Appendix

A1. Proof of Mass Advertising Configuration

Without the option of geofencing, Advertiser A compares no advertising and mass advertising, considering his product valuation, consumer awareness, and transportation costs. We start with the analysis of the no-advertising case.

Suppose Advertiser A chooses not to advertise. The demand function is

$$D_0 = \begin{cases} 0 \quad , & p_0 > V \\ \alpha_0\theta, & V - t < p_0 \le V, \\ \alpha_0 \quad , & p_0 \le V - t \end{cases}$$

where p_0, α_0, θ, V, and t are the product price, initial consumer awareness level, the proportion of consumers in the local market, product valuation, and unit transportation cost, respectively. The profit function of the advertiser is

$$\pi_0 = p_0 D_0 = \begin{cases} 0 \quad , & p_0 > V \\ \alpha_0\theta p_0, & V - t < p_0 \le V. \\ \alpha_0 p_0 \quad , & p_0 \le V - t \end{cases}$$

Maximizing π_0 by choosing p_0, we obtain the optimal price as $p_0^* = V$ and the corresponding profit as $\pi_0^* = \alpha_0\theta V$ if $V \le 2t$; $p_0^* = V - t$ and $\pi_0^* = \alpha_0(V - t)$, otherwise.

Now, suppose Advertiser A adopts mass advertising. The demand function is

$$D_M = \begin{cases} 0 \quad , & p_M > V \\ (\alpha_0 + (1 - \alpha_0)\beta)\theta, & V - t < p_M \le V. \\ \alpha_0 + (1 - \alpha_0)\beta \quad , & p_M \le V - t \end{cases}$$

The profit function of the advertiser is

$$\pi_M = p_M D_M - C_M = \begin{cases} -\beta^2 \quad , & p_M > V \\ (\alpha_0 + (1 - \alpha_0)\beta)\theta p_M - \beta^2, & V - t < p_M \le V \\ (\alpha_0 + (1 - \alpha_0)\beta)p_M - \beta^2 \quad , & p_M \le V - t \end{cases}$$

Maximizing π_M by choosing p_M and β, we obtain the optimal price as $p_M^* = V$, the optimal advertising effort as $\beta^* = \frac{V(1-\alpha_0)}{4}$, and the corresponding profit as $\pi_M^* = \frac{V\left(V(1-\alpha_0)^2+8\alpha_0\right)}{16}$ if $V \le 2t$; $p_M^* = V - t$, $\beta^* =$

$\frac{(V-t)(1-\alpha_0)}{2}$, and $\pi_M^* = \frac{(V-t)\left((V-t)(1-\alpha_0)^2+4\alpha_0\right)}{4}$, otherwise.

Comparing no and mass advertising, we have $\pi_M^* > \pi_0^*$ in any market condition. Advertiser *A* shall choose the strategy that brings him a higher profit. Thus, the advertiser's optimal strategy is $S^* = M$, setting $p_M^* = V$ and $\beta^* = \frac{V(1-\alpha_0)}{4}$ if $V \leq 2t$; $p_M^* = V - t$ and $\beta^* = \frac{(V-t)(1-\alpha_0)}{2}$, otherwise. ☐

A2. Proof of Lemma 1 (Geofencing Configuration)

On the basis of Appendix A.1, we further include geofencing in the strategy set for Advertiser *A* to choose. Suppose the advertiser adopts geofencing. He sends ads to either the local market ($r = L$) with a cost of $C_G^{r=L} = \theta^2$ or both markets ($r = B$) with a cost of $C_G^{r=B} = 1$. The demand function is

$$D_G = \begin{cases} 0 & , \quad p_G > V \\ \theta & , \quad V - t < p_G \leq V \\ \theta + \alpha_0(1-\theta), & p_G \leq V - t \textit{ and } r = L \\ 1 & , \quad p_G \leq V - t \textit{ and } r = B \end{cases}.$$

The advertiser's profit function of geofencing is

$$\pi_G = p_G D_G - C_G = \begin{cases} -\theta^2 & , \quad p_G > V \textit{ and } r = L \\ -1 & , \quad p_G > V \textit{ and } r = B \\ \theta p_G - \theta^2 & , \quad V - t < p_G \leq V \textit{ and } r = L \\ \theta p_G - 1 & , \quad V - t < p_G \leq V \textit{ and } r = B \\ (\theta + \alpha_0(1-\theta))p_G - \theta^2, & p_G \leq V - t \textit{ and } r = L \\ p_G - 1 & , \quad p_G \leq V - t \textit{ and } r = B \end{cases}.$$

Maximizing π_G by choosing p_G and r, we obtain the optimal geofencing configurations, which depend on market conditions regarding the valuation and awareness. Specifically, the optimal strategy is to set a small radius $r^* = L$ and a relatively high price $p_G^* = V$ to obtain a profit of $\pi_G^* = \theta V - \theta^2$ if

(1) $0 < t \leq \frac{1}{4}$, $0 < \alpha_0 < \frac{1+2t}{3+2t}$, and $\frac{1-4t\alpha_0}{2-4\alpha_0} < V < \frac{1}{2}(3+4t)$,

(2) $\frac{1}{4} < t \leq \frac{1}{2}$, $0 < \alpha_0 < \frac{4t-1}{4t}$, and $\frac{1}{2-2\alpha_0} < V < \frac{1}{2}(3+4t)$,

(3) $\frac{1}{4} < t \leq \frac{1}{2}$, $\frac{4t-1}{4t} \leq \alpha_0 < \frac{1+2t}{3+2t}$, and $\frac{1-4t\alpha_0}{2-4\alpha_0} < V < \frac{1}{2}(3+4t)$,

(4) $t > \frac{1}{2}$, $0 < \alpha_0 < \frac{1+2t}{3+2t}$, and $\frac{1}{2-2\alpha_0} < V < \frac{1}{2}(3+4t)$), or

(5) $t > \frac{1}{2}$, $\frac{1+2t}{3+2t} \leq \alpha_0 < \frac{4t-1}{4t}$, and $\frac{1}{2-2\alpha_0} < V < \frac{1-4t\alpha_0}{2-4\alpha_0}$.

