

# Electronic Companion to: Response to Entry in Streaming Markets: A Game-Theoretic Model

## EC.1. Additional Literature Positioning

This appendix provides a fuller discussion of how the paper relates to three literature streams: (i) pricing of information goods and services, (ii) competitive pricing and assortment strategy in content markets, and (iii) business models for digital content platforms.

First, our paper relates to the literature on pricing of information goods and services. This literature has shown how negligible marginal costs create scope for nonlinear pricing, versioning, and alternative tariff structures (Sundararajan 2004). Prior studies have examined the choice between subscription- and usage-based pricing in settings involving piracy (Bhattacharjee et al. 2003), network effects (Bhargava and Choudhary 2004, Niculescu et al. 2012, Zhang and Seidmann 2010), durable goods (Gilbert et al. 2014), and valuation and usage heterogeneity (Bala and Carr 2010). Related work has also studied such pricing choices in duopoly and SaaS contexts (Balasubramanian et al. 2015, Gangwar and Bhargava 2023). Our paper contributes to this stream by examining pricing-model choice under entry competition in a streaming market where content composition and exclusivity matter directly.

Second, our study connects to the literature on assortment, bundling, and competitive strategy in content markets. Prior work has shown the value of bundling and assortment design for information goods (Bakos and Brynjolfsson 1999, Gourville and Soman 2005, Hitt and Chen 2005), and has examined pricing under capacity or congestion considerations (Essegaier et al. 2002, Randhawa and Kumar 2008). Other studies show that recommendations, endorsements, and content design shape demand and platform performance (Fang et al. 2023, Lee et al. 2015, Lin et al. 2017, Peitz and Valletti 2008). Emerging work has also considered competition, bundling, diffusion, and entry-related forces in information markets (Amelio et al. 2020, Bakos and Brynjolfsson 2000, Hosanagar et al. 2010, Liu et al. 2006, Raghunathan and Sarkar 2016, Schmalensee 1981). Our contribution is to bring these ideas into a competitive streaming setting in which the incumbent and entrant differ in content composition and pricing-model flexibility.

Third, our paper relates to research on digital platform business models. This literature has studied multi-homing and platform competition (Bakos and Halaburda 2020), compensation of content producers in two-sided markets (Jain and Qian 2021), and pricing design for digital content and online services (Lambrecht and Misra 2017, Li et al. 2020, Oh et al. 2016). A key limitation of much of this work, however, is that pricing-model choice and content composition are often treated separately. Our paper bridges these perspectives by studying how exclusive versus non-exclusive content, consumer heterogeneity, and entry competition jointly shape pricing choices in streaming markets. This joint focus is increasingly important as streaming platforms compete simultaneously on catalog structure, monetization model, and market position (Canales 2021, Precedence-Research 2023, Statista 2023).

Overall, our paper contributes by linking three themes that are often studied separately: pricing-model choice, content composition, and entry competition. Our analysis shows how the incumbent's response to entry depends not only on whether firms use subscription or usage-based pricing, but also on the strategic role of exclusivity and the market conditions under which entry occurs.

## EC.2. Lemmas

LEMMA EC.1. (a) *When the entrant adopts subscription-based pricing, the incumbent's optimal pricing*

$$\text{strategy is given by: } j^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} S; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ U; \quad \frac{V}{16\gamma} \leq c \leq \frac{32\gamma}{(1+\beta)^2(16-V^2)} \\ S; \quad c > \frac{32\gamma}{(1+\beta)^2(16-V^2)} \end{array} \right\}; \quad 0 \leq \beta \leq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{32\gamma}{(1+\beta)^2(16-V^2)} \\ S; \quad c \geq \frac{32\gamma}{(1+\beta)^2} \end{array} \right\}; \quad \beta \geq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \\ \left\{ \begin{array}{l} S; \quad 0 \leq \beta \leq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2} - 1} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ S; \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad \beta \geq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2} - 1} \end{array} \right\}; \quad \begin{array}{l} 0 \leq V \leq 4 \\ 4 \leq V \leq 8(1+\beta)^2 \\ V \geq 8(1+\beta)^2 \end{array} \end{array} \right.$$

(b) *When the entrant adopts usage-based pricing, the incumbent's optimal pricing strategy is:*

$$j^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \left(\frac{2}{\sqrt{3}} - 1\right) \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} < c < c_L \end{array} \right\}; \quad \left(\frac{2}{\sqrt{3}} - 1\right) < \beta \leq \beta_2 \\ \left\{ \begin{array}{l} U; \quad c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} < c < c_L \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x < x_1 \\ \left\{ \begin{array}{l} U; \quad c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \beta > \beta_2 \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad x_1 < x \leq \frac{1}{2} \\ S; \end{array} \right. \quad \begin{array}{l} 0 \leq V \leq 4 \\ V > 4 \end{array}$$

$$\text{Where } c_L = \frac{(1+\beta)(2x(1+\beta) - \sqrt{4x^2(1+\beta)^2 - 2x})}{2x\gamma}, \quad x_1 = \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}, \quad \text{and } \beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}}$$

