Online Appendix

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This online appendix contains detailed proofs of Propositions 1-5 (Appendix A) and analyses of three important extensions of the main model (Appendices B–D). We have also analyzed other extensions, including multi-homing consumers, consumer annoyance at advertisements, the network effect, heterogeneous content production efficiency, consumers' vertical heterogeneity, subscription revenue model and ISP's price discrimination. The detailed analysis and results of these other extensions are available upon request.

Appendix A: Proofs of Main Model

Proof of Lemma 1

In this proof, we construct an equilibrium that achieves full market coverage (hereinafter referred to as the "fully-covered equilibrium") of subgame N wherein no CP is allowed to subsidize consumers' data usage ($I_{\mathrm{A}} = I_{\mathrm{B}} = 0$). In this equilibrium, although the ISP is free to choose a high data usage price p that makes the market partially covered, we prove that the ISP has no incentive to do so, and hence the market is fully covered.

We construct the fully-covered equilibrium via backward induction. First, we solve for the consumers' optimal choice between CPs A and B given the CPs' content quality and the ISP's data usage price. Second, we deduce the CPs' optimal decisions on content quality and the ISP's optimal pricing scheme in a candidate fully-covered equilibrium. Finally, we show that both the ISP and CPs have no incentive to deviate. In particular, it is unprofitable to deviate to a partially covered market.

In stage 3, if the market is fully covered, each consumer chooses between CPs A and B to maximize their utility. All consumers' decisions can be represented by the marginal consumer x_{m} with the consumers located within $[0, x_{\mathrm{m}}]$ choosing CP A and the consumers located within $[x_{\mathrm{m}}, 1]$ choosing CP B. To determine the marginal consumer x_{m}, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$ (see Equations (1) and (2) in the manuscript) and obtain $x_{\mathrm{m}} = \frac{V_{\mathrm{A}} - V_{\mathrm{B}} + t}{2t}$, which is independent of the ISP's data usage price p. If the price p is low such that $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) \geq 0$, our assumption of a fully covered market is justified; if the ISP chooses a high price p that makes $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) < 0$, some consumers will exit the market and the market is partially covered.

In stage 2, both CPs make investments in content quality and the ISP decides on the usage-based per-packet price p to maximize their respective profits. In a candidate fully-covered equilibrium, we derive the CPs' optimal content quality decisions and then derive the ISP's optimal pricing decision.

First, from Equation (3) in the manuscript, we can write down CP i's profit function as $\pi_{\mathrm{i}} = r\frac{V_{\mathrm{i}}-V_{-\mathrm{i}}+t}{2t} - \frac{\phi}{2}V_{\mathrm{i}}{}^2$. From the first-order condition (FOC) of CP i's profit function π_{i} w.r.t. V_{i} ($i =$ A or B), we determine CP i's optimal content quality decision $V_{\mathrm{i}}^* = \frac{r}{2\phi t}$. Due to the ISP's network management choice (i.e., not allowing data sponsorship), the CPs have no decision on data sponsorship in this subgame. Then, the equilibrium marginal consumer is $x_{\mathrm{m}}^* = \frac{V_{\mathrm{A}}^*-V_{\mathrm{B}}^*+t}{2t} = \frac{1}{2}$ and the CPs' equilibrium profit is $\pi_{\mathrm{i}} = \frac{r}{2} - \frac{r^2}{8\phi t^2}$. The assumption that $r \le 4\phi t^2$ guarantees that $\pi_{\mathrm{i}} \ge 0$.

Second, we derive the ISP's optimal data usage price p^*. The ISP's profit-maximizing problem (see Equation (4) in the manuscript) is equivalent to the problem of maximizing p under the full market coverage assumption. Because the marginal consumer is the one who has the lowest utility among all consumers, the highest price that the ISP can set should let the marginal consumer's utility be zero. Therefore, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}^*) = U_{\mathrm{B}}(x_{\mathrm{m}}^*) = 0$ and obtain the ISP's optimal price $p^* = \frac{1}{2}(V_{\mathrm{A}}^* + V_{\mathrm{B}}^* - t) = \frac{r}{2\phi t} - \frac{t}{2}$.

Finally, we show that both the ISP and CPs have no incentive to deviate. Notice that if the ISP charges a lower price or a CP increases its quality, the market remains fully covered. Such a deviation cannot be profitable from the construction of our equilibrium. Since $U_{\mathrm{A}}(x_{\mathrm{m}}^*) = U_{\mathrm{B}}(x_{\mathrm{m}}^*) = 0$, the market becomes partially covered if the ISP charges a higher price or a CP decreases its quality. Hence, the key is to prove that neither the ISP nor a CP has an incentive to deviate and make the market partially covered.

We first prove that the ISP has no incentive to set a high data usage price that causes a partially covered market under the assumption that $r \ge 2\phi t^2$. From the above analysis, we get the ISP's optimal data usage price $p_{\mathrm{F}}^{\mathrm{N}} = \frac{r}{2\phi t} - \frac{t}{2}$ and its profit $\pi_{\mathrm{ISP\_F}}^{\mathrm{N}} = p_{\mathrm{F}}^{\mathrm{N}} = \frac{r}{2\phi t} - \frac{t}{2}$ in a fully covered market of subgame N. According to our analysis of stage 3, the market is partially covered if $p > p_{\mathrm{F}}^{\mathrm{N}} = \frac{r}{2\phi t} - \frac{t}{2}$. In the following analysis, we calculate the ISP's profit $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p)$ if it deviates to a partially covered market of subgame N, and then compare $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p)$ with $\pi_{\mathrm{ISP\_F}}^{\mathrm{N}}$. In a partially covered market, we use x_{A} (x_{B}) to represent the marginal consumer who is indifferent about whether or not they consume CP A's (CP B's) content. Given the CPs' content quality strategy $V_{\mathrm{i}}^{*} = \frac{r}{2\phi t}$, solving $U_{\mathrm{A}}(x_{\mathrm{A}}) = 0$ and $U_{\mathrm{B}}(x_{\mathrm{B}}) = 0$ yields $x_{\mathrm{A}} = \frac{V_{\mathrm{A}}^{*} - p}{t} = \frac{r - 2\phi t p}{2\phi t^2}$ and $x_{\mathrm{B}} = 1 - \frac{V_{\mathrm{B}}^{*} - p}{t} = 1 - \frac{r - 2\phi t p}{2\phi t^2}$. Then, $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p) = p(x_{\mathrm{A}} + 1 - x_{\mathrm{B}}) = \frac{p(r - 2\phi t p)}{\phi t^2}$. It is straightforward to obtain that $\frac{d\,\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p)}{d\,p} = \frac{r - 4\phi t p}{\phi t^2} \leq 0$ if and only if $p \geq \frac{r}{4\phi t}$. Under the assumption that $r \geq 2\phi t^2$, we have $p_{\mathrm{F}}^{\mathrm{N}} = \frac{r}{2\phi t} - \frac{t}{2} \geq \frac{r}{4\phi t}$, which implies that $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p)$ strictly decreases with p when $p > p_{\mathrm{F}}^{\mathrm{N}}$. As shown in Figure A.1, under the assumption that $r \geq 2\phi t^2$, $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p) < \pi_{\mathrm{ISP\_P}}^{\mathrm{N}}(p_{\mathrm{F}}^{\mathrm{N}}) = \frac{r}{2\phi t} - \frac{t}{2} = \pi_{\mathrm{ISP\_F}}^{\mathrm{N}}$ for any $p > p_{\mathrm{F}}^{\mathrm{N}}$. Therefore, compared to the fully-covered equilibrium of subgame N, the ISP's profit decreases if it chooses a price higher than $p_{\mathrm{F}}^{\mathrm{N}}$ and deviates to a partially covered market.

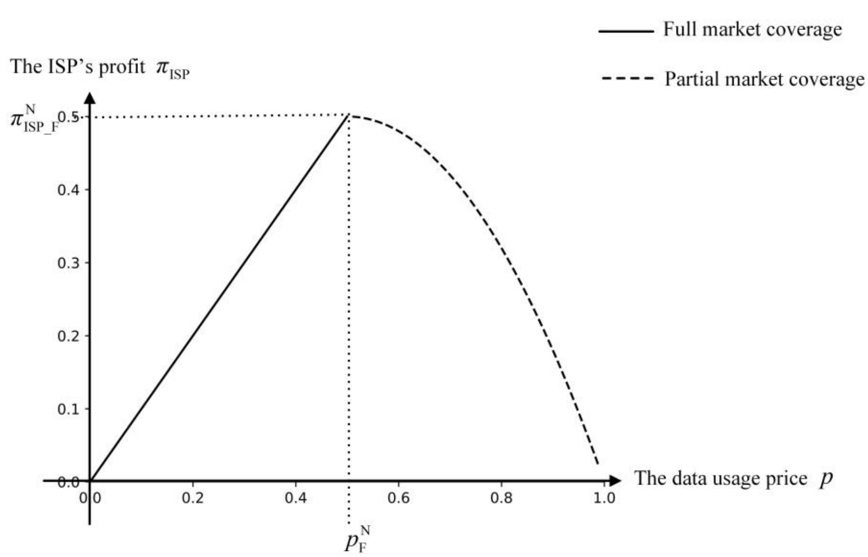

Figure A.1. Illustration of the ISP's optimal pricing decision in subgame N when $r = 2, \phi = 1, t = 1$.

We then show that a CP has no incentive to decrease its quality in a fully-covered equilibrium.

Suppose that CP A decreases its quality from V_{A}^{*} to V_{A}' (i.e., $V_{\mathrm{A}}' < V_{\mathrm{A}}^{*}$). As a result, its market share decreases from x_{m}^{*} to $x_{\mathrm{m}}' = x_{\mathrm{m}}^{*} - \frac{V_{\mathrm{A}}^{*} - V_{\mathrm{A}}'}{t}$, and its profit changes by

$$\Delta\pi_{\mathrm{A}}' = \pi_{\mathrm{A}}^{*} - \pi_{\mathrm{A}}' = -\frac{\phi}{2}\left({V_{\mathrm{A}}^{*}}^{2} - {V_{\mathrm{A}}'}^{2}\right) + r(x_{\mathrm{m}}^{*} - x_{\mathrm{m}}') = (V_{\mathrm{A}}^{*} - V_{\mathrm{A}}')\left(\frac{r}{t} - \frac{\phi}{2}(V_{\mathrm{A}}^{*} + V_{\mathrm{A}}')\right).$$

It is straightforward to show that $\Delta\pi_{\mathrm{A}}' > 0$ since $\frac{r}{t} - \frac{\phi}{2}(V_{\mathrm{A}}^{*} + V_{\mathrm{A}}') > \frac{r}{t} - \frac{\phi}{2} \times 2V_{\mathrm{A}}^{*} = \frac{r}{2t} > 0$. Therefore, compared to the fully-covered equilibrium of subgame N, the CP's profit also decreases if it chooses a lower quality and deviates to a partially covered market.

Proofs of Lemmas 2–5

We prove Lemmas 2–5 by constructing a fully-covered equilibrium of subgame S wherein only one CP (CP A) is allowed to sponsor consumer data traffic. Similar to the proof of Lemma 1, we construct the fully-covered equilibrium via backward induction. First, we solve for the consumers' optimal choices between CPs A and B given the CPs' content quality and data sponsorship and the ISP's data usage price. Second, we deduce the CPs' optimal decisions on content quality and data sponsorship and the ISP's optimal pricing scheme in a candidate fully-covered equilibrium. Finally, we show both the ISP and CPs have no incentive to deviate. In particular, although the ISP and CPs can deviate and make the market partially covered, it is unprofitable for them to do so.

In stage 3, if the market is fully covered, we determine the marginal consumer $x_{\mathrm{m}} = \frac{V_{\mathrm{A}} + s_{\mathrm{A}} - V_{\mathrm{B}} + t}{2t}$ by solving $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$. If the price p is low such that $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) \geq 0$, our assumption of a fully covered market is justified; if the ISP chooses a high price p that makes $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) < 0$, some consumers exit the market and the market is partially covered.

In stage 2, CP A makes decisions on content quality and data sponsorship while CP B only needs to decide on content quality, and the ISP decides the data usage price. In a candidate fully-covered equilibrium, we derive CP B's optimal content quality decision, then derive CP A's optimal decisions on content quality and data sponsorship, and finally derive the ISP's optimal pricing decision. First, CP B's profit function can be written as: $\pi_{\mathrm{B}} = r(1 - \frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}+t}{2t}) - \frac{\phi}{2}{V_{\mathrm{B}}}^2$. By solving the FOC of CP B's profit function π_{B} w.r.t. V_{B}, we get CP B's optimal content quality decision $V_{\mathrm{B}}^* = \frac{r}{2\phi t}$. Next, we analyze CP A's optimal content quality decision V_{A}^* and data sponsorship decision s_{A}^*. To guarantee the existence of the maximum of CP A's profit function $\pi_{\mathrm{A}} = (r - s_{\mathrm{A}})\frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}+t}{2t} - \frac{\phi}{2}{V_{\mathrm{A}}}^2$, we apply the assumption that $\phi > \frac{1}{4t}$, which ensures that the Jacobian $\frac{d^2\,\pi_{\mathrm{A}}}{d\,{V_{\mathrm{A}}}^2} \times \frac{d^2\,\pi_{\mathrm{A}}}{d\,{s_{\mathrm{A}}}^2} - (\frac{d^2\,\pi_{\mathrm{A}}}{d\,V_{\mathrm{A}} d\,s_{\mathrm{A}}})^2 = \frac{\phi}{t} - (\frac{1}{2t})^2 > 0$. Without this assumption, CP A's profit function π_{A} is convex and hence its optimal content quality decision V_{A}^* is infinitely high. Solving the FOC of π_{A} w.r.t. V_{A} and s_{A} yields $V_{\mathrm{A}}^* = \frac{r-s_{\mathrm{A}}^*}{2\phi t}$ and $s_{\mathrm{A}}^* = \frac{1}{2}(V_{\mathrm{B}}^* - V_{\mathrm{A}}^* - t + r) = \frac{1}{2}\left(\frac{r}{2\phi t} - V_{\mathrm{A}}^* - t + r\right)$, respectively. Notice that $V_{\mathrm{A}}^* = \frac{r-s_{\mathrm{A}}^*}{2\phi t}$ and $V_{\mathrm{B}}^* = \frac{r}{2\phi t}$ correspond to Lemma 2.

Jointly solving $V_{\mathrm{A}}^* = \frac{r-s_{\mathrm{A}}^*}{2\phi t}$ and $s_{\mathrm{A}}^* = \frac{1}{2}\left(\frac{r}{2\phi t} - V_{\mathrm{A}}^* - t + r\right)$ yields $s_{\mathrm{A}}^* = \frac{2\phi t(r-t)}{4\phi t-1}$ and $V_{\mathrm{A}}^* = \frac{r+t-\frac{r}{2\phi t}}{4\phi t-1}$. Then, the equilibrium marginal consumer is $x_{\mathrm{m}}^* = \frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*-V_{\mathrm{B}}^*+t}{2t} = \frac{-r+2\phi tr+2\phi t^2}{2t(4\phi t-1)}$. Finally, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}^*) = U_{\mathrm{B}}(x_{\mathrm{m}}^*) = 0$ and obtain the ISP's optimal price $p^* = \frac{1}{2}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* - t) = \frac{-3\phi t^2+t+\phi tr+\frac{3}{2}r-\frac{r}{2t\phi}}{4\phi t-1}$. Notice that $s_{\mathrm{A}}^* = \frac{2\phi t(r-t)}{4\phi t-1} \le 0$ when $r \le t$, given that $\phi > \frac{1}{4t}$. Therefore, when $r \le t$, we have $s_{\mathrm{A}}^* = 0$, and the equilibrium degenerates into that of subgame N. This is the finding in Lemma 3.

We summarize the fully-covered equilibrium of subgame S in Table A.1, which is the same as Table 2 in Lemma 4.

Table A.1. Fully-covered equilibrium of subgame S.					
Assumptions	$2\phi t^2 \le r \le 4\phi t^2$ and $\phi > \frac{1}{4t}$				
Market conditions	Per-packet price p^*	CP A's content quality decision V_A^*	CP A's data sponsorship decision s_A^*	CP B's content quality decision V_B^*	Marginal consumer x_m^*
$r \le t$	$\frac{r}{2\phi t} - \frac{t}{2}$	$\frac{r}{2\phi t}$	0	$\frac{r}{2\phi t}$	$\frac{1}{2}$
$r > t$	$\frac{-3\phi t^2+t+\phi tr+\frac{3}{2}r-\frac{r}{2t\phi}}{4\phi t-1}$	$\frac{r+t-\frac{r}{2\phi t}}{4\phi t-1}$	$\frac{2\phi t(r-t)}{4\phi t-1}$	$\frac{r}{2\phi t}$	$\frac{-r+2\phi tr+2\phi t^2}{2t(4\phi t-1)}$

We check the constraint $0 \le x_m^* \le 1$ under the assumptions that $\phi > \frac{1}{4t}$ and $2\phi t^2 \le r \le 4\phi t^2$. We discuss the following three situations:

(1) When $r \le t$, we have $0 \le x_m^* = \frac{1}{2} \le 1$.

(2) When $r > t$ and $\frac{1}{4t} < \phi \le \frac{1}{2t}$, solving $0 \le x_m^* = \frac{-r+2\phi tr+2\phi t^2}{2t(4\phi t-1)} \le \frac{1}{2}$ yields $t \le r \le \frac{2\phi t^2}{1-2\phi t}$. Since $r \le 4\phi t^2 \le \frac{2\phi t^2}{1-2\phi t}$, we have $0 \le x_m^* \le \frac{1}{2}$.

(3) When $r > t$ and $\phi > \frac{1}{2t}$, solving $\frac{1}{2} \le x_m^* = \frac{-r+2\phi tr+2\phi t^2}{2t(4\phi t-1)} \le 1$ yields $t \le r \le \frac{2t(3\phi t-1)}{2\phi t-1}$. Since $r \le 4\phi t^2 \le \frac{2t(3\phi t-1)}{2\phi t-1}$, we have $\frac{1}{2} \le x_m^* \le 1$.

To sum up, we always have $0 \le x_m^* \le 1$ in subgame S under the assumptions that $\phi > \frac{1}{4t}$ and $2\phi t^2 \le r \le 4\phi t^2$.

Finally, as in the proof of Lemma 1, it suffices to prove that neither the ISP nor the CPs have an incentive to deviate and make the market partially covered. We first prove that the ISP has no incentive to set a high data usage price that causes a partially covered market under the assumption that $r \ge 2\phi t^2$. From the above analysis, we get the ISP's highest profit $\pi_{ISP\_F}^S = p_F^S = \frac{1}{2}(V_A^* + s_A^* + V_B^* - t)$ in a fully covered market of subgame S. According to our analysis of stage 3, the market is partially covered if $p > p_F^S$. In the following analysis, we calculate the ISP's profit $\pi_{ISP\_P}^S(p)$ if it deviates to a partially covered market of subgame S and then compare $\pi_{ISP\_P}^S(p)$ with $\pi_{ISP\_F}^S$. Given the CPs' content quality strategies

$V_{\mathrm{B}}^{*} = \frac{r}{2\phi t}$ and $V_{\mathrm{A}}^{*} = \frac{r - s_{\mathrm{A}}^{*}}{2\phi t}$ and data sponsorship strategy $s_{\mathrm{A}}^{*} = \begin{cases} 0 & \text{when } r \le t \\ \frac{2\phi t(r-t)}{4\phi t - 1} & \text{when } r > t \end{cases}$, solving $U_{\mathrm{A}}(x_{\mathrm{A}}) = 0$ and $U_{\mathrm{B}}(x_{\mathrm{B}}) = 0$ yields $x_{\mathrm{A}} = \frac{V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} - p}{t}$ and $x_{\mathrm{B}} = 1 - \frac{V_{\mathrm{B}}^{*} - p}{t}$. Then, the ISP's profit is $\pi_{\mathrm{ISP\_P}}^{\mathrm{S}}(p) = p(x_{\mathrm{A}} + 1 - x_{\mathrm{B}}) = \frac{p(V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} - 2p)}{t}$ if $p > p_{\mathrm{F}}^{\mathrm{S}}$. Notice that $\pi_{\mathrm{ISP\_F}}^{\mathrm{S}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{S}}(p) = \frac{1}{2t}(2p - t)(2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} - t))$. For any $p > p_{\mathrm{F}}^{\mathrm{S}}$, to show $\pi_{\mathrm{ISP\_F}}^{\mathrm{S}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{S}}(p) > 0$, we prove $2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} - t) > 0$ and $2p - t > 0$ respectively. First, it is straightforward to get that $2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} - t) > 0$ if $p > p_{\mathrm{F}}^{\mathrm{S}} = \frac{1}{2}(V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} - t)$. Second, we can prove that $p_{\mathrm{F}}^{\mathrm{S}} \ge \frac{t}{2}$, which implies that $2p - t > 0$ if $p > p_{\mathrm{F}}^{\mathrm{S}}$. Under the assumption that $r \ge 2\phi t^2$, it is easy to see that $p_{\mathrm{F}}^{\mathrm{S}} - \frac{t}{2} = \frac{1}{2}\left(\frac{r - 2\phi t^2}{\phi t} + \left(1 - \frac{1}{2\phi t}\right) s_{\mathrm{A}}^{*}\right) \ge 0$ when $\phi \ge \frac{1}{2t}$ or $s_{\mathrm{A}}^{*} = 0$ (i.e., $r \le t$). When $\phi < \frac{1}{2t}$ and $r > t$, we also have $p_{\mathrm{F}}^{\mathrm{S}} - \frac{t}{2} = \frac{1}{2(4\phi t - 1)}\left(\frac{r(2(\phi t)^2 + 3\phi t - 1)}{\phi t} - (10\phi t - 3)t\right) \ge \frac{1 - 2\phi t}{2\phi} > 0$. Therefore, as shown in Figure A.2, $\pi_{\mathrm{ISP\_F}}^{\mathrm{S}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{S}}(p) > 0$ for any $p > p_{\mathrm{F}}^{\mathrm{S}}$. Thus, compared to the fully-covered equilibrium of subgame S, the ISP's profit decreases if it chooses a price higher than $p_{\mathrm{F}}^{\mathrm{S}}$ and deviates to a partially covered market.

