alpha)(s-\mu)+t\right]}{(1-\alpha)\beta f_h + (1-\beta) f_l},$$

$$m_1^{11*} = m_2^{11*} = \frac{(1-\alpha)\beta f_h^2 \mu + (1-\beta) f_h f_l \left[(1-\alpha)\mu + \alpha s\right]}{(1-\alpha)\beta f_h + (1-\beta) f_l}.$$

(b) ***Under Case M*3-*M*3** (both e-tailer 1 and e-tailer 2 use *MMCC*):

$$p_1^{33*} = p_2^{33*} = t + s - (1-\alpha)\mu - \frac{\alpha(1-\alpha) f_h f_l \mu}{\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l},$$

$$m_1^{33*} = m_2^{33*} = \frac{\alpha(1-\alpha) f_h f_l \bar{f} \mu}{\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l}.$$

(c) ***Under Case M*1-*M*3** (e-tailer 1 uses *MPCC*, whereas e-tailer 2 uses *MMCC*):

$$p_1^{13*} = \frac{\left[(1-\alpha)h_3 + t h_4\right]\beta f_l + \left[(1-\alpha)(s-\mu)+t\right]h_5 + \beta^2 (f_h - f_l) f_h f_l h_6 + h_7}{4(1-\alpha)(1-\beta)\beta^2 f_h^3 + \beta h_1 f_h^2 f_l + 2(1-\beta) h_2 f_h f_l^2 + 3\beta(1-\beta)^2 f_l^3},$$

$$m_1^{13*} = \frac{(1-\alpha)\bar{f} h_8 \mu + (1-\beta) f_h f_l h_9 s + h_{12}}{4(1-\alpha)(1-\beta)\beta^2 f_h^3 + \beta h_1 f_h^2 f_l + 2(1-\beta) h_2 f_h f_l^2 + 3\beta(1-\beta)^2 f_l^3},$$

$$p_2^{13*} = \frac{\beta\left[s+t-(1-\alpha)\mu\right]h_{10} + \beta f_h f_l^2 h_{11} + \beta^2 f_h f_l (f_h - f_l)\left[h_{11} - 2(1-\alpha)\alpha(\mu+s)\right]}{4(1-\alpha)(1-\beta)\beta^2 f_h^3 + \beta h_1 f_h^2 f_l + 2(1-\beta) h_2 f_h f_l^2 + 3\beta(1-\beta)^2 f_l^3},$$

$$m_2^{13*} = \frac{(1-\alpha) f_h f_l \bar{f} \left\{\left[(2-3\alpha+\beta)\beta f_h + (1-\beta)(2+\beta) f_l\right]\mu + 2(1-\beta)\bar{f} s\right\}}{4(1-\alpha)(1-\beta)\beta^2 f_h^3 + \beta h_1 f_h^2 f_l + 2(1-\beta) h_2 f_h f_l^2 + 3\beta(1-\beta)^2 f_l^3}.$$

where

$$h_1 = 8(1-\alpha)(1-\beta)^2 + 3(\alpha-\beta)^2,$$

$$h_2 = 2(1-\alpha)(1-\beta)^2 + 3\beta(\beta-\alpha),$$

$$h_3 = 3 f_l^2 (s-\mu) + f_h f_l \left[(12-7\alpha)\mu - (12-5\alpha)s\right] + f_h^2 \left[4(2-\alpha)s - (8-6\alpha)\mu\right],$$

$$h_4 = (8 - 8\alpha + 3\alpha^2) f_h^2 - 6(2-\alpha) f_h f_l + 3 f_l^2,$$

$$h_5 = 4(1-\alpha) f_h f_l^2 + 3\beta^3 (f_h - f_l)^2 f_l + 6\beta^2 (f_h - f_l) f_l^2,$$

$$h_6 = (1-\alpha)\left[(12-5\alpha)\mu - (12-\alpha)s\right] - 6(2-\alpha)t,$$

$$h_7 = 2(1-\alpha)\beta^2 f_h (f_h - f_l)(f_h + f_l - \overline{f})\left[2s + 2t - (2-\alpha)\mu\right],$$