The optimal strategy is to set a large radius $r^* = B$ and a relatively low price $p_G^* = V - t$ to obtain a profit of $\pi_G^* = V - t - 1$ if

(1) $0 < \alpha_0 \leq \frac{1}{3}$ and $V \geq \frac{1}{2}(3 + 4t)$,

(2) $\frac{1+2t}{3+2t} \leq \alpha_0 < 1$ and $V > \frac{1+t-t\alpha_0}{1-\alpha_0}$, or

(3) $\frac{1}{3} < \alpha_0 < \frac{1+2t}{3+2t}$ and $V \geq \frac{1}{2}(3 + 4t)$.

If the market condition does not meet any of the above scenarios, the optimal strategy of Advertiser *A* is not to advertise at all (i.e., geofencing not applicable). □

A3. Proof of Proposition 1 (Optimal Advertising Strategy)

Comparing π_G^* in Appendix A.2 and π_M^* in Appendix A.1, we identify the market conditions of geofencing dominating mass advertising (i.e., $\pi_G^* > \pi_M^*$):

(I) $0 < \alpha_0 < \frac{1}{3}(3 - \sqrt{3})$, $\frac{2\alpha_0 - 3\alpha_0^2}{2-4\alpha_0+2\alpha_0^2} < t \leq \frac{2-\sqrt{3}}{1-\alpha_0}$, and

$\frac{1+t-2\alpha_0-2t\alpha_0+t{\alpha_0}^2-\sqrt{2t-2\alpha_0-4t\alpha_0+3{\alpha_0}^2+2t{\alpha_0}^2}}{(1-\alpha_0)^2} < V < \frac{1+t-2\alpha_0-2t\alpha_0+t{\alpha_0}^2+\sqrt{2t-2\alpha_0-4t\alpha_0+3{\alpha_0}^2+2t{\alpha_0}^2}}{(1-\alpha_0)^2}$, or

(II) $\frac{2-\sqrt{3}}{1-\alpha_0} < t \leq \frac{2}{1-\alpha_0}$ and $\frac{4-2\sqrt{3}}{1-\alpha_0} < V < \frac{1+t-2\alpha_0-2t\alpha_0+t{\alpha_0}^2+\sqrt{2t-2\alpha_0-4t\alpha_0+3{\alpha_0}^2+2t{\alpha_0}^2}}{(1-\alpha_0)^2}$. □

A4. Discussion on Fixed Setup Costs

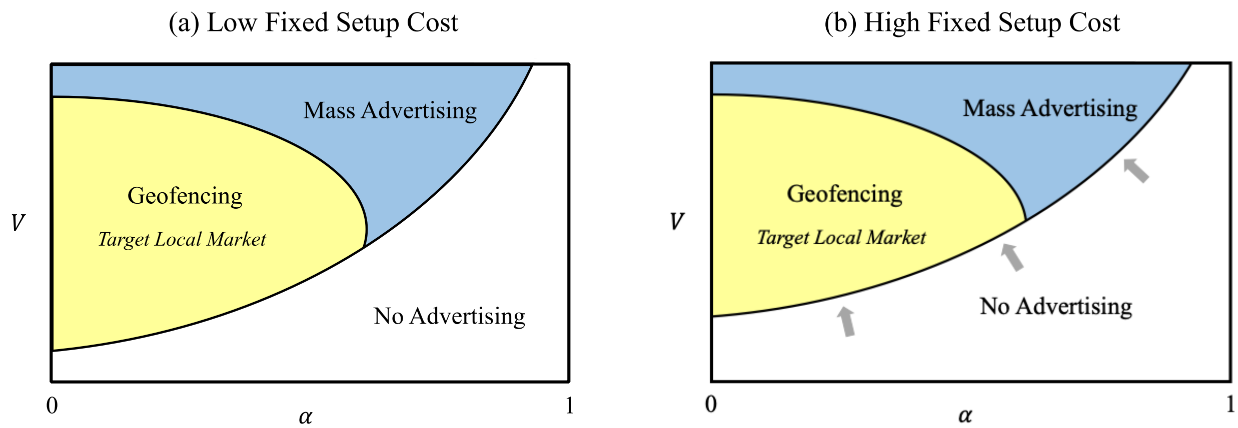
When including fixed cost *c* in the cost structure of advertising, the advertiser sends ads to β consumers in the market under mass advertising with a cost of $c + \beta^2$, or sends ads to the θ consumers in the local area under geofencing advertising a cost of $c + \theta^2$. Fixed cost *c* does not influence optimal choices of price *p* or advertising efforts, but the advertiser's profit under both geofencing and mass advertising strategies. We use the benchmark scenario to prove that it does not qualitatively influence the findings about comparisons between geofencing advertising and mass advertising, but only no advertising strategy.