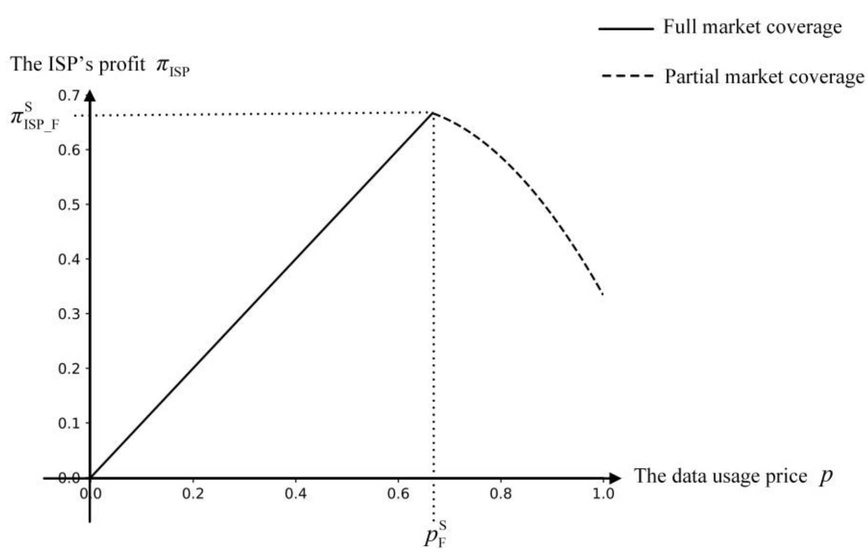

Figure A.2. Illustration of the ISP's optimal pricing decision in subgame S when $r = 2, \phi = 1, t = 1$.

We then show that a CP has no incentive to deviate from a fully-covered equilibrium. For CP A, it can deviate from (s_A^*, V_A^*) to (s_A', V_A') with $V_A' + s_A' < V_A^* + s_A^*$, which makes $U_A(x_m^*) < 0$ and causes a partially covered market. As a result, its market share decreases from $x_m^* = \frac{s_A^*+V_A^*-V_B^*+t}{2t}$ to $x_m' = x_m^* - \frac{V_A^*+s_A^*-V_A'-s_A'}{t}$. In the fully-covered equilibrium of subgame S, CP A's profit is

$$\pi_A^* = (r - s_A^*)\left(\frac{s_A^*+V_A^*-V_B^*+t}{2t}\right) - \frac{\phi}{2}{V_A^*}^2$$

$$= -\frac{\phi}{2}\left(V_A^* - \frac{r-s_A^*}{2\phi t}\right)^2 - \frac{4\phi t-1}{8\phi t^2}\left(s_A^* - \frac{2\phi t(r-t)}{4\phi t-1}\right)^2 + \frac{1}{8\phi t^2}\left(\frac{4\phi^2 t^2(r-t)^2}{4\phi t-1} + r(4\phi t^2 - r)\right).$$

Because $V_A^* = \frac{r-s_A^*}{2\phi t}$ and $s_A^* = \frac{2\phi t(r-t)}{4\phi t-1}$, we further have $\pi_A^* = \frac{1}{8\phi t^2}\left(\frac{4\phi^2 t^2(r-t)^2}{4\phi t-1} + r(4\phi t^2 - r)\right) \geq 0$ under the assumptions $\phi > \frac{1}{4t}$ and $r \leq 4\phi t^2$. If CP A deviates to (s_A', V_A'), its profit becomes

$$\pi_A' = (r - s_A')\left(\frac{s_A^*+V_A^*-V_B^*+t}{2t} - \frac{V_A^*+s_A^*-V_A'-s_A'}{t}\right) - \frac{\phi}{2}{V_A'}^2$$

$$= (r - s_A')\left(\frac{s_A^*+V_A^*-V_B^*+t}{2t} - \frac{V_A^*+s_A^*-V_A'-s_A'}{2t} - \frac{V_A^*+s_A^*-V_A'-s_A'}{2t}\right) - \frac{\phi}{2}{V_A'}^2$$

$$= (r - s_A')\left(\frac{V_A'+s_A'-V_B^*+t}{2t}\right) - \frac{\phi}{2}{V_A'}^2 - \frac{(r-s_A')(V_A^*+s_A^*-V_A'-s_A')}{2t}.$$

By plugging in the expression of V_B^*, the first two terms of the above expression become:

$$(r - s_A')\left(\frac{V_A'+s_A'-V_B^*+t}{2t}\right) - \frac{\phi}{2}{V_A'}^2$$

$$= -\frac{\phi}{2}\left(V_A' - \frac{r-s_A'}{2\phi t}\right)^2 - \frac{4\phi t-1}{8\phi t^2}\left(V_A' - \frac{2\phi t(r-t)}{4\phi t-1}\right)^2 + \frac{1}{8\phi t^2}\left(\frac{4\phi^2 t^2(r-t)^2}{4\phi t-1} + r(4\phi t^2 - r)\right)$$

$$= \pi_A^* - \frac{\phi}{2}\left(V_A' - \frac{r-s_A'}{2\phi t}\right)^2 - \frac{4\phi t-1}{8\phi t^2}\left(V_A' - \frac{2\phi t(r-t)}{4\phi t-1}\right)^2.$$

Then, we can calculate CP A's profit changes as:

$$\Delta\pi_A' = \pi_A^* - \pi_A'$$

$$= \frac{\phi}{2}\left(V_A' - \frac{r-s_A'}{2\phi t}\right)^2 + \frac{4\phi t-1}{8\phi t^2}\left(V_A' - \frac{2\phi t(r-t)}{4\phi t-1}\right)^2 + \frac{(r-s_A')(V_A^*+s_A^*-V_A'-s_A')}{2t}.$$

Notice that when $s_A' > r$, it is straightforward to see from the definition of π_A' that $\pi_A' < 0$ and we trivially have $\pi_A' < \pi_A^*$. When $s_A' \leq r$, we also get $\Delta\pi_A' \geq 0$ because $\frac{(r-s_A')(V_A^*+s_A^*-V_A'-s_A')}{2t} \geq 0$ and $\frac{\phi}{2}\left(V_A' - \frac{r-s_A'}{2\phi t}\right)^2 + \frac{4\phi t-1}{8\phi t^2}\left(V_A' - \frac{2\phi t(r-t)}{4\phi t-1}\right)^2 \geq 0$.

For CP B, since it has no option to subsidize, we follow the proof of Lemma 1 that deviating to a partially covered market is unprofitable. Therefore, we conclude that compared to the fully-covered equilibrium of subgame S, the CP's profit decreases if it deviates to a partially covered market in every situation.

Finally, from Table A.1, it is straightforward to see that subsidizing consumers' data usage will decrease a sponsoring CP's market share if $x_{\mathrm{m}}^{*} < \frac{1}{2}$, which implies $\phi < \frac{1}{2t}$ as $r > t$. This is the finding in Lemma 5.

Proof of Lemma 6

In this proof, we construct a fully-covered equilibrium of subgame SS wherein both CPs are allowed to sponsor consumer data traffic. Similar to the proof of the previous two subgames, we construct a fully-covered equilibrium via backward induction and show that both the ISP and CPs have no incentive to deviate and make the market partially covered.

In stage 3, if the market is fully covered, we determine the marginal consumer $x_{\mathrm{m}} = \frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}-s_{\mathrm{B}}+t}{2t}$ by solving $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$. If the price p is low such that $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) \geq 0$, our assumption of a fully covered market is justified; if the ISP chooses a high price p that makes $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) < 0$, some consumers exit the market and the market is partially covered.

In stage 2, both CPs decide on content quality and data sponsorship and the ISP decides the data usage price. In a candidate fully-covered equilibrium, we derive the CPs' optimal decisions on content quality and data sponsorship and then derive the ISP's optimal pricing decision. First, we derive the CPs' optimal content quality decision V_{i}^{*} and data sponsorship decision s_{i}^{*} based on the profit function $\pi_{\mathrm{i}} = (r - s_{\mathrm{i}})\frac{V_{\mathrm{i}}+s_{\mathrm{i}}-V_{-\mathrm{i}}-s_{-\mathrm{i}}+t}{2t} - \frac{\phi}{2}V_{\mathrm{i}}^{2}$. To guarantee the existence of the maximums of π_{A} and π_{B}, we require that $\phi > \frac{1}{4t}$, which ensures that the Jacobian $\frac{d^2 \pi_i}{d V_i^2} \times \frac{d^2 \pi_i}{d s_i^2} - (\frac{d^2 \pi_i}{d V_i d s_i})^2 = \frac{\phi}{t} - (\frac{1}{2t})^2 > 0$. Solving $\frac{d \pi_A}{d V_A} = 0$ and $\frac{d \pi_B}{d V_B} = 0$ yields $V_A = \frac{r - s_A}{2\phi t}$ and $V_B = \frac{r - s_B}{2\phi t}$ respectively. Thus, the result in Lemma 2 also holds in subgame SS. Solving $\frac{d \pi_A}{d s_A} = 0$ and $\frac{d \pi_B}{d s_B} = 0$ yields $s_A = \frac{1}{2}(V_B + s_B - V_A - t + r)$ and $s_B = \frac{1}{2}(V_A + s_A - V_B - t + r)$ respectively. Jointly solving the four FOCs yields $V_A^* = V_B^* = \frac{1}{2\phi}$, $s_A^* = s_B^* = r - t$. Then, we determine the equilibrium marginal consumer $x_m^* = \frac{V_A^* + s_A^* - V_B^* - s_B^* + t}{2t} = \frac{1}{2}$. Second, we solve $U_A(x_m^*) = U_B(x_m^*) = 0$ and get the ISP's optimal price $p^* = \frac{1}{2}(V_A^* + s_A^* + V_B^* + s_B^* - t) = \frac{1}{2\phi} + r - \frac{3t}{2}$.

Notice that $s_A^* = s_B^* = r - t \le 0$ when $r \le t$. Therefore, when $r \le t$, we have $s_A^* = s_B^* = 0$, and the equilibrium degenerates into that of subgame N as in Lemma 3. We summarize the fully-covered equilibrium of subgame SS in Table A.2, which is the same as Table 3 in Lemma 6.

Table A.2. Fully-covered equilibrium of subgame SS.

Assumptions	$2\phi t^2 \le r \le 4\phi t^2$ and $\phi > \frac{1}{4t}$			
Market conditions	Per-packet price p^*	CPs' content quality decisions $V_A^* (= V_B^*)$	CPs' data sponsorship decisions $s_A^* (= s_B^*)$	Marginal consumer x_m^*
$r \le t$	$\frac{r}{2\phi t} - \frac{t}{2}$	$\frac{r}{2\phi t}$	0	$\frac{1}{2}$
$r > t$	$\frac{1}{2\phi} + r - \frac{3t}{2}$	$\frac{1}{2\phi}$	$r - t$	$\frac{1}{2}$

Finally, we prove that neither the ISP nor the CPs have an incentive to deviate and make the market partially covered. For the ISP, again we show that it has no incentive to set a high data usage price that causes a partially covered market under the assumption that $r \ge 2\phi t^2$. From the above analysis, we get the ISP's highest profit $\pi_{ISP\_F}^{SS} = p_F^{SS} = \frac{1}{2}(V_A^* + s_A^* + V_B^* + s_B^* - t)$ in a fully covered market of subgame SS. According to our analysis of stage 3, the market is partially covered if $p > p_F^{SS}$. In the following analysis, we calculate the ISP's profit $\pi_{ISP\_P}^{SS}(p)$ if it deviates to a partially covered market of subgame SS and then compare it with $\pi_{ISP\_F}^{SS}$. Given the CPs' content quality strategy $V_i^* = \frac{r - s_i^*}{2\phi t}$ and data sponsorship strategy $s_{\mathrm{i}}^{*} = \begin{cases} 0 & \text{when } r \leq t \\ r - t & \text{when } r > t \end{cases}$, solving $U_{\mathrm{A}}(x_{\mathrm{A}}) = 0$ and $U_{\mathrm{B}}(x_{\mathrm{B}}) = 0$ yields $x_{\mathrm{A}} = 1 - x_{\mathrm{B}} = \frac{V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} - p}{t}$. Then, the ISP's profit is $\pi_{\mathrm{ISP\_P}}^{\mathrm{SS}}(p) = p(x_{\mathrm{A}} + 1 - x_{\mathrm{B}}) = \frac{p(V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} + s_{\mathrm{B}}^{*} - 2p)}{t}$ if $p > p_{\mathrm{F}}^{\mathrm{SS}}$. Notice that $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{SS}}(p) = \frac{1}{2t}(2p - t)(2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} + s_{\mathrm{B}}^{*} - t))$. For any $p > p_{\mathrm{F}}^{\mathrm{SS}}$, to show $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{SS}}(p) > 0$, we prove $2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} + s_{\mathrm{B}}^{*} - t) > 0$ and $2p - t > 0$ respectively. First, it is straightforward to get that $2p - (V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} + s_{\mathrm{B}}^{*} - t) > 0$ if $p > p_{\mathrm{F}}^{\mathrm{SS}} = \frac{1}{2}(V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} + V_{\mathrm{B}}^{*} + s_{\mathrm{B}}^{*} - t)$. Second, we can prove that $p_{\mathrm{F}}^{\mathrm{SS}} \geq \frac{t}{2}$. Under the assumption that $r \geq 2\phi t^{2}$, it is easy to see that $p_{\mathrm{F}}^{\mathrm{SS}} - \frac{t}{2} = \frac{r - 2\phi t^{2}}{2\phi t} + \left(1 - \frac{1}{2\phi t}\right) s_{\mathrm{i}}^{*} \geq 0$ when $\phi \geq \frac{1}{2t}$ or $s_{\mathrm{i}}^{*} = 0$ (i.e., $r \leq t$). When $\phi < \frac{1}{2t}$ and $r > t$, we also have $p_{\mathrm{F}}^{\mathrm{SS}} - \frac{t}{2} = r - \frac{(4\phi t - 1)t}{2\phi t} \geq \frac{1 - 2\phi t}{2\phi} > 0$. Therefore, as shown in Figure A.3, $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}} - \pi_{\mathrm{ISP\_P}}^{\mathrm{SS}}(p) > 0$ for any $p > p_{\mathrm{F}}^{\mathrm{SS}}$. Therefore, compared to the fully-covered equilibrium of subgame SS, the ISP's profit decreases if it chooses a price higher than $p_{\mathrm{F}}^{\mathrm{SS}}$ and deviates to a partially covered market.

For the CPs, we want to show that CP i has no incentive to deviate from $(s_{\mathrm{i}}^{*}, V_{\mathrm{i}}^{*})$ to $(s_{\mathrm{i}}', V_{\mathrm{i}}')$ with $V_{\mathrm{i}}' + s_{\mathrm{i}}' < V_{\mathrm{i}}^{*} + s_{\mathrm{i}}^{*}$. Such a deviation makes $U_{\mathrm{A}}(x_{\mathrm{m}}^{*}) < 0$ and causes a partially covered market. Similar to the proofs of Lemmas 2–4, we find the profit change as:

$$\Delta\pi_{\mathrm{A}}' = \pi_{\mathrm{A}}^{*} - \pi_{\mathrm{A}}'$$

$$= \frac{\phi}{2}\left(\mathrm{V}_{\mathrm{A}}' - \frac{r - s_{\mathrm{A}}'}{2\phi t}\right)^{2} + \frac{4\phi t - 1}{8\phi t^{2}}\left(\mathrm{V}_{\mathrm{A}}' - \frac{2\phi t(r - t)}{4\phi t - 1}\right)^{2} + \frac{(r - s_{\mathrm{A}}')(V_{\mathrm{A}}^{*} + s_{\mathrm{A}}^{*} - V_{\mathrm{A}}' - s_{\mathrm{A}}')}{2t} \geq 0.$$

Hence, we conclude that compared to the fully-covered equilibrium of subgame SS, the CP's profit decreases if it deviates to a partially covered market in every situation.

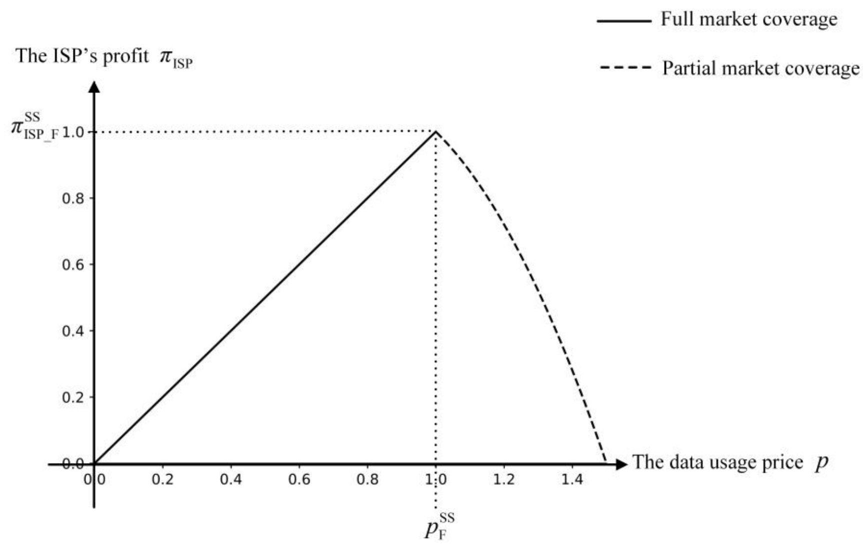

Figure A.3. Illustration of the ISP's optimal pricing decision in subgame SS when $r = 2, \phi = 1, t = 1$.

Proof of Proposition 1

In our model, the ISP's profit π_{ISP} is equal to the data usage price p. According to Lemmas 1, 4, and 6, we have $\pi^{\text{N}}_{\text{ISP}} = p^{\text{N}} = \frac{r}{2\phi t} - \frac{t}{2}$, $\pi^{\text{S}}_{\text{ISP}} = p^{\text{S}} = \begin{cases} \frac{r}{2\phi t} - \frac{t}{2} & \text{when } r \le t \\ \frac{-3\phi t^2 + t + \phi t r + \frac{3}{2}r - \frac{r}{2\phi t}}{4\phi t - 1} & \text{when } r > t \end{cases}$, and $\pi^{\text{SS}}_{\text{ISP}} = p^{\text{SS}} = \begin{cases} \frac{r}{2\phi t} - \frac{t}{2} & \text{when } r \le t \\ \frac{1}{2\phi} + r - \frac{3t}{2} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi^{\text{N}}_{\text{ISP}}$, $\pi^{\text{S}}_{\text{ISP}}$ and $\pi^{\text{SS}}_{\text{ISP}}$ when $r > t$.

First of all, when $r > t$, $\pi^{\text{SS}}_{\text{ISP}} - \pi^{\text{N}}_{\text{ISP}} = \left(\phi t - \frac{1}{2}\right)(r - t)$, which is positive only when $\phi t - \frac{1}{2} > 0$. Second, when $r > t$, the sign of $\pi^{\text{S}}_{\text{ISP}} - \pi^{\text{N}}_{\text{ISP}}$ is the same as that of $(1 - \frac{1}{2\phi t})(r - t)$, which is positive also when $\phi t - \frac{1}{2} > 0$. Finally, when $r > t$, the sign of $\pi^{\text{SS}}_{\text{ISP}} - \pi^{\text{S}}_{\text{ISP}}$ is the same as that of $\left(3\phi t - \frac{5}{2} + \frac{1}{2\phi t}\right)(r - t)$, which is positive when $\phi t - \frac{1}{2} > 0$ or $\frac{1}{4} < \phi t < \frac{1}{3}$.

Therefore, we conclude that under our parameter assumptions, we have $\pi^{\text{N}}_{\text{ISP}} > \pi^{\text{S}}_{\text{ISP}}$ and $\pi^{\text{N}}_{\text{ISP}} > \pi^{\text{SS}}_{\text{ISP}}$ when $\phi < \frac{1}{2t}$, and $\pi^{\text{N}}_{\text{ISP}} < \pi^{\text{S}}_{\text{ISP}} < \pi^{\text{SS}}_{\text{ISP}}$ when $\phi > \frac{1}{2t}$. Then, Proposition 1 is proved.