LEMMA EC.2. *The optimal pricing strategy of the entrant is:*

$$l^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} U; \\ \left\{ \begin{array}{l} U; 0 \leq x \leq \bar{x}_1 \\ S; \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \\ n.a.; \end{array} \right. \left. \begin{array}{l} 0 \leq c \leq c_u \\ c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \\ \left\{ \begin{array}{l} \left\{ \begin{array}{l} U; \\ \left\{ \begin{array}{l} U; 0 \leq x \leq \bar{x}_1 \\ S; \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \\ U; \\ n.a.; \end{array} \right. \left. \begin{array}{l} 0 \leq c \leq c_u \\ c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \end{array} \right\}; \\ \left\{ \begin{array}{l} U; \\ \left\{ \begin{array}{l} U; 0 \leq x \leq \bar{x}_1 \\ S; \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \\ \left\{ \begin{array}{l} U; 0 \leq x \leq x_1 \\ n.a.; x_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \\ n.a.; \end{array} \right. \left. \begin{array}{l} 0 \leq c \leq c_u \\ c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \end{array} \right\}; \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{1}{2\gamma} \\ n.a.; c \geq \frac{1}{2\gamma} \end{array} \right\}; \\ \left\{ \begin{array}{l} S; 0 \leq c \leq \frac{V}{16\gamma} \\ U; \frac{V}{16\gamma} \leq c \leq \frac{1}{2\gamma} \\ n.a.; c \geq \frac{1}{2\gamma} \end{array} \right\}; \\ \left\{ \begin{array}{l} S; 0 \leq c \leq \frac{V}{16\gamma} \\ n.a.; c \geq \frac{V}{16\gamma} \end{array} \right\}; \end{array} \right\}; \quad \left. \begin{array}{l} 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \\ 0 \leq \beta \leq \beta_2 \\ \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \leq V \leq 4 \\ \beta \geq \beta_2 \\ 0 \leq \beta \leq \beta_1 \\ \beta \geq \beta_1 \\ 4 \leq V \leq 8 \\ V \geq 8 \end{array} \right\}$$

where  $c_u = \frac{(1+\beta)^2}{\gamma} - \frac{(1+\beta)\sqrt{V^2+64\beta(2+\beta)}}{8\gamma}$ ,  $\bar{x}_1 = \frac{4(4c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{V^2(1+\beta)^4+16c^2\gamma(-2(1+\beta)^2+c\gamma)^2})}{8\gamma}$ ,

$x_1 = \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}$ ,  $\beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}}$  and  $\beta_1 = \sqrt{\frac{V^2(1+\beta)^2}{16V^2 - (V^2 - 64c^2\gamma^2)}} - 1$ .

LEMMA EC.3. *The complete Nash equilibrium of the game is:*

$$(j^*, l^*) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (U, U); \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} (U, U); \quad 0 \leq x \leq \bar{x}_1 \\ (S, S); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ (U, n.a.); \quad c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \\ \left\{ \begin{array}{l} (U, U); \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} (U, U); \quad 0 \leq x \leq \bar{x}_1 \\ (U, S); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ (S, U); \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ (U, n.a.); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_2 \\ \left\{ \begin{array}{l} (U, U); \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} (U, U); \quad 0 \leq x \leq \bar{x}_1 \\ (U, S); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ \left\{ \begin{array}{l} (S, U); \quad 0 \leq x \leq x_1 \\ (U, n.a.); \quad x_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ (U, n.a.); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \beta \geq \beta_2 \\ \left\{ \begin{array}{l} (S, U); \quad 0 \leq c \leq \frac{1}{2\gamma} \\ (S, n.a.); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left\{ \begin{array}{l} (U, S); \quad 0 \leq c \leq \frac{V}{16\gamma} \\ (S, U); \quad \frac{V}{16\gamma} \leq c \leq \frac{1}{2\gamma} \\ (S, n.a.); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \beta \geq \beta_1 \\ \left\{ \begin{array}{l} (S, S); \quad 0 \leq c \leq \frac{V}{16\gamma} \\ (S, n.a.); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left\{ \begin{array}{l} (U, S); \quad 0 \leq c \leq \frac{V}{16\gamma} \\ (S, n.a.); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad \beta \geq \beta_1 \\ (S, S); \quad 0 \leq c \leq \frac{V}{16\gamma} \\ (S, n.a.); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad V \geq 8(1+\beta)^2 \end{array} \right\}; \quad \begin{array}{l} 4 \leq V \leq 8 \\ 8 \leq V \leq 8(1+\beta)^2 \\ V \geq 8(1+\beta)^2 \end{array}$$

where  $c_u = \frac{(1+\beta)^2}{\gamma} - \frac{(1+\beta)\sqrt{V^2+64\beta(2+\beta)}}{8\gamma}$ ,  $\bar{x}_1 = \frac{4(4c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{V^2(1+\beta)^4+16c^2\gamma(-2(1+\beta)^2+c\gamma)})}{V^2(1+\beta)^2}$ ,

$x_1 = \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}$ ,  $\beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}}$  and  $\beta_1 = \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)}} - 1$ .

LEMMA EC.4. *The equilibrium pricing strategies of the platforms are given by:*

$$(j^*, l^*) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (U, U); \quad 0 \leq c \leq \frac{(16 - V^2)}{32\gamma} \\ (U, S); \quad \frac{(16 - V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ (U, U); \quad \frac{1}{2\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., U); \quad c > \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq V \leq 4 \\ \left\{ \begin{array}{l} (U, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \\ \left\{ \begin{array}{l} (S, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ (U, S); \quad \frac{(1 + \beta)^2 V}{16\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \leq V \leq 8 \\ \left\{ \begin{array}{l} (S, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2 V}{16\gamma} \end{array} \right\}; \quad V \geq 8 \end{array} \right.$$

### EC.3. Tables

$k_E$	Entrant's content-attractiveness index (catalog appeal; decision variable)
$k_I$	Incumbent's content-attractiveness index (catalog appeal; decision variable)
$\beta$	Exogenous exclusivity (licensed rights & curation; baseline platform affinity)
$\alpha$	Strategic exclusivity (owned/controlled; Stage 0 costly investment in the extension)
$\gamma$	Content similarity index / multi-homing disutility
$x$	Utility-to-consumption factor (consumption intensity; $0 \leq x \leq \frac{1}{2}$ )
$V$	Upper bound of content valuation
$T \in [0, V]$	Consumer valuation (type)
$c$	Marginal cost for content-attractiveness investments
$l \in \{S, U\}$	Entrant's pricing model (S: subscription; U: usage-based)
$j \in \{S, U\}$	Incumbent's pricing model (S: subscription; U: usage-based)
$p_E$	Entrant's per-usage price
$p_I$	Incumbent's per-usage price
$r_E$	Entrant's subscription fee
$r_I$	Incumbent's subscription fee
$\pi_E^{lj}$	Entrant's profits under strategy $j$ and $l$
$\pi_I^{lj}$	Incumbent's profits under strategy $j$ and $l$