$$h_8 = 2\beta(1-\beta) f_h^2 + \left[2 - 3\alpha + \beta - 4\beta(1-\beta)\right] f_h f_l + (1-\beta)(1+2\beta) f_l^2,$$

$$h_9 = \overline{f}^2 - 3\alpha^2 f_h f_l - 4\alpha\beta^2 (f_h - f_l)^2 + \alpha f_l (3 f_h - f_l) + \alpha\beta\left(3 f_h^2 - 8 f_h f_l + 5 f_l^2\right),$$

$$h_{10} = 3 f_l^3 + \beta^2 (f_h - f_l)^2 \left[3 f_l - 4(1-\alpha) f_h\right] + 2\beta (f_h - f_l)\left[3 f_l^2 + 2(1-\alpha) f_h^2\right],$$

$$h_{11} = (1-\alpha)(12-5\alpha)\mu - 2\left(6 - 5\alpha + 2\alpha^2\right)s - 6(2-\alpha)t,$$

$$h_{12} = 2(1-\alpha) f_h f_l^2 \left[(2-\alpha)(s-\mu) + 2t\right] + \beta f_h^2 f_l \left[-2(1-\alpha)(4-3\alpha)\mu + \left(8 - 10\alpha + 5\alpha^2\right)s + \left(8 - 8\alpha + 3\alpha^2\right)t\right].$$

The profit functions of these cases are very complicated to compare analytically. Thus, we use numerical simulation to find the equilibrium under different settings of parameters. Specifically, we take the following values for each parameter:

t : Arithmetic sequence from 0.05 to 1 with 0.05 as the common difference

k : Arithmetic sequence from 0.05 to 0.95 with 0.05 as the common difference

α : Arithmetic sequence from 0.1 to 0.9 with 0.1 as the common difference

β : Arithmetic sequence from 0.1 to 0.9 with 0.1 as the common difference

μ : Arithmetic sequence from 0.1 to 4.0 with 0.1 as the common difference

f_h : Arithmetic sequence from 1.2 to 5.0 with 0.2 as the common difference

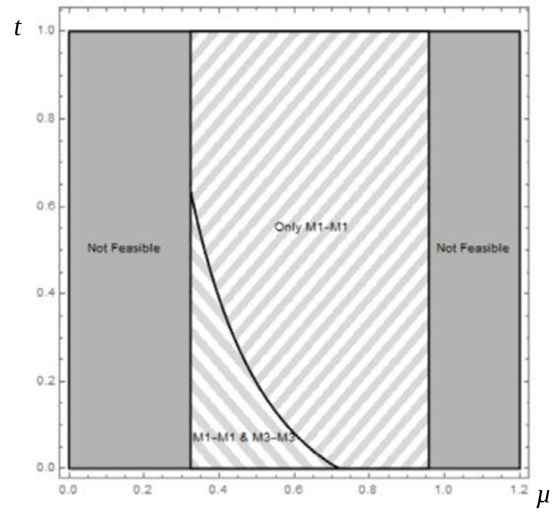
$f_l = 1$

$s = 1$

With the above settings for each parameter, we simulate 24,624,000 cases. Out of all cases, 277,860 have feasible parameters, and *M*1-*M*1 is always an equilibrium for these cases. 26,651 cases also have *M*3-*M*3 as an equilibrium. *M*1-*M*3 and *M*3-*M*1 are not equilibria in any cases. Figure A4 demonstrates our results of the numerical simulation where we fix α, β, f_h, and k. The regions marked with “not feasible” mean that both *MPCC* and *MMCC* are not feasible strategies with the values of parameters. In the green area, both e-tailers operating *MPCC* configuration is the only equilibrium. In the yellow area, both e-tailers operating *MPCC* and both e-tailers operating *MMCC* are the two equilibria, but operating *MPCC* is the better equilibrium for both. This can be proved by subtracting the optimal profit of case *M*3-*M*3 from that of case *M*1-*M*1:

$$\pi_1^{11*} - \pi_1^{33*} = \frac{\alpha(1-\alpha)^2 f_h f_l \overline{f} \mu}{2(\beta(1-\beta)(f_h - f_l)^2 + (1-\alpha) f_h f_l)} > 0.$$

