Appendix

A. Complete Information Scenario

A.1. Proof of Lemma 1

Lemma 1. *If $H - c_H > M - c_M$, the high-quality firm spends more on social media marketing than the mid-quality firm, in the complete-information case.*

Proof. It can be directly derived from Table 1, by comparing $-1 + \sqrt{(H - c_H)(1 - r)}$ and $-1 + \sqrt{(M - c_M)(1 - r)}$.

A.2. Proof of Lemma 2

Lemma 2. *When consumers are homogeneous, price alone cannot serve as a quality signal.*

Proof. When consumers are homogeneous, the maximum price that can be charged is the consumers' reservation price. For the high-/mid-/low-quality firm, the maximum price that it can charge is H, M and L, respectively. Suppose a separating equilibrium exists in which the firm uses price alone to signal its type. If the high-quality firm sets its price higher than the mid-/low-quality firm, the mid-/low-quality firm has an incentive to mimic and pretend to be the high-quality firm by charging the same price as the high-quality firm does. Conversely, if the mid-quality or low-quality firm charges a higher price than the high-quality firm, the high-quality firm would also mimic by increasing its own price and make more profit. Therefore, in a homogeneous consumer market, prices alone cannot be used as signals to signal the quality.

B. Benchmark Case

In the benchmark case, we consider the case in which social media marketing spending not only signals product quality, but also serves the purpose of increasing consumers' awareness of the product, in the spirit of Zhao (2000).

B.1. Proof of Proposition 1

Proposition 1. *In the separating equilibrium, the high-quality, mid-quality and low-quality firms spend on social media marketing at levels of* $s_H^* = \frac{M(H-1-r)-c_M(1-r)+\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)}}{2M}$, $s_M^* = M - 1$ *and* $s_L^* = 0$, *respectively. The equilibrium prices are H, M and L, respectively.*

Proof. The low-quality firm needs to solve the following profit maximization program.

$$\max_{s_L, p(L)} \pi(L, L; p(L), s_L) = [r + (1 - r) * \frac{s_L}{1 + s_L}]\mathbb{1}(p(L) \leq L)(p(L) - c_L) - s_L \tag{B.1}$$

$$s.t.\ \pi(L,L;p(L),s_L) \geq \pi(M,L;p^*(M),s_M^*) \tag{B.2}$$

$$\pi(L,L;p(L),s_L) \geq \pi(H,L;p^*(H),s_H^*) \tag{B.3}$$

$$\pi(M,M;p^*(M),s_M^*) \geq \pi(L,M;p^*(L),s_L^*) \tag{B.4}$$

$$\pi(H,H;p^*(H),s_H^*) \geq \pi(L,H;p^*(L),s_L^*), \tag{B.5}$$

In the separating equilibrium, the low-quality firm's strategy is simply its complete information optimal strategy $(p^*(L),s_L^*)$, where $p^*(L)=L=1$ and $s_L^*=0$. $\pi(L,L;p^*(L),s_L^*)=r$.

The mid-quality firm needs to solve the following profit maximization program.

$$\max_{s_M,p(M)} \pi(M,M;p(M),s_M) = [r+(1-r)*\frac{s_M}{1+s_M}]\mathbb{1}(p(M)\leq M)[p(M)-c_M]-s_M \tag{B.6}$$

$$s.t.\ \pi(L,L;p^*(L),s_L^*) \geq \pi(M,L;p^*(M),s_M^*) \tag{B.7}$$

$$\pi(H,H;p^*(H),s_H^*) \geq \pi(M,H;p^*(M),s_M^*) \tag{B.8}$$

$$\pi(M,M;p(M),s_M) \geq \pi(L,M;p^*(L),s_L^*) \tag{B.9}$$

$$\pi(M,M;p(M),s_M) \geq \pi(H,M;p^*(H),s_H^*), \tag{B.10}$$

The first condition has to bind in the least-cost separation, so that the mid-quality firm will signal its quality by choosing such a strategy that the low-quality firm would rather reveal its true quality than mimic the mid-quality firm's strategy. Therefore, we have $r=\pi^*(L,L;p^*(L),s_L^*)=\pi(M,L;p^*(M),s_M^*)=[r+(1-r)*\frac{s_M^*}{1+s_M^*}]M-s_M^*$. Solving this equation, we obtain $s_M^*=M-1$ (we discard the other impossible root $-r$), and $p^*(M)=M$.

Next, we solve the high-quality firm's profit maximization problem:

$$\max_{s_H,p(H)} \pi(H,H;p(H),s_H) = [r+(1-r)*\frac{s_H}{1+s_H}]\mathbb{1}(p(H)\leq H)[p(H)-c_H)]-s_H \tag{B.11}$$

$$s.t.\pi(H,M;p(H),s_H) \leq \pi(M,M;p^*(M),s_M^*) \tag{B.12}$$

$$\pi(M,H;p^*(M),s_M^*) \leq \pi(H,H;p(H),s_H) \tag{B.13}$$

$$\pi(H,L;p(H),s_H) \leq \pi(L,L;p^*(L),s_L^*) \tag{B.14}$$

$$\pi(L,H;p^*(L),s_L^*) \leq \pi(H,H;p(H),s_H), \tag{B.15}$$

In the separating equilibrium, the high-quality firm's optimal price is $p^*(H)=H$, which is consumer's reservation price for the high-quality product. At the least-cost separation (Zhao 2000, Kalra, Rajiv, and

Srinivasan 1998), the mimicking constraint of the mid-quality firm is binding so that the mid-quality firm would rather reveal its true quality than mimic the high-quality firm's strategy. In other words, $r - c_M(-1 + M + r))/M = \pi^*(M, M; p^*(M), s_M^*) = \pi(H, M; p^*(H), s_H^*) = [r + (1-r) * \frac{s_H}{1+s_H}][p(H) - c_M)] - s_H$. Solving for s_H^*, we obtain $s_H^* = \frac{M(H-1-r)-c_M(1-r)+\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)}}{2M}$.

Taken together, in the separating equilibrium, the high-quality, mid-quality and low-quality firms spend on social media marketing at levels of $s_H^* = \frac{M(H-1-r)-c_M(1-r)+\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)}}{2M}$, $s_M^* = M - 1$ and $s_L^* = 0$, respectively.

In the separating equilibrium described in Proposition 1, consumer belief (e.g., the probability that the product is of the true quality) is given by $\mu(q|s,p) = 1$ if $(s,p) = (s_q^*, q)$, where $q \in \{L, M, H\}$, and for other $(s,p) \neq (s_q^*, q)$, $\mu(L|s,p) = 1$.

B.2. Proof of Corollary 1

Corollary 1. *In the separating equilibrium, both the mid-quality and high-quality firms spend more on social media marketing than their optimal spending levels in the complete-information scenario.*

Proof. Since the curve of $\pi(H, M; p(H), s_H)$ lies to the right of $\pi(H, H; p(H), s_H)$ (or $\sqrt{(H - c_H)(1-r)} < \sqrt{(H - c_M)(1-r)}$), we have $s_H^* > -1 + \sqrt{(H - c_H)(1-r)}$. That is the high-quality firm invests more than its optimal SMM spending under the complete-information scenario.

It is also straightforward to see from earlier, that $-1 + \sqrt{(M - c_M)(1-r)} < -1 + \sqrt{(M - c_M)M} < -1 + \sqrt{M^2} = M - 1$. Therefore, in order to deter the low-quality firm from mimicking, the mid-quality firm has to increase its SMM spending to a level such that the low-quality firm does not have an incentive to mimic.

B.3. Proof of Proposition 2

Proposition 2. *There is no partial pooling where the high-quality firm and the low-quality firm pool together, e.g. $\Lambda = \{L, H\}$, while the mid-quality firm invests in social media marketing and separates. The partial-pooling equilibria where either the high-quality and the mid-quality firms pool together or the mid-quality and the low-quality firms pool together do not exist, either.*

Proof. Let's first show that the partial-pooling equilibria where either the high-quality and the mid-quality firms pool together or the mid-quality and the low-quality firms pool together do not exist. Suppose in the partial pooling equilibrium, the high- and mid-quality firms choose the same level of s, say s_{HM}^*. Consider a consumer that observes this level of SMM spending, and the consumer's expected perceived quality based on their prior belief will then be $E(q)$. Therefore, $p_\Lambda^* = E(q) = \overline{q}$, where we use $\overline{q}$ to simplify the notation.

Thus, the ex-ante expected gross payoff from choosing s^*_{HM} for a firm of type H is:

$\pi^*(\Lambda, H; p^*_\Lambda, s^*_{HM}) = [r + (1-r) * \frac{s^*_{HM}}{1+s^*_{HM}}][p^*_\Lambda - c_H] - s^*_{HM}$.

Clearly, $M \leq p^*_\Lambda \leq H$. Since $\pi(\Lambda, H; p_\Lambda, s_{HM})$ is continuous in s, there exists an ε such that:

$\pi^*(H, H; p^*(H), s^*_{HM} + \varepsilon) > \pi^*(\Lambda, H; p^*_\Lambda, s^*_{HM})$.

which in turn implies that H will deviate. A similar line of reasoning can be used to eliminate $\{M, L\}$ pooling as well as the pooling equilibrium where $\{H, M, L\}$ pool together.

Next, let's suppose that the partial-pooling equilibrium where the high-quality and low-quality firms pool together, and the mid-quality firm separates, exists. We use s_{HL} to denote the level of SMM spending at which the high-quality and low-quality firms pool, and the mid-quality firm separates itself by investing s_M in social media marketing. Suppose there exist $s^*_{HL} \geq 0$ and $s^*_M \geq 0$, in the partial-pooling equilibrium, $\Lambda = \{L, H\}$, we have the following conditions:

$$\pi(\Lambda, H; p^*_\Lambda, s^*_{HL}) > \pi(M, H; p^*(M), s^*_M)$$

that is,

$$[r + (1-r) * \frac{s^*_{HL}}{1+s^*_{HL}}](p^*_\Lambda - c_H) - s^*_{HL} > [r + (1-r) * \frac{s^*_M}{1+s^*_M}](M - c_H) - s^*_M$$

Also, we know from above that, in the mid-quality profit maximization problem, $\pi(\Lambda, L; p^*_\Lambda, s^*_{HL}) \geq \pi(M, L; p^*_M, s^*_M)$ has to bind. In other words, $\pi(\Lambda, L; p^*_\Lambda, s^*_{HL}) = [r + (1-r) * \frac{s^*_M}{1+s^*_M}]M - s^*_M$. We can denote $A = r + (1-r) * \frac{s^*_M}{1+s^*_M}$, $B = r + (1-r) * \frac{s^*_{HL}}{1+s^*_{HL}}$, and substitute these into the inequality above. Then $\pi(\Lambda, H; p^*_\Lambda, s^*_{HL}) > \pi(M, H; p^*(M), s^*_M)$ is equivalent to:

$$B(\overline{q} - c_H) - s^*_{HL} > A(M - c_H) - s^*_M = \pi(\Lambda, L; p^*_\Lambda, s^*_{HL}) - AC_H = B\overline{q} - s^*_{HL} - Ac_H$$

.

Then we have $B < A$.

Also, in the partial-pooling equilibrium, the mid-quality firm has no incentive to mimic the high-quality and low-quality firms. In other words, we have the following condition:

$$\pi(M, M; p^*(M), s^*_M) \geq \pi(\Lambda, M; p^*_\Lambda, s^*_{HL})$$

Or,

$$A(M - c_M) - s^*_M \geq B(\overline{q} - c_M) - s^*_{HL}$$

Because we have

$$B\overline{q} - s^*_{HL} = \pi(\Lambda, L; p^*_\Lambda, s^*_{HL}) = AM - s^*_M$$

, we substitute this into the inequality above and have:

$$A(M - c_M) - s^*_M \geq AM - s^*_M - Bc_M$$

.

Then we have $A \leq B$, which is a conflict to what we obtained earlier, $B < A$.

Q.E.D.

C. The Information-Revelation Role of Social Media Marketing

C.1. Proof of Proposition 3

The separating equilibrium is similar to the one we obtain in Proposition 1.