**Table EC.1 Key Notations**

Strategy	Entrant-Usage	Entrant-Subscription
Incumbent-Usage	$(i) \pi_I^{U^V} = \begin{cases} \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}; & 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ 0; & c > \frac{(1+\beta)^2}{2\gamma} \end{cases}$ $, \quad p_I = \begin{cases} \frac{1}{x}; & 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \text{n.a.}; & c > \frac{(1+\beta)^2}{2\gamma} \end{cases}$ $k_I = \begin{cases} \frac{(1+\beta)}{2c}; & 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \text{n.a.}; & c > \frac{(1+\beta)^2}{2\gamma} \end{cases}$ $(ii) \pi_I^{U^V} = \frac{(1+\beta)^2}{4c}, p_I = \frac{1}{x}, k_I = \frac{(1+\beta)}{2c}$	$(i) \pi_I^{S^V} = \frac{(1+\beta)^2}{4c} - \gamma + \frac{\gamma r_E}{k_E V}, p_I = \frac{1}{x}, k_I = \frac{(1+\beta)}{2c}$ $(ii) \pi_I^{S^V} = \frac{(1+\beta)^2}{4c}, p_I = \frac{1}{x}, k_I = \frac{(1+\beta)}{2c}$
Incumbent-Subscription	$\pi_I^{S^S} = \frac{V}{2} \left( \frac{(1+\beta)^2 V}{8c} - \gamma(1-p_E x) \right) - \frac{(1+\beta)^2 V^2}{8c},$ $r_I = \frac{V}{2} \left( \frac{(1+\beta)^2 V}{8c} - \gamma(1-p_E x) \right), k_I = \frac{(1+\beta)V}{8c}$	$\pi_I^{S^S} = \begin{cases} \left\{ \begin{array}{l} \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4}; & 0 \leq c \leq c_1 \\ \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{4ck_E^4 V^2}; & c \geq c_1 \end{array} \right\}; & 0 \leq r_E \leq \frac{(k_E - \gamma)V}{2} \\ \left\{ \begin{array}{l} \frac{r_E^2 (1+\beta)^2 ((k_E - \gamma)V - r_E)^2}{4ck_E^4 V^2} - \frac{\gamma r_E ((k_E - \gamma)V - r_E)}{V(k_E - \gamma)^2}; & 0 \leq c \leq c_2 \\ \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{4ck_E^4 V^2}; & c \geq c_2 \end{array} \right\}; & \frac{(k_E - \gamma)V}{2} \leq r_E \leq \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \\ \left\{ \begin{array}{l} \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{4ck_E^4 V^2}; & \\ \frac{(1+\beta)^2 V^2}{16c}; & \end{array} \right\}; & \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \leq r_E \leq \frac{k_E V}{2} \\ & \frac{k_E V}{2} \leq r_E \leq k_E V \end{cases}$ $r_I = \begin{cases} \left\{ \begin{array}{l} \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{2}; & 0 \leq c \leq c_1 \\ \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{2ck_E^2 V}; & c \geq c_1 \end{array} \right\}; & 0 \leq r_E \leq \frac{(k_E - \gamma)V}{2} \\ \left\{ \begin{array}{l} \frac{r_E^2 (1+\beta)^2 ((k_E - \gamma)V - r_E)^2}{2c(k_E - \gamma)^3 V} - \frac{\gamma r_E}{(k_E - \gamma)}; & 0 \leq c \leq c_2 \\ \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{2ck_E^2 V}; & c \geq c_2 \end{array} \right\}; & \frac{(k_E - \gamma)V}{2} \leq r_E \leq \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \\ \left\{ \begin{array}{l} \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{2ck_E^2 V}; & \\ \frac{(1+\beta)^2 V^2}{16c}; & \end{array} \right\}; & \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \leq r_E \leq \frac{k_E V}{2} \\ & \frac{k_E V}{2} \leq r_E \leq k_E V \end{cases}, \quad k_I =$ $\left\{ \begin{array}{l} \frac{(1+\beta)V}{8c}; & 0 \leq c \leq c_1 \\ \frac{r_E(1+\beta)(k_E V - r_E)}{2ck_E^2 V}; & c \geq c_1 \end{array} \right\}; & 0 \leq r_E \leq \frac{(k_E - \gamma)V}{2}$ $\left\{ \begin{array}{l} \frac{r_E(1+\beta)((k_E - \gamma)V - r_E)}{2c(k_E - \gamma)^2 V}; & 0 \leq c \leq c_2 \\ \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{4ck_E^4 V^2}; & c \geq c_2 \end{array} \right\}; & \frac{(k_E - \gamma)V}{2} \leq r_E \leq \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)}$ $\left\{ \begin{array}{l} \frac{r_E^2 (1+\beta)^2 (k_E V - r_E)^2}{4ck_E^4 V^2}; & \\ \frac{(1+\beta)V}{8c}; & \end{array} \right\}; & \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \leq r_E \leq \frac{k_E V}{2}$ $\frac{k_E V}{2} \leq r_E \leq k_E V$

$$c_1 = \frac{(k_E V - 2r_E)^2 (1+\beta)^2 (k_E^2 V^2 - 4r_E^2 + 4r_E k_E V)}{16k_E^4 V^3 \gamma}$$

$$c_2 = \frac{r_E (1+\beta)^2 (k_E^2 V + r_E \gamma - k_E (2r_E + V \gamma)) (2k_E^3 V - r_E \gamma^2 + k_E \gamma (2r_E + V \gamma) - k_E^2 (2r_E + 3V \gamma))}{4k_E^4 V ((k_E - \gamma)V - r_E) (k_E - \gamma)^2}$$