When geofencing advertising is either not available or not in consideration, the advertiser would like to adopt mass advertising determined by the fixed cost and product valuation. When $0 < c \leq \frac{t^2(1-\alpha)^2}{4}$, the advertiser would like to adopt mass ads if $V > \frac{4\sqrt{c}}{1-\alpha}$, no ads otherwise. Specifically, if $\frac{4\sqrt{c}}{1-\alpha} < V \leq 2t$, the advertiser sets $p_M^* = V$ and $\beta^* = \frac{V(1-\alpha)}{4}$ to obtain profit $\pi_M^* = \frac{V\left(V(1-\alpha)^2+8\alpha\right)}{16} - c$; if $V > 2t$, the advertiser sets $p_M^* = V - t$ and $\beta^* = \frac{(V-t)(1-\alpha)}{4}$ to obtain profit $\pi_M^* = \frac{(V-t)\left(V(1-\alpha)^2-4\alpha-t(1-\alpha)^2\right)}{4} - c$. When $c > \frac{t^2(1-\alpha)^2}{4}$, the advertiser would like to adopt mass ads if $V > t + \frac{2\sqrt{c}}{1-\alpha}$, no ads otherwise.

From the results, we can see that besides a discussion on the threshold $\frac{t^2(1-\alpha)^2}{4}$ of fixed cost c that determines the choice between mass advertising and no advertising, other analysis on the configuration of mass advertising stays the same as our analysis in the manuscript with zero fixed cost.

Similarly, when geofencing advertising is available, Lemma 1 about the qualitative results on geofencing configuration is the same. The advertiser would adopt geofencing advertising, configuring it as $r^* = L$ and $p_G^* = V$ if product valuation is moderate; $r^* = B$ and $p_G^* = V - t$ if product valuation is high. The only difference is the specific threshold for “moderate” product valuation that determines the choice between geofencing advertising and no advertising. Consequently, with respect to the optimal advertising strategy, the comparative analysis between geofencing advertising and mass advertising qualitatively stays the same, except that there is a threshold driven by the level of fixed cost c under which the advertiser would like to adopt no ads strategy. Figure A1 plots the regimes of advertising modes that roughly remain the same with an additional blank area for no advertising from the right-upper to the left-bottom edge. ☐

Figure A1. Optimal Advertising Strategy with Fixed Setup Costs



Note. The yellow, blue, and white colors code geofencing, mass advertising, and no advertising as the optimal strategy, respectively. The solid line draws the Geofencing-Mass-No Advertising boundary when θ = ½ and t = 1. Panels a and b showcase the optimal strategies when c = 0.2 and 0.4, respectively.

A5. Proofs of Proposition 2 (Economic Benefits of Geofencing) and Corollary 1 (Price Comparison between Geofencing and Mass Advertising)

Geofencing has two benefits compared to mass advertising. (I) *Profit-margin enhancement* if $V > 2t$. Under this market condition, $p_G^* = V > p_M^* = V - t$, meaning that geofencing enables Advertiser *A* to charge a higher product price (resulting in a higher profit margin) than mass advertising. (II) *Cost savings* if $V < 2t$. Under this market condition, there is no price difference between the two ($p_G^* = p_M^*$). However, geofencing costs less than mass advertising ($C_G = \theta^2 < C_M = \left(\frac{V(1-\alpha_0)}{4}\right)^2$). □

A6. Proof of Mass Advertising Configuration under Location-Dependent Awareness

Appendix A.1 to A.5 have analyzed advertising strategies under uniform consumer awareness. Now, we extend the basic model by substituting uniform awareness with location-dependent one. In the location-dependent awareness model, the initial consumer awareness is a function of consumer location, denoted as $\alpha_0^l(x)$ that can be expressed as

$$\alpha_0^l(x) = \begin{cases} \alpha_0\ , & x = L \\ k\alpha_0, & x = R \end{cases},$$

where $0 \le k \le 1$. Suppose Advertiser *A* chooses not to advertise. The demand function is

$$D_0^l = \begin{cases} 0 & , \quad p_0 > V \\ \alpha_0\theta & , \quad V - t < p_0 \le V \\ \alpha_0\theta + k\alpha_0(1-\theta), & \quad p_0 \le V - t \end{cases}.$$

The profit function of the advertiser is

$$\pi_0^l = p_0^l D_0^l = \begin{cases} 0 & , \quad p_0^l > V \\ \alpha_0\theta p_0^l & , \quad V - t < p_0^l \le V \\ (\alpha_0\theta + k\alpha_0(1-\theta))p_0^l & , \quad p_0^l \le V - t \end{cases}.$$

Maximizing π_0^l by choosing p_0^l, we obtain the optimal price as ${p_0^l}^* = V$ and the corresponding profit as $\pi_0^* = \alpha_0\theta V$ if $V \le \frac{(1+k)t}{k}$; $p_0^* = V - t$ and $\pi_0^* = (\alpha_0\theta + k\alpha_0(1-\theta))(V-t)$, otherwise.