Proof of Proposition 2

To prove Proposition 2, we compare CP A's profits in the equilibria under different subgames. According to Lemmas 1, 4, and 6, the CP A's profit in subgame N is $\pi_{\mathrm{A}}^{\mathrm{N}} = \pi_{\mathrm{B}}^{\mathrm{N}} = \frac{r}{2} - \frac{r^2}{8\phi t^2}$, that in subgame S is $\pi_{\mathrm{A}}^{\mathrm{S}} = \begin{cases} \frac{r}{2} - \frac{r^2}{8\phi t^2} & \text{when } r \leq t \\ \frac{(-r+2\phi tr+2\phi t^2)^2}{8\phi t^2(4\phi t-1)} & \text{when } r > t \end{cases}$, and that in subgame SS is $\pi_{\mathrm{A}}^{\mathrm{SS}} = \pi_{\mathrm{B}}^{\mathrm{SS}} = \begin{cases} \frac{r}{2} - \frac{r^2}{8\phi t^2} & \text{when } r \leq t \\ \frac{4\phi t-1}{8\phi} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi_{\mathrm{A}}^{\mathrm{N}}$, $\pi_{\mathrm{A}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{SS}}$ when $r > t$.

First of all, when $r > t$, $\pi_{\mathrm{A}}^{\mathrm{N}} - \pi_{\mathrm{A}}^{\mathrm{S}} = \frac{-4\phi^2 t^2 (r-t)^2}{8\phi t^2(4\phi t-1)} < 0$ as $\phi > \frac{1}{4t}$. Second, when $r > t$, the sign of $\pi_{\mathrm{A}}^{\mathrm{SS}} - \pi_{\mathrm{A}}^{\mathrm{S}}$ is the same as that of $(1-2\phi t)(r-t)(6\phi t^2 - t + 2\phi tr - r)$. As $r \leq 4\phi t^2$ and $\phi > \frac{1}{4t}$, $6\phi t^2 - t + 2\phi tr - r > (4\phi t^2 - r) + (4\phi t^2 - t) > 0$. Thus, we have $\pi_{\mathrm{A}}^{\mathrm{SS}} - \pi_{\mathrm{A}}^{\mathrm{S}} > 0$ only when $1 - 2\phi t > 0$. Finally, when $r > t$, $\pi_{\mathrm{A}}^{\mathrm{SS}} - \pi_{\mathrm{A}}^{\mathrm{N}} = \frac{(r-4\phi t^2+t)(r-t)}{8\phi t^2}$, which is positive only when $r > 4\phi t^2 - t$.

Therefore, we conclude that under the assumptions that $r > t$, $4\phi t^2 \geq r \geq 2\phi t^2$, and $\phi > \frac{1}{4t}$, we have $\pi_{\mathrm{A}}^{\mathrm{N}} < \pi_{\mathrm{A}}^{\mathrm{S}} < \pi_{\mathrm{A}}^{\mathrm{SS}}$ when $\phi < \frac{1}{2t}$. Then, Proposition 2.1 is proved. When $\phi > \frac{1}{2t}$, we find $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{N}}$, and $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$. In addition, $\pi_{\mathrm{A}}^{\mathrm{SS}} > \pi_{\mathrm{A}}^{\mathrm{N}}$ only if $r > 4\phi t^2 - t$. Then, Proposition 2.2 is proved.

Proof of Proposition 3

To prove Proposition 3, we consider two different cases. When the CP is allowed to sponsor in subgame S (i.e., CP A), then we just need to show that $\pi_{\mathrm{A}}^{\mathrm{S}} < \pi_{\mathrm{A}}^{\mathrm{SS}}$ when $\phi < \frac{1}{2t}$; and $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$ when $\phi > \frac{1}{2t}$. This has already been shown by Proposition 2. Hence, we just need to consider the other case when the CP is not allowed to sponsor in subgame S (i.e., CP B), and show that $\pi_{\mathrm{B}}^{\mathrm{N}} < \pi_{\mathrm{B}}^{\mathrm{S}}$ when $\phi < \frac{1}{2t}$; and $\pi_{\mathrm{B}}^{\mathrm{N}} > \pi_{\mathrm{B}}^{\mathrm{S}}$ when $\phi > \frac{1}{2t}$.

We compare CP B's equilibrium profits in different subgames. According to Lemmas 1, 4, and 6, CP B's equilibrium profit in subgame N is $\pi_{\mathrm{B}}^{\mathrm{N}} = \frac{r}{2} - \frac{r^2}{8\phi t^2}$, that in subgame S is $\pi_{\mathrm{B}}^{\mathrm{S}} =$

$\begin{cases} \frac{r}{2} - \frac{r^2}{8\phi t^2} & \text{when } r \le t \\ \frac{3\phi t^2 r - tr - \phi t r^2 + \frac{r^2}{8\phi t}}{t(4\phi t - 1)} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi_{\mathrm{B}}^{\mathrm{N}}$ and $\pi_{\mathrm{B}}^{\mathrm{S}}$ when $r > t$. When $r > t$, the sign of $\pi_{\mathrm{B}}^{\mathrm{N}} - \pi_{\mathrm{B}}^{\mathrm{S}}$ is the same as that of $\left(\frac{1}{2} - \phi t\right)(r - t)$, which is positive only when $\phi t - \frac{1}{2} < 0$. Therefore, we conclude that under our parameter assumptions, we have $\pi_{\mathrm{B}}^{\mathrm{N}} < \pi_{\mathrm{B}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{S}} < \pi_{\mathrm{A}}^{\mathrm{SS}}$ when $\phi < \frac{1}{2t}$. Therefore, when the content production efficiency is high (i.e., $\phi < \frac{1}{2t}$), a CP always benefits from its rival's data sponsorship regardless of whether it has the sponsoring option. Then, Proposition 3.1 is proved. When $\phi > \frac{1}{2t}$, we find $\pi_{\mathrm{B}}^{\mathrm{N}} > \pi_{\mathrm{B}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$. Therefore, a CP in this circumstance suffers from its rival's data sponsorship regardless of whether it has the sponsoring option. Then, Proposition 3.2 is proved.

Proof of Proposition 4

Notice that in all of the three subgames, $U_{\mathrm{A}}(x) = U_{\mathrm{A}}(x_{\mathrm{m}}) + t(x_{\mathrm{m}} - x)$ for $x < x_{\mathrm{m}}$ and $U_{\mathrm{B}}(x) = U_{\mathrm{B}}(x_{\mathrm{m}}) + t(x - x_{\mathrm{m}})$ for $x > x_{\mathrm{m}}$. Hence, consumer surplus is calculated as

$$CS = \int_0^{x_{\mathrm{m}}} U_{\mathrm{A}}(x)\ dx + \int_{x_{\mathrm{m}}}^1 U_{\mathrm{B}}(x)\ dx = t(x_{\mathrm{m}} - \tfrac{1}{2})^2 + \tfrac{t}{4},$$

which immediately implies that consumer surplus increases as the marginal consumer x_{m} deviates from $\frac{1}{2}$. When $x_{\mathrm{m}} = \frac{1}{2}$, consumer surplus reaches its minimum. According to Lemmas 1, 4, and 6, $x_{\mathrm{m}} = \frac{1}{2}$ in subgames N and SS, and $x_{\mathrm{m}} \begin{cases} = \frac{1}{2} & \text{when } \phi = \frac{1}{2t} \text{ or } r \le t \\ \neq \frac{1}{2} & \text{otherwise} \end{cases}$ in subgame S. Therefore, we have $CS^{\mathrm{N}} = CS^{\mathrm{SS}} \le CS^{\mathrm{S}}$.

Proof of Proposition 5

To prove Proposition 5, we compare equilibrium social welfare in different subgames. Social welfare is defined as $SW = \pi_{\mathrm{ISP}} + \pi_{\mathrm{A}} + \pi_{\mathrm{B}} + CS$. According to the proofs of Propositions 1–4, social welfare in subgame N is $SW^{\mathrm{N}} = \frac{r}{2\phi t} - \frac{t}{4} + r - \frac{r^2}{4\phi t^2}$, that in subgame S is $SW^{\mathrm{S}} = \begin{cases} \frac{r}{2\phi t} - \frac{t}{4} + r - \frac{r^2}{4\phi t^2} & \text{when } r \le t \\ \frac{r}{2\phi t} - \frac{t}{4} + r - \frac{r^2}{4\phi t^2} + \frac{(-4\phi^2 t^2 + 6\phi t - 1)(r-t)^2}{4t(4\phi t - 1)^2} & \text{when } r > t \end{cases}$, and that in subgame SS is $SW^{\mathrm{SS}} = \begin{cases} \frac{r}{2\phi t} - \frac{t}{4} + r - \frac{r^2}{4\phi t^2} & \text{when } r \le t \\ \frac{r}{2\phi t} - \frac{t}{4} + r - \frac{r^2}{4\phi t^2} + \frac{(r-t)^2}{4\phi t^2} & \text{when } r > t \end{cases}$. Hence, we just need to compare SW^{N}, SW^{S} and SW^{SS} when $r > t$.

First, comparing SW^{N} and SW^{SS}, we have $SW^{\mathrm{SS}} - SW^{\mathrm{N}} = \frac{(r-t)^2}{4\phi t^2} > 0$ when $r > t$. Second, comparing SW^{N} and SW^{S}, we have $SW^{\mathrm{S}} - SW^{\mathrm{N}} = \frac{(-4\phi^2 t^2 + 6\phi t - 1)(r-t)^2}{4t(4\phi t - 1)^2}$ when $r > t$. Hence, the sign of $SW^{\mathrm{S}} - SW^{\mathrm{N}}$ is the same as that of $-4(\phi t)^2 + 6\phi t - 1$, which is positive when $\frac{1}{4t} < \phi < \frac{3+\sqrt{5}}{4t}$. Therefore, $SW^{\mathrm{S}} - SW^{\mathrm{N}} > 0$ only when $\frac{1}{4t} < \phi < \frac{3+\sqrt{5}}{4t}$. Finally, comparing SW^{S} and SW^{SS}, we have $SW^{\mathrm{SS}} - SW^{\mathrm{S}} = \frac{(r-t)^2}{4\phi t^2} - \frac{(-4\phi^2 t^2 + 6\phi t - 1)(r-t)^2}{4t(4\phi t - 1)^2}$ when $r > t$. The sign of $SW^{\mathrm{SS}} - SW^{\mathrm{S}}$ is the same as that of $\frac{(r-t)^2}{4\phi t^2} - \frac{(-4\phi^2 t^2 + 6\phi t - 1)(r-t)^2}{4t(4\phi t - 1)^2}$, which is proportional to a cubic function $f(\phi t) = 4(\phi t)^3 + 10(\phi t)^2 - 7\phi t + 1$.

Below we provide a detailed analysis of function $f(\phi t)$. First, it's straightforward to obtain: $f'(\phi t) = 12(\phi t)^2 + 20\phi t - 7$ and $f''(\phi t) = 24\phi t + 20$. Since $\phi t \ge 0$, we have result (i): $f'(\phi t) < 0$ if and only if $\phi t < \frac{\sqrt{46}-5}{6}$, and result (ii) $f''(\phi t) > 0$. This implies that $f(\phi t)$ is a strictly convex function that decreases initially and then increases as ϕt increases. Second, it's easy to verify that $f\left(\frac{1}{4}\right) = -\frac{1}{16} < 0$ and $f(1) = 8 > 0$. As we require $\phi > \frac{1}{4t}$ to avoid the trivial case where CPs over-invest in content production and produce content of infinitely high quality (please refer to page 11 of the main manuscript for the statement of the assumption and refer to page 15, footnote 4 of the main manuscript for the detailed explanation, "without this assumption, CP A's profit function is convex and hence will produce content of infinitely high quality, as shown by Online Appendix A."), we can obtain that $f(\phi t) < 0$ for

$\frac{1}{4} < \phi t < \frac{\sqrt{46}-5}{6}$ because $f(\phi t)$ is a strictly decreasing function when $\phi t < \frac{\sqrt{46}-5}{6}$ (see result (i)). In addition, since $f(\phi t)$ is a strictly increasing and continuous function for $\phi t > \frac{\sqrt{46}-5}{6}$ and $f(1) = 8 > 0$. Hence, there must exist a unique cutoff $\underline{\phi t}$ such that $f(\phi t) > 0$ for $\phi t > \underline{\phi t}$ and $f(\phi t) < 0$ for $\frac{1}{4} < \phi t < \underline{\phi t}$. We use Matlab to solve $f(\phi t) = 4(\phi t)^3 + 10(\phi t)^2 - 7\phi t + 1 = 0$ numerically, as illustrated in Figure A.4 below. We have verified that the solution is approximately 0.3785 (as shown in Figure A.4).[1] Thus, we have $SW^{\text{SS}} > SW^{\text{S}}$ only when $\phi > \underline{\phi} \approx \frac{0.3785}{t}$.

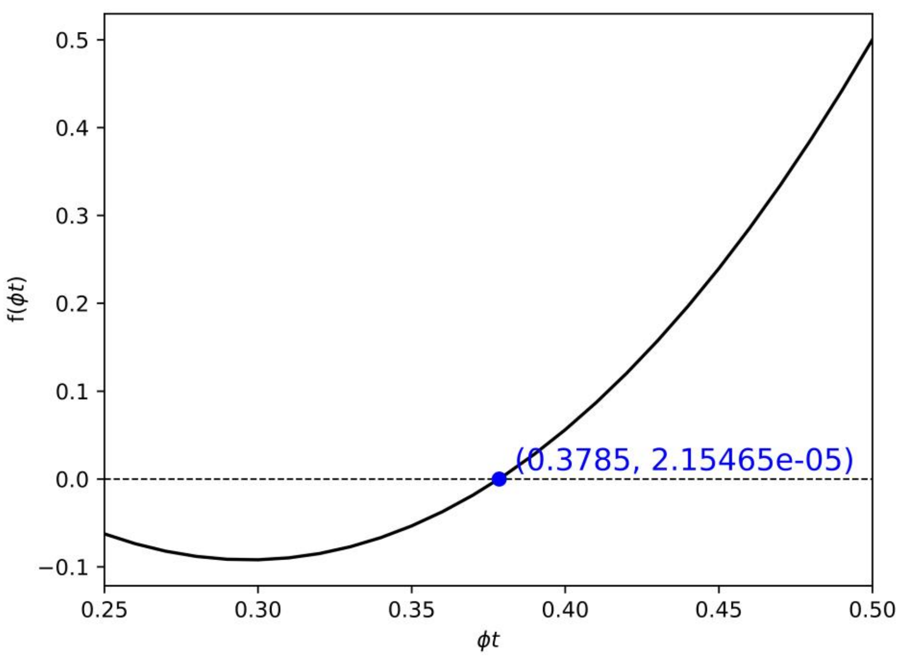

Figure A.4. Illustration of $f(\phi t) = 4(\phi t)^3 + 10(\phi t)^2 - 7\phi t + 1$ for $\frac{1}{4} < \phi t < \frac{1}{2}$.

As $\frac{1}{4} < \underline{\phi} \approx 0.3785 < \frac{3+\sqrt{5}}{4}$, we denote $\overline{\phi} = \frac{3+\sqrt{5}}{4t}$ and obtain the three cases shown in Proposition 5:

(i) When $\frac{1}{4t} < \phi < \underline{\phi}$, we have $SW^{\text{S}} > SW^{\text{SS}} > SW^{\text{N}}$;

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[1] The equation $f(\phi t) = 4(\phi t)^3 + 10(\phi t)^2 - 7\phi t + 1 = 0$ has two additional roots: one lies between 0 and $\frac{1}{4}$ and the other is negative. Obviously, neither of these two roots fall within the required region where $\phi t > \frac{1}{4}$.

(ii) When $\underline{\phi} < \phi < \overline{\phi}$, we have $SW^{\mathrm{SS}} > SW^{\mathrm{S}} > SW^{\mathrm{N}}$;

(iii) When $\overline{\phi} < \phi$, we have $SW^{\mathrm{SS}} > SW^{\mathrm{N}} > SW^{\mathrm{S}}$.

Appendix B: Equilibrium Analysis under the Assumption of Exogenous Content Quality

In Appendix B, we deduce the equilibria under the assumption that the content quality is fixed at V exogenously (i.e., $V_{\mathrm{A}} = V_{\mathrm{B}} = V$).

B.1. No Data Sponsorship (Subgame N)

We deduce the equilibrium of subgame N wherein no CP is allowed to subsidize consumers' data usage ($I_{\mathrm{A}} = I_{\mathrm{B}} = 0$).

In stage 3, to determine the marginal consumer x_{m}, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$ (see Equations (1) and (2) in the manuscript) and obtain $x_{\mathrm{m}} = \frac{V_{\mathrm{A}} - V_{\mathrm{B}} + t}{2t} = \frac{1}{2}$.

In stage 2, because CPs cannot endogenously determine content quality, CPs have no actions in this stage and the ISP decides the data usage price. We solve $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) = 0$ and obtain the ISP's optimal price $p^* = V - \frac{t}{2}$. To ensure $p^* \geq 0$, we apply the assumption that $V \geq \frac{t}{2}$. This assumption guarantees that internet access is appealing to consumers and worth paying for. The equilibrium of subgame N is summarized as follows.

LEMMA 1B (EQUILIBRIUM OF SUBGAME N). *In subgame N, wherein no CP is allowed to subsidize consumers' data usage, the equilibrium under the assumption of exogenous content quality is as follows: the optimal per-packet price set by the ISP is $p^* = V - \frac{t}{2}$, and the equilibrium marginal consumer is $x_{\mathrm{m}}^* = \frac{1}{2}$.*

B.2. Only One CP Sponsors Consumers (Subgame S)

We deduce the equilibrium of subgame S wherein only one CP is allowed to sponsor consumer data traffic. Without loss of generality, suppose that only CP A has the option to sponsor consumers ($I_{\mathrm{A}} = 1$ and $I_{\mathrm{B}} = 0$).

In stage 3, to determine the marginal consumer, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$ and obtain $x_{\mathrm{m}} = \frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}+t}{2t} = \frac{s_{\mathrm{A}}+t}{2t}$.

In stage 2, CP A decides on the amount of data sponsorship per consumer s_{A} and the ISP decides the data usage price p. We derive CP A's optimal decision on data sponsorship and then derive the ISP's optimal pricing decision. First, CP A's profit function can be written as: $\pi_{\mathrm{A}} = \frac{(r-s_{\mathrm{A}})(s_{\mathrm{A}}+t)}{2t} - \frac{\phi}{2}{V_{\mathrm{A}}}^2$. By solving the FOC of CP A's profit function π_{A} w.r.t. s_{A}, we determine CP A's optimal data sponsorship decision $s_{\mathrm{A}}^* = \frac{1}{2}(V_{\mathrm{B}} - V_{\mathrm{A}} - t + r) = \frac{r-t}{2}$. Then, the equilibrium marginal consumer is $x_{\mathrm{m}}^* = \frac{s_{\mathrm{A}}^*+t}{2t} = \frac{r+t}{4t}$. To guarantee $x_{\mathrm{m}}^* \leq 1$, we assume that $r \leq 3t$. Next, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}^*) = U_{\mathrm{B}}(x_{\mathrm{m}}^*) = 0$ and obtain the ISP's optimal price $p^* = V - \frac{3t-r}{4}$.

Notice that $s_{\mathrm{A}}^* = \frac{r-t}{2} \leq 0$ when $r \leq t$. Therefore, when $r \leq t$, we have $s_{\mathrm{A}}^* = 0$, and the equilibrium degenerates into that of subgame N. Then, the equilibrium of subgame S is summarized as follows.

LEMMA 2B (EQUILIBRIUM OF SUBGAME S). *In subgame S, wherein only one CP is allowed to sponsor consumers' data usage, the equilibrium under the assumption of exogenous content quality is as follows:*

Table B.1. Equilibrium of subgame S in the extension of exogenous content quality.

Assumptions	$V \geq \frac{t}{2}$ and $r \leq 3t$		
Market conditions	Per-packet price p^*	CP A's data sponsorship decision s_{A}^*	Marginal consumer x_{m}^*
$r \leq t$	$V - \frac{t}{2}$	0	$\frac{1}{2}$
$r > t$	$V - \frac{3t-r}{4}$	$\frac{r-t}{2}$	$\frac{r+t}{4t}$

B.3. Both CPs Sponsor Consumers (Subgame SS)

We deduce the equilibrium of subgame SS wherein both CPs are allowed to sponsor consumer data traffic.

In stage 3, to determine the marginal consumer, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}})$ and obtain $x_{\mathrm{m}} = \frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}-s_{\mathrm{B}}+t}{2t} = \frac{s_{\mathrm{A}}-s_{\mathrm{B}}+t}{2t}$.