Table EC.2 Stage 4 Results

Strategy	Entrant-Usage	Entrant-Subscription
Incumbent-Usage	$\pi_E^{UV} = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \pi_{E1}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1 \\ \left\{ \begin{array}{l} \pi_{E1}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} \pi_{E1}; 0 \leq c \leq c_L \\ 0; c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \end{array} \right\}; \quad \beta > \frac{2}{\sqrt{3}} - 1$	$\pi_E^{SV} = \frac{V^2}{64c}, k_E = \frac{V}{8c}, r_E = \frac{V^2}{16c}$
	$k_E = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \frac{1}{2cx}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{2c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1 \\ \left\{ \begin{array}{l} \frac{1}{2cx}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{2c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} \frac{1}{2cx}; 0 \leq c \leq c_L \\ \text{n.a.}; c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{2c}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \end{array} \right\}; \quad \beta > \frac{2}{\sqrt{3}} - 1$	
	$p_E = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \frac{c^2 x \gamma^2 + (1+\beta)^2(1-2cx\gamma)}{x(1+\beta)^2(1-cx\gamma)}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{x}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1 \\ \left\{ \begin{array}{l} \frac{c^2 x \gamma^2 + (1+\beta)^2(1-2cx\gamma)}{x(1+\beta)^2(1-cx\gamma)}; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{x}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} \frac{c^2 x \gamma^2 + (1+\beta)^2(1-2cx\gamma)}{x(1+\beta)^2(1-cx\gamma)}; 0 \leq c \leq c_L \\ \text{n.a.}; c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{x}; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \end{array} \right\}; \quad \beta > \frac{2}{\sqrt{3}} - 1$	
Incumbent-Subscription	$\pi_E^{VS} = \begin{cases} \frac{1}{4c} - \frac{\gamma}{2}; & 0 \leq c \leq \frac{1}{2\gamma} \\ 0; & c > \frac{1}{2\gamma} \end{cases}, p_E = \begin{cases} \frac{1}{x}; & 0 \leq c \leq \frac{1}{2\gamma} \\ \text{n.a.}; & c > \frac{1}{2\gamma} \end{cases}, k_E = \begin{cases} \frac{1}{2c}; & 0 \leq c \leq \frac{1}{2\gamma} \\ \text{n.a.}; & c > \frac{1}{2\gamma} \end{cases}$	$\pi_E^{SS} = \begin{cases} \frac{V^2 - \gamma V}{64c} - \frac{\gamma V}{4}; & 0 \leq c \leq \frac{V}{16\gamma} \\ 0; & c \geq \frac{V}{16\gamma} \end{cases},$ $r_E = \begin{cases} \frac{(V - 8c\gamma)V}{16c}; & 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; & c \geq \frac{V}{16\gamma} \end{cases},$ $k_E = \begin{cases} \frac{V}{8c}; & 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; & c \geq \frac{V}{16\gamma} \end{cases}$

Where  $\pi_{E1} = \frac{(1+\beta)^2 - 4cx\gamma(1+\beta)^2 + 2c^2x\gamma^2}{4cx^2(1+\beta)^2}$ , and  $c_L = \frac{(1+\beta)(2x(1+\beta) - \sqrt{4x^2(1+\beta)^2 - 2x})}{2x\gamma}$

**Table EC.3 Stage 3 Results**







## EC.4. Figures

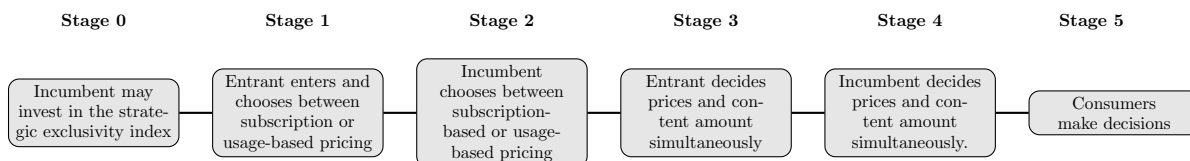


Figure EC.1 Game set-up and decision-making sequence (Section 5.1)

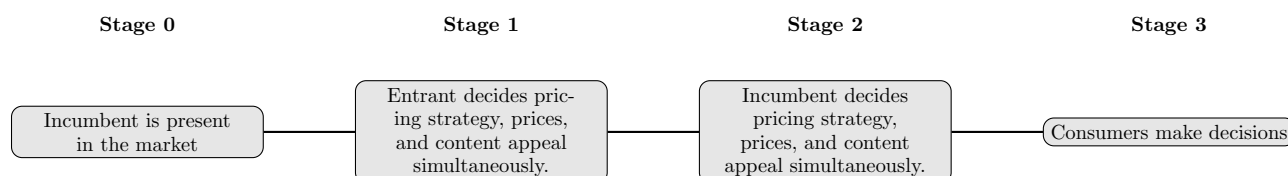


Figure EC.2 Alternative game set-up and decision-making sequence (Section 5.2.1)

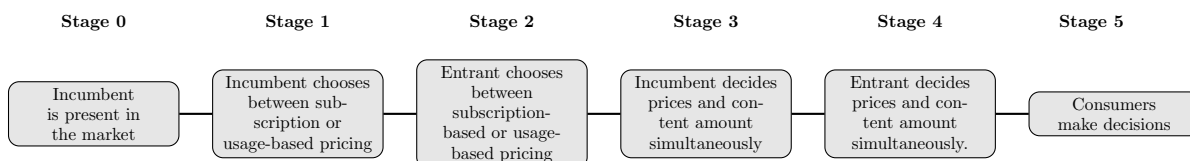


Figure EC.3 Game set-up and decision-making sequence (Section 5.2.2)

## EC.5. Model Assumptions

While solving the model, the following assumptions are made for the feasibility of the solution that would make practical sense.

$$(i) \quad 0 \leq p_I \leq \frac{1}{x}, 0 \leq p_E \leq \frac{1}{x}, 0 \leq r_E \leq k_E V, 0 \leq r_I \leq k_I (1 + \beta) V$$

$$(ii) \quad k_E \geq \gamma, k_I \geq \gamma.$$

Assumption (i) is a required condition to ensure that there is at least one consumer present in the market. Suppose  $p_E > \frac{1}{x}$ , then there will be no consumer who would opt for the entrant's usage plan. Similarly, assumption (ii) is required for the presence of at least one consumer for multi-home. Suppose  $k_E \leq \gamma$ ; there will be no consumers who would opt for multi-home. Given these assumptions in place, we now solve the game (defined in Figure 1) through backward induction.