Proposition 3. *In the separating equilibrium, the high-quality, mid-quality and low-quality firms spend on social media marketing at levels of* $s^*_H = \frac{M(H-1-r)-c_M(1-r)+\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)}}{2M}$, $s^*_M = M-1$ *and* $s^*_L = 0$*, respectively. The equilibrium prices are* H*,* M *and* L*, respectively.*

Proof. In a separating equilibrium, the firm could use SMM spending to signal its quality type. Suppose there exists a separating equilibrium, in which these three types spend at different levels of social media marketing. Let's denote the equilibrium SMM spending s_L, s_M and s_H. All three types maximize their profits by charging their complete information optimal prices $p(q) = q$, where $q \in \{L, M, H\}$. Since the low-quality firm's profit is always decreasing in s, that is, $\partial\pi(L, L; P(L), s)s = \frac{-s^2-2s-r}{(1+s)^2} < 0$. Therefore, the low-quality firm's complete-information optimal level of SMM spending is 0, and so is its optimal level of SMM spending in the separating equilibrium.

The mid-quality firm needs to solve the following profit maximization program.

$$\max_{s_M, p(M)} \pi(M, M; p(M), s_M) = [r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)[p(M) - c_M)] - s_M$$

$$s.t. \pi(M, L; p(M), s_M) \leq \pi^*(L, L; p^*(L), s^*_L)$$

$$\pi(L,M;p(L),s_L) \leq \pi(M,M;p(M),s_M)$$

$$\pi(H,M;p(H),s_H) \leq \pi(M,M;p(M),s_M)$$

$$\pi(M,H;p(M),s_M) \leq \pi^*(H,H;p^*(H),s^*_H)$$

where $\pi^*(L,L;p^*(L),s^*_L) = r$, $\pi(L,M;p(L),s_L) = [r + (1-r) * \frac{s_L}{1+s_L}][p(L) - c_M)] - s_L = r(1-c_M)$ and $\pi(M,L;p(M),s_M) = [r + (1-r) * \frac{s_M}{1+s_M}][p(M) - c_L] - s_M = [r + (1-r) * \frac{s_M}{1+s_M}]p(M) - s_M$.

Ignoring the second and third constraints, the Lagarangean function for the mid-quality firm's maximization program is given by:

$$L = ([r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)[p(M) - c_M)] - s_M) + \tag{C.1}$$

$$\gamma_1[[r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)p(M) - s_M - r] + \gamma_2[r(1-c_M) - \pi(M,M;p(M),s_M)] \tag{C.2}$$

The Kuhn-Tucker conditions are: $\frac{\partial L}{\partial s_M} = 0$, $[r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)p(M) - s_M \leq r$, $r(1-c_M) \leq \pi(M,M;p(M),s_M)$, $\gamma_1 \leq 0$, $\gamma_2 \leq 0$, $s_M \frac{\partial L}{\partial s_M} = 0$, $\gamma_1 \frac{\partial L}{\partial \gamma_1} = 0$, $\gamma_2 \frac{\partial L}{\partial \gamma_2} = 0$, $s_M \geq 0$.

At the least-cost quality separation (Zhao 2000, Kalra, Rajiv, and Srinivasan 1998), the mimicking constraint of the low-quality firm is binding while that of the mid-quality firm is not. This implies $\gamma_1 < 0$, $\frac{\partial L}{\partial \gamma_1} = 0$, $\gamma_2 = 0$, and $\frac{\partial L}{\partial \gamma_2} < 0$. That is, $[r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)p(M) - s_M - r = 0$; $\gamma_2 = 0$, and $r(1-c_M) - \pi(M,M;p(M),s_M) < 0$; Solving these for s_M and discarding an implausible solution $(-r)$, we get:

$$s^*_M = M - 1, P^*(M) = M$$

When $s_M > s^*_M$, $[r + (1-r) * \frac{s_M}{1+s_M}]p(M) - s_M < r$. The non-mimicking constraint for the low-quality firm is statisfied.

Also, the low-quality firm's optimal SMM spending and price are

$$s^*_L = 0, P^*(L) = 1$$

We also have, $\pi^*(M,M;p^*(M),s^*_M) = r - \frac{c_M(-1+M+r)}{M}$

Note that in the complete information case, the mid-quality firm's optimal SMM spending is $-1 + \sqrt{(M-c_M)(1-r)}$. It is straightforward to see that $-1 + \sqrt{(M-c_M)(1-r)} < -1 + \sqrt{(M-c_M)M} < -1 + \sqrt{M^2} = M - 1$. Therefore, in order to deter the low-quality firm from mimicking, the mid-quality firm has to increase its SMM spending to a level such that the low-quality firm does not have an incentive to mimic.

Then, we solve the following profit maximization program for the high-quality firm.

$$\max_{s_H, p(H)} \pi(H,H;p(H),s_H) = [r+(1-r)*\frac{s_H}{1+s_H}]\mathbb{1}(p(H)\le H)[p(H)-c_H)]-s_H$$

$$s.t.\pi(H,M;p(H),s_H)\le \pi^*(M,M;p^*(M),s_M^*)$$

$$\pi(M,H;p(M),s_M)\le \pi(H,H;p(H),s_H)$$

$$\pi(H,L;p(H),s_H)\le \pi^*(L,L;p^*(L),s_L^*)$$

$$\pi(L,H;p(L),s_L)\le \pi(H,H;p(H),s_H)$$

where $\pi^*(L,L;p^*(L),s_L^*) = r,\ \pi^*(M,M;p^*(M),s_M^*) = r - \frac{c_M(-1+M+r)}{M},\ \pi(H,M;p(H),s_H) = [r+(1-r)*\frac{s_H}{1+s_H}][p(H)-c_M)]-s_H,\ \pi(M,H;p(M),s_M) = [r+(1-r)*\frac{s_M}{1+s_M}][p(M)-c_H)]-s_M,\ \pi(H,L;p(H),s_H) = [r+(1-r)*\frac{s_H}{1+s_H}][p(H)-c_L]-s_H = [r+(1-r)*\frac{s_H}{1+s_H}]p(H)-s_H$, and $\pi(L,H;p(L),s_L) = [r+(1-r)*\frac{s_L}{1+s_L}][p(L)-c_H)]-s_L = r(1-c_H)$.

Ignoring the second and fourth constraints, the Lagarangean function for the high-quality firm's maximization program is given by:

$$L = [r+(1-r)*\frac{s}{1+s}]\mathbb{1}(p(H)\le H)[p(H)-c_H)]-s_H+ \tag{C.3}$$

$$\gamma_1([r+(1-r)*\frac{s_H}{1+s_H}]\mathbb{1}(p(H)\le H)[p(H)-c_M]-s_H-\pi^*(M,M;p^*(M),s_M^*)+ \tag{C.4}$$

$$\gamma_2([r+(1-r)*\frac{s_H}{1+s_H}]\mathbb{1}(p(H)\le H)p(H)-s_H-\pi^*(L,L;p^*(L),s_L^*) \tag{C.5}$$

After writing the Kuhn-Tucker conditions, we know among these four constraints in this optimization problem, the mimicking constraint of the mid-quality firm is binding while the others are not. This is because the high-quality firm needs to invest at a level higher than the mid-quality firm's optimal SMM spending (s_M^*) in order to deter it from mimicking. Thus, the low-quality firm finds it even less profitable to mimic and pretend to be the high-quality firm. This implies that

$$[r+(1-r)*\frac{s_H}{1+s_H}]\mathbb{1}(p(H)\le H)[p(H)-c_M]-s_H-\pi^*(M,M;p^*(M),s_M^*)=0.$$

Therefore, we have $s_H^* = \frac{M(H-1-r)-c_M(1-r)+\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)}}{2M}$ (we discard the other root because it is negative.),[1] and $P^*(H)=H$. When $s_H > s_H^*$, the mid-quality firm finds it not profitable to

[1] Note that $\sqrt{[c_M(r-1)-M(-1+H+r)]^2+4c_M M(H-M)} > M(H-1-r)-c_M(1-r)$. Therefore, even when $M(H-1-r)-c_M(1-r)<0$, that is $H<1+r+\frac{c_M}{M}(1-r)$, we still have $s_H^*>0$.

mimic the high-quality firm by investing at s_H. Therefore, the incentive compatibility (IC) constraint is satisfied.

In the separating equilibrium described in Proposition 1, consumer belief (e.g., the probability that the product is of the true quality) is given by $\mu(q|s,p)=1$ if $(s,p)=(s_q^*,q)$, where $q\in\{L,M,H\}$, and for other $(s,p)\neq(s_q^*,q)$, $\mu(L|s,p)=1$.

C.2. Proof of Corollary 3

This corollary is similar to the one we obtained in Corollary 1.

Corollary 2. *The partial-pooling equilibria where either the high-quality and the mid-quality firms pool together or the mid-quality and the low-quality firms pool together do not exist.*

Proof. Suppose in the partial pooling equilibrium, the high- and mid-quality firms choose the same level of s, say s_{HM}^*. Consider a consumer that observes this level of SMM spending. The consumer's belief that the firm is type H will then be $\frac{\lambda+s_{HM}^*}{1+s_{HM}^*}$.

Thus, the ex-ante expected gross payoff from choosing s_{HM}^* for a firm of type H is:

$\pi^*(\Lambda,H;p_\Lambda^{H*},s_{HM}^*)=[r+(1-r)*\frac{s_{HM}^*}{1+s_{HM}^*}][p_\Lambda^{H*}-c_H]-s_{HM}^*$.

Clearly, $p_\Lambda^H<H$. Since $\pi(\Lambda,H;p_\Lambda^H,s_{HM})$ is continuous in s, there exists an ε such that:

$\pi^*(H,H;p^*(H),s_{HM}^*+\varepsilon)>\pi^*(\Lambda,H;p_\Lambda^{H*},s_{HM}^*)$.

which in turn implies that H will deviate. A similar line of reasoning can be used to eliminate $\{M,L\}$ pooling as well as the pooling equilibrium where $\{H,M,L\}$ pool together.

C.3. Proof of Proposition 4

Proposition 4. *When $\underline{H}\leq H\leq\overline{H}$, a partial-pooling equilibrium exists where the high-quality and low-quality firms pool together and spend zero on social media marketing, whereas the mid-quality firm separates, namely, $(s_\Lambda^*,p_\Lambda^*)=(0,A)$, $(s_M^*,p_M^*)=(s_M^*,M)$, where*

$$s_M^*=\begin{cases}1/2(-1+M-Ar+\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}), & \text{if } A\leq M\\ 1/2(-1+M-Ar-\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}), & \text{if } M<A\leq\frac{1+M-2\sqrt{M-Mr}}{r}\\ -1+\sqrt{(M-c_M)(1-r)}, & \text{if } A>\frac{1+M-2\sqrt{M-Mr}}{r}\end{cases}\tag{C.6}$$

$A=\lambda+(1-\lambda)H$, $\underline{H}=\frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_H)-s_M^*)+c_H-\lambda]$ *and* $\overline{H}=\frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_M)-s_M^*)+c_M-\lambda]$.

Proof. In a partial-pooling equilibrium, in which $\Lambda = \{H, L, \}$, the high-quality and low-quality firms spend the same amount on SMM while the mid-quality firm spends at a different level to signal its quality type. Let's denote the equilibrium SMM spending and prices as $(s_{HL},\, p_\Lambda)$ and $(s_M,\, p_M)$.