In stage 2, both CPs make decisions on data sponsorship and the ISP decides the data usage price. We derive the CPs' optimal decisions on data sponsorship and then derive the ISP's optimal pricing decision. First, CP i's profit function can be written as: $\pi_{\mathrm{i}} = \frac{(r-s_{\mathrm{i}})(s_{\mathrm{i}}-s_{-\mathrm{i}}+t)}{2t} - \frac{\phi}{2}{V_{\mathrm{i}}}^2$. We solve the FOC of π_{A} w.r.t. s_{A} (i.e., $\frac{d\,\pi_{\mathrm{A}}}{d\,s_{\mathrm{A}}} = \frac{1}{2t}(s_{\mathrm{B}} - t + r_{\mathrm{A}} - 2s_{\mathrm{A}}) = 0$) and get $s_{\mathrm{A}} = \frac{1}{2}(s_{\mathrm{B}} - t + r)$. In the same way, we solve the FOC of π_{B} w.r.t. s_{B} and get $s_{\mathrm{B}} = \frac{1}{2}(s_{\mathrm{A}} - t + r)$. Jointly solving the two FOCs yields the CPs' optimal data sponsorship decision $s_{\mathrm{A}}^* = s_{\mathrm{B}}^* = r - t$. Then, the equilibrium marginal consumer is $x_{\mathrm{m}}^* = \frac{s_{\mathrm{A}}^*-s_{\mathrm{B}}^*+t}{2t} = \frac{1}{2}$. Next, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}^*) = U_{\mathrm{B}}(x_{\mathrm{m}}^*) = 0$ and obtain the ISP's optimal price $p^* = V + r - \frac{3t}{2}$.

Notice that $s_{\mathrm{A}}^* = s_{\mathrm{B}}^* = r - t > 0$ only when $r > t$. Therefore, when $r \leq t$, both CPs ignore the sponsoring options, and this equilibrium degenerates into that of subgame N. Then, the equilibrium of subgame SS is summarized as follows.

LEMMA 3B (EQUILIBRIUM OF SUBGAME SS). *In subgame SS, wherein both CPs are allowed to subsidize consumers' data usage, the equilibrium under the assumption of exogenous content quality is as follows:*

Table B.2. Equilibrium of subgame SS in the extension of exogenous content quality.			
Market conditions	$V \geq \frac{t}{2}$ and $r \leq 3t$		
Cases	Per-packet price p^*	CPs' data sponsorship decisions (s_A^*, s_B^*)	Marginal consumer x_m^*
$r \leq t$	$V - \frac{t}{2}$	(0,0)	$\frac{1}{2}$
$r > t$	$V + r - \frac{3t}{2}$	$(r-t, r-t)$	$\frac{1}{2}$

B.4. Welfare Analysis

We derive model predictions regarding Propositions 1-5 in this extension and arrive at Propositions 1B-5B. As summarized by Table 4 in the main text, Propositions 1–3 remain robust in this extension only when content production efficiency is low; when it is high, endogenizing their content provision decisions leads to the opposite conclusion. Proposition 4 keeps the same while Proposition 5 significantly changes in this extension. We present Propositions 1B-5B and their proofs as follows.

PROPOSITION 1B (ISP'S OPTIMAL NETWORK MANAGEMENT CHOICE). *In the SPNE, if content service quality is assumed to be exogenous, the ISP always benefits from the CPs' data sponsorship, and its optimal network management choice is to allow both CPs to subsidize consumers' data usage (i.e., $\pi_{\text{ISP}}^{\text{N}} < \pi_{\text{ISP}}^{\text{S}} < \pi_{\text{ISP}}^{\text{SS}}$).*

PROOF OF PROPOSITION 1B. In the model of exogenous content quality, the ISP's profit π_{ISP} is equal to the per-packet price p. According to Lemmas 1B–3B, we have $\pi_{\text{ISP}}^{\text{N}} = p^{\text{N}} = V - \frac{t}{2}$, $\pi_{\text{ISP}}^{\text{S}} = p^{\text{S}} = \begin{cases} V - \frac{t}{2} & \text{when } r \leq t \\ V - \frac{3t-r}{4} & \text{when } r > t \end{cases}$, and $\pi_{\text{ISP}}^{\text{SS}} = p^{\text{SS}} = \begin{cases} V - \frac{t}{2} & \text{when } r \leq t \\ V + r - \frac{3t}{2} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi_{\text{ISP}}^{\text{N}}$, $\pi_{\text{ISP}}^{\text{S}}$ and $\pi_{\text{ISP}}^{\text{SS}}$ when $r > t$. First of all, when $r > t$, $\pi_{\text{ISP}}^{\text{S}} - \pi_{\text{ISP}}^{\text{N}} = \frac{r-t}{4} > 0$. Second, when $r > t$, $\pi_{\text{ISP}}^{\text{SS}} - \pi_{\text{ISP}}^{\text{S}} = \frac{5r-3t}{4} > 0$. Therefore, we have $\pi_{\text{ISP}}^{\text{N}} < \pi_{\text{ISP}}^{\text{S}} < \pi_{\text{ISP}}^{\text{SS}}$ when $r > t$. Then, Proposition

1B is proved.

PROPOSITION 2B (IMPLICATION OF SUBSIDIZING CONSUMERS). *Considering CP A, which is allowed to sponsor in subgame S: If the quality of content services is assumed to be exogenous to CPs, CP A achieves maximal profit in subgame S, where it sponsors consumers alone, and it achieves a lower profit in subgame SS than in subgame N (i.e.,* $\pi_{\mathrm{A}}^{\mathrm{SS}} < \pi_{\mathrm{A}}^{\mathrm{N}} < \pi_{\mathrm{A}}^{\mathrm{S}}$*).*

PROOF OF PROPOSITION 2B. According to Lemmas 1B–3B, CP A's equilibrium profit in subgame N is $\pi_{\mathrm{A}}^{\mathrm{N}} = \frac{r}{2} - \frac{\phi V^2}{2}$, that in subgame S is $\pi_{\mathrm{A}}^{\mathrm{S}} = \begin{cases} \frac{r}{2} - \frac{\phi V^2}{2} & \text{when } r \leq t \\ \frac{(r+t)^2}{8t} - \frac{\phi V^2}{2} & \text{when } r > t \end{cases}$, and that in subgame SS is $\pi_{\mathrm{A}}^{\mathrm{SS}} = \begin{cases} \frac{r}{2} - \frac{\phi V^2}{2} & \text{when } r \leq t \\ \frac{r+t}{4} - \frac{\phi V^2}{2} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi_{\mathrm{A}}^{\mathrm{N}}$, $\pi_{\mathrm{A}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{SS}}$ when $r > t$. First of all, when $r > t$, $\pi_{\mathrm{A}}^{\mathrm{N}} - \pi_{\mathrm{A}}^{\mathrm{SS}} = \frac{r-t}{4} > 0$. Second, when $r > t$, $\pi_{\mathrm{A}}^{\mathrm{S}} - \pi_{\mathrm{A}}^{\mathrm{N}} = \frac{(r-t)^2}{8t} > 0$. Therefore, we have $\pi_{\mathrm{A}}^{\mathrm{SS}} < \pi_{\mathrm{A}}^{\mathrm{N}} < \pi_{\mathrm{A}}^{\mathrm{S}}$ when $r > t$. Then, Proposition 2B is proved.

PROPOSITION 3B (IMPACT OF THE RIVAL'S DATA SPONSORSHIP). *If the quality of content services is assumed to be exogenous to CPs, compared with the scenario in which the rival has no sponsoring option, a CP always suffers from its rival's data sponsorship, regardless of whether it has the option to sponsor.*

PROOF OF PROPOSITION 3B. To prove Proposition 3B, we consider two different cases. When the CP is allowed to sponsor in subgame S (i.e., CP A), then we just need to show that $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$. This has already been shown by Proposition 2B. Hence, we just need to consider the other case when the CP is not allowed to sponsor in subgame S (i.e., CP B), and show that $\pi_{\mathrm{B}}^{\mathrm{N}} > \pi_{\mathrm{B}}^{\mathrm{S}}$.

We compare CP B's equilibrium profits in subgames N and S. According to Lemmas 1B–3B, CP

B's equilibrium profit in subgame N is $\pi_{\mathrm{B}}^{\mathrm{N}} = \frac{r}{2} - \frac{\phi V^2}{2}$ and that in subgame S is $\pi_{\mathrm{B}}^{\mathrm{S}} = \begin{cases} \frac{r}{2} - \frac{\phi V^2}{2} & \text{when } r \le t \\ \frac{r(3t-r)}{4t} - \frac{\phi V^2}{2} & \text{when } r > t \end{cases}$. Hence, we just need to compare $\pi_{\mathrm{B}}^{\mathrm{N}}$ and $\pi_{\mathrm{B}}^{\mathrm{S}}$ when $r > t$. When $r > t$, $\pi_{\mathrm{B}}^{\mathrm{N}} - \pi_{\mathrm{B}}^{\mathrm{S}} = \frac{r(r-t)}{4t} > 0$. Therefore, we conclude that under our parameter assumptions, we have $\pi_{\mathrm{B}}^{\mathrm{N}} > \pi_{\mathrm{B}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$. Then a CP always suffers from its rival's data sponsorship regardless of whether it has the option to sponsor. Proposition 3B is proved.

PROPOSITION 4B (CONSUMER SURPLUS). *If the quality of content services is assumed to be exogenous to CPs, compared to the baseline scenario without data sponsorship (subgame N), (1) allowing both CPs to sponsor consumers does not change consumer surplus, whereas (2) allowing only one CP to sponsor consumers always increases consumer surplus.*

PROOF OF PROPOSITION 4B. Notice that in all of the three subgames, $U_{\mathrm{A}}(x) = U_{\mathrm{A}}(x_{\mathrm{m}}) + t(x_{\mathrm{m}} - x)$ for $x < x_{\mathrm{m}}$ and $U_{\mathrm{B}}(x) = U_{\mathrm{B}}(x_{\mathrm{m}}) + t(x - x_{\mathrm{m}})$ for $x > x_{\mathrm{m}}$. Hence, consumer surplus is calculated as $CS = \int_0^{x_{\mathrm{m}}} U_{\mathrm{A}}(x)\ dx + \int_{x_{\mathrm{m}}}^1 U_{\mathrm{B}}(x)\ dx = t(x_{\mathrm{m}} - \frac{1}{2})^2 + \frac{t}{4}$. It is easy to see that consumer surplus increases as the marginal consumer x_{m} deviates from $\frac{1}{2}$. When $x_{\mathrm{m}} = \frac{1}{2}$, consumer surplus reaches its minimum. According to Lemmas 1B–3B, $x_{\mathrm{m}} = \frac{1}{2}$ in subgames N and SS, and $x_{\mathrm{m}} = \frac{r+t}{4t} > \frac{1}{2}$ in subgame S when $r > t$. Therefore, we have $CS^{\mathrm{N}} = CS^{\mathrm{SS}} < CS^{\mathrm{S}}$ when $r > t$.

PROPOSITION 5B (SOCIAL WELFARE). *If the quality of content services is assumed to be exogenous to CPs, compared to the baseline scenario without data sponsorship (subgame N), (1) allowing both CPs to sponsor consumers does not change social welfare, whereas (2) allowing only one CP to sponsor consumers always decreases social welfare.*

PROOF OF PROPOSITION 5B. To prove Proposition 5B, we compare equilibrium social welfare in different subgames under the assumption of exogenous content quality. Social welfare is defined as $SW = \pi_{\mathrm{ISP}} + \pi_{\mathrm{A}} + \pi_{\mathrm{B}} + CS$. According to the proofs of Propositions 1B–4B, $SW^{\mathrm{N}} = SW^{\mathrm{SS}} = V - \frac{t}{4} + r - \phi V^2$, $SW^{\mathrm{S}} = V - \frac{t}{2}\left(x_{\mathrm{m}}^{\mathrm{S}}\right)^2 - \frac{t}{2}\left(1 - x_{\mathrm{m}}^{\mathrm{S}}\right)^2 + r - \phi V^2$, where $x_{\mathrm{m}}^{\mathrm{S}} = \begin{cases} \frac{1}{2} & \text{when } r \le t \\ \frac{r+t}{4t} & \text{when } r > t \end{cases}$. When $r > t$, it is easy to prove that $SW^{\mathrm{S}} - SW^{\mathrm{SS}} = -\frac{t}{4}\left(1 - 2x_{\mathrm{m}}^{\mathrm{S}}\right)^2 < 0$ because $x_{\mathrm{m}}^{\mathrm{S}} = \frac{r+t}{4t} > \frac{1}{2}$. Then, we have $SW^{\mathrm{S}} < SW^{\mathrm{SS}} = SW^{\mathrm{N}}$ when $r > t$. Proposition 5B is proved.

Appendix C: Consumers' Decisions Regarding Data Usage

In Appendix C, we relax the assumption of a fixed amount of data usage and analyze an extension in which consumers decide the amount of their data usage with each CP. This model extension interprets the ISP's market demand as consumers' data usage, which increases with CPs' content quality and data sponsorship but decreases with the ISP's data usage price. Thus, this extension captures the fact that consumers consume more data if the content is of a superior quality.

In this extension, consumers decide how many data packets to spend with each CP. The utility $U_{\mathrm{i}}(x, \lambda)$ that a consumer at location $x \in [0,1]$ derives from spending λ data packets on CP i's content is formulated as

$$U_{\mathrm{A}}(x, \lambda) = (\alpha + V_{\mathrm{A}} - tx - p + I_{\mathrm{A}} s_{\mathrm{A}})\lambda - \frac{\delta}{n+1}\lambda^{n+1},$$

$$\text{and } U_{\mathrm{B}}(x, \lambda) = (\alpha + V_{\mathrm{B}} - t(1-x) - p + I_{\mathrm{B}} s_{\mathrm{B}})\lambda - \frac{\delta}{n+1}\lambda^{n+1}.$$

α represents the intrinsic value of the online services provided by CPs ($\alpha > 0$). Each unit of a data packet used by consumer x generates the marginal value $\alpha + V_{\mathrm{A}} - tx$ from CP A's content or $\alpha + V_{\mathrm{B}} - t(1-x)$ from CP B's content. Under a sponsored data service ($I_{\mathrm{i}} = 1$), CP i provides data sponsorship s_{i} for each unit of data packets in its content services. The consumption of λ units of data packets incurs the time cost $\frac{\delta}{n+1}\lambda^{n+1}$ and the internet access fee λp. As $U_{\mathrm{i}}(x, \lambda)$ is a concave function of λ, we solve the FOC of $\mathrm{U}_{\mathrm{i}}(x, \lambda)$ w.r.t. λ to obtain the optimal data usage of consumer x:

$$\lambda_{\mathrm{A}}(x) = \left(\frac{\alpha + V_{\mathrm{A}} - tx - p + I_{\mathrm{A}} s_{\mathrm{A}}}{\delta}\right)^{\frac{1}{n}},$$

$$\text{and } \lambda_{\mathrm{B}}(x) = \left(\frac{\alpha + V_{\mathrm{B}} - t(1-x) - p + I_{\mathrm{B}} s_{\mathrm{B}}}{\delta}\right)^{\frac{1}{n}}.$$

A consumer's optimal data usage for CP i (i.e., $\lambda_{\mathrm{i}}(x)$) increases with CP i's content quality V_{i} and data sponsorship s_{i} and decreases with the ISP's data usage price p. Compared with a consumer with a lower x, a consumer with a higher x has the incentive to spend more data packets with CP B and fewer data packets with CP A (i.e., $\lambda_{\mathrm{B}}(x_1) \leq \lambda_{\mathrm{B}}(x_2)$ and $\lambda_{\mathrm{A}}(x_1) \geq \lambda_{\mathrm{A}}(x_2)$ if $x_1 \leq x_2$). Consistent with the main model, this extension maintains the assumption of full market coverage. The sufficient and necessary condition for this assumption is $U_{\mathrm{A}}\big(x_{\mathrm{m}}, \lambda_{\mathrm{A}}(x_{\mathrm{m}})\big) = U_{\mathrm{B}}\big(x_{\mathrm{m}}, \lambda_{\mathrm{B}}(x_{\mathrm{m}})\big) \geq 0$. Building upon the main model, we assume each CP's profit is proportional to the data traffic generated by its consumer base. Then the CPs' profit functions in this extension are formulated as

$$\pi_{\mathrm{A}} = (r - I_{\mathrm{A}} s_{\mathrm{A}}) \int_0^{x_{\mathrm{m}}} \lambda_{\mathrm{A}}(x)\, dx - \frac{\phi}{2} {V_{\mathrm{A}}}^2$$

$$\text{and } \pi_{\mathrm{B}} = (r - I_{\mathrm{B}} s_{\mathrm{B}}) \int_{x_{\mathrm{m}}}^1 \lambda_{\mathrm{B}}(x)\, dx - \frac{\phi}{2} {V_{\mathrm{B}}}^2.$$

To ensure the CPs' optimal decisions on content quality exist and π_{i} is a concave function of V_{i}, we assume n is sufficiently large such that $n > 2$. Accordingly, the ISP's profit function in this extension becomes $\pi_{\mathrm{ISP}} = p(\int_0^{x_{\mathrm{m}}} \lambda_{\mathrm{A}}(x)\, dx + \int_{x_{\mathrm{m}}}^1 \lambda_{\mathrm{B}}(x)\, dx)$. In contrast to the main model, in which the ISP's market demand is fixed at 1 unit, this extension interprets the ISP's market demand as consumers' total data usage. This usage decreases with the ISP's data usage price and increases with the CPs' content quality and data sponsorship.

The game sequence is as follows. In stage 1, the ISP chooses a network management option, i.e., it decides the values of I_{A} and I_{B}. In stage 2, the ISP announces the per-packet price p to the end consumers; meanwhile, CPs A and B determine the quality of their content services (V_{A} and V_{B}) and the data sponsorship per packet (s_{A} and s_{B}). In stage 3, each consumer chooses to consume content from either CP A or CP B and determines their data usage $\lambda_{\mathrm{i}}(x)$.

Because of the complexity of this extended model, we derive the equilibria through a numerical example in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8, 1.4]$. In this numerical analysis, we derive the ISP's optimal data usage price by solving $U_{\mathrm{A}}\left(x_{\mathrm{m}}, \lambda_{\mathrm{A}}(x_{\mathrm{m}})\right) = U_{\mathrm{B}}\left(x_{\mathrm{m}}, \lambda_{\mathrm{B}}(x_{\mathrm{m}})\right) = 0$, which yields $p^* = \alpha + V_{\mathrm{A}} + I_{\mathrm{A}} s_{\mathrm{A}} - t x_{\mathrm{m}} = \alpha + V_{\mathrm{B}} + I_{\mathrm{B}} s_{\mathrm{B}} - t(1 - x_{\mathrm{m}})$. We denote the optimal price derived from FOC is p^{FOC}. As shown in Figure C.1, the sufficient condition of p^* being the ISP's optimal choice is $\frac{d\,\pi_{\mathrm{ISP}}}{d\,p}|_{p=p^*} > 0$. Under this condition, we have $p^{\mathrm{FOC}} > p^*$, meaning that p^{FOC} will cause negative marginal consumer utility.

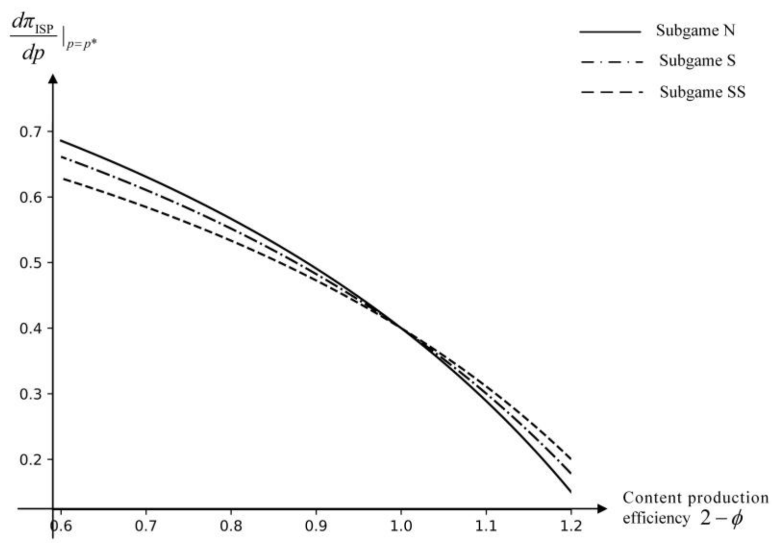

Figure C.1. The derivative of π_{ISP} w.r.t. p in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

C.1. No Data Sponsorship (Subgame N)

We construct an equilibrium of subgame N wherein no CP is allowed to subsidize consumers' data usage ($I_{\mathrm{A}} = I_{\mathrm{B}} = 0$) and the market is fully covered. We construct the equilibrium via backward induction. First, given the CPs' content quality V_{i} and the ISP's data usage price p, we solve for the consumers' optimal choice between CPs A and B and their optimal data usage $\lambda_{\mathrm{i}}(x)$. Second, we deduce the CPs' optimal decisions on content quality V_{i}^* and the ISP's optimal pricing scheme p^* in the fully-covered equilibrium.