Furthermore, in our model  $k_j$  is a reduced-form perceived value index that scales valuation across a heterogeneous population. Practically, when a platform improves its catalog, the addressable audience may broaden (some consumers gain a lot, others a little), so average utility rises and cross-sectional dispersion can widen; this reflects demand heterogeneity rather than outcome uncertainty for a given user.

Furthermore, market outcomes depend on “who becomes a user.” To reflect this reality, in our model, not everyone has to be a customer on any of the platforms (i.e., the market is not covered). Prices and  $k_j$  jointly

determine participation thresholds; a higher  $k_j$  may support higher prices and thus a tighter realized user base (selection), even if the unconditional spread in  $k_j T$  is wider. Our results are driven by these threshold shifts and by relative scales (e.g.,  $k_I/k_E, \frac{p}{k}$ ), not by variance per se.

Formally, let  $\tau \equiv \frac{T}{V} \sim U[0, 1]$ . Platform- $j$  valuation is  $U_j = k_j V \tau$ , i.e., a linear dilation of the same base distribution. All cutoffs and comparative statics use monotone thresholds in  $\tau$ ; we never invoke  $\text{Var}(U_j)$  separately. If one prefers a “mean-shift without spread change” representation, an equivalent reparameterization is  $U_j = \mu_j + \lambda \tau$  with a common slope and platform-specific mean  $\mu_j \propto k_j$ ; the equilibrium thresholds and insights are qualitative unchanged.

## EC.6. Model Solution

In this section, we present the results for all five stages by solving the game through backward induction.

### EC.6.1. Stage 5 Decisions:

In this stage, each consumer chooses one of the following options: (i) consume content only from I, (ii) consume content only from E, (iii) consume content from both I and E (Multi-home). Each consumer experiences a utility for all three scenarios, conditional on the incumbent’s and entrant’s choice of pricing strategy. The utility structure is defined in Table 1. While solving the model, we consider the following cases based on the pricing strategies of the incumbent ( $I$ ) and the entrant ( $E$ ). A brief summary of consumer segmentation is outlined below, and a detailed analysis is presented after this.

1. **Both  $I$  and  $E$  adopt usage-based pricing:** When the incumbent’s per-usage price ( $p_I$ ) is low ( $0 \leq p_I \leq \frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)}$ ), all consumers with valuations between 0 and  $V$  choose to multi-home if the entrant’s per-usage price is low ( $0 \leq p_E \leq C$ ). For higher values of the entrant’s per-usage price, all consumers choose  $I$  only. When the incumbent’s price is higher, consumers opt for  $E$  only, multi-home, or  $I$  only when the entrant’s per-usage price is in the low ( $0 \leq p_E \leq B$ ), intermediary ( $B \leq p_E \leq C$ ), or high ( $C \leq p_E \leq \frac{1}{x}$ ) ranges, respectively.
2. **Both  $I$  and  $E$  adopt subscription-based pricing:** If the entrant’s subscription fee meets the condition ( $0 \leq r_E \leq (k_E - \gamma)V$ ), consumers with valuations within the range of  $T_1$  to  $T_4$  will choose the incumbent’s subscription. Consumers with valuations exceeding  $T_4$  will engage in multi-homing, provided the incumbent’s subscription price is sufficiently low, specifically  $0 \leq r_I \leq B$ . When the incumbent’s price is between  $B$  and  $D$ , consumers with valuations between  $T_2$  and  $T_6$  will opt for the entrant’s subscription, those with valuations between  $T_6$  and  $T_4$  will opt for the incumbent’s subscription, and those with valuations above  $T_4$  will engage in multi-homing. When the incumbent’s subscription price is between  $D$  and  $H$ , consumers with valuations between  $T_2$  and  $T_5$  will opt for the entrant’s subscription, and consumers with valuations higher than  $T_5$  will opt for multi-homing. However, when the incumbent’s price is significantly high (i.e.,  $H \leq r_I \leq k_I(1+\beta)V$ ), all consumers with valuations between  $T_2$  and  $V$  will opt for the entrant’s subscription. All other consumers will abstain from the market.

When the entrant’s subscription fee is high enough (i.e.,  $(k_E - \gamma)V \leq r_E \leq k_E V$ ), consumers with valuations higher than  $T_1$  ( $T_2$ ) will opt for the incumbent’s (entrant’s) subscription if the incumbent’s

subscription price is low (high) enough. When the incumbent's subscription price is in the intermediate range (i.e.,  $B \leq r_I \leq G$ ), consumers with valuations between  $T_2$  and  $T_6$  will opt for the entrant's subscription, and consumers with valuations higher than  $T_6$  will opt for the incumbent's subscription. All other consumers will abstain from the market.

3. **I adopts usage-based and E adopts subscription-based pricing:** When the entrant's subscription price ( $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$ ) is low, consumers with valuations between 0 and  $T_4$  choose the incumbent's usage-based plan, while others choose to multi-home. For higher subscription prices, all consumers with valuations between 0 and  $V$  opt for the incumbent's usage plan.
4. **I adopts subscription-based and E adopts usage-based pricing:** When the incumbent's subscription price ( $0 \leq r_I \leq (k_I(1 + \beta) - \gamma(1 - p_E x))V$ ) is low, consumers with valuations between 0 and  $T_4$  choose the entrant's usage-based plan, while others choose to multi-home. For higher subscription prices, all consumers with valuations between 0 and  $V$  select the entrant's usage plan.