The low-quality firm needs to solve the following profit maximization program.

$$\max_{s_{HL}, p_\Lambda^L} \pi(\Lambda, L; p_\Lambda^L, s_{HL}) = [r + (1-r) * \frac{s_{HL}}{1+s_{HL}}]\mathbb{1}(p_\Lambda^L \leq \frac{\lambda+s}{1+s} + \frac{1-\lambda}{1+s}H)[p_\Lambda^L - c_L] - s_{HL} \tag{C.7}$$

$$s.t.\ \pi(\Lambda, L; p_\Lambda^L, s_{HL}) \geq \pi(M, L; p(M), s_M) \tag{C.8}$$

$$\pi^*(M, M; p^*(M), s_M^*) \geq \pi(\Lambda, M; p_\Lambda^M, s_{HL}), \tag{C.9}$$

Because consumers' posterior assessment of the probability of dealing with a mid-quality firm is 1 after observing $(s_M,\ p_M)$, the firm could set a price at $p^*(M) = M$. If the product price does not exceed M, consumers will buy, and otherwise not. The high-quality and low-quality firms employ the same strategies, and thus consumers turn to rely on the external information. For the low-quality firm, the external information signal indicates with a probability of $\lambda' = \frac{\lambda+s}{1+s}$ that it is the low-quality firm. Therefore, consumers' reservation price for the product is $p_\Lambda^{*L} = \frac{\lambda+s}{1+s} + \frac{1-\lambda}{1+s}H$. Similarly, for the high-quality firm, consumers' reservation price based on the external information signal is $p_\Lambda^{*H} = \frac{\lambda+s}{1+s}H + \frac{1-\lambda}{1+s}$. We note that $p_\Lambda^{*L} \leq p_\Lambda^{*H}$ due to $\lambda \geq \frac{1}{2}$ when $s = 0$. Therefore, $p_\Lambda^* = p_\Lambda^{*L}$, because a price higher than this would result in zero sales for the low-quality firm.

Note that $\partial p_\Lambda^{L*}/\partial s = \frac{(1-\lambda)(1-H)}{(1+s)^2} < 0$. While SMM spending expands the market size, it also imposes some pricing pressure for the low-quality firm since the price it can charge declines. $\partial \pi(\Lambda, L; p_\Lambda^{L*}, s)/\partial s = -((1 - r + s + rs + 3s^2 + s^3 - H(-1+\lambda)(-1+2r+s) + \lambda(-1+2r+s))/(1+s)^3) < 0$ for all $s > 0$ if $r \geq \frac{\lambda+(1-\lambda)H-1}{2[\lambda+(1-\lambda)H]-1} = \frac{A-1}{2A-1}$, where $A = \lambda + (1-\lambda)H$.[2] This implies that when the informed consumer base is large enough, the market expansion effect is dominated by the information revelation effect. Therefore, the low-quality firm finds it not profitable to spend on social media marketing. The low-quality firm's first best option is not to spend on social media marketing.

The mid-quality firm needs to spend s_M in order to deter the low-quality firm from mimicking. Its profit maximization problem is:

[2] For $r < \frac{\lambda+(1-\lambda)H-1}{2[\lambda+(1-\lambda)H]-1}$, $\pi(\Lambda, L; p_\Lambda(L), s_{HL})$ initially increases and then quickly decreases after s reaches a level even smaller than $1-2r$. For the consistency and simplicity of the model, we focus on the case when $r \geq \frac{\lambda+(1-\lambda)H-1}{2[\lambda+(1-\lambda)H]-1}$.

$$\max_{s_M, p(M)} \pi(M, M; p(M), s_M) = [r + (1-r) * \frac{s_M}{1+s_M}] \mathbb{1}(p(M) \leq M)[p(M) - c_M] - s_M \tag{C.10}$$

$$s.t.\ \pi^*(\Lambda, L; p^*_\Lambda, 0) \geq \pi(M, L; p(M), s_M) \tag{C.11}$$

$$\pi(M, M; p(M), s_M) \geq \pi(\Lambda, M; p^*_\Lambda, 0) \tag{C.12}$$

The lagarangean function for the mid-quality firm's maximization program is given by:

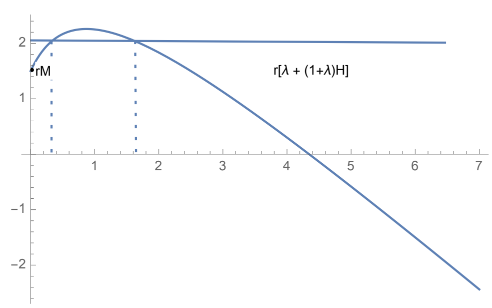
$$L = [r + (1-r) * \frac{s_M}{1+s_M}] \mathbb{1}(p(M) \leq M)[p(M) - c_M] - s_M + \tag{C.13}$$

$$\gamma_1(\pi^*(\Lambda, L; p^*_\Lambda, 0) - \pi(M, L; p(M), s_M)) + \tag{C.14}$$

$$\gamma_2(\pi(M, M; p(M), s_M) - \pi(\Lambda, M; p^*_\Lambda, 0)) \tag{C.15}$$

Among these two constraints, at the least-cost quality separation, the mimicking-constraint of the low-quality is binding, while that of the mid-quality firm is not. This implies that $\gamma_1 < 0$, $\pi^*(\Lambda, L; p^*_\Lambda, 0) - \pi(M, L; p(M), s_M) = 0$; $\gamma_2 = 0$ and $\pi(M, M; p(M), s_M) - \pi(\Lambda, M; p^*_\Lambda, 0) \neq 0$. Solving the equation of $r[\lambda + (1-\lambda)H] = [r + (1-r) * \frac{s_M}{1+s_M}] * M - s_M$, which is depicted in Figure 1 below, we get:

Figure 1: Profit curve of $\pi(M, L; p(M), s_M)$, when $M = 5$, $r = 0.3$



$$s^*_M = \begin{cases} 1/2(-1 + M - Ar + \sqrt{(-1 + M - Ar)^2 + 4(-Ar + Mr)}), & \text{if } A \leq M \\ 1/2(-1 + M - Ar \pm \sqrt{(-1 + M - Ar)^2 + 4(-Ar + Mr)}), & \text{if } M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r} \\ \text{any positive s}, & \text{if } A > \frac{1+M-2\sqrt{M-Mr}}{r} \end{cases} \tag{C.16}$$

where $A = [\lambda + (1-\lambda)H]$. We can interpret A as the reservation price for the low-quality under pooling, given the presence of existing social media information. To further eliminate solutions considering that the mid-quality firm needs to maximize its profit, we derive the mid-quality firm's optimal level of SMM spending:

$$s_M^* = \begin{cases} 1/2(-1+M-Ar+\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}), & \text{if } A \leq M \\ 1/2(-1+M-Ar-\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}), & \text{if } M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r} \\ -1+\sqrt{(M-c_M)(1-r)}, & \text{if } A > \frac{1+M-2\sqrt{M-Mr}}{r} \end{cases} \quad \text{(C.17)}$$

Note that $\pi(M,M;p(M),0) < \pi(M,L;p(M),0)$. When C.11 is satisfied, that is, $\pi^*(\Lambda,L;p_\Lambda^*,0) \geq \pi^*(M,L;p^*(M),s_M^*)$, we have $\pi^*(\Lambda,L;p_\Lambda^*,0) \geq \pi^*(M,M;p^*(M),s_M^*)$.

Now, we need the following incentive compatibility constraints for the high-quality firm to be satisfied, which are: (1) the high-quality firm does not find it profitable to mimic the mid-quality firm by spending s_M^* and (2) the mid-quality firm does not find it profitable to pool with the high-quality and low-quality firms and spend zero on social media marketing. Mathematically, these conditions are:

$$\pi^*(\Lambda,H;p_\Lambda^*,0) \geq \pi(M,H;p^*(M),s_M^*)$$

$$\pi^*(M,M;p^*(M),s_M^*) \geq \pi(\Lambda,M;p_\Lambda^*,0)$$

That is,

$$r[\lambda+(1-\lambda)H-c_H] \geq [r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_H)-s_M^* \quad \text{(C.18)}$$

$$[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_M)-s_M^* \geq r[\lambda+(1-\lambda)H-c_M] \quad \text{(C.19)}$$

As shown earlier, we have s_M^* by solving $r[\lambda+(1-\lambda)H] = [r+(1-r)*\frac{s_M^*}{1+s_M^*}]*M-s_M^*$. We substitute s_M^* to C.18 and C.19 above, and we have the following condition:

$$\underline{H} \leq H \leq \overline{H}$$

where $\underline{H} = \frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_H)-s_M^*)+c_H-\lambda]$ and $\overline{H} = \frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_M)-s_M^*)+c_M-\lambda]$. Because $s_M^* > 0$ and $c_H > c_M$, we have $\underline{H} < \overline{H}$.

This condition suggests that H has to lie within a moderate range. If it is too high, it might incentivize the mid-quality firm to pool with the high-quality and low-quality firm. If it is too low, pooling with the low-quality firm does not guarantee enough profit for the high-quality firm. Therefore, the high-quality firm might be better off with signaling.

As we could see, when A is relatively small (or $A \le M$), the mid-quality firm needs to spend more on social media marketing, compared to the case where A is relatively large (or $M < A \le \frac{1+M-2\sqrt{M-Mr}}{r}$). Since A is decreasing in λ, when there are more informed consumers, the benefit of pooling with the high-quality firm decreases for the low-quality firm because a significant fraction of consumers know the exact quality of the firm. Therefore, the low-quality firm has less of an incentive to pool with the high-quality firm and more incentive to imitate the mid-quality firm. Hence, the mid-quality firm needs to spend more on social media marketing in order to deter the low-quality firm from mimicking to a level that the signaling cost of the low-quality firm is too high. As A further increases (or $A > \frac{1+M-2\sqrt{M-Mr}}{r}$), the pooling benefit for the low-quality firm is always greater than imitating the mid-quality firm. Therefore, the mid-quality firm could choose its first optimal level of SMM spending to maximize its profit as in a complete-information monopoly case.

In this partial-pooling equilibrium, consumer belief (e.g., the probability that the product is of the true quality) is given by $\mu(M|s,p)=1$ if $(s,p)=(s_M^*,M)$, $\mu(q|s,p)=\lambda$ if $(s,p)=(0,A)$ where $q\in\{L,H\}$, and for other $(s,p)\ne\{(0,A),(s_M^*,M)\}$, $\mu(L|s,p)=1$.

C.4. Proof of Proposition 5

Proposition 5. *Denote* $\lambda_1=\frac{H-M}{H-1}$.

(1) When $\lambda>\lambda_1=\frac{H-M}{H-1}$*, the mid-quality firm spends more on social media marketing in the partial-pooling equilibrium as the precision of the external signal increases, that is,* $\partial s_M^*/\partial\lambda>0$.

(2) When $\lambda<\lambda_1$*, the mid-quality firm spends less on social media marketing in the partial-pooling equilibrium as the precision of the external signal increases, that is,* $\partial s_M^*/\partial\lambda\le 0$.

Proof. It is straightforward to show parts (1) and (2) from Equation C.17. It is easy to see that $\partial A/\partial\lambda<0$, $\partial s_M^*/\partial A<0$ if $A\le M$, $\partial s_M^*/\partial A>0$ if $M<A<\frac{1+M-2\sqrt{M-Mr}}{r}$ and $\partial s_M^*/\partial A=0$ if $A>\frac{1+M-2\sqrt{M-Mr}}{r}$.

C.5. Proof of Proposition 6

Proposition 6. *In the partial-pooling equilibrium, the mid-quality firm spends less on social media marketing as more consumers become aware of the product, that is,* $\partial s_M^*/\partial r<0$.

Proof. When $A>\frac{1+M-2\sqrt{M-Mr}}{r}$, it is straightforward to see that $\partial s_M^*/\partial r<0$.

When $A\le M$, we have $s_M^*=1/2(-1+M-Ar+\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)})$. Take the derivative with respect to r, and we have $\partial s_M^*/\partial r=\frac{1}{2}[-A+\frac{1}{\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}}(A^2r-MA-A+2M)]$. Since $A^2((-1+M-Ar)^2+4(-Ar+Mr))-(A^2r-MA-A+2M)^2=4M(M-A)(1-A)<0$, we have $\partial s_M^*/\partial r<0$.

When $M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r}$, $\partial s_M^*/\partial r = \frac{1}{2}[-A - \frac{1}{\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}}(A^2r - MA - A + 2M)]$.