Figure A4. Equilibrium Strategies for the Two Competing E-tailers [5]



D. Analysis on Asymmetric Misfit Costs

In this Appendix, we conduct a robustness check by relaxing the independence assumption between misfit cost and order frequency. Based on our basic model, we further suppose that occasional shoppers incur a higher misfit cost t ($t > 1$), relative to that of frequent shoppers which can be normalized to 1. Accordingly, we can rewrite the consumer base for frequent and occasional shopper segment in general forms as $Q^h = v - p - s$ and $Q^l = (v - p - s)/t$, respectively. Using the same notational rules, we then can formulate the e-tailer's problem under *MPCC* configuration in the presence of asymmetric misfit costs as:

[5] In Figure A4, the parameters used for plotting are $\alpha = 0.6$, $\beta = 0.3$, $f_h = 1.5$, $f_l = 1$, $s = 1$, $k = 0.1$.

$$\max_{p'_{M1},m'_{M1}} \pi'_{M1} = (p'_{M1}-s)Q_M^h \alpha\beta f_h + m'_{M1}Q_M^h\alpha\beta + (p'_{M1}-s+\mu)(1-\alpha)\left[Q_C^h\beta f_h + Q_C^l(1-\beta) f_l\right] + p'_{M1}Q_P^l\alpha(1-\beta) f_l,$$

Subject to $\max(f_l s, f_h k) < m'_{M1} < f_h s$. Following the same procedure, we can derive the optimal pricing terms (p'^*_{M1}, m'^*_{M1}) and profits π'^*_{M1} as long as market parameters fall into region R'_{M1}, where

$$p'^*_{M1} = \frac{1}{2}\frac{(v+s-\mu-k)(1-\alpha)\beta f_h t + \left[s+v-(\mu+k)(1-\alpha)-2s\alpha\right](1-\beta) f_l}{(1-\alpha)\beta f_h t + (1-\beta) f_l},$$

$$m'^*_{M1} = \frac{f_h}{2}\frac{(\mu+k)(1-\alpha)\beta f_h t + \left[(\mu+k)(1-\alpha)+2s\alpha\right](1-\beta) f_l}{(1-\alpha)\beta f_h t + (1-\beta) f_l},$$

$$\pi'^*_{M1} = \frac{\overline{f}\left[c_1(1-\alpha)\beta f_h t + c_2(1-\beta) f_l\right]}{4\left[(1-\alpha)\beta f_h t + (1-\beta) f_l\right]},$$

$$R'_{M1} = \left\{(t,s,\mu,k,\alpha,\beta,f_l,f_h)\,\middle|\,(w_1-w_2)\mu + 2w_2 s > (w_1+w_2)k, \frac{2s(f_l w_1 - f_h w_2)}{f_h(w_1-w_2)} < \mu + k < 2s\right\},$$

$$w_1 = (1-\alpha)\beta f_h t + (1-\beta) f_l,\ w_2 = \alpha(1-\beta) f_l.$$

Similarly, we can solve e-tailer's problem under *MPMC* and *MMCC* configurations. For *MPMC* configuration, the e-tailer's problem becomes:

$$\max_{p'_{M2},\, m'_{M2}} \pi'_{M2} = Q_M^h\beta\left[(p'_{M2}-s) f_h + m'_{M2}\right] + (p'_{M2}-s+\mu)Q_C^l(1-\alpha)(1-\beta) f_l + p'_{M2}Q_P^l\alpha(1-\beta) f_l,$$

Subject to $f_l s < m'_{M1} < f_h k$. Optimal pricing terms (p'^*_{M2}, m'^*_{M2}) and profits π'^*_{M2} and feasible region R'_{M2} are:

$$p'^*_{M2} = \frac{1}{2}\left[s+v-(\mu+k)(1-\alpha)-2s\alpha\right],$$

$$m'^*_{M2} = \frac{f_h}{2}\left[(\mu+k)(1-\alpha)+2s\alpha\right],$$

$$\pi'^*_{M2} = \frac{(v-s)^2\beta f_h t + c_3(1-\beta) f_l}{4t},$$

$$R'_{M2} = \left\{(t,s,\mu,k,\alpha,f_h,f_l)\,\middle|\,(1+\alpha)k-(1-\alpha)\mu > 2\alpha s,\ \mu+k > \frac{2s(f_l - \alpha f_h t)}{(1-\alpha) f_h t}\right\}.$$