**I-Usage, E-Subscription:** When the incumbent adopts a usage-based strategy and the entrant adopts a subscription-based strategy, we solve for individual rationality (I-R), and incentive compatibility (I-C) constraints by comparing the utilities (defined in Table 1) to arrive at the final decision.

**I-R constraints:**

- (a) I-only  $k_I T(1 + \beta)(1 - p_I x) \geq 0 \Rightarrow p_I \leq \frac{1}{x}$  (which holds).
- (b) E-only  $k_E T - r_E \geq 0 \Rightarrow T \geq \frac{r_E}{k_E}$ . And  $\frac{r_E}{k_E} \leq V \Rightarrow r_E \leq k_E V$ .
- (c) Multi-home

$$\begin{aligned} & (k_I T(1 + \beta) - \gamma T)(1 - p_I x) + k_E T - r_E \geq 0 \\ \Rightarrow & T((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E) \geq r_E \\ \Rightarrow & T \geq \frac{r_E}{((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E)} \end{aligned}$$

Given the assumptions above,  $((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E) \geq 0$ . And  $\frac{r_E}{((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E)} \leq V \Rightarrow r_E \leq ((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E)V$ . Also,  $((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E) \geq k_E$ . It implies  $\frac{r_E}{((k_I(1 + \beta) - \gamma)(1 - p_I x) + k_E)} \leq V$  for  $0 \leq r_E \leq k_E V$ .

**I-C constraints:**

- (a) Comparison of I-only with E -only

$$\begin{aligned} & k_I T(1 + \beta)(1 - p_I x) \geq k_E T - r_E \\ & T(k_E - k_I(1 + \beta)(1 - p_I x)) \leq r_E \\ & T \leq \frac{r_E}{k_E - k_I(1 + \beta)(1 - p_I x)} \text{ if } k_E - k_I(1 + \beta)(1 - p_I x) \geq 0 \end{aligned}$$

$k_E - k_I(1 + \beta)(1 - p_I x) \geq 0 \Rightarrow k_E \geq k_I(1 + \beta)(1 - p_I x)$ . Whenever  $k_E - k_I(1 + \beta)(1 - p_I x) \leq 0$ , the consumer will choose I-only over E-only. This implies that equation (1) holds for all  $T \geq 0$ .

(b) Comparison of I-only with Multi-home

$$\begin{aligned} k_I T(1 + \beta)(1 - p_I x) &\geq (k_I T(1 + \beta) - \gamma T)(1 - p_I x) + k_E T - r_E \\ 0 &\geq -\gamma T(1 - p_I x) + k_E T - r_E \\ T(k_E - \gamma(1 - p_I x)) &\leq r_E \\ T &\leq \frac{r_E}{k_E - \gamma(1 - p_I x)} \end{aligned}$$

And,  $\frac{r_E}{k_E - \gamma(1 - p_I x)} \geq 0 \Rightarrow (k_E - \gamma(1 - p_I x)) \geq 0$ . This inequality holds as  $k_E \geq \gamma$ . And,  $\frac{r_E}{k_E - \gamma(1 - p_I x)} \leq V \Rightarrow r_E \leq (k_E - \gamma(1 - p_I x))V$ .

(c) Comparison of E-only with Multi-home

$$\begin{aligned} k_E T - r_E &\geq (k_I T(1 + \beta) - \gamma T)(1 - p_I x) + k_E T - r_E \\ 0 &\geq T(k_I(1 + \beta) - \gamma)(1 - p_I x) \end{aligned}$$

This inequality does not hold as  $(k_I(1 + \beta) - \gamma) \geq 0$ . It implies that the utility under multi-home is greater than the utility under E-only.

Let us consider  $T_1 = \frac{r_E}{k_E}$ ,  $T_2 = \frac{r_E}{k_E + (k_I(1 + \beta) - \gamma)(1 - p_I x)}$ ,  $T_3 = \frac{r_E}{k_E - k_I(1 + \beta)(1 - p_I x)}$ ,  $T_4 = \frac{r_E}{k_E - \gamma(1 - p_I x)}$ . Market segmentation is as follows:

(a)  $0 \leq k_E \leq k_I(1 + \beta)(1 - p_I x)$  We have  $T_3 \leq 0, T_2 \leq T_1 \leq T_4$

(i)  $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$ , Consumers with usage with  $(0, T_4)$  consumer I-only, and others  $(T \in (T_4, V))$  do multi-home.

(ii)  $(k_E - \gamma(1 - p_I x))V \leq r_E \leq k_E V$ , all consumers with usage between 0 and V consume I-only.

(b)  $k_I(1 + \beta)(1 - p_I x) \leq k_E \leq k_I$  We have  $T_3 \geq 0, T_2 \leq T_1 \leq T_4 \leq T_3$

(i)  $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$ , Consumers with usage with  $(0, T_4)$  consume I-only and others  $(T \in (T_4, V))$  do multi-home.

(ii)  $(k_E - \gamma(1 - p_I x))V \leq r_E \leq k_E V$ , all consumers with usage between 0 and V consume I-only.

Combining all of this, we have

(i)  $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$ , Consumers with usage with  $(0, T_4)$  consume I-only and others  $(T \in (T_4, V))$  do multi-home.

$$\begin{aligned} \pi_I^{S^U} &= p_I [k_I(1 + \beta)x \frac{T_4}{V} + (k_I(1 + \beta) - \gamma)x \frac{(V - T_4)}{V}] - ck_I^2 \\ \pi_E^{S^U} &= \frac{r_e(V - T_4)}{V} - ck_E^2 \end{aligned}$$

(ii)  $(k_E - \gamma(1 - p_I x))V \leq r_E \leq k_E V$ , all consumers with usage between 0 and V consume I-only.

$$\begin{aligned} \pi_I^{S^U} &= p_I k_I(1 + \beta)x - ck_I^2 \\ \pi_E^{S^U} &= 0 \end{aligned}$$

**I-Subscription, E-Subscription:** We now solve for the consumer decisions when both incumbent and entrant opt for a subscription-based pricing strategy. By comparing the utilities (defined in Table 1), we arrive at the decision.