Now, suppose Advertiser *A* adopts mass advertising. The demand function is

$$D_M^l = \begin{cases} 0 & , \quad p_M^l > V \\ (\alpha_0 + (1-\alpha_0)\beta^l)\theta & , \quad V - t < p_M^l \le V \\ (\alpha_0 + (1-\alpha_0)\beta^l)\theta + (k\alpha_0 + (1-k\alpha_0)\beta^l)(1-\theta), & \quad p_M^l \le V - t \end{cases}.$$

The profit function of the advertiser is

$$\begin{aligned} \pi_M^l &= p_M^l D_M^l - {\beta^l}^2 \\ &= \begin{cases} -(\beta^l)^2 & , \quad p_M^l > V \\ (\alpha_0 + (1-\alpha_0)\beta^l)\theta p_M^l - {\beta^l}^2 & , \quad V - t < p_M^l \le V \\ (\alpha_0 + (1-\alpha_0)\beta^l)\theta p_M^l + (k\alpha_0 + (1-k\alpha_0)\beta^l)(1-\theta)p_M^l - {\beta^l}^2, & \quad p_M^l \le V - t \end{cases}. \end{aligned}$$

Maximizing π_M^l by choosing p_M^l and β^l, we obtain the optimal price as ${p_M^l}^* = V$, the optimal advertising effort as ${\beta^l}^* = \frac{V(1-\alpha_0)}{4}$, and the corresponding profit as ${\pi_M^l}^* = \frac{V(V(1-\alpha_0)^2+8\alpha_0)}{16}$ if $V \le \bar{\bar{V}}$; ${p_M^l}^* = V - 1$, ${\beta^l}^* = \frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}$, and ${\pi_M^l}^* = \frac{(V-1)\left((1+k)\alpha_0(12-\alpha_0-k\alpha_0)+V(2-\alpha_0-k\alpha_0)^2-4\right)}{16}$, otherwise, where

$\bar{\bar{V}} = \frac{4-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+\sqrt{4+12(1-k)\alpha_0+(-3-6k+17k^2)\alpha_0^2+6(-1+k^2)\alpha_0^3+(1+k)^2\alpha_0^4}}{3-2(1+2k)\alpha_0+k(2+k)\alpha_0^2}$.

Comparing no and mass advertising, we have $\pi_M^* > \pi_0^*$ in any market condition. Advertiser *A* shall choose the strategy that brings him a higher profit. Thus, the advertiser's optimal strategy is ${S^l}^* = M$, setting ${p_M^l}^* = V$ and ${\beta^l}^* = \frac{V(1-\alpha_0)}{4}$ if $V \le \bar{\bar{V}}$; ${p_M^l}^* = V - 1$ and ${\beta^l}^* = \frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}$, otherwise. ☐

A7. Lemma 2 (Geofencing Configuration under Location-Dependent Awareness)

Suppose the advertiser adopts geofencing. He sends ads to either the local market ($r^l = L$) with a cost of $C_G^{l,r=L} = \theta^2$ or both markets ($r^l = B$) with a cost of $C_G^{l,r=B} = 1$. The demand function is

$$D_G^l = \begin{cases} 0 & , \quad p_G^l > V \\ \theta & , \quad V - t < p_G^l \leq V \\ \theta + k\alpha_0(1-\theta), & p_G^l \leq V - t \text{ and } r^l = L \\ 1 & , \quad p_G^l \leq V - t \text{ and } r^l = B \end{cases}.$$

The advertiser's profit function of geofencing is

$$\pi_G^l = p_G^l D_G^l - C_G^l = \begin{cases} -\theta^2 & , \quad p_G^l > V \text{ and } r^l = L \\ -1 & , \quad p_G^l > V \text{ and } r^l = B \\ \theta p_G^l - \theta^2 & , \quad V - t < p_G^l \leq V \text{ and } r^l = L \\ \theta p_G^l - 1 & , \quad V - t < p_G^l \leq V \text{ and } r^l = B \\ (\theta + k\alpha_0(1-\theta))p_G^l - \theta^2, & p_G^l \leq V - t \text{ and } r^l = L \\ p_G^l - 1 & , \quad p_G^l \leq V - t \text{ and } r^l = B \end{cases}.$$

Maximizing π_G^l by choosing p_G^l and r^l, we obtain the optimal geofencing configurations as follows. Advertiser A should set a small radius ${r^l}^* = L$ and a relatively high price ${p_G^l}^* = V$ to obtain a profit of ${\pi_G^l}^* = \theta V - \theta^2$ if

(1) $0 < k \leq \frac{2}{5}$, $0 < \alpha_0 < \frac{6}{7}$, and $\frac{1}{2-2\alpha_0} < V < \frac{7}{2}$,

(2) $\frac{2}{5} < k < 1$, $0 < \alpha_0 \leq \frac{6}{5+5k}$, and $\frac{1}{2-2\alpha_0} < V < \frac{7}{2}$, or

(3) $\frac{2}{5} < k < 1$, $\frac{6}{5+5k} < \alpha_0 < \frac{2+k}{2+2k}$, and $\frac{1}{2-2\alpha_0} < V < \frac{1-2\alpha_0-2k\alpha_0}{2-2\alpha_0-2k\alpha_0}$.