In stage 3, we determine the marginal consumer $x_{\mathrm{m}} = \frac{V_{\mathrm{A}} - V_{\mathrm{B}} + t}{2t}$ by solving $U_{\mathrm{A}}(x_{\mathrm{m}}, \lambda) =$

$U_{\mathrm{B}}(x_{\mathrm{m}}, \lambda)$. The consumers located at $x \in [0, x_{\mathrm{m}}]$ choose CP A and consume $\lambda_{\mathrm{A}}(x) = (\frac{\alpha + V_{\mathrm{A}} - tx - p}{\delta})^{\frac{1}{n}}$ units of data packets. The consumers located at $x \in [x_{\mathrm{m}}, 1]$ choose CP B and consume $\lambda_{\mathrm{B}}(x) = (\frac{\alpha + V_{\mathrm{B}} - t(1-x) - p}{\delta})^{\frac{1}{n}}$ units of data packets.

In stage 2, both CPs make investments in content quality V_{i} and the ISP decides on the usage-based per-packet price p to maximize their respective profits. We first derive the FOCs of the CPs' profit functions w.r.t. content quality, then derive the ISP's optimal pricing decision, and finally determine the equilibrium of subgame N. First, CP i's profit function can be written as: $\pi_{\mathrm{i}} = \frac{rn}{t\delta^{\frac{1}{n}}(n+1)}((\alpha + V_{\mathrm{i}} - p)^{\frac{1+n}{n}} - (\alpha + \frac{V_{\mathrm{i}} + V_{-\mathrm{i}} - t}{2} - p)^{\frac{1+n}{n}}) - \frac{\phi}{2} {V_{\mathrm{i}}}^2$. By solving the FOC of CP i's profit function π_{i} w.r.t. V_{i} ($i =$ A or B), we get

$$\frac{r}{t\delta^{\frac{1}{n}}}\left((\alpha + V_{\mathrm{i}}^* - p^*)^{\frac{1}{n}} - \frac{1}{2}\left(\alpha + \frac{V_{\mathrm{i}}^* + V_{-\mathrm{i}}^* - t}{2} - p^*\right)^{\frac{1}{n}}\right) - \phi V_{\mathrm{i}}^* = 0. \quad \text{(C-1)}$$

Second, we solve $U_{\mathrm{A}}(x_{\mathrm{m}}, \lambda_{\mathrm{A}}(x_{\mathrm{m}})) = U_{\mathrm{B}}(x_{\mathrm{m}}, \lambda_{\mathrm{B}}(x_{\mathrm{m}})) = \frac{n}{\delta^{\frac{1}{n}}(n+1)}(\alpha + \frac{V_{\mathrm{A}}^* + V_{\mathrm{B}}^* - t}{2} - p)^{\frac{1+n}{n}} = 0$ and obtain the ISP's optimal price

$$p^* = \alpha + \frac{1}{2}(V_{\mathrm{A}}^* + V_{\mathrm{B}}^* - t). \quad \text{(C-2)}$$

Finally, we can determine (p^*, V_{A}^*, V_{B}^*) by jointly solving Equations (C-1) and (C-2).

C.2. Only One CP Sponsors Consumers (Subgame S)

We construct an equilibrium of subgame S wherein only one CP is allowed to sponsor consumer data traffic and the market is fully covered. Without loss of generality, suppose that only CP A has the option to sponsor consumers ($I_{\mathrm{A}} = 1$ and $I_{\mathrm{B}} = 0$). Similar to the analysis of subgame N, we construct the fully-covered equilibrium via backward induction. First, given the CPs' content quality V_{i} and data sponsorship s_{i} and the ISP's data usage price p, we solve for the consumers' optimal choice between CPs A and B and their optimal data usage $\lambda_{\mathrm{i}}(x)$. Second, we deduce the CPs' optimal decisions on content quality V_{i}^{*} and data sponsorship s_{i}^{*} and the ISP's optimal pricing scheme p^{*} in the fully-covered equilibrium.

In stage 3, we determine the marginal consumer $x_{\mathrm{m}}=\frac{V_{\mathrm{A}}+s_{\mathrm{A}}-V_{\mathrm{B}}+t}{2t}$ by solving $U_{\mathrm{A}}(x_{\mathrm{m}},\lambda)=U_{\mathrm{B}}(x_{\mathrm{m}},\lambda)$. The consumers located at $x\in[0,x_{\mathrm{m}}]$ choose CP A and consume $\lambda_{\mathrm{A}}(x)=(\frac{\alpha+V_{\mathrm{A}}-tx-p+s_{\mathrm{A}}}{\delta})^{\frac{1}{n}}$ units of data packets. The consumers located at $x\in[x_{\mathrm{m}},1]$ choose CP B and consume $\lambda_{\mathrm{B}}(x)=(\frac{\alpha+V_{\mathrm{B}}-t(1-x)-p}{\delta})^{\frac{1}{n}}$ units of data packets.

In stage 2, CP A makes decisions on content quality and data sponsorship while CP B only needs to decide on content quality, and the ISP decides the data usage price. We first derive the FOCs of the CPs' profit functions w.r.t. content quality and data sponsorship, then derive the ISP's optimal pricing decision, and finally determine the equilibrium of subgame S. First, we derive the FOCs of the CPs' profit functions w.r.t. V_{i} and s_{i}. CP A's profit function can be written as: $\pi_{\mathrm{A}}=\frac{(r-s_{\mathrm{A}})n}{t\delta^{\frac{1}{n}}(n+1)}((\alpha+V_{\mathrm{A}}+s_{\mathrm{A}}-p)^{\frac{1+n}{n}}-(\alpha+\frac{V_{\mathrm{A}}+s_{\mathrm{A}}+V_{\mathrm{B}}-t}{2}-p)^{\frac{1+n}{n}})-\frac{\phi}{2}{V_{\mathrm{A}}}^{2}$. Solving the FOCs of π_{A} w.r.t. V_{A} and s_{A} yields

$$\frac{r-s_{\mathrm{A}}^{*}}{t\delta^{\frac{1}{n}}}\left((\alpha+V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}-p^{*})^{\frac{1}{n}}-\frac{1}{2}\left(\alpha+\frac{V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}-t}{2}-p^{*}\right)^{\frac{1}{n}}\right)-\phi V_{\mathrm{A}}^{*}=0 \tag{C-3}$$

$$\text{and } (r-s_{\mathrm{A}}^{*})\left((\alpha+V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}-p^{*})^{\frac{1}{n}}-\frac{1}{2}\left(\alpha+\frac{V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}-t}{2}-p^{*}\right)^{\frac{1}{n}}\right)$$

$$-\frac{n}{n+1}\left((\alpha+V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}-p^{*})^{\frac{1+n}{n}}-\left(\alpha+\frac{V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}-t}{2}-p^{*}\right)^{\frac{1+n}{n}}\right)=0. \tag{C-4}$$

CP B's profit function can be written as: $\pi_{\mathrm{B}}=\frac{rn}{t\delta^{\frac{1}{n}}(n+1)}((\alpha+V_{\mathrm{B}}-p)^{\frac{1+n}{n}}-(\alpha+\frac{V_{\mathrm{A}}+s_{\mathrm{A}}+V_{\mathrm{B}}-t}{2}-p)^{\frac{1+n}{n}})-\frac{\phi}{2}{V_{\mathrm{B}}}^{2}$. Solving the FOC of π_{B} w.r.t. V_{B} yields

$$\frac{r}{t\delta^{\frac{1}{n}}}\left((\alpha+V_{\mathrm{B}}^{*}-p^{*})^{\frac{1}{n}}-\frac{1}{2}\left(\alpha+\frac{V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}-t}{2}-p^{*}\right)^{\frac{1}{n}}\right)-\phi V_{\mathrm{B}}^{*}=0. \tag{C-5}$$

Next, we solve $U_A(x_m, \lambda_A(x_m)) = U_B(x_m, \lambda_B(x_m)) = \frac{n}{\delta^{\frac{1}{n}}(n+1)}(\alpha + \frac{V_A^* + s_A^* + V_B^* - t}{2} - p)^{\frac{1+n}{n}} = 0$ and obtain the ISP's optimal price

$$p^* = \alpha + \frac{1}{2}(V_A^* + s_A^* + V_B^* - t). \tag{C-6}$$

Finally, we can determine (p^*, V_A^*, s_A^*, V_B^*) by jointly solving Equations (C-3), (C-4), (C-5), and (C-6).

C.3. Both CPs Sponsor Consumers (Subgame SS)

We construct a fully-covered equilibrium of subgame SS wherein both CPs are allowed to sponsor consumer data traffic. Similar to the analysis of the previous two cases, we construct the fully-covered equilibrium via backward induction.

In stage 3, we determine the marginal consumer $x_m = \frac{V_A + s_A - V_B - s_B + t}{2t}$ by solving $U_A(x_m, \lambda) = U_B(x_m, \lambda)$. The consumers located at $x \in [0, x_m]$ choose CP A and consume $\lambda_A(x) = (\frac{\alpha + V_A - tx - p + s_A}{\delta})^{\frac{1}{n}}$ units of data packets. The consumers located at $x \in [x_m, 1]$ choose CP B and consume $\lambda_B(x) = (\frac{\alpha + V_B - t(1-x) - p + s_B}{\delta})^{\frac{1}{n}}$ units of data packets.

In stage 2, both CPs make decisions on content quality and data sponsorship and the ISP decides the data usage price. In a candidate fully-covered equilibrium, we first derive the FOCs of the CPs' profit functions w.r.t. content quality and data sponsorship, then derive the ISP's optimal pricing decision, and finally determine the equilibrium of subgame S. First, we derive the FOCs of CP i' profit function w.r.t. V_i and s_i (i =A or B). CP i's profit function can be written as: $\pi_i = \frac{(r - s_i)n}{t\delta^{\frac{1}{n}}(n+1)}((\alpha + V_i + s_i - p)^{\frac{1+n}{n}} - (\alpha + \frac{V_i + s_i + V_{-i} + s_{-i} - t}{2} - p)^{\frac{1+n}{n}}) - \frac{\phi}{2}V_i^2$. Solving $\frac{d\,\pi_i}{d\,V_i} = 0$ and $\frac{d\,\pi_i}{d\,s_i} = 0$ yields

$$\frac{r - s_i^*}{t\delta^{\frac{1}{n}}}\left((\alpha + V_i^* + s_i^* - p^*)^{\frac{1}{n}} - \frac{1}{2}\left(\alpha + \frac{V_i^* + s_i^* + V_{-i}^* + s_{-i}^* - t}{2} - p^*\right)^{\frac{1}{n}}\right) - \phi V_i^* = 0 \tag{C-7}$$

and $(r - s_i^*)\left((\alpha + V_i^* + s_i^* - p^*)^{\frac{1}{n}} - \frac{1}{2}\left(\alpha + \frac{V_i^* + s_i^* + V_{-i}^* + s_{-i}^* - t}{2} - p^*\right)^{\frac{1}{n}}\right)$

$$-\frac{n}{n+1}\left((\alpha+V_{\mathrm{i}}^{*}+s_{\mathrm{i}}^{*}-p^{*})^{\frac{1+n}{n}}-\left(\alpha+\frac{V_{\mathrm{i}}^{*}+s_{\mathrm{i}}^{*}+V_{-\mathrm{i}}^{*}+s_{-\mathrm{i}}^{*}-t}{2}-p^{*}\right)^{\frac{1+n}{n}}\right)=0 \text{ respectively.} \quad \text{(C-8)}$$

Second, we solve $U_{\mathrm{A}}(x_{\mathrm{m}},\lambda_{\mathrm{A}}(x_{\mathrm{m}}))=U_{\mathrm{B}}(x_{\mathrm{m}},\lambda_{\mathrm{B}}(x_{\mathrm{m}}))=\frac{n}{\delta^{\frac{1}{n}}(n+1)}(\alpha+\frac{V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}+s_{\mathrm{B}}^{*}-t}{2}-p)^{\frac{1+n}{n}}=0$ and obtain the ISP's optimal price

$$p^{*}=\alpha+\frac{1}{2}(V_{\mathrm{A}}^{*}+s_{\mathrm{A}}^{*}+V_{\mathrm{B}}^{*}+s_{\mathrm{B}}^{*}-t). \quad \text{(C-9)}$$

Finally, we can determine (p^{*}, V_{i}^{*}, s_{i}^{*}) by jointly solving Equations (C-7), (C-8), and (C-9).

C.4. Welfare Analysis

In the following discussion, we compare model predictions in this extension with those of the main model. Due to the complexity of this extended model, we could not derive analytical solutions. To derive model predictions and form a comparison with Propositions 1–5 in our main model, we conducted numerical analyses by setting $r=0.5$, $t=1$, $\delta=0.5$, $n=4$, $\alpha=0.2$, and $\phi\in[0.8,1.4]$. These parameter values satisfy the assumption of $n>2$ and ensure that the ISP's profit, the CPs' profits, and consumer utility remain non-negative in the equilibria across all subgames. Our main insights are robust under diverse sets of parameter values, and those results are available upon request.

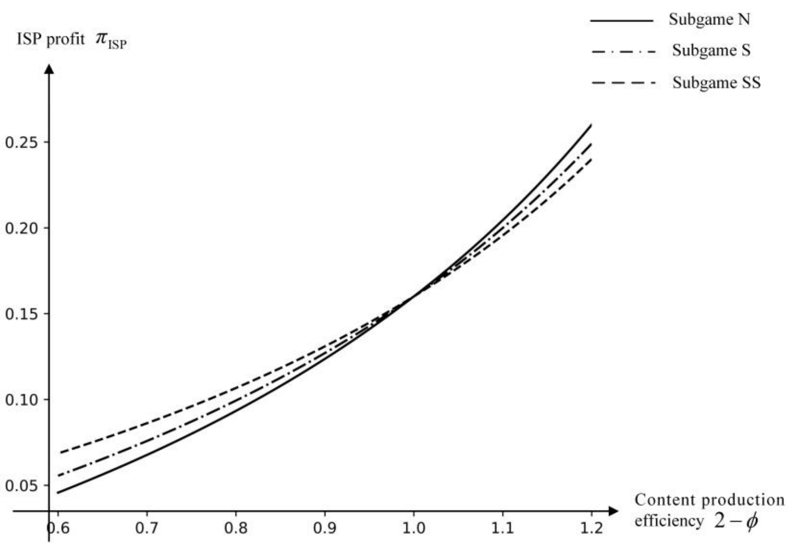

Figure C.2. The ISP's profit in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

First, we derive the ISP's optimal network management choice in this extension and examine the robustness of Proposition 1 in this extension. As shown in Figure C.2, we compare the ISP's profits in the equilibria under different subgames and get Result 1C, which is consistent with Proposition 1.

RESULT 1C (ISP'S OPTIMAL NETWORK MANAGEMENT CHOICE). *In the SPNE*

(1) When CPs' content production efficiency is high (i.e., $\phi < 1$), the ISP suffers from the CPs' data sponsorship, and its optimal network management choice is to not introduce sponsored data services (i.e., $\pi_{\text{ISP}}^{\text{N}} > \pi_{\text{ISP}}^{\text{S}} > \pi_{\text{ISP}}^{\text{SS}}$);

(2) When CPs' content production efficiency is low (i.e., $\phi > 1$), the ISP benefits from the CPs' data sponsorship, and its optimal network management choice is to allow both CPs to subsidize consumers' data usage (i.e., $\pi_{\text{ISP}}^{\text{N}} < \pi_{\text{ISP}}^{\text{S}} < \pi_{\text{ISP}}^{\text{SS}}$).

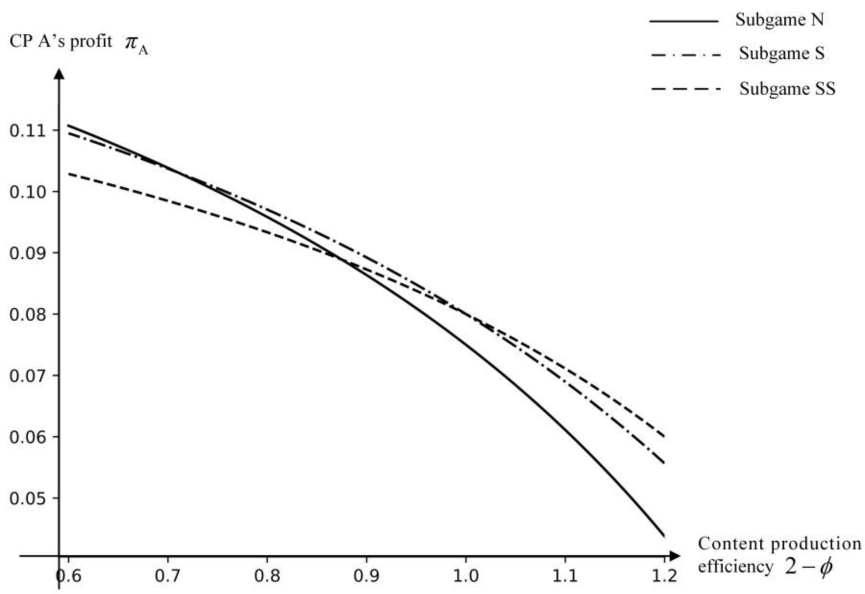

Figure C.3. CP A's profit in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

Second, we examine the robustness of Proposition 2 in this extension and explore whether a CP can benefit from subsidizing consumers' data usage compared with subgame N. As shown in Figure C.3, we compare CP A's profits in the equilibria under different subgames and get Result 2C. When $\phi < 1.2895$, Result 2C is almost the same as Proposition 2. When $\phi > 1.2895$, Result 2C shows a novel finding that no data sponsorship maximizes a CP's profit. The underlying rationale is that endogenizing consumers' data usage makes CPs' competition more intense than the case of fixed data usage (i.e., the CPs not only compete for the number of consumers but also compete in consumers' data usage). As a result, when content production efficiency is low ($\phi > 1.2895$), a CP suffers from sponsored data services in both subgames S and SS compared to subgame N because of intense competition to subsidize consumers.

RESULT 2C (IMPLICATION OF SUBSIDIZING CONSUMERS). *Considering CP A, which is allowed to sponsor in subgame S:*

(1) When content production efficiency is high (i.e., $\phi < 1$), CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers, and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers (i.e., $\pi_{\mathrm{A}}^{\mathrm{N}} < \pi_{\mathrm{A}}^{\mathrm{S}} < \pi_{\mathrm{A}}^{\mathrm{SS}}$);

(2) When content production efficiency is medium (i.e., $1 < \phi < 1.2895$), CP A achieves maximal profit in subgame S, where it sponsors consumers alone (i.e., $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{N}}$ and $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$), and it achieves a lower profit in subgame SS than in subgame N (i.e., $\pi_{\mathrm{A}}^{\mathrm{SS}} < \pi_{\mathrm{A}}^{\mathrm{N}}$) if the content production efficiency is lower than a threshold (i.e., $\phi > 1.1250$);

(3) When content production efficiency is low (i.e., $\phi > 1.2895$), CP A achieves maximal profit in subgame N (i.e., $\pi_{\mathrm{A}}^{\mathrm{N}} > \pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$).

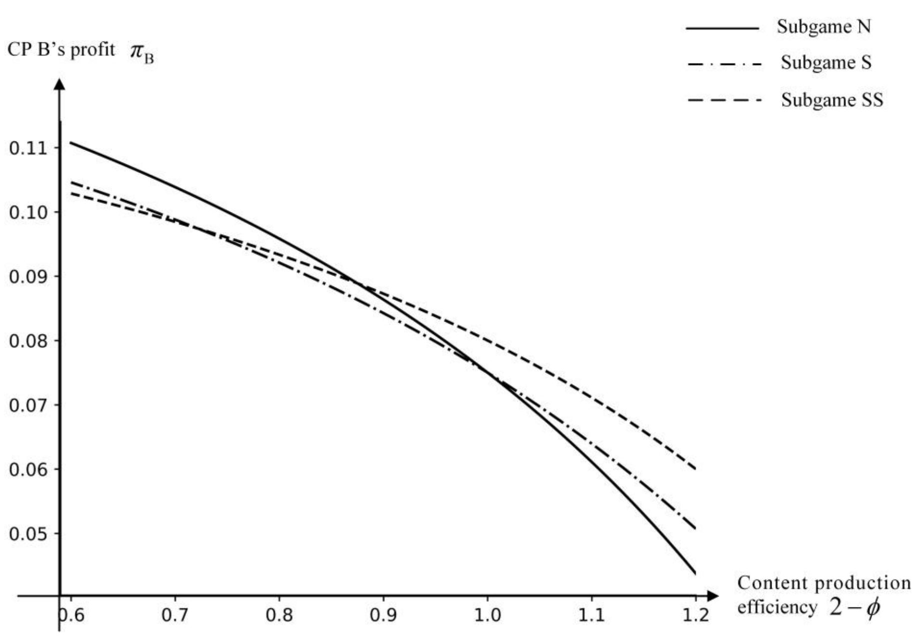

Figure C.4. CP B's profit in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

Third, we examine the robustness of Proposition 3 and explore whether a CP suffers because of its rival's data sponsorship in this extension. As shown in Figure C.4, we compare CP B's profits in the equilibria under different subgames and further get Result 3C, which is consistent with Proposition 3.