For *MMCC* configuration, the e-tailer's problem becomes:

$$\max_{p'_{M3},\, m'_{M3}} \pi'_{M3} = \alpha\left\{(p'_{M3}-s)\left[Q_M^h \beta f_h + Q_M^l (1-\beta) f_l\right] + m'_{M3}\left[Q_M^h \beta + Q_M^l (1-\beta)\right]\right\}$$
$$+ (1-\alpha)(p'_{M3}-s+\mu)(Q_C^h \beta f_h + Q_C^l (1-\beta) f_l),$$

Subject to $f_h k < m'_{M1} < f_l s$. Optimal pricing terms (p'^*_{M3}, m'^*_{M3}) and profits π'^*_{M3} and feasible region R'_{M3} are:

$$p'^*_{M3} = \frac{1}{2}\left\{v + s - \frac{(k+\mu)(1-\alpha)\left[(1-\beta) f_h t + \beta f_l\right]\left[\beta f_h t + (1-\beta) f_l\right]}{\beta(1-\beta)(f_h t - f_l)^2 + (1-\alpha) f_h f_l t}\right\},$$

$$m'^*_{M3} = \frac{1}{2}\frac{(k+\mu)(1-\alpha) f_h f_l \left[\beta f_h t + (1-\beta) f_l\right]}{\beta(1-\beta)(f_h t - f_l)^2 + (1-\alpha) f_h f_l t},$$

$$\pi'^*_{M3} = \frac{\overline{f}}{4}\left\{(1-\alpha)^2 k^2 - 2(1-\alpha)\left[v - s + (1+\alpha)\mu\right]k + \left[v - s + (1-\alpha)\mu\right]^2 + w_3\right\}, \text{ where}$$

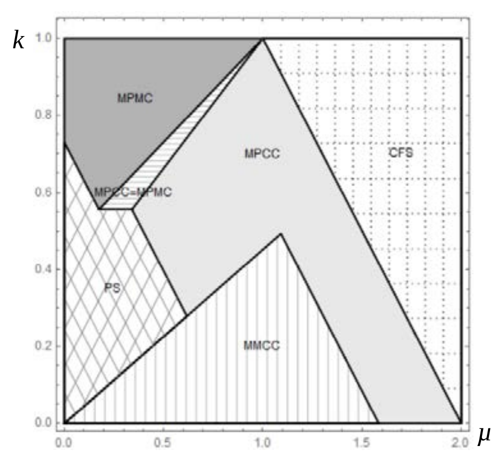
$$w_3 = \frac{\alpha(1-\alpha)^2 (k+\mu)^2 f_h f_l t}{\beta(1-\beta)(f_h t - f_l)^2 + (1-\alpha) f_h f_l t},$$

$$R'_{M3} = \left\{(t, s, \mu, k, \alpha, \beta, f_h, f_l) \middle| \mu > \left(\frac{2w_4}{f_l} - 1\right)k,\ \mu + k < \frac{2 w_4 s}{f_h t}\right\},$$

$$w_4 = \frac{(1-\beta)\beta(f_h t - f_l)^2 + (1-\alpha) f_h f_l t}{(1-\alpha)\left[\beta f_h t + (1-\beta) f_l\right]}.$$

The strategy equilibrium is depicted in Figure A5, with a similar pattern to the analysis of the basic model.

Figure A5. Best Choice of Shipping Strategy for the E-tailer (with Asymmetric Misfit Costs) [6]



[6] In Figure A5, the parameters used for plotting are: $\alpha = 0.3$, $\beta = 0.3$, $f_h = 1.5$, $f_l = 1$, $s = 1$, $v = 2$, $t = 1.2$.