He sets a large radius ${r^l}^* = B$ and a relatively low price ${p_G^l}^* = V - 1$ to obtain a profit of ${\pi_G^l}^* = V - 2$ if

(1) $0 < k \leq \frac{2}{5}$, $0 < \alpha_0 \leq \frac{6}{7}$, and $V > \frac{7}{2}$,

(2) $0 < k \leq \frac{1}{3}$, $\frac{6}{7} < \alpha_0 < 1$, and $V > \frac{4}{2-\alpha_0}$,

(3) $\frac{1}{3} < k < \frac{2}{5}$, $\frac{6}{7} < \alpha_0 \leq \frac{2-2k}{1+k}$, and $V > \frac{4}{2-\alpha_0}$,

(4) $\frac{1}{3} < k < \frac{2}{5}$, $\frac{2-2k}{1+k} < \alpha_0 < 1$, and $V > \frac{4-\alpha_0-k\alpha_0}{2-\alpha_0-k\alpha_0}$,

(5) $\frac{2}{5} \leq k < 1$, $0 < \alpha_0 \leq \frac{6}{5+5k}$, and $V > \frac{7}{2}$, or

(6) $\frac{2}{5} \leq k < 1$, $\frac{6}{5+5k} < \alpha_0 < 1$, and $V > \frac{4-\alpha_0-k\alpha_0}{-2-\alpha_0-k\alpha_0}$.

If the market condition does not meet any of the above scenarios, the optimal strategy of Advertiser A is not to advertise at all (i.e., geofencing not applicable). □

A8. Proof of Proposition 3 (Optimal Advertising Strategy under Location-Dependent Awareness)

Comparing ${\pi_G^l}^*$ in Appendix A.7 and ${\pi_M^l}^*$ in Appendix A.6, we identify the market conditions of geofencing dominating mass advertising (i.e., ${\pi_G^l}^* > {\pi_M^l}^*$) as $\underline{\alpha} < \alpha_0 \leq \bar{\alpha}$, $0 < k < \bar{k}$, and $\underline{V} < V < \bar{V}$, where

$\underline{\alpha} = \sqrt{3} - 1$; $\bar{\alpha}$ is the 4th root of the polynomial equality of α_0 (due to the complexity and space limit, we do not write the full expression of $\bar{\alpha}$): $\alpha_0^8 - 12\alpha_0^7 - 34\alpha_0^6 + 588\alpha_0^5 - 823\alpha_0^4 - 1176\alpha_0^3 + 2752\alpha_0^2 - 1344\alpha_0 + 64 = 0$; $\bar{k}$ is the 2nd root of the polynomial equation of k:

$64 - 1344\alpha_0 + 2752{\alpha_0}^2 - 1176{\alpha_0}^3 - 823{\alpha_0}^4 + 588{\alpha_0}^5 - 34{\alpha_0}^6 - 12{\alpha_0}^7 + {\alpha_0}^8 + (-576\alpha_0 + 3200{\alpha_0}^2 - 4104{\alpha_0}^3 + 708{\alpha_0}^4 + 960{\alpha_0}^5 - 232{\alpha_0}^6 - 24{\alpha_0}^7 + 4{\alpha_0}^8)k + (576{\alpha_0}^2 - 2376{\alpha_0}^3 + 2294{\alpha_0}^4 - 72{\alpha_0}^5 - 332{\alpha_0}^6 + 6{\alpha_0}^8)k^2 + (-216{\alpha_0}^3 + 708{\alpha_0}^4 - 480{\alpha_0}^5 - 104{\alpha_0}^6 + 24{\alpha_0}^7 + 4{\alpha_0}^8)k^3 + (9{\alpha_0}^4 - 36{\alpha_0}^5 + 30{\alpha_0}^6 + 12{\alpha_0}^7 + {\alpha_0}^8)k^4 = 0$;

$\underline{V} = \frac{4-2\sqrt{3}}{1-\alpha_0}$; $\bar{V} = \frac{-8(1+k)\alpha_0+(1+k)^2\alpha_0^2+2(4+\sqrt{8-12(1+k)\alpha_0+5(1+k)^2\alpha_0^2})}{(-2+\alpha_0+k\alpha_0)^2}$. □

A9. Proof of Lemma 3 (Flexible Geofencing Configuration)

Appendix A.1 to A.8 analyze the optimal advertising strategy with the assumption that Advertiser A uses his store as the center of the fencing zone. Now, we consider flexible geofencing that allows him to choose his POI flexibly. In other words, the advertiser could target the local ($r = L$), remote ($r = R$), or both markets ($r = B$) with a cost of $C_F^{l,r=L} = \theta^2$, $C_F^{l,r=R} = (1-\theta)^2$, or $C_F^{l,r=B} = 1$. Since uniform awareness is a special case of location-dependent awareness (i.e., uniform awareness is equivalent to location-dependent awareness when $k = 1$), we focus on location-dependent awareness here.