RESULT 3C (IMPACT OF THE RIVAL'S DATA SPONSORSHIP). *Compared with the scenario* *in which the rival has no sponsoring option,*

(1) When content production efficiency is high (i.e., $\phi < 1$), a CP always benefits from its rival's data sponsorship through higher profits (i.e., $\pi_{\mathrm{A}}^{\mathrm{S}} < \pi_{\mathrm{A}}^{\mathrm{SS}}$ and $\pi_{\mathrm{B}}^{\mathrm{N}} < \pi_{\mathrm{B}}^{\mathrm{S}}$), regardless of whether it has the option to sponsor.

(2) When content production efficiency is low (i.e., $\phi > 1$), a CP always suffers from its rival's data sponsorship (i.e., $\pi_{\mathrm{A}}^{\mathrm{S}} > \pi_{\mathrm{A}}^{\mathrm{SS}}$ and $\pi_{\mathrm{B}}^{\mathrm{N}} > \pi_{\mathrm{B}}^{\mathrm{S}}$), regardless of whether it has the option to sponsor.

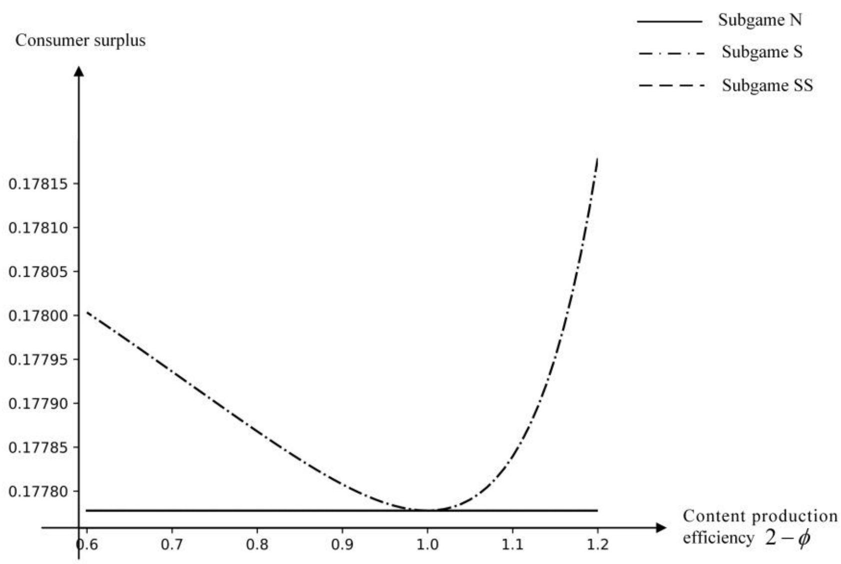

Figure C.5. Consumer surplus in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

Fourth, we examine the robustness of Proposition 4 in this extension and explore the impact of sponsored data services on consumer surplus. As shown in Figure C.5, we compare consumer surplus in the equilibria under different subgames and further get Result 4C, which is consistent with Proposition 4.

RESULT 4C (CONSUMER SURPLUS). *Compared to the baseline scenario without data sponsorship (subgame N), (1) allowing both CPs to sponsor consumers does not change consumer surplus, but (2) allowing only one CP to sponsor consumers always increases consumer surplus (i.e., $CS^{\mathrm{N}} = CS^{\mathrm{SS}} \leq CS^{\mathrm{S}}$).*

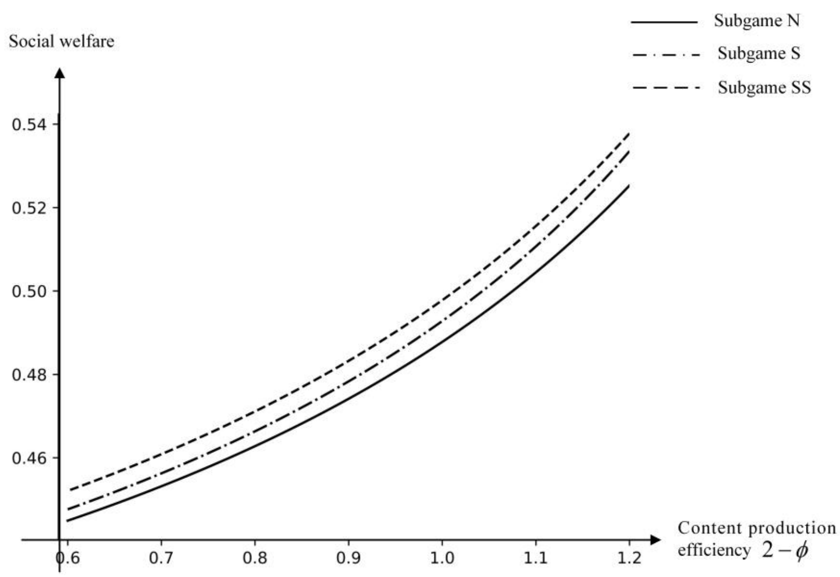

Figure C.6. Social welfare in the equilibria of the numerical case in which $r = 0.5$, $t = 1$, $\delta = 0.5$, $n = 4$, $\alpha = 0.2$, and $\phi \in [0.8,1.4]$.

Finally, we examine the robustness of Proposition 5 in this extension and explore the impact of sponsored data services on social welfare. As shown in Figure C.6, we compare social welfare in the equilibria under different subgames and get Result 5C.

RESULT 5C (SOCIAL WELFARE). *Compared to the baseline scenario without data sponsorship (subgame N), sponsored data services increase social welfare and allowing both CPs to sponsor consumers maximizes it (i.e., $SW^{\mathrm{N}} < SW^{\mathrm{S}} < SW^{\mathrm{SS}}$).*

In contrast to the main model, where allowing only one CP to sponsor consumers maximizes social welfare when content production efficiency is sufficiently high, the extension incorporating endogenous data usage shows that permitting both CPs' data sponsorship always maximizes social welfare. In the main model, when content production efficiency is sufficiently high, allowing only one CP to sponsor consumers results in a greater reduction in the cost of content production compared with subgame SS, which overwhelms its effect on increasing consumers' fit costs and thus maximizes social welfare. However, in this extension, the effect of data sponsorship on reducing content quality is attenuated. This is because, in the extended model, CPs' competition is more intense, and the decline in a CP's content quality not only reduces its consumer base but also diminishes consumers' data usage. As the impact of data sponsorship on content quality declines, the influence of increased consumers' fit costs becomes more pronounced, such that allowing only one CP to sponsor data results in lower social welfare than permitting both CPs to engage in data sponsorship.

We summarize the comparison between the main model and this extension in Table C.1. The comparison between Propositions 1-5 and Results 1C-5C reveals that the majority of the predictions from the main model (i.e., Propositions 1–4) remain unchanged in this extension, while the prediction regarding social welfare (i.e., Proposition 5) exhibits only minor adjustments. Thus, we can conclude that the predictions of the main model remain robust in this extended framework.

Table C.1. Propositions in the main model and predictions in the extension of endogenous data usage.			
Proposition	Content of the proposition	Predictions in the extension based on numerical analyses	Comparison with the proposition
Proposition 1 (ISP's optimal network management choice)	In the SPNE, (1) when CPs' content production efficiency is high, the ISP suffers from the CPs' data sponsorship, and its optimal network management choice is to not introduce sponsored data services; (2) when CPs' content production efficiency is low, the ISP benefits from the CPs' data sponsorship, and its optimal network management choice is to allow both CPs to subsidize consumers' data usage.	In the SPNE, (1) when CPs' content production efficiency is high ($\phi \leq 1$), the ISP suffers from the CPs' data sponsorship, and its optimal network management choice is to not introduce sponsored data services; (2) when CPs' content production efficiency is low ($\phi > 1$), the ISP benefits from the CPs' data sponsorship, and its optimal network management choice is to allow both CPs to subsidize consumers' data usage.	This prediction is consistent with Proposition 1.
Proposition 2 (Implication of subsidizing consumers)	Considering CP A, which is allowed to sponsor in subgame S: (1) when content production efficiency is high, CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers, and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers; (2) when content production efficiency is low, CP A achieves maximal profit in subgame S, where it sponsors consumers alone, and it achieves a lower profit in subgame SS than in subgame N if the advertising revenue rate is sufficiently low.	Considering CP A, which is allowed to sponsor in subgame S: (1) when content production efficiency is high ($\phi < 1$), CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers, and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers; (2) when content production efficiency is medium ($1 < \phi < 1.2895$), CP A achieves maximal profit in subgame S, where it sponsors consumers alone, and it achieves a lower profit in subgame SS than in subgame N if the content production efficiency is lower than a threshold ($\phi > 1.1250$); (3) when content production efficiency is low ($\phi > 1.2895$), CP A achieves maximal profit in subgame N.	This prediction is consistent with Proposition 2.
Proposition 3 (Impact of the rival's data	Compared to the scenario in which the rival has no sponsoring option, (1) when content production efficiency	Compared to the scenario in which the rival has no sponsoring option, (1) when content production efficiency	This prediction is consistent with Proposition 3.

sponsorship)	is high, a CP always benefits from its rival's data sponsorship through higher profits, regardless of whether it has the option to sponsor; (2) when content production efficiency is low, a CP always suffers from its rival's data sponsorship, regardless of whether it has the option to sponsor.	is high ($\phi < 1$), a CP always benefits from its rival's data sponsorship through higher profits, regardless of whether it has the option to sponsor; (2) when content production efficiency is low ($\phi > 1$), a CP always suffers from its rival's data sponsorship, regardless of whether it has the option to sponsor.	
Proposition 4 (Consumer surplus)	Compared to subgame N, (1) allowing both CPs to sponsor consumers does not change consumer surplus, whereas (2) allowing only one CP to sponsor consumers always increases consumer surplus.	Compared to subgame N, (1) allowing both CPs to sponsor consumers does not change consumer surplus, whereas (2) allowing only one CP to sponsor consumers always increases consumer surplus.	This prediction is consistent with Proposition 4.
Proposition 5 (Social welfare)	(1) When CPs' content production efficiency is extremely high, allowing only one CP to sponsor consumers maximizes social welfare while no data sponsorship minimizes it; (2) when CPs' content production efficiency is intermediate, allowing both CPs to sponsor consumers maximizes social welfare while no data sponsorship minimizes it; (3) when CPs' content production efficiency is extremely low, allowing both CPs to sponsor consumers maximizes social welfare while allowing only one CP to sponsor consumers minimizes it.	Compared to subgame N, sponsored data services always increase social welfare and allowing both CPs to sponsor consumers maximizes it.	(1) Consistent with the main model, allowing both CPs to sponsor consumers always increases social welfare in this extension. (2) Different from the main model, allowing both CPs' data sponsorship always maximizes social welfare in this extension.

Appendix D: Partial Market Coverage

In Appendix D, we extend the main model analysis by considering an equilibrium that achieves partial market coverage (hereinafter referred to as the "partially-covered equilibrium") under the condition of $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$. Under this assumption, the CPs only serve a portion of consumers as local monopolists.[2] In this extension, the size of a CP's consumer base increases with its content quality and data sponsorship but decreases with the ISP's data usage price. Therefore, this model extension captures the influence of the data usage price, content quality, and data sponsorship on the market demand.

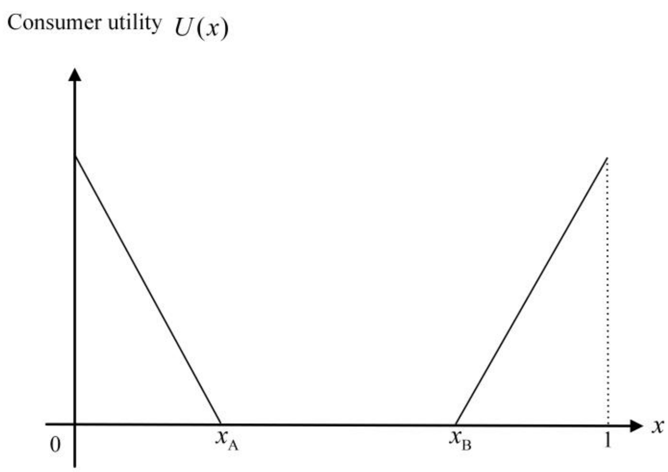

Figure D.1. Consumer utility in the partially-covered equilibria.

Figure D.1 illustrates the partially-covered equilibria. x_A (x_B) represents the marginal consumer who is indifferent about whether or not to consume CP A's (CP B's) content. In the equilibria, a consumer with $x \in [0, x_A)$ chooses CP A and a consumer with $x \in (x_B, 1]$ chooses CP B. A consumer with $x \in [x_A, x_B]$ does not participate in the market because of the high fit costs. Then, CP A's market share is x_A

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[2] When $r \geq 2\phi t^2$, as shown in Appendix A, we can construct the fully-covered equilibria that always dominate the cases of a partially covered market. By contrast, when $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$, we can construct the partially-covered equilibria that always dominate the cases of a fully covered market. In addition, when $\min\left\{\phi t^2, \frac{3\phi t-1}{2\phi}\right\} \leq r < 2\phi t^2$, we cannot find pure-strategy equilibrium in either a fully or partially covered market.

and CP B's market share is $1-x_{\mathrm{B}}$. Solving $U_{\mathrm{A}}(x_{\mathrm{A}})=0$ and $U_{\mathrm{B}}(x_{\mathrm{B}})=0$ yields $x_{\mathrm{A}}=\frac{V_{\mathrm{A}}+I_{\mathrm{A}}s_{\mathrm{A}}-p}{t}$ and $x_{\mathrm{B}}=1-\frac{V_{\mathrm{B}}+I_{\mathrm{B}}s_{\mathrm{B}}-p}{t}$. The assumption that $r<\min\{\phi t^2,\frac{3\phi t-1}{2\phi}\}$ guarantees $x_{\mathrm{A}}<x_{\mathrm{B}}$ in the equilibrium. The CPs' profits are formulated as

$$\pi_{\mathrm{A}}=(r-I_{\mathrm{A}}s_{\mathrm{A}})x_{\mathrm{A}}-\frac{\phi}{2}{V_{\mathrm{A}}}^2,$$

$$\pi_{\mathrm{B}}=(r-I_{\mathrm{B}}s_{\mathrm{B}})(1-x_{\mathrm{B}})-\frac{\phi}{2}{V_{\mathrm{B}}}^2.$$

The ISP's revenue is $\pi_{\mathrm{ISP}}=p(x_{\mathrm{A}}+1-x_{\mathrm{B}})=\frac{1}{t}p(V_{\mathrm{A}}+I_{\mathrm{A}}s_{\mathrm{A}}+V_{\mathrm{B}}+I_{\mathrm{B}}s_{\mathrm{B}}-2p)$. Given the sponsoring option I_{i} and the CPs' decisions of V_{i} and s_{i}, the ISP's profit-maximizing problem is

$$\max_{p}\pi_{\mathrm{ISP}}=\frac{1}{t}p(V_{\mathrm{A}}+I_{\mathrm{A}}s_{\mathrm{A}}+V_{\mathrm{B}}+I_{\mathrm{B}}s_{\mathrm{B}}-2p)$$

$$\text{s.t. } x_{\mathrm{A}}<x_{\mathrm{B}}.$$

In the following analysis, we construct the partially-covered equilibria via backward induction. First, we solve for the consumers' optimal choice between participating and not participating in the market given the CPs' content quality and data sponsorship and the ISP's data usage price. Second, we deduce the CPs' optimal decisions on content quality and data sponsorship and the ISP's optimal pricing scheme in a candidate partially-covered equilibrium. Finally, we calculate the ISP's maximal profit in a fully covered market and show that it is lower than the ISP's profit in the candidate partially-covered equilibrium under the assumption that $r<\min\{\phi t^2,\frac{3\phi t-1}{2\phi}\}$. Therefore, although the ISP is free to choose a low data usage price p that makes the market fully covered, it has no incentive to do so, and serving a partially covered market is its optimal choice.

D.1. No Data Sponsorship (Subgame N)

In this section, we construct a partially-covered equilibrium of subgame N wherein no CP is allowed to subsidize consumers' data usage ($I_{\mathrm{A}}=I_{\mathrm{B}}=0$).

In stage 3, if the market is partially covered, we solve $U_{\mathrm{A}}(x_{\mathrm{A}}) = 0$ and $U_{\mathrm{B}}(x_{\mathrm{B}}) = 0$ and determine the marginal consumers $x_{\mathrm{A}} = \frac{V_{\mathrm{A}}-p}{t}$ and $x_{\mathrm{B}} = 1 - \frac{V_{\mathrm{B}}-p}{t}$. If the price p is high such that $x_{\mathrm{A}} < x_{\mathrm{B}}$, our assumption of a partially covered market is justified; if the ISP chooses a low price p that makes $x_{\mathrm{A}} \geq x_{\mathrm{B}}$, then all consumers participate in the market and the market is fully covered.

In stage 2, both CPs make investments in content quality and the ISP decides on the usage-based per-packet price p to maximize their respective profits. In a candidate partially-covered equilibrium, we derive the CPs' optimal decisions on content quality and then derive the ISP's optimal pricing decision. First, we can write down CP i's profit function as $\pi_{\mathrm{i}} = r\frac{V_{\mathrm{i}}-p}{t} - \frac{\phi}{2}V_{\mathrm{i}}^2$. From the FOC of CP i's profit function π_{i} w.r.t. V_{i} (i = A or B), we determine CP i's optimal content quality decision $V_{\mathrm{i}}^* = \frac{r}{\phi t}$. Then, $x_{\mathrm{A}}^* = \frac{V_{\mathrm{A}}^*-p}{t} = \frac{r-\phi tp}{\phi t^2}$, $x_{\mathrm{B}}^* = 1 - \frac{V_{\mathrm{B}}^*-p}{t} = 1 - \frac{r-\phi tp}{\phi t^2}$, and $\pi_{\mathrm{ISP}} = p(x_{\mathrm{A}}^* + 1 - x_{\mathrm{B}}^*) = \frac{2p(r-\phi tp)}{\phi t^2}$. Second, we solve the FOC of π_{ISP} w.r.t. p and obtain the ISP's optimal price $p^* = \frac{r}{2\phi t}$. Then, $x_{\mathrm{A}}^* = \frac{r-\phi tp^*}{\phi t^2} = \frac{r}{2\phi t^2}$ and $x_{\mathrm{B}}^* = 1 - \frac{r-\phi tp^*}{\phi t^2} = 1 - \frac{r}{2\phi t^2}$. The assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$ guarantees that $x_{\mathrm{A}}^* < x_{\mathrm{B}}^*$.

Finally, we prove that the ISP has no incentive to set a low data usage price that causes a fully covered market under the assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$. From the above analysis, we get the ISP's data usage price $p_{\mathrm{P}}^{\mathrm{N}} = \frac{r}{2\phi t}$ and its profit $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}} = p_{\mathrm{P}}^{\mathrm{N}}(x_{\mathrm{A}}^* + 1 - x_{\mathrm{B}}^*) = \frac{r^2}{2\phi^2 t^3}$ in a candidate partial-coverage equilibrium of subgame N. According to our analysis of stage 3, the market is fully covered if the data usage price p is low such that $x_{\mathrm{A}} \geq x_{\mathrm{B}}$. In the following analysis, we calculate the ISP's maximal profit $\pi_{\mathrm{ISP\_F}}^{\mathrm{N}}$ in a fully covered market of subgame N and then compare $\pi_{\mathrm{ISP\_F}}^{\mathrm{N}}$ with $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}}$. Given the CPs' content quality strategy $V_{\mathrm{i}}^* = \frac{r}{\phi t}$, solving $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) = 0$ yields the ISP's optimal data usage price $p_{\mathrm{F}}^{\mathrm{N}} = \frac{1}{2}(V_{\mathrm{A}}^* + V_{\mathrm{B}}^* - \mathrm{t}) = \frac{r}{\phi t} - \frac{t}{2}$ in a fully covered market. Then, $\pi_{\mathrm{ISP\_F}}^{\mathrm{N}} = p_{\mathrm{F}}^{\mathrm{N}} = \frac{r}{\phi t} - \frac{t}{2}$. As shown in

Figure D.2, the market is fully covered if $p \leq p_{\mathrm{F}}^{\mathrm{N}}$. It is easy to see that $\pi_{\mathrm{ISP\_P}}^{\mathrm{N}} - \pi_{\mathrm{ISP\_F}}^{\mathrm{N}} = \frac{1}{2t}\left(\frac{r}{\phi t} - t\right)^2 \geq 0$. Therefore, compared to the partially-covered equilibrium of subgame N, the ISP's profit decreases if it chooses a price lower than $p_{\mathrm{F}}^{\mathrm{N}}$ and deviates to a fully covered market.