Because $M < A < \frac{1+M-2\sqrt{M-Mr}}{r}$, we have $A^2r - MA - A + 2M < A^2r - MA + A < A[2 - 2\sqrt{M(1-r)}] < 0$. From the result above, we know $A^2((-1+M-Ar)^2 + 4(-Ar+Mr)) > (A^2r - MA - A + 2M)^2$ when $M < A$. Therefore, we have $\partial s_M^*/\partial r = \frac{1}{2}[-A - \frac{1}{\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)}}(A^2r - MA - A + 2M)] < 0$.

C.6. Proof of Proposition 7

Proposition 7. *The mid-quality firm spends less on social media marketing in the partial-pooling equilibrium than in the separating equilibrium, that is,* $s_M^* < s_M^{*sep}$.

Proof. Recall that $s_M^{*sep} = M - 1$. When $A > \frac{1+M-2\sqrt{M-Mr}}{r}$, it is straightforward to see that $s_M^* = -1 + \sqrt{(M-c_M)(1-r)} < s_M^{*sep}$.

When $A \leq M$, we have $s_M^* = 1/2(-1 + M - Ar + \sqrt{(-1+M-Ar)^2 + 4(-Ar+Mr)})$. Let's compare s_M^* and s_M^{*sep}. $s_M^* - s_M^{*sep} = 1/2(1 - M - Ar + \sqrt{(-1+M-Ar)^2 + 4(-Ar+Mr)})$. Because $(1-M-Ar)^2 - (-1 + M - Ar)^2 - 4(-Ar+Mr) = 4r(M(1-A) - A) < 0$ given $A > 1$, we have $s_M^* < s_M^{*sep}$.

When $M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r}$, we have $s_M^* = 1/2(-1 + M - Ar - \sqrt{(-1+M-Ar)^2 + 4(-Ar+Mr)})$. Clearly, $s_M^* < s_M^{*sep}$.

C.7. Proof of Proposition 8

Definition 1. Let $v^*(q)$ be a vector of PBE payoffs for the sender. For any $s \in S$, define

$$J(s) = \{q | v^*(q) > \pi(H, q; p(H), s)\} \tag{C.20}$$

A pure strategy PBE satisfies the intuitive criterion iff there does not exist $s \in S$, and $q \in Q$, s.t.

$$v^*(q) < \pi(min\{Q \backslash J(S)\}, q; p((min\{Q \backslash J(S)\}), s) \tag{C.21}$$

for $\{Q \backslash J(s)\} \neq \emptyset$.

Proposition 8. *The partial-pooling equilibrium survives the intuitive criterion iff* $\dot{s}_M^* > \dot{s}_H^*$ *(*$\dot{s}_M^*$ *is the critical value that makes* $\pi(M, M; p(M), s_M^*) = \pi(H, M; p(H), \dot{s}_M^*)$*, and* $\dot{s}_H^*$ *is the critical value that makes* $\pi(\Lambda, H; p_\Lambda^*, 0) = \pi(H, H; p(H), \dot{s}_H^*)$*).*

Proof. ($\Rightarrow$) In order to characterize the condition on whether the partial-pooling equilibrium (0, s_M^*, 0) satisfies the intuitive criterion, we denote $\dot{s}_L^*$, $\dot{s}_M^*$, $\dot{s}_H^*$ that solve the following equations:

$$\pi^*(\Lambda, L; p_\Lambda^*, 0) = \pi(H, L; p(H), \dot{s}_L^*) \tag{C.22}$$

$$\pi^*(M,M;p^*(M),s_M^*)=\pi(H,M;p(H),\dot{s}_M^*) \tag{C.23}$$

$$\pi^*(\Lambda,H;p_\Lambda^*,0)=\pi(H,H;p(H),\dot{s}_H^*) \tag{C.24}$$

Or equivalently,

$$[r+(1-r)*\frac{\dot{s}_L^*}{1+\dot{s}_L^*}]H-\dot{s}_L^*=r[\lambda+(1-\lambda)H]$$

$$[r+(1-r)*\frac{\dot{s}_M^*}{1+\dot{s}_M^*}](H-c_M)-\dot{s}_M^*=[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_M)-s_M^*$$

$$[r+(1-r)*\frac{\dot{s}_H^*}{1+\dot{s}_H^*}](H-c_H)-\dot{s}_H^*=r[\lambda+(1-\lambda)H-c_H]$$

These critical values represent the threshold signal that agents of any type would ever be willing to send under the best possible beliefs (if they were believed to be the high-quality type with probability 1).

Given the partial-pooling equilibrium $S=(0,s_M^*,0),P=(p_\Lambda^*,M,p_\Lambda^*)$, $J(S)$ is the set of types that satisfy

$$\begin{cases} \pi^*(\Lambda,q;p_\Lambda^*,0)>\pi(H,q;p(H),s) & \text{for } q\in\{L,H\} \\ \pi^*(M,M;p^*(M),s_M^*)>\pi(H,M;p(H),s) & \text{for } q=M \end{cases}$$

Case 1. When $M>A$, $s_M^*=1/2(-1+M-Ar+\sqrt{(-1+M-Ar)^2+4(-Ar+Mr)})$. This implies that in the neighborhood of s_M^*, $\partial\pi(M,L;p(M),s)/\partial s\le 0$.

Due to single-crossing property, we have $\dot{s}_L^*<\dot{s}_M^*$.

There are three mutually exclusive cases:

(a) $\dot{s}_L^*<\dot{s}_M^*<\dot{s}_H^*$

(b) $\dot{s}_L^*<\dot{s}_H^*<\dot{s}_M^*$

(c) $\dot{s}_H^*<\dot{s}_L^*<\dot{s}_M^*$

Under case (a):

$$J(S)=\begin{cases} \emptyset & if s\le\dot{s}_L^* \\ \{L\} & if \dot{s}_L^*<s\le\dot{s}_M^* \\ \{L,M\} & if \dot{s}_M^*<s\le\dot{s}_H^* \\ \{L,M,H\} & if s>\dot{s}_H^* \end{cases}$$

Only the second and third cases impose any restrictions on equilibrium beliefs. Let's first consider the third case. For signals $s\in(\dot{s}_M^*,\dot{s}_H^*)$, the right-hand side of C.21 becomes $\pi(H,q;p(H),s)$. By definition, the high-quality firm is indifferent from spending $\dot{s}_H^*$ for a payoff of $\pi(H,H;p(H),\dot{s}_H^*)$ and spending 0 on social media marketing. The belief satisfying the intuitive criterion puts the probability of 1 on $q=H$. Since $s<\dot{s}_H^*$ and in the neighborhood of $\dot{s}_H^*$, $\partial\pi(H,H;p(H),\dot{s}_H^*)/\partial s<0$, the high-quality firm strictly prefers s to s_H^*. Therefore, this case fails the intuitive criterion.

Under case (b):

$$J(S)=\begin{cases}\emptyset & if s\leq \dot{s}_L^* \\ \{L\} & if \dot{s}_L^*<s\leq \dot{s}_H^* \\ \{L,H\} & if \dot{s}_H^*<s\leq \dot{s}_M^* \\ \{L,M,H\} & if s>\dot{s}_M^*\end{cases}$$

Again, only the second and third cases impose any restrictions on equilibrium beliefs. In both cases, the right-hand side of C.21 becomes $\pi(M,q;p(M),s)$.

Because, $[r+(1-r)*\frac{\dot{s}_L^*}{1+\dot{s}_L^*}]H-\dot{s}_L^*=r[\lambda+(1-\lambda)H]=[r+(1-r)*\frac{s_M^*}{1+s_M^*}]M-s_M^*$, we could derive $s_M^*<\dot{s}_L^*$. We know $\dot{s}_L^*<\dot{s}_H^*$ given by the assumption. Next, we will check whether any types have incentives to deviate from the partial-pooling equilibrium.

For the low-quality type, by the equilibrium definition, we have

$$\pi(\Lambda,L;p_\Lambda,0)>\pi(M,L;p(M),s_M^*)$$

Because $s>\dot{s}_L^*>s_M^*$, and $\partial\pi(M,L;p(M),s)/\partial s<0$ in the neighborhood of s_M^*, we have $\pi(M,L;p(M),s_M^*)>\pi(M,L;p(M),s)$. Therefore, the low-quality type won't deviate to s.

For the mid-quality type, by $\partial\pi(M,M;p(M),s)/\partial s<0$ in the neighborhood of s_M^* and $s>s_M^*$, we have

$$\pi(M,M;p(M),s_M^*)>\pi(M,M;p(M),s)$$

For the high-quality type, by the equilibrium definition, we have

$$\pi(\Lambda,H;p_\Lambda^*,0)>\pi(M,H;p(M),s_M^*)$$

Since $\partial\pi(M,H;p(M),s)/\partial s<0$ in the neighborhood of s_M^* and $s>s_M^*$, we have

$$\pi(M,H;p(M),s_M^*)>\pi(M,H;p(M),s)$$

Therefore, no type will ever deviate to s.

Under case (c):

$$J(S) = \begin{cases} \emptyset & if s \leq \dot{s}_L^* \\ \{H\} & if \dot{s}_H^* < s \leq \dot{s}_L^* \\ \{L, H\} & if \dot{s}_L^* < s \leq \dot{s}_M^* \\ \{L, M, H\} & if s > \dot{s}_M^* \end{cases}$$

Again, only the second and third cases impose any restrictions on equilibrium beliefs. For the second case, the right-hand side of C.21 becomes $\pi(L, q; p(L), s)$. No type will have an incentive to deviate to such an s.

For the third case, the right-hand side of C.21 becomes $\pi(M, q; p(M), s)$. Since $s > \dot{s}_L^* > s_M^*$, no type will deviate to s (following a similar argument to case (b)).

To summarize, when $\dot{s}_M^* > \dot{s}_H^*$, this partial-pooling equilibrium survives the intuitive criterion.

Case 2. When $M < A$, $s_M^* = 1/2(-1 + M - Ar - \sqrt{(-1 + M - Ar)^2 + 4(-Ar + Mr)})$ or $s_M^* = -1 + \sqrt{(M - c_M)(1 - r)}$. In both cases, because $\pi(H, L; p(H), 0) = rH > rA$, we have $\dot{s}_L^* > s_M^*$. We also have $\partial\pi(H, L; p(H), s)/\partial s < 0$ in the neighborhood of $\dot{s}_L^*$ (similar to case 1). Again, due to single-crossing property, we have $\dot{s}_L^* < \dot{s}_M^*$. Similar to case 1, we can derive that when $\dot{s}_M^* > \dot{s}_H^*$, this partial-pooling equilibrium survives the intuitive criterion.

We could solve for $\dot{s}_M^*$ and $\dot{s}_H^*$, using C.23 and C.24, and get: $\dot{s}_H^* = 1/2(-1 - c_H + H + \sqrt{(-1 - c_H + H - rB)^2 + 4(-c_H r + Hr - rB)} - rB)$ where $B = \lambda H + (1 - \lambda)$ and $\dot{s}_M^* = 1/2(-1 - c_M + H + \sqrt{(-1 - C - c_M + H)^2 + 4(-C - c_M r + Hr)} - C)$ where $C = Ar - \frac{c_M}{M}(Ar + s_M^*)$.

C.8. Proof of Proposition 9

Proposition 9. *If the partial-pooling equilibrium survives the intuitive criterion, it Pareto-dominates the separating equilibrium.*

Proof. Let's compare the optimal payoff each type of firms receives in the separating equilibrium and the partial-pooling equilibrium.

For the low-quality firm, in both equilibria, it spends zero on social media marketing. However, by pooling with the high-quality firm, the low-quality firm could charge a higher price than in the separating equilibrium. In other words, $r \leq r[\lambda + (1 - \lambda)H]$. Therefore, the low-quality firm is better off in the partial-pooling equilibrium.