The demand function of flexible geofencing is

$$D_F^l = \begin{cases} 0 & , \quad p_F^l > V \\ \theta & , \quad V - t < p_F^l \leq V \text{ and } r_F^l = L \text{ or } B \\ \alpha_0 \theta & , \quad V - t < p_F^l \leq V \text{ and } r_F^l = R \\ \theta + k\alpha_0(1-\theta) & , \quad p_F^l \leq V - t \text{ and } r_F^l = L \\ \alpha_0\theta + (1-\theta) & , \quad p_F^l \leq V - t \text{ and } r_F^l = R \\ 1 & , \quad p_F^l \leq V - t \text{ and } r_F^l = B \end{cases}.$$

The advertiser's profit function of flexible geofencing is

$$\pi_F^l = p_F^l D_F^l - C_F^l = \begin{cases} -\theta^2 & , \quad p_F^l > V \text{ and } r_F^l = L \\ -(1-\theta)^2 & , \quad p_F^l > V \text{ and } r_F^l = R \\ -1 & , \quad p_F^l > V \text{ and } r_F^l = B \\ \theta p_F^l - \theta^2 & , \quad V - t < p_F^l \leq V \text{ and } r_F^l = L \\ \alpha_0 \theta p_F^l - (1-\theta)^2 & , \quad V - t < p_F^l \leq V \text{ and } r_F^l = R \\ \theta p_F^l - 1 & , \quad V - t < p_F^l \leq V \text{ and } r_F^l = B \\ \left(\theta + k\alpha_0(1-\theta)\right) p_F^l - \theta^2 & , \quad p_F^l \leq V - t \text{ and } r_F^l = L \\ \left(\alpha_0\theta + (1-\theta)\right) p_F^l - (1-\theta)^2 & , \quad p_F^l \leq V - t \text{ and } r_F^l = R \\ p_F^l - 1 & , \quad p_F^l \leq V - t \text{ and } r_F^l = B \end{cases}.$$

Maximizing π_F^l by choosing p_F^l and r_F^l, we obtain the optimal geofencing configurations as follows.

Advertiser *A* should set ${r_F^l}^* = L$ and ${p_F^l}^* = V$ to obtain a profit of ${\pi_F^{l,r=L}}^* = \theta V - \theta^2$ if

(1) $0 < k \leq 1$, $0 < \alpha_0 \leq \frac{2}{5}$, and $\frac{1}{2-2\alpha_0} \leq V < \frac{7}{2}$,

(2) $0 < k \leq \frac{1}{4}\left(-1+\sqrt{17}\right)$, $\frac{2}{5} < \alpha_0 < \frac{1}{4}\left(-1+\sqrt{17}\right)$, and $\frac{1}{2-2\alpha_0} \leq V < \frac{1+\alpha_0}{\alpha_0}$,

(3) $\frac{1}{4}\left(-1+\sqrt{17}\right) < k \leq 1$, $\frac{2}{5} < \alpha_0 \leq \frac{2}{1+2k}$, and $\frac{1}{2-2\alpha_0} \leq V < \frac{1+\alpha_0}{\alpha_0}$, or

(4) $\frac{1}{4}\left(-1+\sqrt{17}\right) < k \leq 1$, $\frac{2}{1+2k} < \alpha_0 < \frac{2+k}{2+2k}$, and $\frac{1}{2-2\alpha_0} \leq V \leq \frac{-1+2\alpha_0+2k\alpha_0}{-2+2\alpha_0+2k\alpha_0}$.

Advertise *A* should set ${r_F^l}^* = R$ and ${p_F^l}^* = V - 1$ to obtain a profit of ${\pi_F^{l,r=R}}^* = \left(\alpha_0\theta + (1-\theta)\right)(V-1) -$

$(1-\theta)^2$ if

(1) $0 < k \leq \frac{2}{3}$, $\frac{1}{4}\left(-1+\sqrt{17}\right) < \alpha_0 < 1$, and $\frac{1}{2}(3+2\alpha_0) \leq V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$,

(2) $0 < k < \frac{1}{4}\left(-1+\sqrt{17}\right)$, $\frac{2}{5} < \alpha_0 < \frac{1}{4}\left(-1+\sqrt{17}\right)$, and $\frac{1+\alpha_0}{\alpha_0} < V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$,

(3) $\frac{2}{3} < k < \frac{1}{4}\left(-1+\sqrt{17}\right)$, $\frac{1}{4}\left(-1+\sqrt{17}\right) < \alpha_0 < \frac{2-k}{2k}$, and $\frac{1}{2}(3+2\alpha_0) \le V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$,

(4) $\frac{2}{3} < k < \frac{1}{4}\left(-1+\sqrt{17}\right)$, $\frac{2-k}{2k} < \alpha_0 < 1$, and $\frac{-3+2k\alpha_0}{-2+2k\alpha_0} \le V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$,

(5) $\frac{1}{4}\left(-1+\sqrt{17}\right) < k \le 1$, $\frac{2}{5} < \alpha_0 < \frac{2}{1+2k}$, and $\frac{1+\alpha_0}{\alpha_0} < V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$, or

(6) $\frac{1}{4}\left(-1+\sqrt{17}\right) < k \le 1$, $\frac{2}{1+2k} < \alpha_0 < 1$, and $\frac{-3+2k\alpha_0}{-2+2k\alpha_0} \le V < \frac{-5+2\alpha_0}{-2+2\alpha_0}$.

Advertise *A* should set ${r_F^l}^* = B$ and ${p_F^l}^* = V-1$ to obtain a profit of ${\pi_F^{l,r=B}}^* = V-2$ if (1) $0 < \alpha_0 \le \frac{2}{5}$ and $V > \frac{7}{2}$, or (2) $\frac{2}{5} < \alpha_0 < 1$ and $V > \frac{-5+2\alpha_0}{-2+2\alpha_0}$.