**I-R constraints:**

- (a) I-only  $k_I T(1 + \beta) - r_I \geq 0 \Rightarrow T \geq \frac{r_I}{k_I(1 + \beta)}$  ( $= T_1$ ). And  $\frac{r_I}{k_I(1 + \beta)} \leq V \Rightarrow r_I \leq k_I(1 + \beta)V$ .
- (b) E-only  $k_E T - r_E \geq 0 \Rightarrow T \geq \frac{r_E}{k_E}$  ( $= T_2$ ). And  $\frac{r_E}{k_E} \leq V \Rightarrow r_E \leq k_E V$ .
- (c) Multi-home

$$\begin{aligned} k_I T(1 + \beta) + k_E T - \gamma T - r_I - r_E &\geq 0 \\ \Rightarrow T(k_I(1 + \beta) + k_E - \gamma) &\geq r_I + r_E \\ \Rightarrow T &\geq \frac{r_I + r_E}{(k_I(1 + \beta) + k_E - \gamma)} \quad (= T_3) \end{aligned}$$

And  $\frac{r_I + r_E}{(k_I(1 + \beta) + k_E - \gamma)} \leq V \Rightarrow r_I \leq (k_I(1 + \beta) + k_E - \gamma)V - r_E$  ( $= F$ ). And  $(k_I(1 + \beta) + k_E - \gamma)V - r_E \geq 0 \Rightarrow r_E \leq (k_E V + (k_I(1 + \beta) - \gamma)V)$  ( $\geq k_E V$ ). And  $(k_I(1 + \beta) + k_E - \gamma)V - r_E \geq k_I(1 + \beta)V \Rightarrow r_E \leq (k_E - \gamma)V$ . This implies that  $T_3 =$

$$\left\{ \begin{array}{l} \leq V ; \\ \left\{ \begin{array}{l} \leq V ; \quad 0 \leq r_I \leq (k_I(1 + \beta) + k_E - \gamma)V - r_E \\ \geq V ; \quad (k_I(1 + \beta) + k_E - \gamma)V - r_E \leq r_I \leq k_I(1 + \beta)V \end{array} \right\} ; \quad 0 \leq r_E \leq (k_E - \gamma)V \\ \geq V ; \quad (k_E - \gamma)V \leq r_E \leq k_E V \end{array} \right\}.$$

**I-C constraints:**

- (a) Comparison of I-only with Multi-home

$$\begin{aligned} k_I T(1 + \beta) - r_I &\geq k_I T(1 + \beta) + k_E T - \gamma T - r_I - r_E \\ 0 &\geq (k_E - \gamma)T - r_E \\ T &\leq \frac{r_E}{k_E - \gamma} \quad (= T_4) \end{aligned}$$

Check if  $T_4 \leq V \Rightarrow r_E \leq (k_E - \gamma)V$ . Also  $T_4 \geq 0$  (Because  $k_E \geq \gamma$ ). So  $T_4 =$

$$\left\{ \begin{array}{l} \leq V ; \quad 0 \leq r_E \leq (k_E - \gamma)V \\ \geq V ; \quad (k_E - \gamma)V \leq r_E \leq k_E V \end{array} \right\}$$

- (b) Comparison of E-only with Multi-home

$$\begin{aligned} k_E T - r_E &\geq k_I T(1 + \beta) + k_E T - \gamma T - r_I - r_E \\ 0 &\geq T k_I(1 + \beta) - \gamma T - r_I \\ T &\leq \frac{r_I}{k_I(1 + \beta) - \gamma} \quad (= T_5) \end{aligned}$$

Check if  $T_5 \leq V \Rightarrow r_I \leq (k_I(1 + \beta) - \gamma)V$  ( $= H$ ). Also,  $T_5 \geq 0$  (Because  $k_I \geq \gamma$ ). So  $T_5 =$

$$\left\{ \begin{array}{l} \leq V ; \quad 0 \leq r_I \leq (k_I(1 + \beta) - \gamma)V \\ \geq V ; \quad (k_I(1 + \beta) - \gamma)V \leq r_I \leq k_I(1 + \beta)V \end{array} \right\}$$

- (c) Comparison of I-only with E -only

$$\begin{aligned} k_I T(1 + \beta) - r_I &\geq k_E T - r_E \\ T(k_I(1 + \beta) - k_E) &\geq r_I - r_E \\ T &\geq \left\{ \begin{array}{l} 0 ; \\ \frac{r_I - r_E}{k_I(1 + \beta) - k_E} \quad (= T_6) ; \quad r_I \geq r_E \end{array} \right\} \end{aligned}$$

Check if  $T_6 \leq V \Rightarrow r_I \leq r_E + (k_I(1 + \beta) - k_E)V$  ( $= G$ ). And  $r_E + (k_I(1 + \beta) - k_E)V \leq k_I(1 + \beta)V \Rightarrow r_E \leq k_E V$  (it holds). So we have  $T_6 =$

$$\left\{ \begin{array}{l} \leq V ; \quad 0 \leq r_I \leq r_E + (k_I(1 + \beta) - k_E)V \\ \geq V ; \quad r_E + (k_I(1 + \beta) - k_E)V \leq r_I \leq k_I(1 + \beta)V \end{array} \right\}.$$