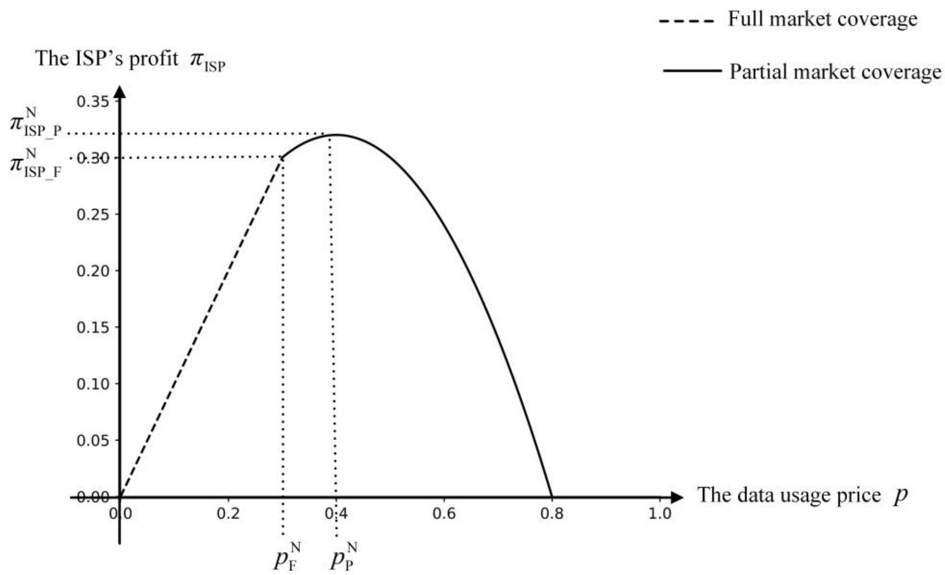

Figure D.2. Illustration of the ISP's optimal pricing decision in subgame N when $r = 0.8, \phi = 1, t = 1$.

D.2. Only One CP Sponsors Consumers (Subgame S)

In this section, we construct a partially-covered equilibrium of subgame S wherein only one CP is allowed to sponsor consumer data traffic. Without loss of generality, suppose that only CP A has the option to sponsor consumers ($I_{\mathrm{A}} = 1$ and $I_{\mathrm{B}} = 0$).

In stage 3, if the market is partially covered, we solve $U_{\mathrm{A}}(x_{\mathrm{A}}) = 0$ and $U_{\mathrm{B}}(x_{\mathrm{B}}) = 0$ and determine the marginal consumers $x_{\mathrm{A}} = \frac{V_{\mathrm{A}} + s_{\mathrm{A}} - p}{t}$ and $x_{\mathrm{B}} = 1 - \frac{V_{\mathrm{B}} - p}{t}$. If the price p is high such that $x_{\mathrm{A}} < x_{\mathrm{B}}$, our assumption of a partially covered market is justified; if the ISP chooses a low price p that makes $x_{\mathrm{A}} \geq x_{\mathrm{B}}$, all consumers participate in the market and the market is fully covered.

In stage 2, CP A makes decisions regarding content quality and data sponsorship while CP B only needs to make a decision about content quality, and the ISP decides the data usage price. In a candidate partially-covered equilibrium, we analyze the CPs' optimal decisions on content quality and data sponsorship given the data usage price p, then derive the ISP's optimal pricing decision given the CPs' decisions, and finally make a joint analysis of the CPs' and the ISP's optimal decisions to determine the equilibrium. First, given the data usage price p, we derive the CPs' optimal content quality decisions V_{A}^* and V_{B}^* and data sponsorship decision s_{A}^*. CP B's profit function can be written as: $\pi_{\mathrm{B}} = r\frac{V_{\mathrm{B}}-p}{t} - \frac{\phi}{2}{V_{\mathrm{B}}}^2$. Solving the FOC of π_{B} w.r.t. V_{B} yields $V_{\mathrm{B}}^* = \frac{r}{\phi t}$. To guarantee the existence of the maximum of CP A's profit function $\pi_{\mathrm{A}} = \frac{(r-s_{\mathrm{A}})(V_{\mathrm{A}}-p)}{t} - \frac{\phi}{2}{V_{\mathrm{A}}}^2$, we apply the assumption $\phi > \frac{1}{2t}$, which ensures that the Jacobian $\frac{d^2\,\pi_{\mathrm{A}}}{d\,{V_{\mathrm{A}}}^2} \times \frac{d^2\,\pi_{\mathrm{A}}}{d\,{s_{\mathrm{A}}}^2} - (\frac{d^2\,\pi_{\mathrm{A}}}{dV_{\mathrm{A}}d\,s_{\mathrm{A}}})^2 = \frac{2\phi}{t} - \frac{1}{t^2} > 0$. Solving $\frac{d\,\pi_{\mathrm{A}}}{d\,V_{\mathrm{A}}} = 0$ and $\frac{d\,\pi_{\mathrm{A}}}{d\,s_{\mathrm{A}}} = 0$ yields $V_{\mathrm{A}} = \frac{r-s_{\mathrm{A}}}{\phi t}$ and $s_{\mathrm{A}} = \frac{r-V_{\mathrm{A}}+p}{2}$, respectively. Jointly solving $V_{\mathrm{A}} = \frac{r-s_{\mathrm{A}}}{\phi t}$ and $s_{\mathrm{A}} = \frac{r-V_{\mathrm{A}}+p}{2}$ yields $s_{\mathrm{A}}^* = \frac{(\phi t-1)r+\phi tp}{2\phi t-1}$ and $V_{\mathrm{A}}^* = \frac{r-p}{2\phi t-1}$. Second, given the CPs' optimal content quality decisions V_{A}^* and V_{B}^* and data sponsorship decision s_{A}^*, we derive the ISP's optimal data usage price p^*. The ISP's profit is $\pi_{\mathrm{ISP}} = p(x_{\mathrm{A}} + 1 - x_{\mathrm{B}}) = p(\frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*+V_{\mathrm{B}}^*-2p}{t})$. We solve the FOC of π_{ISP} w.r.t. p and obtain the ISP's optimal price $p^* = \frac{1}{4}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^*) = \frac{1}{4}\left(\frac{2r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{A}}^*\right)$. Finally, we jointly solve $s_{\mathrm{A}}^* = \frac{(\phi t-1)r+\phi tp^*}{2\phi t-1}$ and $p^* = \frac{1}{4}\left(\frac{2r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{A}}^*\right)$ and get $s_{\mathrm{A}}^* = \frac{(4\phi t-2)r}{7\phi t-3}$ and $p^* = \frac{((\phi t)^2+2\phi t-1)r}{(7\phi t-3)\phi t}$. Then, $V_{\mathrm{A}}^* = \frac{r-p^*}{2\phi t-1} = \frac{(3\phi t-1)r}{(7\phi t-3)\phi t}$, $x_{\mathrm{A}}^* = \frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*-p^*}{t} = \frac{(3\phi t-1)r}{(7\phi t-3)t}$, and $x_{\mathrm{B}}^* = 1 - \frac{V_{\mathrm{B}}^*-p^*}{t} = 1 - \frac{(-(\phi t)^2+5\phi t-2)r}{(7\phi t-3)\phi t^2}$.

Notice that CP B's profit $\pi_{\mathrm{B}}^{\mathrm{S}} = \frac{r^2(1-\phi t)}{2\phi t^2(3\phi t+1)} < 0$ when $\phi > \frac{1}{t}$. Therefore, when $\phi > \frac{1}{t}$, CP B exits from the market and $\pi_{\mathrm{ISP}} = px_{\mathrm{A}} = p(\frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*-p}{t})$. Solving the FOC of π_{ISP} w.r.t. p yields $p^* = \frac{1}{2}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^*) = \frac{1}{2}\left(\frac{r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{A}}^*\right)$. Then, we jointly solve $s_{\mathrm{A}}^* = \frac{(\phi t-1)r+\phi tp^*}{2\phi t-1}$ and $p^* = \frac{1}{2}\left(\frac{r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{A}}^*\right)$ and get $s_{\mathrm{A}}^* = \frac{(2\phi t-1)r}{3\phi t-1}$ and $p^* = \frac{\phi tr}{3\phi t-1}$. Accordingly, we have $V_{\mathrm{A}}^* = \frac{r-p^*}{2\phi t-1} = \frac{r}{3\phi t-1}$ and $x_{\mathrm{A}}^* = \frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*-p^*}{t} = \frac{\phi r}{3\phi t-1}$. We summarize the equilibria of subgame S when $\phi \leq \frac{1}{t}$ and $\phi > \frac{1}{t}$ in Table D.1. The assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$ guarantees that $x_A^* < x_B^*$ when $\phi \leq \frac{1}{t}$ and $\phi > \frac{1}{t}$.

Table D.1. Partially-covered equilibrium of subgame S.				
Assumptions	$r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$ and $\phi > \frac{1}{2t}$			
Market conditions	ISP's data usage pricing p^*	CPs' content quality decisions (V_A^*, V_B^*)	CPs' data sponsorship decisions (s_A^*, s_B^*)	Marginal consumers (x_A^*, x_B^*)
$\phi \leq \frac{1}{t}$	$\frac{((\phi t)^2+2\phi t-1)r}{(7\phi t-3)\phi t}$	$(\frac{(3\phi t-1)r}{(7\phi t-3)\phi t}, \frac{r}{\phi t})$	$(\frac{(4\phi t-2)r}{7\phi t-3}, 0)$	$(\frac{(3\phi t-1)r}{(7\phi t-3)t}, 1 - \frac{(-(\phi t)^2+5\phi t-2)r}{(7\phi t-3)\phi t^2})$
$\phi > \frac{1}{t}$	$\frac{\phi tr}{3\phi t-1}$	$(\frac{r}{3\phi t-1}, 0)$	$(\frac{(2\phi t-1)r}{3\phi t-1}, 0)$	$(\frac{\phi r}{3\phi t-1}, 1)$

Finally, we prove that the ISP has no incentive to set a low data usage price that causes a fully covered market under the assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$. From the above analysis, we get the ISP's data usage price $p_P^S = \frac{1}{4}(V_A^* + s_A^* + V_B^*) = \frac{((\phi t)^2+2\phi t-1)r}{(7\phi t-3)\phi t}$ and its profit $\pi_{ISP\_P}^S = p_P^S(x_A^* + 1 - x_B^*) = \frac{1}{8t}(V_A^* + s_A^* + V_B^*)^2$ in a candidate partially-covered equilibrium of subgame S. According to our analysis of stage 3, the market is fully covered if the data usage price p is low such that $x_A \geq x_B$. In the following analysis, we calculate the ISP's maximal profit $\pi_{ISP\_F}^S$ in a fully covered market of subgame S and then compare $\pi_{ISP\_F}^S$ with $\pi_{ISP\_P}^S$. Given the CPs' content quality strategies V_A^* and V_B^* and data sponsorship strategy s_A^*, we have $x_m = \frac{V_A^*+s_A^*-V_B^*+t}{2t}$. Solving $U_A(x_m) = U_B(x_m) = 0$ yields the ISP's optimal data usage price $p_F^S = \frac{1}{2}(V_A^* + V_B^* + s_A^* - t)$ in a fully covered market. Then, $\pi_{ISP\_F}^S = p_F^S = \frac{1}{2}(V_A^* + V_B^* + s_A^* - t)$. As shown in Figure D.3, the market is fully covered if $p \leq p_F^S$. It is easy to see that $\pi_{ISP\_P}^S - \pi_{ISP\_F}^S = \frac{1}{8t}(V_A^* + V_B^* + s_A^* - 2t)^2 \geq 0$. Therefore, compared to the partially-covered equilibrium of subgame S, the ISP's profit decreases if it chooses a price lower than p_F^S and deviates to a fully covered market.

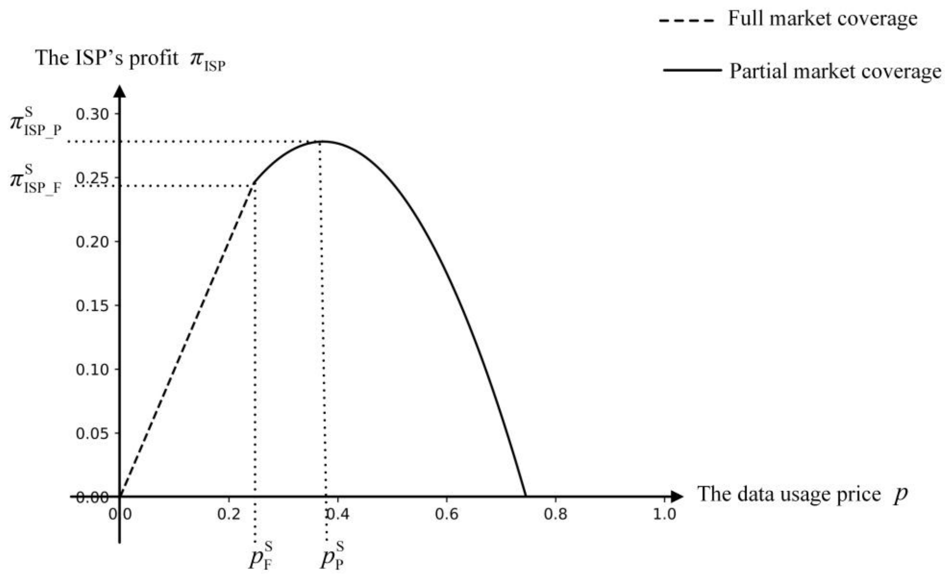

Figure D.3. Illustration of the ISP's optimal pricing decision in subgame S when $r = 0.8, \phi = 1, t = 1$.

D.3. Both CPs Sponsor Consumers (Subgame SS)

Finally, we construct the partially-covered equilibrium of subgame SS wherein both CPs are allowed to sponsor consumer data traffic.

In stage 3, if the market is partially covered, we solve $U_{\text{A}}(x_{\text{A}}) = 0$ and $U_{\text{B}}(x_{\text{B}}) = 0$ and determine the marginal consumer consumers $x_{\text{A}} = \frac{V_{\text{A}} + s_{\text{A}} - p}{t}$ and $x_{\text{B}} = 1 - \frac{V_{\text{B}} + s_{\text{B}} - p}{t}$. If the price p is high such that $x_{\text{A}} < x_{\text{B}}$, our assumption of a partially covered market is justified; if the ISP chooses a low price p that makes $x_{\text{A}} \geq x_{\text{B}}$, then all consumers participate in the market and the market is fully covered.

In stage 2, both CPs make decisions regarding content quality and data sponsorship and the ISP decides the data usage price. In a candidate partially-covered equilibrium, we analyze the CPs' optimal decisions on content quality and data sponsorship given the data usage price p, then derive the ISP's optimal pricing decision given the CPs' decisions, and finally make a joint analysis of the CPs' and the ISP's optimal decisions to determine the equilibrium. First, given the data usage price p, we derive the

CPs' optimal content quality decision V_{i}^* and data sponsorship decision s_{i}^* ($i = \mathrm{A}$ or B) based on the profit function $\pi_{\mathrm{i}} = (r - s_{\mathrm{i}})\frac{V_{\mathrm{i}}+s_{\mathrm{i}}-p}{t} - \frac{\phi}{2}V_{\mathrm{i}}{}^2$. To guarantee the existence of the maximums of π_{i}, we apply the assumption that $\phi > \frac{1}{2t}$, which ensures that the Jacobian $\frac{d^2\,\pi_{\mathrm{i}}}{d\,V_{\mathrm{i}}{}^2} \times \frac{d^2\,\pi_{\mathrm{i}}}{d\,s_{\mathrm{i}}{}^2} - (\frac{d^2\,\pi_{\mathrm{i}}}{d\,V_{\mathrm{i}} d\,s_{\mathrm{i}}})^2 = \frac{2\phi}{t} - \frac{1}{t^2} > 0$. Solving $\frac{d\,\pi_{\mathrm{A}}}{d\,V_{\mathrm{A}}} = 0$ and $\frac{d\,\pi_{\mathrm{B}}}{d\,V_{\mathrm{B}}} = 0$ yields $V_{\mathrm{A}} = \frac{r-s_{\mathrm{A}}}{\phi t}$ and $V_{\mathrm{B}} = \frac{r-s_{\mathrm{B}}}{\phi t}$, respectively. Solving $\frac{d\,\pi_{\mathrm{A}}}{d\,s_{\mathrm{A}}} = 0$ and $\frac{d\,\pi_{\mathrm{B}}}{d\,s_{\mathrm{B}}} = 0$ yields $s_{\mathrm{A}} = \frac{r-V_{\mathrm{A}}+p}{2}$ and $s_{\mathrm{B}} = \frac{r-V_{\mathrm{B}}+p}{2}$, respectively. Jointly solving the four FOCs yields $V_{\mathrm{A}}^* = V_{\mathrm{B}}^* = \frac{r-p}{2\phi t-1}$ and $s_{\mathrm{A}}^* = s_{\mathrm{B}}^* = \frac{(\phi t-1)r+\phi tp}{2\phi t-1}$. Second, given the CPs' optimal content quality decision V_{i}^* and data sponsorship decision s_{i}^*, we derive the ISP's optimal data usage price p^*. The ISP's profit is $\pi_{\mathrm{ISP}} = p(x_{\mathrm{A}} + 1 - x_{\mathrm{B}}) = p(\frac{V_{\mathrm{A}}^*+s_{\mathrm{A}}^*+V_{\mathrm{B}}^*+s_{\mathrm{B}}^*-2p}{t})$. We solve the FOC of π_{ISP} w.r.t. p and obtain the ISP's optimal price $p^* = \frac{1}{4}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^*) = \frac{1}{2}\left(\frac{r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{i}}^*\right)$. Finally, we jointly solve $s_{\mathrm{i}}^* = \frac{(\phi t-1)r+\phi tp^*}{2\phi t-1}$ and $p^* = \frac{1}{2}\left(\frac{r}{\phi t} + (1 - \frac{1}{\phi t})s_{\mathrm{i}}^*\right)$ and get $s_{\mathrm{i}}^* = \frac{(2\phi t-1)r}{3\phi t-1}$ and $p^* = \frac{\phi tr}{3\phi t-1}$. Then, $V_{\mathrm{i}}^* = \frac{r-p^*}{2\phi t-1} = \frac{r}{3\phi t-1}$, $x_{\mathrm{A}}^* = 1 - x_{\mathrm{B}}^* = \frac{V_{\mathrm{i}}^*+s_{\mathrm{i}}^*-p^*}{t} = \frac{\phi r}{3\phi t-1}$. The assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$ guarantees that $x_{\mathrm{A}}^* < x_{\mathrm{B}}^*$.

Finally, we prove that the ISP has no incentive to set a low data usage price that causes a fully covered market under the assumption that $r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$. From the above analysis, we get the ISP's data usage price $p_{\mathrm{P}}^{\mathrm{SS}} = \frac{1}{4}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^*) = \frac{\phi tr}{3\phi t-1}$ and its profit $\pi_{\mathrm{ISP\_P}}^{\mathrm{SS}} = p_{\mathrm{P}}^{\mathrm{S}}(x_{\mathrm{A}}^* + 1 - x_{\mathrm{B}}^*) = \frac{1}{8t}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^*)^2$ in a partially-covered equilibrium of subgame SS. According to our analysis of stage 3, the market is fully covered if the data usage price p is low such that $x_{\mathrm{A}} \geq x_{\mathrm{B}}$. In the following analysis, we calculate the ISP's maximal profit $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}}$ in a fully covered market of subgame SS and then compare $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}}$ with $\pi_{\mathrm{ISP\_P}}^{\mathrm{SS}}$. Given the CPs' content quality strategy V_{i}^* and data sponsorship strategy s_{i}^*, solving $U_{\mathrm{A}}(x_{\mathrm{m}}) = U_{\mathrm{B}}(x_{\mathrm{m}}) = 0$ yields the ISP's optimal data usage price $p_{\mathrm{F}}^{\mathrm{SS}} = \frac{1}{2}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^* - \mathrm{t})$ in a fully covered market. Then, $\pi_{\mathrm{ISP\_F}}^{\mathrm{SS}} = p_{\mathrm{F}}^{\mathrm{SS}} = \frac{1}{2}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^* - \mathrm{t})$. As shown in Figure D.4, the market is fully covered if $p \leq p_{\mathrm{F}}^{\mathrm{SS}}$. It is easy to see that $\pi_{\mathrm{ISP\_P}}^{\mathrm{SS}} - \pi_{\mathrm{ISP\_F}}^{\mathrm{SS}} = \frac{1}{8t}(V_{\mathrm{A}}^* + s_{\mathrm{A}}^* + V_{\mathrm{B}}^* + s_{\mathrm{B}}^* - 2t)^2 \geq 0$. Therefore, compared to the partially-covered equilibrium of subgame SS, the ISP's profit decreases if it chooses a price lower than $p_{\mathrm{F}}^{\mathrm{SS}}$ and deviates to a fully covered market.

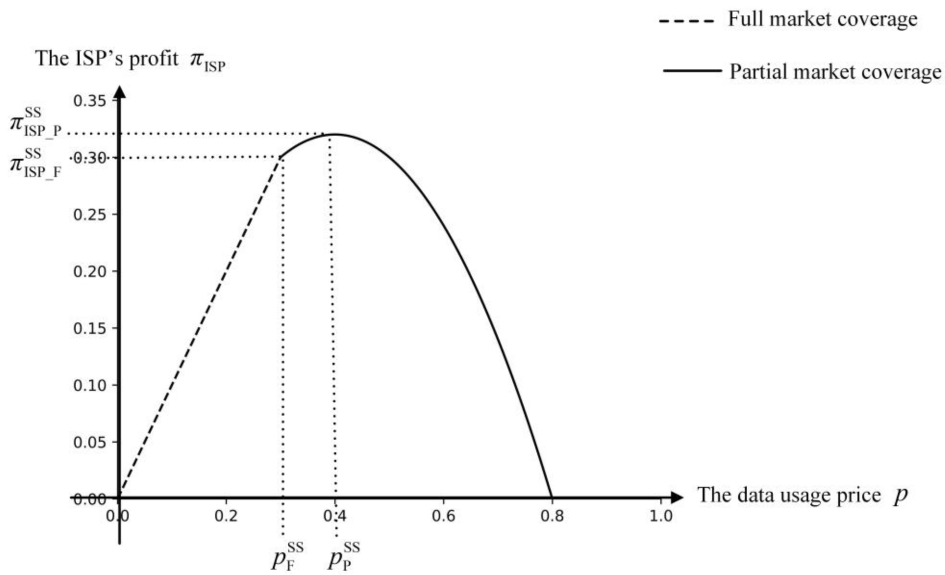

Figure D.4. Illustration of the ISP's optimal pricing decision in subgame SS when $r = 0.8, \phi = 1, t = 1$.