If the market condition does not meet any of the above scenarios, the optimal strategy of Advertiser *A* is not to advertise at all (i.e., flexible geofencing not applicable). ☐

A10. Proof of Proposition 4 (Optimal Advertising Strategy with Flexible Geofencing)

Comparing ${\pi_F^l}^*$ in Appendix A.9 and ${\pi_M^l}^*$ in Appendix A.8, we identify the market conditions of flexible geofencing dominating mass advertising (i.e., ${\pi_F^l}^* > {\pi_M^l}^*$). Specifically, Advertiser *A* should apply flexible geofencing, setting ${r_F^l}^* = L$ and ${p_F^l}^* = V$ if

(1) $0 < \alpha_0 \le \frac{2\left(3+2\sqrt{1-k}+k\right)}{5+10k+k^2}$ and

$$\frac{4-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+\sqrt{4-12(-1+k)\alpha_0+(-3-6k+17k^2)\alpha_0^2+6(-1+k^2)\alpha_0^3+(1+k)^2\alpha_0^4}}{3-2(1+2k)\alpha_0+k(2+k)\alpha_0^2} < V \le$$

$$\frac{-8(1+k)\alpha_0+(1+k)^2\alpha_0^2+2(4+\sqrt{8-12(1+k)\alpha_0+5(1+k)^2\alpha_0^2})}{(-2+\alpha_0+k\alpha_0)^2},$$

(2) $0 < k \le \bar{\bar{k}}$, $\frac{2\left(3+2\sqrt{1-k}+k\right)}{5+10k+k^2} \le \alpha_0 < \bar{\bar{\alpha}}$, and

$$\frac{4-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+\sqrt{4-12(-1+k)\alpha_0+(-3-6k+17k^2)\alpha_0^2+6(-1+k^2)\alpha_0^3+(1+k)^2\alpha_0^4}}{3-2(1+2k)\alpha_0+k(2+k)\alpha_0^2} < V < \frac{1+\alpha_0}{\alpha_0}, \text{ or}$$

(3) $\bar{\bar{k}} < k \le 1$, $\frac{2\left(3+2\sqrt{1-k}+k\right)}{5+10k+k^2} < \alpha_0 < \underline{\underline{\alpha}}$, and

$\frac{4-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+\sqrt{4-12(-1+k)\alpha_0+(-3-6k+17k^2)\alpha_0^2+6(-1+k^2)\alpha_0^3+(1+k)^2\alpha_0^4}}{3-2(1+2k)\alpha_0+k(2+k)\alpha_0^2} < V \leq$

$\frac{-8(1+k)\alpha_0+(1+k)^2\alpha_0^2+2(4+\sqrt{8-12(1+k)\alpha_0+5(1+k)^2\alpha_0^2})}{(-2+\alpha_0+k\alpha_0)^2}$, where

$\bar{\bar{k}}$ is the 6th root of the polynomial equation of k: $-61247 - 60408\,k + 116700\,k^2 + 120376\,k^3 + 20614\,k^4 - 456\,k^5 - 164\,k^6 + 8\,k^7 + k^8 = 0$; $\bar{\bar{\alpha}}$ is the 2nd root of the polynomial equation of α_0: $-3 + (4+4k)\alpha_0 + (-3-10k-k^2)\alpha_0^2 + 8\alpha_0^3 + \alpha_0^4 = 0$; $\underline{\underline{\alpha}}$ is the 4th root of the polynomial equation of α_0: $64 + (-1344-576k)\alpha_0 + (2752+3200k+576k^2)\alpha_0^2 + (-1176-4104k-2376k^2-216k^3)\alpha_0^3 + (-823+708k+2294k^2+708k^3+9k^4)\alpha_0^4 + (588+960k-72k^2-480k^3-36k^4)\alpha_0^5 + (-34-232k-332k^2-104k^3+30k^4)\alpha_0^6 + (-12-24k+24k^3+12k^4)\alpha_0^7 + (1+4k+6k^2+4k^3+k^4)\alpha_0^8 = 0$.

Advertiser *A* should apply flexible geofencing, setting $r_F^l = R$ and $p_F^l = V - 1$ if

(1) $0 < k \leq \bar{\bar{k}}$, $\frac{6-4\sqrt{1-k}+2k}{5+10k+k^2} < \alpha_0 \leq \underline{A}$, and $\frac{1+\alpha_0}{\alpha_0} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$,

(2) $0 < k \leq \frac{1}{3}\left(4\sqrt{3}-5\right)$, $\underline{A} < \alpha_0 < 1$, and

$\frac{2\left(2-\sqrt{1+4\alpha_0+\alpha_0^2-2\alpha_0^3}\right)}{(1-\alpha_0)^2} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$,

(3) $\frac{1}{3}\left(4\sqrt{3}-5\right) < k \leq \bar{\bar{k}}$, $\underline{A} < \alpha_0 \leq \bar{A}$, and

$\frac{2\left(2-\sqrt{1+4\alpha_0+\alpha_0^2-2\alpha_0^3}\right)}{(1-\alpha_0)^2} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$,

(4) $\frac{1}{3}\left(4\sqrt{3}-5\right) < k \leq \bar{\bar{k}}$, $\bar{A} < a < 1$, and

$\frac{8-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2-2\sqrt{(1-k)\alpha_0(4-\alpha_0-3k\alpha_0)}}{(2-\alpha_0-k\alpha_0)^2} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$,

(5) $\bar{\bar{k}} < k < 1$, $\frac{6-4\sqrt{1-k}+2k}{5+10k+k^2} < \alpha_0 \leq \frac{2\left(3+2\sqrt{1-k}+k\right)}{5+10k+k^2}$, and