For the mid-quality firm, $s_M^{*sep} = M - 1$ and $\pi^*(M, M; p^*(M), s_M^{*sep})^{*sep} = r - \frac{c_M(-1+M+r)}{M}$ (note: $\pi^*(M, M; p^*(M), s_M^{*sep})^{*sep} \geq 0$, when $\frac{c_M(M-1)}{M-c_M} \leq r \leq 1$).[3] In all three cases, whether $M \geq A$, $M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r}$ or $A > \frac{1+M-2\sqrt{M-Mr}}{r}$, the mid-quality firm makes a higher profit in the partial-pooling equilibrium than in the separating equilibrium. First, let's consider $A > \frac{1+M-2\sqrt{M-Mr}}{r}$, $s_M^* = -1 + \sqrt{(M-c_M)(1-r)}$ is the complete-information profit-maximizing level of SMM spending. Therefore, the mid-quality firm makes a higher profit in the partial-pooling equilibrium. When $M < A \leq \frac{1+M-2\sqrt{M-Mr}}{r}$, $s_M^* = 1/2(-1 + M - Ar - \sqrt{(-1+M-Ar)^2 + 4(-Ar+Mr)})$. Due to the fact that $\pi(M, M; p(M), s)$ lies below $\pi(M, L; p(M), s)$ and $r[\lambda + (1-\lambda)H] > r$, we have $\pi^*(M, M; p^*(M), s_M^*) > \pi^*(M, M; p^*(M), s_M^{*sep})$.

Similarly, when $M \geq A$, $s_M^* = 1/2(-1 + M - Ar + \sqrt{(-1+M-Ar)^2 + 4(-Ar+Mr)})$, by the same argument as above, we have $\pi^*(M, M; p^*(M), s_M^*) > \pi^*(M, M; p^*(M), s_M^{*sep})$. Therefore, the mid-quality firm is better off in the partial-pooling equilibrium.

Lastly, let's compare the high-quality firm's equilibrium payoff under both equilibria.

Let's suppose the partial-pooling equilibrium does not Pareto-dominate the separating equilibrium. Since we have shown above that both the low-quality and mid-quality firm benefit in the partial pooling equilibrium, it must be the high-quality firm who is worse off. Let's suppose that the high-quality firm deviates and sends s_H^{*sep} - its equilibrium of SMM spending in separating equilibrium. Consumers would believe it is from the high-quality firm with probability 1 because the low-quality and mid-quality firm would never be willing to spend this amount. Since the high-quality firm receives a payoff greater than its partial pooling payoff, the high-quality firm has an incentive to deviate from the partial-pooling equilibrium by playing according to the separating equilibrium. Then, the partial-pooling equilibrium must not pass the Intuitive Criterion. Therefore, we have $\pi^*(H, H; p^*(H), s_H^{*sep}) < r(\lambda + (1-\lambda)H - c_H)$, that is the high-quality firm is also better off in the partial-pooling equilibrium. Q.E.D.

D. A Continuous-Type Model

D.1. Proof of Proposition 10

Proposition 10. *In the continuum-of-type model,*

(1) there is a unique separating perfect Bayesian equilibrium, in which the lowest-type firm spend its full-information optimal level of s on SMM, that is $s_{\underline{q}}^* = s_{\underline{q}}^c$, *and for all* $q \in (\underline{q}, \overline{q}]$, *the firm spends* s_q^* *on SMM. The firm charges* $p^*(q) = q$.

[3] For notation simplicity, we use the superscript "sep" to denote the equilibrium choices in the separating equilibrium

(2) a partial-pooling equilibrium exists, in which firm types $q \in [\underline{q}, q_1) or (q_2, \overline{q}]$ *choose* $(0,\ p^*_\Lambda)$ *and firm types* $q \in [q_1, q_2]$ *choose* $(s^*_q,\ q)$*.*

Proof. It has been shown in the paper.

E. Extension to a Two-Period Model

In this section, we extend our model to a two-period one. In the two-period model, a unit mass of customers enters the market at the beginning of period 1 and lives for two periods. As discussed earlier, a monopoly firm carries a product of quality unknown to the consumers. In period 1, the firm first learn about its own type, and makes its SMM spending which will remain the same in period 2. Among the aware consumers, they receive an external information signal (e.g. online WOM) the precision of which will be affected by the amount of SMM spending, as well as observe the firms' spending in SMM, and in turn they form an expectation of the product quality. The best price that the firm can set is equal to consumers' reservation value. In period 2, the true quality is revealed to consumers who purchased the product in the period 1. Therefore, if the price is higher than their willingness to pay given the true quality, some consumers will choose not to buy anymore in period 2. Note that each customer consumes one product at most in each period and we assume all the decisions are made in period 1. The separating equilibrium remains the same in that the true quality can be correctly inferred from the SMM spending in the period 1 which leads to no discrepancy between the expected quality and the true quality. For instance, in the separating equilibrium, the low-quality firm needs to solve the following optimizing program by maximizing the sum of profits across two periods (see below), which is equivalent to maximizing one-period's profit.

$$\max_{s_L, p(L)} \pi(L, L; p(L), s_L) = 2[r + (1-r) * \frac{s_L}{1+s_L}](p(L) - c_L) - 2s_L \tag{E.1}$$

$$s.t.\ \pi(L, L; p(L), s_L) \geq \pi(M, L; p^*(M), s^*_M) \tag{E.2}$$

$$\pi(L, L; p(L), s_L) \geq \pi(H, L; p^*(H), s^*_H) \tag{E.3}$$

$$\pi(M, M; p^*(M), s^*_M) \geq \pi(L, M; p^*(L), s^*_L) \tag{E.4}$$

$$\pi(H, H; p^*(H), s^*_H) \geq \pi(L, H; p^*(L), s^*_L), \tag{E.5}$$

Therefore, we can derive the same separating equilibrium as in the one-period model.

Next, we turn our attention to the partial-pooling equilibrium. In the partial-pooling equilibrium, the low-quality firm and the high-quality firm spend the same amount on social media marketing. Although the low-quality firm is able to gain some pricing advantage by pooling with high-quality firm in the first period, its true quality will be revealed to consumers in period two through consumption. Therefore, the low-quality firm maximizes profits from both periods by choosing the optimal SMM spending and price:

$$\max_{s_{HL},p_\Lambda^L} \pi(\Lambda,L;p_\Lambda^L,s_{HL}) = [r+(1-r)*\frac{s_{HL}}{1+s_{HL}}]\mathbb{1}(p_\Lambda^L \le \frac{\lambda+s}{1+s}+\frac{1-\lambda}{1+s}H)[p_\Lambda^L - c_L] - s_{HL} + \mathbb{1}(p_\Lambda^L \le L)[r+(1-r)*\frac{s_{HL}}{1+s_{HL}}][p_\Lambda^L - c_L] - s_{HL} \tag{E.6}$$

$$s.t.\ \pi(\Lambda,L;p_\Lambda^L,s_{HL}) \ge \pi(M,L;p(M),s_M) \tag{E.7}$$

$$\pi^*(M,M;p^*(M),s_M^*) \ge \pi(\Lambda,M;p_\Lambda^M,s_{HL}), \tag{E.8}$$

The low-quality firm faces a pricing choice between L and $\frac{\lambda+s}{1+s}+\frac{1-\lambda}{1+s}H$. By charging the former one, the low-quality firm could secure sales from both periods, whereas the latter price is greater than consumer's willingness to pay in period 2, which is L. If the low-quality firm chooses L, the maximization function above turns to $\max_{s_{HL}} \pi(\Lambda,L;p_\Lambda^L,s_{HL}) = 2[r+(1-r)*\frac{s_{HL}}{1+s_{HL}}][L-c_L]-2s_{HL}$, subject to conditions A.7 and A.8. Given that $\frac{\partial\pi(\Lambda,L;p_\Lambda^L,s_{HL})}{\partial s_{HL}} < 0$ for $s>0$, the optimal $s_{HL}^*=0$ and $\pi^*(\Lambda,L;p_\Lambda^L,s_{HL})=2r$

On the other hand, if the low-quality firm chooses $\frac{\lambda+s}{1+s}+\frac{1-\lambda}{1+s}H$, the maximization problem becomes $\max_{s_{HL}} \pi(\Lambda,L;p_\Lambda^L,s_{HL}) = [r+(1-r)*\frac{s_{HL}}{1+s_{HL}}][\frac{\lambda+s}{1+s}+\frac{1-\lambda}{1+s}H-c_L]-2s_{HL}$ subject to conditions A.7 and A.8. $\frac{\partial\pi(\Lambda,L;p_\Lambda^L,s_{HL})}{\partial s_{HL}} < 0$ for all $s>0$ if $r > \frac{\lambda+(1-\lambda)H-2}{2[\lambda+(1-\lambda)H]-1}$. Therefore, when $r > \frac{\lambda+(1-\lambda)H-1}{2[\lambda+(1-\lambda)H]-1} = \frac{A-1}{2A-1}$, $\frac{\partial\pi(\Lambda,L;p_\Lambda^L,s_{HL})}{\partial s_{HL}} < 0$ still holds. The optimal $s_{HL}^*=0$ and $\pi^*(\Lambda,L;p_\Lambda^L,s_{HL}) = r(\lambda+(1-\lambda)H)$. By comparing profits under two pricing decisions, we can derive that when $\lambda < \frac{H-2}{H-1}$, the low-quality firm is better off choosing $p_\Lambda^{L*} = \lambda+(1-\lambda)H$. Otherwise, it sets price at L.

In this partial-pooling equilibrium, the mid-quality firm tries to separate from both the low-quality and high-quality firm by choosing a different amount of sMM spending. Because consumers successfully infer the true type in the first period for the mid-quality firm, the same group of consumers will continue purchase the product in period 2. Therefore, the mid-quality firm maximizes its profit from both periods as follows:

$$\max_{s_M,p(M)} \pi(M,M;p(M),s_M) = 2[r+(1-r)*\frac{s_M}{1+s_M}]\mathbb{1}(p(M)\le M)[p(M)-c_M]-2s_M \tag{E.9}$$

$$s.t.\ \pi^*(\Lambda,L;p_\Lambda^{L*},0) \ge \pi(M,L;p(M),s_M) \tag{E.10}$$

$$\pi(M,M;p(M),s_M) \ge \pi(\Lambda,M;p_\Lambda^{M*},0) \tag{E.11}$$

The lagarangean function for the mid-quality firm's maximization program is given by:

$$L = 2[r + (1-r) * \frac{s_M}{1+s_M}]\mathbb{1}(p(M) \leq M)[p(M) - c_M] - 2s_M + \tag{E.12}$$

$$\gamma_1(\pi^*(\Lambda, L; p_\Lambda^{L*}, 0) - \pi(M, L; p(M), s_M)) + \tag{E.13}$$

$$\gamma_2(\pi(M, M; p(M), s_M) - \pi(\Lambda, M; p_\Lambda^{M*}, 0)) \tag{E.14}$$

Among these two constraints, at the least-cost quality separation, the mimicking-constraint of the low-quality is binding, while that of the mid-quality firm is not. This implies that $\gamma_1 < 0$, $\pi^*(\Lambda, L; p_\Lambda^{L*}, 0) - \pi(M, L; p(M), s_M) = 0$; $\gamma_2 = 0$ and $\pi(M, M; p(M), s_M) - \pi(\Lambda, M; p_\Lambda^{M*}, 0) \neq 0$. Because the true type of the low-quality firm will be revealed in period 2, the low-quality firm can only gain profits in period 1 by mimicking the mid-quality firm's SMM spending. Therefore, $\pi(M, L; p(M), s_M) = [r + (1-r) * \frac{s_M}{1+s_M}] * M - 2s_M$.

When $\lambda \geq \frac{H-2}{H-1}$, we need to solve the equation of $2r = [r + (1-r) * \frac{s_M}{1+s_M}] * M - 2s_M$. After discarding an implausible root $(-r)$, the optimal $s_M^* = \frac{M}{2} - 1$ if $M \geq 2$, or $s_M^* = \sqrt{(M - c_M)(1-r)} - 1$ otherwise.