D.4. Welfare Analysis

Table D.2. Partially-covered equilibria of different subgames.				
Assumptions	$r < \min\{\phi t^2, \frac{3\phi t-1}{2\phi}\}$ and $\phi > \frac{1}{2t}$			
Subgames	ISP's data usage pricing p^*	CPs' content quality decisions $(V_{\mathrm{A}}^*, V_{\mathrm{B}}^*)$	CPs' data sponsorship decisions $(s_{\mathrm{A}}^*, s_{\mathrm{B}}^*)$	Marginal consumers $(x_{\mathrm{A}}^*, x_{\mathrm{B}}^*)$
Subgame N				
$\forall\, \phi$	$\frac{r}{2\phi t}$	$(\frac{r}{\phi t}, \frac{r}{\phi t})$	$(0, 0)$	$(\frac{\mathrm{r}}{2\phi \mathrm{t}^2}, 1 - \frac{\mathrm{r}}{2\phi \mathrm{t}^2})$
Subgame S				
$\phi \leq \frac{1}{t}$	$\frac{((\phi t)^2+2\phi t-1)r}{(7\phi t-3)\phi t}$	$(\frac{(3\phi t-1)r}{(7\phi t-3)\phi t}, \frac{r}{\phi t})$	$(\frac{(4\phi t-2)r}{7\phi t-3}, 0)$	$(\frac{(3\phi t-1)r}{(7\phi t-3)t}, 1 - \frac{(-(\phi t)^2+5\phi t-2)r}{(7\phi t-3)\phi t^2})$
$\phi > \frac{1}{t}$	$\frac{\phi t r}{3\phi t-1}$	$(\frac{r}{3\phi t-1}, 0)$	$(\frac{(2\phi t-1)r}{3\phi t-1}, 0)$	$(\frac{\phi r}{3\phi t-1}, 1)$

Subgame SS				
$\forall\, \phi$	$\frac{\phi t r}{3\phi t-1}$	$(\frac{r}{3\phi t-1}, \frac{r}{3\phi t-1})$	$(\frac{(2\phi t-1)r}{3\phi t-1}, \frac{(2\phi t-1)r}{3\phi t-1})$	$(\frac{\phi r}{3\phi t-1}, 1-\frac{\phi r}{3\phi t-1})$

The partially-covered equilibria of the three subgames are summarized in Table D.2. In the following analysis, we derive the SPNE and examine the robustness of Propositions 1–5 in this extension.

First, we derive the SPNE and examine the robustness of Proposition 1 in this extension. In subgame N, $\pi_{\text{ISP}}^{\text{N}} = \frac{r^2}{2\phi^2 t^3}$, and in subgame S, $\pi_{\text{ISP}}^{\text{S}} = \begin{cases} \frac{2\left((\phi t)^2+2\phi t-1\right)^2 r^2}{(7\phi t-3)^2\phi^2 t^3} & if\ \phi \le \frac{1}{t} \\ \frac{r^2\phi^2 t}{(3\phi t-1)^2} & if\ \phi > \frac{1}{t} \end{cases}$. In subgame SS, $\pi_{\text{ISP}}^{\text{SS}} = \frac{2r^2\phi^2 t}{(3\phi t-1)^2}$. First of all, the sign of $\pi_{\text{ISP}}^{\text{SS}} - \pi_{\text{ISP}}^{\text{N}}$ is the same as that of $(2\phi t-1)(\phi t-1)$, which is positive only when $\phi t - 1 > 0$ under the assumption $\phi > \frac{1}{2t}$. Second, when $\phi \le \frac{1}{t}$, the sign of $\pi_{\text{ISP}}^{\text{S}} - \pi_{\text{ISP}}^{\text{N}}$ is the same as that of $(2\phi t-1)(\phi t-1)$, which is not positive under the assumption $\phi > \frac{1}{2t}$; when $\phi > \frac{1}{t}$, the sign of $\pi_{\text{ISP}}^{\text{S}} - \pi_{\text{ISP}}^{\text{N}}$ is the same as that of $(\phi t)^2 - 3\phi t + 1$, which is positive only when $\phi > \frac{3+\sqrt{5}}{2t}$. Finally, when $\frac{1}{2t} < \phi < \frac{1}{t}$, the sign of $\pi_{\text{ISP}}^{\text{SS}} - \pi_{\text{ISP}}^{\text{S}}$ is the same as that of $(\phi t-1)(4(\phi t)^2 - 4\phi t + 1)$, which is negative; when $\phi > \frac{1}{t}$, $\pi_{\text{ISP}}^{\text{SS}} - \pi_{\text{ISP}}^{\text{S}} = \frac{r^2\phi^2 t}{(3\phi t-1)^2} > 0$. Therefore, when CPs' content production efficiency is high (i.e., $\frac{1}{2t} < \phi < \frac{1}{t}$), we have $\pi_{\text{ISP}}^{\text{SS}} < \pi_{\text{ISP}}^{\text{S}} < \pi_{\text{ISP}}^{\text{N}}$ and the ISP's optimal network management choice is to not introduce sponsored data services. When CPs' content production efficiency is low (i.e., $\phi > \frac{1}{t}$), we have $\pi_{\text{ISP}}^{\text{SS}} > \pi_{\text{ISP}}^{\text{S}}$ and $\pi_{\text{ISP}}^{\text{SS}} > \pi_{\text{ISP}}^{\text{N}}$, and the ISP's optimal network management choice is to allow both CPs to subsidize consumers' data usage. The ISP's optimal network management choice in this extension is consistent with that in Proposition 1.

Second, we compare CP A's profits in the equilibria under different subgames and examine the robustness of Proposition 2 in this extension. In subgame N, $\pi_{\text{A}}^{\text{N}} = \pi_{\text{B}}^{\text{N}} = 0$. In subgame S, $\pi_{\text{A}}^{\text{S}} =$

$\begin{cases} \frac{r^2(3\phi t-1)^2(2\phi t-1)}{2\phi t^2(7\phi t-3)^2} & if\ \phi \leq \frac{1}{t} \\ \frac{r^2\phi(2\phi t-1)}{2(3\phi t-1)^2} & if\ \phi > \frac{1}{t} \end{cases}$. In subgame SS, $\pi_{\mathrm{A}}^{\mathrm{SS}} = \pi_{\mathrm{B}}^{\mathrm{SS}} = \frac{r^2\phi(2\phi t-1)}{2(3\phi t-1)^2}$. First of all, $\pi_{\mathrm{A}}^{\mathrm{S}} - \pi_{\mathrm{A}}^{\mathrm{N}} = \begin{cases} \frac{r^2(3\phi t-1)^2(2\phi t-1)}{2\phi t^2(7\phi t-3)^2} & if\ \phi \leq \frac{1}{t} \\ \frac{r^2\phi(2\phi t-1)}{2(3\phi t-1)^2} & if\ \phi > \frac{1}{t} \end{cases}$, which is positive under the assumption of $\phi > \frac{1}{2t}$. Second, when $\phi < \frac{1}{t}$, the sign of $\pi_{\mathrm{A}}^{\mathrm{SS}} - \pi_{\mathrm{A}}^{\mathrm{S}}$ is the same as that of $-(2\phi t - 1)(\phi t - 1)$, which is also positive under the assumption of $\phi > \frac{1}{2t}$; when $\phi > \frac{1}{t}$, $\pi_{\mathrm{A}}^{\mathrm{SS}} - \pi_{\mathrm{A}}^{\mathrm{S}} = 0$. Therefore, we have $\pi_{\mathrm{A}}^{\mathrm{N}} < \pi_{\mathrm{A}}^{\mathrm{S}} \leq \pi_{\mathrm{A}}^{\mathrm{SS}}$. Then CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers, and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers. The prediction of CP A's maximal profit in this extension is partially consistent with Proposition 2 regarding high content production efficiency ($\phi < \frac{1}{2t}$).

Third, we analyze whether a CP suffers because of its rival's data sponsorship and examine the robustness of Proposition 3 in this extension. When the CP is allowed to sponsor in subgame S (i.e., CP A), then we just need to show that $\pi_{\mathrm{A}}^{\mathrm{S}} \leq \pi_{\mathrm{A}}^{\mathrm{SS}}$, which has already been proved. Hence, we just need to consider the other case when the CP is not allowed to sponsor in subgame S (i.e., CP B), and show that $\pi_{\mathrm{B}}^{\mathrm{N}} \leq \pi_{\mathrm{B}}^{\mathrm{S}}$. CP B's equilibrium profit in subgame N is $\pi_{\mathrm{B}}^{\mathrm{N}} = 0$ and that in subgame S is $\pi_{\mathrm{B}}^{\mathrm{S}} = \begin{cases} \frac{r^2(1-\phi t)}{2\phi t^2(3\phi t+1)} & if\ \phi \leq \frac{1}{t} \\ 0 & if\ \phi > \frac{1}{t} \end{cases}$. When $\phi < \frac{1}{t}$, $\pi_{\mathrm{B}}^{\mathrm{S}} - \pi_{\mathrm{B}}^{\mathrm{N}} = \frac{r^2(1-\phi t)}{2\phi t^2(3\phi t+1)} > 0$; When $\phi > \frac{1}{t}$, $\pi_{\mathrm{B}}^{\mathrm{S}} - \pi_{\mathrm{B}}^{\mathrm{N}} = 0$. Therefore, we conclude that under our parameter assumptions, we have $\pi_{\mathrm{B}}^{\mathrm{N}} \leq \pi_{\mathrm{B}}^{\mathrm{S}}$ and $\pi_{\mathrm{A}}^{\mathrm{S}} \leq \pi_{\mathrm{A}}^{\mathrm{SS}}$. Then a CP always benefits from its rival's data sponsorship regardless of whether it has the option to sponsor. This prediction is partially consistent with Proposition 3 regarding high content production efficiency ($\phi < \frac{1}{2t}$).

Fourth, we analyze the impact of sponsored data services on consumer surplus and examine the robustness of Proposition 4 in this extension. In subgame N, $CS^{\text{N}} = \frac{t}{2}(x_{\text{A}}^{\text{N}})^2 + \frac{t}{2}(1 - x_{\text{B}}^{\text{N}})^2 = \frac{r^2}{4\phi^2 t^3}$. In subgame S, $CS^{\text{S}} = \begin{cases} \frac{t}{2}(x_{\text{A}}^{\text{S}})^2 + \frac{t}{2}(1 - x_{\text{B}}^{\text{S}})^2 & if\ \phi \le \frac{1}{t} \\ \frac{t}{2}(x_{\text{A}}^{\text{S}})^2 & if\ \phi > \frac{1}{t} \end{cases} = \begin{cases} \frac{r^2((3\phi^2t^2 - \phi t)^2 + (\phi^2 t^2 - 5\phi t + 2)^2)}{2\phi^2 t^3 (7\phi t - 3)^2} & if\ \phi \le \frac{1}{t} \\ \frac{\phi^2 r^2 t}{2(3\phi t - 1)^2} & if\ \phi > \frac{1}{t} \end{cases}$. In subgame SS, $CS^{\text{SS}} = \frac{t}{2}(x_{\text{A}}^{\text{SS}})^2 + \frac{t}{2}(1 - x_{\text{B}}^{\text{SS}})^2 = \frac{\phi^2 r^2 t}{(3\phi t - 1)^2}$. First of all, the sign of $CS^{\text{SS}} - CS^{\text{N}}$ is the same as that of $(2\phi t - 1)(\phi t - 1)$, which is positive only when $\phi t - 1 > 0$ under the assumption of $\phi > \frac{1}{2t}$. Second, when $\phi > \frac{1}{t}$, $CS^{\text{SS}} - CS^{\text{S}} = \frac{\phi^2 r^2 t}{2(3\phi t - 1)^2} > 0$. Finally, when $\frac{1}{2t} < \phi < \frac{1}{t}$, the sign of $x_{\text{A}}^{\text{N}} - x_{\text{A}}^{\text{S}}$ is the same as that of $-(2\phi t - 1)(\phi t - 1)$, which is positive, and the sign of $x_{\text{B}}^{\text{N}} - x_{\text{B}}^{\text{S}}$ is the same as that of $-(2(\phi t)^2 + 2\phi t - 1)$, which is negative. Because of $x_{\text{A}}^{\text{N}} - x_{\text{A}}^{\text{S}} > 0$ and $x_{\text{B}}^{\text{N}} - x_{\text{B}}^{\text{S}} < 0$, we have $CS^{\text{N}} = \frac{t}{2}\left(x_{\text{A}}^{\text{N}}\right)^2 + \frac{t}{2}\left(1 - x_{\text{B}}^{\text{N}}\right)^2 > CS^{\text{S}} = \frac{t}{2}\left(x_{\text{A}}^{\text{S}}\right)^2 + \frac{t}{2}\left(1 - x_{\text{B}}^{\text{S}}\right)^2$ when $\frac{1}{2t} < \phi < \frac{1}{t}$. Therefore, when CPs' content production efficiency is low (i.e., $\phi > \frac{1}{t}$), we have $CS^{\text{SS}} > CS^{\text{S}}$ and $CS^{\text{SS}} > CS^{\text{N}}$, and then allowing both CPs to sponsor consumers maximizes consumer surplus. When CPs' content production efficiency is high (i.e., $\phi < \frac{1}{t}$), we have $CS^{\text{N}} > CS^{\text{SS}}$ and $CS^{\text{N}} > CS^{\text{S}}$, and not allowing sponsored data services maximizes consumer surplus. Compared to the prediction from Proposition 4, the scenario of a partially covered market causes major changes in the prediction of consumer surplus.

Finally, we analyze the impact of sponsored data services on social welfare and examine the robustness of Proposition 5 in this extension. Because of the complexity of calculating social welfare, we compare social welfare in different subgames via a numerical example where $r = 0.5, t = 1$, and $\phi \in [0.6,1.2]$. These parameter values satisfy the assumptions of $r < \min\{\phi t^2, \frac{3\phi t - 1}{2\phi}\}$ and $\phi > \frac{1}{2t}$. The prediction of social welfare is robust under diverse sets of parameter values, and those results are available upon request. As shown in Figure D.5, when CPs' content production efficiency is extremely high (i.e., $\phi < 0.7597$), not allowing sponsored data services maximizes social welfare, while allowing only one CP to sponsor consumers minimizes it. When CPs' content production efficiency is intermediate (i.e., $0.7597 < \phi < 1$), allowing both CPs to sponsor consumers maximizes social welfare, while no data sponsorship or allowing only one CP to sponsor consumers minimizes it. When CPs' content production efficiency is extremely low (i.e., $\phi > 1$), allowing both CPs to sponsor consumers maximizes social welfare, while allowing only one CP to sponsor consumers minimizes it. Compared to Proposition 5, the prediction of social welfare has some differences in this extension. Consistent with Proposition 5, allowing both CPs to sponsor consumers maximizes social welfare in this extension when CPs' content production efficiency is lower than a threshold ($\phi > 0.7597$). Different from Proposition 5, no data sponsorship maximizes social welfare in this extension when CPs' content production efficiency is higher than a threshold ($\phi < 0.7597$).

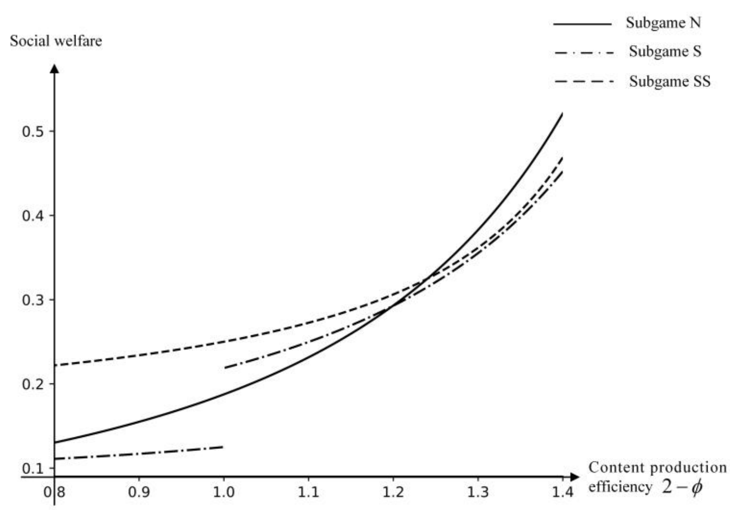

Figure D.5. Social welfare in the equilibria of the numerical case where $r = 0.5, t = 1$, and $\phi \in [0.6,1.2]$.

To sum up, most predictions from the main model (i.e., those regarding Propositions 1–3 and 5) are consistent or partially consistent with those in this extension. We summarize the comparison between the main model and this extension in Table D.3.

Table D.3. Propositions in the main model and predictions in a partially covered market.			
Proposition	Content of the proposition	Predictions in a partially covered market	Comparison with the proposition
Proposition 1 (ISP's optimal network management choice)	In the SPNE, (1) when CPs' content production efficiency is high, the ISP suffers from the CPs' data sponsorship, and its optimal network management choice is to not introduce sponsored data services; (2) when CPs' content production efficiency is low, the ISP benefits from the CPs' data sponsorship, and its optimal network management choice is to allow both CPs to subsidize consumers' data usage.	In the SPNE, (1) when CPs' content production efficiency is high, the ISP suffers from the CPs' data sponsorship, and its optimal network management choice is to not introduce sponsored data services; (2) when CPs' content production efficiency is low, the ISP's optimal network management choice is to allow both CPs to subsidize consumers' data usage.	This prediction is consistent with Proposition 1.
Proposition 2 (Implication of subsidizing consumers)	Considering CP A, which is allowed to sponsor in subgame S: (1) when the content production efficiency is high, CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers; (2) when the content production efficiency is low, CP A achieves maximal profit in subgame S, where it sponsors consumers alone, and it achieves a lower profit in subgame SS than in subgame N if the advertising revenue rate is sufficiently low.	Considering CP A, which is allowed to sponsor in subgame S: CP A benefits from subsidizing consumers' data usage regardless of whether the other CP sponsors consumers, and it achieves maximal profit in subgame SS, where both CPs are allowed to sponsor consumers.	This prediction is partially consistent with Proposition 2 regarding high content production efficiency.
Proposition 3 (Impact of the rival's data sponsorship)	Compared to the scenario in which the rival has no sponsoring option, (1) when content production efficiency is high, a CP always benefits from its rival's data sponsorship through higher profits, regardless of whether it has the option to sponsor; (2) when content production efficiency is low, a CP always suffers from its rival's data sponsorship, regardless of whether it has the	Compared to the scenario in which the rival has no sponsoring option, a CP benefits from its rival's data sponsorship regardless of whether it has the option to sponsor.	This prediction is partially consistent with Proposition 3 regarding high content production efficiency.

	option to sponsor.		
Proposition 4 (Consumer surplus)	Compared to subgame N, (1) allowing both CPs to sponsor consumers does not change consumer surplus, whereas (2) allowing only one CP to sponsor consumers always increases consumer surplus.	(1) Not allowing data sponsorship maximizes consumer surplus when CPs' content production efficiency is high, and (2) allowing both CPs to sponsor consumers maximizes it when the content production efficiency is low.	This prediction is different from Proposition 4.
Proposition 5 (Social welfare)	(1) When CPs' content production efficiency is extremely high, allowing only one CP to sponsor consumers maximizes social welfare while no data sponsorship minimizes it; (2) when CPs' content production efficiency is intermediate, allowing both CPs to sponsor consumers maximizes social welfare while no data sponsorship minimizes it; (3) when CPs' content production efficiency is extremely low, allowing both CPs to sponsor consumers maximizes social welfare while allowing only one CP to sponsor consumers minimizes it.	(1) When CPs' content production efficiency is extremely high, no data sponsorship maximizes social welfare while allowing only one CP to sponsor consumers minimizes it; (2) when CPs' content production efficiency is intermediate, allowing both CPs to sponsor consumers maximizes social welfare while no data sponsorship or allowing only one CP to sponsor consumers minimizes it; (3) when CPs' content production efficiency is extremely low, allowing both CPs to sponsor consumers maximizes social welfare while allowing only one CP to sponsor consumers minimizes it.	(1) Consistent with the main model, allowing both CPs to sponsor consumers maximizes social welfare in this extension when CPs' content production efficiency is lower than a threshold. (2) Different from the main model, no data sponsorship maximizes social welfare in this extension when CPs' content production efficiency is higher than a threshold.