$\frac{1+\alpha_0}{\alpha_0} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$, or

(6) $\bar{\bar{k}} < k < 1$, $\frac{2(3+2\sqrt{1-k}+k)}{5+10k+k^2} < \alpha_0 < 1$, and

$\frac{8-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2-2\sqrt{(1-k)\alpha_0(4-\alpha_0-3k\alpha_0)}}{(2-\alpha_0-k\alpha_0)^2} < V < \frac{-4(1+2k)\alpha_0+(1+k)^2\alpha_0^2+2\left(4+\sqrt{(-1+k)\alpha_0(-4+\alpha_0+3k\alpha_0)}\right)}{(2-\alpha_0-k\alpha_0)^2}$,

where $\underline{A}$ is the 4th root of the polynomial equation of α_0: $1 - 8\alpha_0 + 2\alpha_0^2 + 8\alpha_0^3 + \alpha_0^4 = 0$; $\bar{A}$ is the 1st root of the polynomial equation of α_0: $-256 + 256k + (-560 + 992k - 176k^2)\alpha_0 + (120 + 664k - 952k^2 + 40k^3)\alpha_0^2 + (505 - 380k - 458k^2 + 68k^3 + 25k^4)\alpha_0^3 + (-180 - 320k + 344k^2 + 32k^3 + 60k^4)\alpha_0^4 + (-2 - 8k + 36k^2 + 24k^3 + 46k^4)\alpha_0^5 + (4 + 8k + 16k^2 + 24k^3 + 12k^4)\alpha_0^6 + (1 + 4k + 6k^2 + 4k^3 + k^4)\alpha_0^7 = 0$.

Otherwise, Advertiser *A* should adopt mass advertising as described in Appendix A.6. ☐

A11. Proof of Proposition 5 (Social Welfare)

We compare social welfare under the two advertising paradigms (i.e., with and without flexible geofencing).

Under the traditional paradigm where Advertiser *A* chooses between typical geofencing and mass advertising, social welfare is calculated as $W^l = D_S^{l^*}U(x) + \pi_S^{l^*}$ where $S \in \{G, M\}$. According to Proposition 3, the advertiser adopts geofencing if $\underline{\alpha} < \alpha_0 \leq \bar{\alpha}$, $0 < k < \bar{k}$, and $\underline{V} < V < \bar{V}$, and mass advertising, otherwise. Thus, social welfare without the consideration of flexible geofencing W^l would be $\theta V - \theta^2$ if $\underline{\alpha} < \alpha_0 \leq \bar{\alpha}$, $0 < k < \bar{k}$, and $\underline{V} < V < \bar{V}$; otherwise, W^l is

$\left(\alpha_0 + (1-\alpha_0)\left(\frac{V(1-\alpha_0)}{4}\right)\right)\theta - \left(\frac{V(1-\alpha_0)}{4}\right)^2$ when $V \leq \bar{\bar{V}}$, or

$\left(\alpha_0 + (1-\alpha_0)\left(\frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}\right)\right)\theta + \left(\alpha_0 + (1-\alpha_0)\left(\frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}\right)\right)\theta(V-1) + (k\alpha_0 + (1 - k\alpha_0)\frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}(1-\theta)(V-1) - \left(\frac{(V-1)(2-\alpha_0-\alpha_0 k)}{4}\right)^2$ when $V > \bar{\bar{V}}$.

Under our proposed paradigm where Advertiser *A* chooses between flexible geofencing and mass advertising, social welfare is $W_F^l = D_S^{l^*}U(x) + \pi_S^{l^*}$ where $S \in \{F, M\}$. To avoid repeating the discussion, we specifically focus on the scenario where the advertising strategy changes due to flexible geofencing (i.e., Advertiser A switches from $r^l = B$ in typical geofencing to $r^l = R$ in flexible geofencing). In this scenario, social welfare $W_F^{l,r=R}$ is $\alpha_0\theta + (1-\theta) + (\alpha_0\theta + (1-\theta))(V-1) - (1-\theta)^2$.

Comparing $W_F^{l,r=R}$ with W^l in the same market condition, we have $W_F^{l,r=R} > W^l$ if

(1) $\bar{\bar{V}} < V < -\frac{4(-1+k\alpha)}{(-2+\alpha+k\alpha)^2} +$

$\sqrt{\frac{16-16\alpha_0-48k\alpha_0+20\alpha_0^2+40k\alpha_0^2+36k^2\alpha_0^2-8\alpha_0^3-24k\alpha_0^3-24k^2\alpha_0^3-8k^3\alpha_0^3+\alpha_0^4+4k\alpha_0^4+6k^2\alpha_0^4+4k^3\alpha_0^4+k^4\alpha_0^4}{(-2+\alpha_0+k\alpha_0)^4}}$, or

(2) $0 < k \leq \frac{1}{3}(-5+4\sqrt{3})$, $\underline{A} < \alpha_0 < 1$, and $\frac{4}{(-1+\alpha_0)^2} - 2\sqrt{-\frac{-1-4\alpha_0-\alpha_0^2+2\alpha_0^3}{(-1+\alpha_0)^4}} < V < \bar{\bar{V}}$. In sum, the paradigm with flexible geofencing leads to higher (or equal) social welfare, in general. ☐