When $\lambda < \frac{H-2}{H-1}$, we need to solve the equation of $r[\lambda + (1-\lambda)H] = [r + (1-r) * \frac{s_M}{1+s_M}] * M - 2s_M$. Similar to the one-period model, we derive the optimal SMM spending by the mid-quality firm:

$$s_M^* = \begin{cases} 1/4(-2 + M - Ar - \sqrt{(-2 + M - Ar)^2 + 8(-Ar + Mr)}), & \text{if } A \leq M \\ 1/4(-2 + M - Ar + \sqrt{(-2 + M - Ar)^2 + 8(-Ar + Mr)}), & \text{if } M < A \leq \frac{2+M-2\sqrt{2M(1-r)}}{r} \\ -1 + \sqrt{(M - c_M)(1-r)}, & \text{if } A > \frac{2+M-2\sqrt{2M(1-r)}}{r} \end{cases} \tag{E.15}$$

where $A = [\lambda + (1-\lambda)H]$.

Now, we need the following incentive compatibility constraints for the high-quality firm to be satisfied, which are: (1) the high-quality firm does not find it profitable to mimic the mid-quality firm by spending s_M^* and (2) the mid-quality firm does not find it profitable to pool with the high-quality and low-quality firms and spend zero on social media marketing. Mathematically, these conditions are:

$$\pi^*(\Lambda, H; p_\Lambda^{H*}, 0) \geq \pi(M, H; p^*(M), s_M^*)$$

$$\pi^*(M, M; p^*(M), s_M^*) \geq \pi(\Lambda, M; p_\Lambda^{M*}, 0)$$

In the first period, if the high-quality firm mimics the mid-quality firm by choosing $(s, p) = (s_M^*, M)$, consumers would mistake it for the mid-quality firm. Although its true type is revealed after consumption in period two, consumers would not buy the product in the first period if the price is greater than M. Therefore, the high-quality firm is better off setting price at M when it mimics the mid-quality firm.

Then the first inequality above becomes,

$$\begin{cases} 2r[\lambda+(1-\lambda)H-c_H] \geq 2[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_H)-2s_M^* & \text{if } \lambda<\frac{H-2}{H-1} \\ 2r[L-c_H] \geq 2[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_H)-2s_M^* & \text{if } \lambda\geq\frac{H-2}{H-1} \end{cases} \tag{E.16}$$

Similarly, when the mid-quality firm mimics the high-quality firm by spending zero on SMM and charging a price of $\lambda+(1-\lambda)H$ or L. Then, $\pi^*(M,M;p^*(M),s_M^*) \geq \pi(\Lambda,M;p_\Lambda^*,0)$ becomes

$$\begin{cases} 2[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_M)-2s_M^* \geq r[\lambda+(1-\lambda)H-c_M]+\mathbb{1}(\lambda+(1-\lambda)H\leq M)r[\lambda+(1-\lambda)H-c_M] & \text{if } \lambda<\frac{H-2}{H-1} \\ 2[r+(1-r)*\frac{s_M^*}{1+s_M^*}](M-c_M)-2s_M^* \geq 2r[L-c_M] & \text{if } \lambda\geq\frac{H-2}{H-1} \end{cases} \tag{E.17}$$

As shown earlier, we have solved s_M^*. We substitute this to E.16 and E.17 above, and we have the following condition:

$$\underline{H} \leq H \leq \overline{H}$$

where $\underline{H}=\frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_H)-s_M^*)+c_H-\lambda]$ and

$$\overline{H}=\begin{cases} \frac{1}{1-\lambda}[\frac{1}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_M)-s_M^*)+c_M-\lambda] & \text{if } \frac{H-M}{H-1}\leq\lambda<\frac{H-2}{H-1} \\ \frac{1}{1-\lambda}[\frac{2}{r}(\frac{r+s_M^*}{1+s_M^*}(M-c_M)-s_M^*)+c_M-\lambda] & \text{if } \frac{H-M}{H-1}\leq\lambda<min(\frac{H-2}{H-1},\frac{H-M}{H-1}) \end{cases} \tag{E.18}$$

Note that when $\lambda\geq\frac{H-2}{H-1}$, E.16 always holds whereas E.17 doesn't. Therefore, there is no partial-pooling equilibrium, if $\lambda\geq\frac{H-2}{H-1}$.

F. Extension to a Noisy Signaling Model

Although the notion that consumers can perfectly observe quality signals such as advertising spending has been widely accepted in the signaling literature, such an assumption seems somewhat strong in most real world applications, and in particular in our setting. The reasons are multifold. First, SMM spending is typically part of a firm's overall marketing expenses, which can be difficult to clearly observe or estimate from the firm's disclosed financial information. Consumers may then turn to the social media activities to infer how much the firm spendings on SMM. Second, much social media marketing takes place over various social media platforms, and not everyone is "tuned in" to those marketing activities. Third, different firms may have varying preferences towards those social media platforms, resulting in differences in their allocation of resources. It may also cost differently to conduct marketing activities on those social media platforms. In light of those reasons, it seems reasonable to assume that consumers cannot tell how much firms spending on SMM exactly. In this section, we focus on the case when the assumption of no noise is dropped.

Our findings contribute to the relatively under-explored noisy signaling literature.

Consider the case where the signal that consumers observe does not reveal the firm's SMM spending perfectly. We assume consumers observe a noisy signal s with additive noise, given by:

$$s = a + \epsilon$$

where ϵ is an unobserved noise term, which is distributed independently of a according to a normal distribution, $\epsilon \sim N(0, \sigma^2)$. Therefore $s \sim N(0, \sigma^2)$. We assume that the noise term is realized after the firm has decided on a, the SMM spending. We denote a_H^c, a_M^c, a_L^c as the consumers' conjectures about the unobserved SMM spending made by the high-quality, mid-quality and low quality firms, respectively. The timing of the game is as follows. The firm, which is aware of its true quality, decides on SMM spending a. The SMM spending a again changes the precision of the external signal received by the consumers to $\mu_1 = \frac{\lambda + a}{1+a}$. The realized signal on SMM spending is s, which contains a noise term. The firm observes the realized signal s, and then rationally sets the price that maximize its profits based on consumers' updated beliefs. Next, consumers receive the external information signal (e.g. from online word-of-mouth), observe the realized signal s on the firm's action on SMM spending, as well as the price. Consumers then make purchase decisions.

Consumers have to use statistical inference to update their beliefs about the action taken, and learn about the firm's type. Take the high-quality firm for an example, with $f(s|a) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s-a)^2}{2\sigma^2})$, consumers belief is updated based on Bayes' Rule:

$$\begin{aligned} \mu_2(s|a_H^c, a_M^c, a_L^c) &= \frac{\mu_1 f(s|a = a_H^c)}{\mu_1 f(s|a = a_H^c) + \frac{1-\mu_1}{2} f(s|a = a_M^c) + \frac{1-\mu_1}{2} f(s|a = a_L^c)} \\ &= \frac{LR_1}{LR_1 + \frac{1}{2} exp(\frac{(a_H^c - a_M^c)(a_H^c + a_M^c - 2s)}{2\sigma^2}) + \frac{1}{2} exp(\frac{(a_H^c - a_L^c)(a_H^c + a_L^c - 2s)}{2\sigma^2})} \end{aligned} \tag{F.1}$$

where $\mu_1 = \frac{\lambda + a_H}{1 + a_H}$, LR_1 denotes the likelihood ratio of beliefs after the consumer receives the external signal, $\frac{\mu_1}{1-\mu_1}$, a_H^c, a_M^c, a_L^c denote the consumer's conjectures about the type-dependent actions taken by the high-quality, mid-quality and low-quality firms respectively.

Following Jeitschko and Normann (2009), we employ a switching strategy, such that the consumer changes his willingness to pay to H, if the posterior belief $\mu_2(s|a_H^c, a_M^c, a_L^c)$ is greater than θ_1. Substituting this to F.1, we can derive the consumer's best response given the conjecture a_H^c, a_M^c, a_L^c, and the observed signal s. That is, consumer is willing to pay H for the product if and only if the observed SMM spending is greater than a threshold, $s \geq \widetilde{s}_1$, where $\widetilde{s}_1$ solves the equation $\frac{2(1-\theta_1)LR_1}{\theta_1} = exp(\frac{(a_H^c - a_M^c)(a_H^c + a_M^c - 2s)}{2\sigma^2}) + exp(\frac{(a_H^c - a_L^c)(a_H^c + a_L^c - 2s)}{2\sigma^2})$, with $LR_1 = \frac{\lambda + a_H}{1 + a_H}$.

Similarly, the updated consumer belief after observing the signal s on the firm being a low-quality one, is:

$$\begin{aligned}\mu_2(s|a_H^c, a_M^c, a_L^c) &= \frac{\frac{1-\mu_1}{2} f(s|a = a_L^c)}{\mu_1 f(s|a = a_H^c) + \frac{1-\mu_1}{2} f(s|a = a_M^c) + \frac{1-\mu_1}{2} f(s|a = a_L^c)} \\ &= \frac{1}{LR_1 * exp(\frac{(a_L^c - a_H^c)(a_L^c + a_H^c - 2s)}{2\sigma^2} + \frac{1}{2} exp(\frac{(a_L^c - a_M^c)(a_L^c + a_M^c - 2s)}{2\sigma^2}) + \frac{1}{2}}\end{aligned} \tag{F.2}$$

We assume if the posterior belief $\mu_2(s|a_H^c, a_M^c, a_L^c)$ is greater than θ_1, consumers tend to believe it is a low-quality firm. Substituting this into F.2, we obtain the consumer's best response: consumer is willing to pay L if and only if $s < \widetilde{s}_2$, where $\widetilde{s}_2$ solves the equation $\frac{1}{\theta_1} = 2LR_1 exp(\frac{(a_L^c - a_H^c)(a_L^c + a_H^c - 2s)}{2\sigma^2}) + exp(\frac{(a_L^c - a_M^c)(a_L^c + a_M^c - 2s)}{2\sigma^2}) + 1$.

In the rest of scenarios, given the conjecture a_H^c, a_M^c, a_L^c, the consumer's willingness to pay is M, if $\widetilde{s}_2 < s < \widetilde{s}_1$.

Next, the firm, upon observing the realized s and knowing the consumer's best response strategy, needs to choose a_H (e.g. the amount of SMM spending) to maximize its profit[4]:

$$\begin{aligned}\max_{a_H} \pi_H &= \frac{r + a_H}{1 + a_H}((H - c_H)Pr(s \geq \widetilde{s}_1) + (M - c_H)Pr(\widetilde{s}_2 \leq s \leq \widetilde{s}_1) + (L - c_H)Pr(s \leq \widetilde{s}_2)) - a_H \\ &= \frac{r + a_H}{1 + a_H}[(H - c_H)\int_{\widetilde{s}_1}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_H)^2}{2\sigma^2}) ds + (M - c_H)\int_{\widetilde{s}_2}^{\widetilde{s}_1} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_H)^2}{2\sigma^2}) ds \\ &+ (L - c_H)\int_0^{\widetilde{s}_2} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_H)^2}{2\sigma^2}) ds] - a_H\end{aligned} \tag{F.3}$$

Take the first-order derivative with respect to a_H, we have

[4] In cases where consumers are only willing to pay for M or L, e.g. $s < \widetilde{s}_1$, it will set its price equal to M or L. Therefore, the price is contingent on the realized signal s.

$$\begin{aligned}&\frac{1-r}{(1+a_H)^2}[(H-c_H)\int_{\widetilde{s}_1}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds+(M-c_H)\int_{\widetilde{s}_2}^{\widetilde{s}_1}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds\\&+(L-c_H)\int_0^{\widetilde{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds]+\frac{r+a_H}{1+a_H}[\frac{H-c_H}{\sigma\sqrt{2\pi}}\int_{\widetilde{s}_1}^{\infty}\frac{s-a_H}{\sigma^2}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds\\&+\frac{M-c_H}{\sigma\sqrt{2\pi}}\int_{\widetilde{s}_2}^{\widetilde{s}_1}\frac{s-a_H}{\sigma^2}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds+\frac{L-c_H}{\sigma\sqrt{2\pi}}\int_0^{\widetilde{s}_2}\frac{s-a_H}{\sigma^2}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds]-1=0\end{aligned}\tag{F.4}$$

That is:

$$\begin{aligned}&\frac{1-r}{(1+a_H)^2}[(H-c_H)\int_{\widetilde{s}_1}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds+(M-c_H)\int_{\widetilde{s}_2}^{\widetilde{s}_1}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds\\&+(L-c_H)\int_0^{\widetilde{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H)^2}{2\sigma^2})ds]=\frac{r+a_H}{1+a_H}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_1-a_H)^2}{2\sigma^2})\\&+\frac{M-c_H}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_2-a_H)^2}{2\sigma^2})+\frac{L-c_H}{\sigma\sqrt{2\pi}}(exp(-\frac{a_H^2}{2\sigma^2})-exp(-\frac{(\widetilde{s}_2-a_H)^2}{2\sigma^2})]+1\end{aligned}\tag{F.5}$$

Similarly, the mid-quality firm's and low-quality firm's best actions are implied by:

$$\begin{aligned}&\frac{1-r}{(1+a_M)^2}[(H-c_M)\int_{\widetilde{s}_3}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M)^2}{2\sigma^2})ds+(M-c_M)\int_{\widetilde{s}_4}^{\widetilde{s}_3}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M)^2}{2\sigma^2})ds\\&+(L-c_M)\int_0^{\widetilde{s}_4}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M)^2}{2\sigma^2})ds]=\frac{r+a_M}{1+a_M}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_3-a_M)^2}{2\sigma^2})\\&+\frac{M-c_M}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_4-a_M)^2}{2\sigma^2})+\frac{L-c_M}{\sigma\sqrt{2\pi}}(exp(-\frac{a_M^2}{2\sigma^2})-exp(-\frac{(\widetilde{s}_4-a_M)^2}{2\sigma^2})]+1\end{aligned}\tag{F.6}$$

$$\begin{aligned}&\frac{1-r}{(1+a_L)^2}[(H-c_L)\int_{\widetilde{s}_5}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L)^2}{2\sigma^2})ds+(M-c_L)\int_{\widetilde{s}_6}^{\widetilde{s}_5}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L)^2}{2\sigma^2})ds\\&+(L-c_L)\int_0^{\widetilde{s}_6}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L)^2}{2\sigma^2})ds]=\frac{r+a_L}{1+a_L}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_5-a_L)^2}{2\sigma^2})\\&+\frac{M-c_L}{\sigma\sqrt{2\pi}}exp(-\frac{(\widetilde{s}_6-a_L)^2}{2\sigma^2})+\frac{L-c_L}{\sigma\sqrt{2\pi}}(exp(-\frac{a_L^2}{2\sigma^2})-exp(-\frac{(\widetilde{s}_6-a_L)^2}{2\sigma^2})]+1\end{aligned}\tag{F.7}$$

where $\widetilde{s}_3$, $\widetilde{s}_4$, $\widetilde{s}_5$, $\widetilde{s}_6$, solve equations $\frac{1}{\theta_1}=1+2LR_2exp(\frac{(a_H^c-a_M^c)(a_M^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_H^c-a_L^c)(a_L^c+a_H^c-2s)}{2\sigma^2})$, $\frac{1}{\theta_1}=2LR_2exp(\frac{(a_L^c-a_M^c)(a_L^c+a_M^c-2s)}{2\sigma^2})+exp(\frac{(a_L^c-a_H^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+1$, $\frac{1}{\theta_1}=1+2LR_3exp(\frac{(a_H^c-a_L^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_H^c-a_M^c)(a_M^c+a_H^c-2s)}{2\sigma^2})$, and $\frac{2LR_3}{\theta_1}=exp(\frac{(a_L^c-a_H^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_L^c-a_M^c)(a_L^c+a_M^c-2s)}{2\sigma^2})+2LR_3$, and $LR_2=\frac{\lambda+a_M}{1+a_M}$, $LR_3=\frac{\lambda+a_L}{1+a_L}$ In equilibrium, consumers' belief on firms' SMM spending is consistent with firms' actions, that is, $a_H^c=a_H^*$, $a_M^c=a_M^*$, and $a_L^c=a_L^*$.

Therefore, we have the following separating equilibrium.

Proposition 11. *In the noisy signaling case, in the separating equilibrium, the high-quality, mid-quality and low-quality firms' equilibrium actions, a_H^*, a_M^*, and a_L^* are implied by the following equations:*

$$\begin{aligned}
&\frac{1-r}{(1+a_H^*)^2}[(H-c_H)\int_{\tilde{s}_1}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H^*)^2}{2\sigma^2})ds+(M-c_H)\int_{\tilde{s}_2}^{\tilde{s}_1}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H^*)^2}{2\sigma^2})ds\\
&+(L-c_H)\int_0^{\tilde{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_H^*)^2}{2\sigma^2})ds]=\frac{r+a_H^*}{1+a_H^*}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_1-a_H^*)^2}{2\sigma^2})\\
&+\frac{M-c_H}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_2-a_H^*)^2}{2\sigma^2})+\frac{L-c_H}{\sigma\sqrt{2\pi}}(exp(-\frac{(a_H^*)^2}{2\sigma^2})-exp(-\frac{(\tilde{s}_2-a_H^*)^2}{2\sigma^2})]+1
\end{aligned}$$

$$\begin{aligned}
&\frac{1-r}{(1+a_M^*)^2}[(H-c_M)\int_{\tilde{s}_3}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M^*)^2}{2\sigma^2})ds+(M-c_M)\int_{\tilde{s}_4}^{\tilde{s}_3}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M^*)^2}{2\sigma^2})ds\\
&+(L-c_M)\int_0^{\tilde{s}_4}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M^*)^2}{2\sigma^2})ds]=\frac{r+a_M^*}{1+a_M^*}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_3-a_M^*)^2}{2\sigma^2})\\
&+\frac{M-c_M}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_4-a_M^*)^2}{2\sigma^2})+\frac{L-c_M}{\sigma\sqrt{2\pi}}(exp(-\frac{(a_M^*)^2}{2\sigma^2})-exp(-\frac{(\tilde{s}_4-a_M^*)^2}{2\sigma^2})]+1
\end{aligned}$$

$$\begin{aligned}
&\frac{1-r}{(1+a_L^*)^2}[(H-c_L)\int_{\tilde{s}_5}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L^*)^2}{2\sigma^2})ds+(M-c_L)\int_{\tilde{s}_6}^{\tilde{s}_5}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L^*)^2}{2\sigma^2})ds\\
&+(L-c_L)\int_0^{\tilde{s}_6}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_L^*)^2}{2\sigma^2})ds]=\frac{r+a_L^*}{1+a_L}[\frac{H-M}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_5-a_L^*)^2}{2\sigma^2})\\
&+\frac{M-c_L}{\sigma\sqrt{2\pi}}exp(-\frac{(\tilde{s}_6-a_L^*)^2}{2\sigma^2})+\frac{L-c_L}{\sigma\sqrt{2\pi}}(exp(-\frac{(a_L^*)^2}{2\sigma^2})-exp(-\frac{(\tilde{s}_6-a_L^*)^2}{2\sigma^2})]+1
\end{aligned}$$

where $\tilde{s}_1$, $\tilde{s}_2$, $\tilde{s}_3$, $\tilde{s}_4$, $\tilde{s}_5$, $\tilde{s}_6$ are defined as critical values that solve equations $\frac{1}{\theta_1}=1+2LR_2exp(\frac{(a_H^c-a_M^c)(a_M^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_H^c-a_L^c)(a_L^c+a_H^c-2s)}{2\sigma^2})$, $\frac{1}{\theta_1}=2LR_2exp(\frac{(a_L^c-a_M^c)(a_L^c+a_M^c-2s)}{2\sigma^2})+exp(\frac{(a_L^c-a_H^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+1$, $\frac{1}{\theta_1}=1+2LR_3exp(\frac{(a_H^c-a_L^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_H^c-a_M^c)(a_M^c+a_H^c-2s)}{2\sigma^2})$, *and* $\frac{2LR_3}{\theta_1}=exp(\frac{(a_L^c-a_H^c)(a_L^c+a_H^c-2s)}{2\sigma^2})+exp(\frac{(a_L^c-a_M^c)(a_L^c+a_M^c-2s)}{2\sigma^2})+2LR_3$, *and* $LR_2=\frac{\lambda+a_M}{1+a_M}$, $LR_3=\frac{\lambda+a_L}{1+a_L}$

F.1. Partial-pooling Equilibrium

Suppose there is a partial-pooling equilibrium, wherein the high-quality firm and low-quality firm pool together at $a_H=a_L=a_{HL}$, whereas the mid-quality firm spends a_M on SMM.

The signal s follows a normal distribution with density of $f(s|a)=\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a)^2}{2\sigma^2})$. Again, after observing the signal s, if the firm is a mid-quality one, consumers' belief that it is a mid-quality firm is updated based on Bayes' Rule:

$$\mu_2(s|a_{HL}^c,a_M^c)=\frac{\mu_1 f(s|a=a_M^c)}{\frac{1-\mu_1}{2}f(s|a=a_{HL}^c)+\mu_1 f(s|a=a_M^c)+\frac{1-\mu_1}{2}f(s|a=a_{HL}^c)} \tag{F.8}$$

$$= \frac{LR_1}{LR_1 + exp(\frac{(a^c_M - a^c_{HL})(a^c_{HL} + a^c_M - 2s)}{2\sigma^2})}$$

where $\mu_1 = \frac{\lambda + a_M}{1 + a_M}$, LR_1 denotes the likelihood ratio of beliefs after the consumer receives the external signal, $\frac{\mu_1}{1-\mu_1}$, a^c_{HL}, a^c_M denote the consumer's conjectures about the type-dependent actions taken by the high-/low-quality and mid-quality firms respectively.

Again, consumers employs a switching strategy, such that the consumer changes his willingness to pay to M, if the posterior belief $\mu_2(s|a^c_{HL}, a^c_M)$ is greater than θ_1. Substituting this to F.8, we can derive the consumer's best response given the conjecture a^c_{HL}, a^c_M, and the observed signal s. That is, consumer is willing to pay M for the product if and only if the observed SMM spending is greater than a threshold, $s \geq \overline{s}_1$, where $\overline{s}_1 = \frac{a^c_M + a^c_{HL}}{2} - ln(\frac{1-\theta_1}{\theta_1} LR_1)\frac{\sigma^2}{a^c_M - a^c_{HL}}$, with $LR_1 = \frac{\lambda + a_M}{1 + a_M}$.

If $s \leq \overline{s}_1$, consumers believe that it is more likely to come from the high-/low-quality firm, rather than the mid-quality firm. In equilibrium, consumers know that the high-quality firm and low-quality firm pool together on their SMM spending. Therefore, consumers' belief relies on the external signal which suggests with a probability of $\frac{\lambda + a_{HL}}{1 + a_{HL}}$, this is a high-quality product, and a probability of $\frac{1-\lambda}{1 + a_{HL}}$, this is a low-quality product.

The mid-quality firm needs to solve the following profit maximization program.

$$\begin{aligned}
\max_{a_M} \pi_M &= \frac{r + a_M}{1 + a_M}[(M - c_M)Pr(s \geq \overline{s}_1) + (\frac{\lambda + a_{HL}}{1 + a_{HL}} L + \frac{1-\lambda}{1 + a_{HL}} H - c_M)Pr(0 \leq s \leq \overline{s}_1)] - a_M \\
&= \frac{r + a_M}{1 + a_M}[(M - c_M)\int_{\overline{s}_1}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds \\
&+ (\frac{\lambda + a_{HL}}{1 + a_{HL}} L + \frac{1-\lambda}{1 + a_{HL}} H - c_M)\int_0^{\overline{s}_1} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds] - a_M
\end{aligned} \tag{F.9}$$

Take the first-order derivative with respect to a_M, we have

$$\begin{aligned}
&\frac{1-r}{(1 + a_M)^2}[(M - c_M)\int_{\overline{s}_1}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds + (\frac{\lambda + a_{HL}}{1 + a_{HL}} L + \frac{1-\lambda}{1 + a_{HL}} H - c_M)\int_0^{\overline{s}_1} \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds] \\
&+ \frac{r + a_M}{1 + a_M}[\frac{M - c_M}{\sigma\sqrt{2\pi}}\int_{\overline{s}_1}^{\infty} \frac{s - a_M}{\sigma^2} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds + \frac{\frac{\lambda + a_{HL}}{1 + a_{HL}} L + \frac{1-\lambda}{1 + a_{HL}} H - c_M}{\sigma\sqrt{2\pi}}\int_0^{\overline{s}_1} \frac{s - a_M}{\sigma^2} exp(-\frac{(s - a_M)^2}{2\sigma^2})ds] - 1 = 0
\end{aligned}$$

which can be simplified to the following:

$$\frac{1-r}{(1+a_M)^2}[(M-c_M)\int_{\bar{s}_1}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M)^2}{2\sigma^2})ds+(\frac{\lambda+a_{HL}}{1+a_{HL}}L+\frac{1-\lambda}{1+a_{HL}}H-c_M)\int_0^{\bar{s}_1}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M)^2}{2\sigma^2})ds]$$
$$=\frac{r+a_M}{1+a_M}[\frac{M-\frac{\lambda+a_M^*}{1+a_M^*}L-\frac{1-\lambda}{1+a_M^*}H}{\sigma\sqrt{2\pi}}exp(-\frac{(\bar{s}_1-a_M)^2}{2\sigma^2})+\frac{\frac{\lambda+a_{HL}}{1+a_{HL}}L+\frac{1-\lambda}{1+a_{HL}}H-c_M}{\sigma\sqrt{2\pi}}exp(-\frac{a_M^2}{2\sigma^2})]+1 \tag{F.10}$$

As for the low-quality firm, it's profit maximization problem is:

$$\max_{a_{HL}}\pi_L=\frac{r+a_{HL}}{1+a_{HL}}[(M-c_L)Pr(s\geq\bar{s}_2)+(\frac{\lambda+a_{HL}}{1+a_{HL}}L+\frac{1-\lambda}{1+a_{HL}}H-c_L)Pr(0\leq s\leq\bar{s}_2)]-a_{HL}$$
$$=\frac{r+a_{HL}}{1+a_{HL}}[(M-c_L)\int_{\bar{s}_2}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL})^2}{2\sigma^2})ds+(\frac{\lambda+a_{HL}}{1+a_{HL}}L+\frac{1-\lambda}{1+a_{HL}}H-c_L)$$
$$\int_0^{\bar{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL})^2}{2\sigma^2})ds]-a_{HL} \tag{F.11}$$

Taking the first derivative with respect to a_{HL}, we have the optimal a_{HL}^* implied by the following:

$$\frac{1-r}{(1+a_{HL}^*)^2}[(M-c_L)\int_{\bar{s}_2}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL}^*)^2}{2\sigma^2})ds+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_L)\int_0^{\bar{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL}^*)^2}{2\sigma^2})ds]$$
$$=\frac{r+a_{HL}^*}{1+a_{HL}^*}[\frac{M-\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L-\frac{1-\lambda}{1+a_{HL}^*}H}{\sigma\sqrt{2\pi}}exp(-\frac{(\bar{s}_2-a_{HL}^*)^2}{2\sigma^2})+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_L)\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{a_{HL}^{*2}}{2\sigma^2})]+1 \tag{F.12}$$

where $\bar{s}_2$ is the critical value that satisfies the equation, $\frac{1}{\theta_1}=(2LR_2+1)exp(\frac{(a_M^c-a_{HL}^c)(a_{HL}^c+a_M^c-2s)}{2\sigma^2})+1$, where $LR_2=\frac{\lambda+a_{HL}}{1+a_{HL}}$.

Now we need the following incentive compatibility constraints for the high-quality firm to be satisfied:

$$\pi(M,H;p^*(M),a_M^*)\leq\pi(\Lambda,H;p_\Lambda^*,a_{HL}^*) \tag{F.13}$$

$$\pi(\Lambda,M;p_\Lambda^*,a_{HL}^*)\leq\pi(M,M;p^*(M),a_M^*) \tag{F.14}$$

$$\frac{r+a_M^*}{1+a_M^*}[(M-c_H)Pr(s\geq\bar{s}_1)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_H)Pr(0\leq s\leq\bar{s}_1)]-a_M^*\leq$$
$$\frac{r+a_{HL}^*}{1+a_{HL}^*}[(M-c_H)Pr(s\geq\bar{s}_2)+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_H)Pr(0\leq s\leq\bar{s}_2)]-a_{HL}^* \tag{F.15}$$
$$\frac{r+a_{HL}^*}{1+a_{HL}^*}[(M-c_M)Pr(s\geq\bar{s}_2)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_H)Pr(0\leq s\leq\bar{s}_2)]-a_{HL}^*\leq$$
$$\frac{r+a_M^*}{1+a_M^*}[(M-c_M)Pr(s\geq\bar{s}_1)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_M)Pr(0\leq s\leq\bar{s}_1)]-a_M^* \tag{F.16}$$

Therefore, we have the following equilibrium.

Proposition 12. *In the noisy signaling case, the partial-pooling equilibrium exists if the following conditions are satisfied.*

$$\frac{r+a_M^*}{1+a_M^*}[(M-c_H)Pr(s\geq\overline{s}_1)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_H)Pr(0\leq s\leq\overline{s}_1)]-a_M^*\leq$$
$$\frac{r+a_{HL}^*}{1+a_{HL}^*}[(M-c_H)Pr(s\geq\overline{s}_2)+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_H)Pr(0\leq s\leq\overline{s}_2)]-a_{HL}^*$$
$$\frac{r+a_{HL}^*}{1+a_{HL}^*}[(M-c_M)Pr(s\geq\overline{s}_2)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_H)Pr(0\leq s\leq\overline{s}_2)]-a_{HL}^*\leq$$
$$\frac{r+a_M^*}{1+a_M^*}[(M-c_M)Pr(s\geq\overline{s}_1)+(\frac{\lambda+a_M^*}{1+a_M^*}L+\frac{1-\lambda}{1+a_M^*}H-c_M)Pr(0\leq s\leq\overline{s}_1)]-a_M^*$$

where the high-quality, mid-quality and low-quality firms' equilibrium actions, a_{HL}^* *and* a_M^* *are implied by the following equations:*

$$\frac{1-r}{(1+a_M^*)^2}[(M-c_M)\int_{\overline{s}_1}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M^*)^2}{2\sigma^2})ds+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_M)\int_0^{\overline{s}_1}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_M^*)^2}{2\sigma^2})ds]$$
$$=\frac{r+a_M^*}{1+a_M^*}[\frac{M-\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L-\frac{1-\lambda}{1+a_{HL}^*}H}{\sigma\sqrt{2\pi}}exp(-\frac{(\overline{s}_1-a_M^*)^2}{2\sigma^2})+\frac{\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_M}{\sigma\sqrt{2\pi}}exp(-\frac{a_M^{*2}}{2\sigma^2})]+1$$

$$\frac{1-r}{(1+a_{HL}^*)^2}[(M-c_L)\int_{\overline{s}_2}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL}^*)^2}{2\sigma^2})ds+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_L)\int_0^{\overline{s}_2}\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(s-a_{HL}^*)^2}{2\sigma^2})ds]$$
$$=\frac{r+a_{HL}^*}{1+a_{HL}^*}[\frac{M-\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L-\frac{1-\lambda}{1+a_{HL}^*}H}{\sigma\sqrt{2\pi}}exp(-\frac{(\overline{s}_2-a_{HL}^*)^2}{2\sigma^2})+(\frac{\lambda+a_{HL}^*}{1+a_{HL}^*}L+\frac{1-\lambda}{1+a_{HL}^*}H-c_L)\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{a_{HL}^{*2}}{2\sigma^2})]+1$$

$\overline{s}_1$ *and* $\overline{s}_2$ *are critical values that solve the equations* $\overline{s}_1=\frac{a_M^c+a_{HL}^c}{2}-ln(\frac{1-\theta_1}{\theta_1}LR_1)\frac{\sigma^2}{a_M^c-a_{HL}^c}$, $\frac{1}{\theta_1}=(2LR_2+1)exp(\frac{(a_M^c-a_{HL}^c)(a_{HL}^c+a_M^c-2s)}{2\sigma^2})+1$, *where* $LR_1=\frac{\lambda+a_M}{1+a_M}$ *and* $LR_2=\frac{\lambda+a_{HL}}{1+a_{HL}}$.

G. Explanation on the information updating process

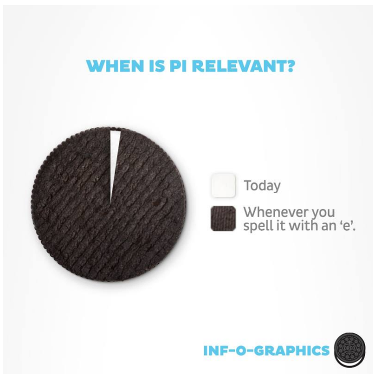
Following prior literature, we assume that consumers are not fully informed about the product's quality before the purchase. However, there exists some external information signal that consumers may receive about the product's quality. For instance, third-party reviews, and social media activities surrounding the product may reveal some quality information, yet not perfectly accurate. Such external information signal comes with an initial precision of λ, that is with a probability of λ, the external information signal is the same as the firm's type, and with a probability of $1-\lambda$, the signal is wrong. Therefore, consumers' prior belief about the product quality follows a binomial distribution – $p(q|q)=\lambda$ and $p(q'|q)=1-\lambda$, for $q' \neq q$. Because *Beta* distribution is a conjugate prior for the binomial distribution, this means that if the likelihood function is binomial and the prior distribution is a *Beta* distribution, then the posterior distribution is also a *Beta* distribution.

We could think of this belief updating process as a Bayesian process. We first specify the prior distribution as a Beta distribution, $Beta(\lambda, 1-\lambda)$. The mean of this Beta distribution is λ. For a general Beta distribution, $Beta(\alpha,\beta)$, Bayesian updating means taking the prior, $Beta(\alpha,\beta)$, and adding the successes from the data, x, to α and the failures, $n-x$, to β. The posterior becomes $Beta(\alpha+x, \beta+n-x)$. Social media marketing allows firms to better facilitate information transmission among consumers on social media channels, which in turn results in an increase in the informativeness of the external information signal. For instance, prior consumer may utilize social media platforms to share their usage experience with prospective consumers, or provide endorsement for the product. Firms' social media marketing efforts could help more consumers to share their experience and reveal the true quality of the product to a potential consumer. In other words, we could think firms' social media marketing efforts always lead to more social media content (or reviews) that truthfully reveals the firm's type. In particular, we assume successes from the data are perfectly linear to the social media marketing spending, in fact, equal to s. After we couple our prior beliefs with the data we have observed and update our beliefs according to the Bayes' rule, we derive the posterior, which is $Beta(\lambda+s, 1-\lambda)$. The mean of this posterior distribution is $\frac{\lambda+s}{1+s}$.

We have also considered other alternative specifications of the prior and the likelihood, such as $Beta(n\lambda, n(1-\lambda))$ and $\frac{ns}{1+s}$ of successes and $\frac{n}{1+s}$ of failures. Results are similar.

H. Examples of Social Media Marketing Activities

Figure 2: Illustration of a Range of Marketing Activities on Social Media Platforms

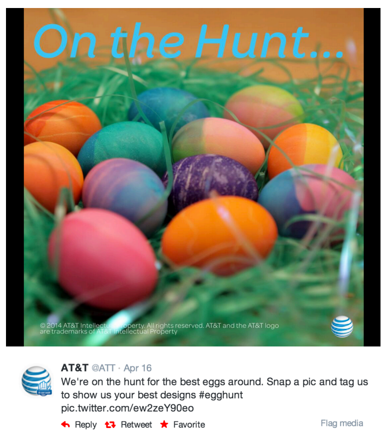


(a) Oreo

(b) Oreo

(c) Amazon

(d) Amazon



(e) AT&T

(f) Cheerios