$$\begin{aligned}
\frac{r_E}{k_E} &\leq \frac{r_E}{k_E - \gamma} \\
\frac{r_I}{k_I(1+\beta)} &\leq \frac{r_I}{k_I(1+\beta) - \gamma} \\
\frac{r_I + r_E}{k_I(1+\beta) + k_E - \gamma} &\leq \frac{r_E}{k_E} \Rightarrow r_I \leq \frac{r_E(k_I(1+\beta) - \gamma)}{k_E} \quad (= A) \\
\frac{r_I + r_E}{k_I(1+\beta) + k_E - \gamma} &\leq \frac{r_I}{k_I(1+\beta)} \Rightarrow r_I \geq \frac{r_E k_I(1+\beta)}{k_E - \gamma} \quad (= E) \\
\text{Also } \frac{k_I(1+\beta)}{k_E - \gamma} &\geq 1 \Rightarrow (k_I - k_E) + k_I\beta + \gamma \geq 0 \text{ (it holds)} \\
\frac{r_I + r_E}{k_I(1+\beta) + k_E - \gamma} &\leq \frac{r_I - r_E}{k_I(1+\beta) - k_E} \Rightarrow r_I \geq \frac{r_E(2k_I(1+\beta) - \gamma)}{2k_E - \gamma} \quad (= C) \\
\text{Also } \frac{2k_I(1+\beta) - \gamma}{2k_E - \gamma} &\geq 1 \Rightarrow k_I(1+\beta) \geq k_E \text{ (it holds)} \\
\frac{r_I + r_E}{k_I(1+\beta) + k_E - \gamma} &\geq \frac{r_I}{k_I(1+\beta) - \gamma} \Rightarrow r_I \leq \frac{r_E(k_I(1+\beta) - \gamma)}{k_E} \\
\frac{r_I + r_E}{k_I(1+\beta) + k_E - \gamma} &\leq \frac{r_E}{k_E - \gamma} \Rightarrow r_I \leq \frac{r_E k_I(1+\beta)}{k_E - \gamma} \\
\text{Also } \frac{k_I(1+\beta)}{k_E - \gamma} &\geq 1 \Rightarrow (k_I(1+\beta) - k_E + \gamma) \geq 0 \text{ (it holds)} \\
\frac{r_I - r_E}{k_I(1+\beta) - k_E} &\leq \frac{r_E}{k_E - \gamma} \Rightarrow r_I \leq \frac{r_E(k_I(1+\beta) - \gamma)}{k_E - \gamma} \quad (= D) \\
\text{And } \frac{k_I(1+\beta) - \gamma}{k_E - \gamma} &\geq 1 \Rightarrow k_I(1+\beta) \geq k_E \text{ (it holds)} \\
\frac{r_I - r_E}{k_I(1+\beta) - k_E} &\geq \frac{r_I}{k_I(1+\beta) - \gamma} \Rightarrow r_I \geq \frac{r_E(k_I(1+\beta) - \gamma)}{k_E - \gamma} \\
\frac{r_I - r_E}{k_I(1+\beta) - k_E} &\leq \frac{r_I}{k_I(1+\beta)} \Rightarrow r_I \leq \frac{r_E k_I(1+\beta)}{k_E} \quad (= B) \\
\frac{r_I - r_E}{k_I(1+\beta) - k_E} &\leq \frac{r_E}{k_E} \Rightarrow r_I \leq \frac{r_E k_I(1+\beta)}{k_E} \\
\frac{r_I}{k_I(1+\beta)} &\leq \frac{r_E}{k_E} \Rightarrow r_I \leq \frac{r_E k_I(1+\beta)}{k_E} \\
\frac{r_I}{k_I(1+\beta) - \gamma} &\leq \frac{r_E}{k_E - \gamma} \Rightarrow r_I \leq \frac{r_E(k_I(1+\beta) - \gamma)}{k_E - \gamma} \\
\frac{r_E}{k_E} &\leq \frac{r_I}{k_I(1+\beta) - \gamma} \Rightarrow r_I \geq \frac{(k_I(1+\beta) - \gamma)r_E}{k_E} \\
\frac{r_E}{k_E - \gamma} &\leq \frac{r_I}{k_I(1+\beta)} \Rightarrow r_I \geq \frac{r_E k_I(1+\beta)}{k_E - \gamma}
\end{aligned}$$

Market segmentation is as follows:

(a)  $0 \leq r_E \leq (k_E - \gamma)V$

- (i)  $0 \leq r_I \leq B$ , consumers with usage with  $(T_1, T_4)$  consume I-only, and consumers with usage between  $(T \in (T_4, V))$  do multi-home. Others abstain from the market.

$$\begin{aligned}
\pi_I^{SS} &= \frac{r_I(V - T_1)}{V} - ck_I^2 \\
\pi_E^{SS} &= \frac{r_E(V - T_4)}{V} - ck_E^2
\end{aligned}$$

- (ii)  $B \leq r_I \leq D$ , consumers with usage with  $(T_2, T_6)$  consume E-only, and consumers with usage between  $(T_6, T_4)$  consume I-only, and consumers with usage between  $(T \in (T_4, V))$  do multi-home. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= \frac{r_I(V - T_6)}{V} - ck_I^2 \\ \pi_E^{SS} &= \frac{r_E(V - T_4 + T_6 - T_2)}{V} - ck_E^2\end{aligned}$$

- (iii)  $D \leq r_I \leq H$ , consumers with usage with  $(T_2, T_5)$  consume E-only, and consumers with usage between  $(T \in (T_5, V))$  do multi-home. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= \frac{r_I(V - T_5)}{V} - ck_I^2 \\ \pi_E^{SS} &= \frac{r_E(V - T_2)}{V} - ck_E^2\end{aligned}$$

- (iv)  $H \leq r_I \leq k_I(1 + \beta)V$ , consumers with usage between  $(T \in (T_2, V))$  consume E-only. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= 0 \\ \pi_E^{SS} &= \frac{r_E(V - T_2)}{V} - ck_E^2\end{aligned}$$

- (b)  $(k_E - \gamma)V \leq r_E \leq k_EV$

- (i)  $0 \leq r_I \leq B$ , consumers with usage with  $(T_1, V)$  consume I-only. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= \frac{r_I(V - T_1)}{V} - ck_I^2 \\ \pi_E^{SS} &= 0\end{aligned}$$

- (ii)  $B \leq r_I \leq G$ , consumers with usage with  $(T_2, T_6)$  consume E-only, and consumers with usage between  $(T \in (T_6, V))$  do multi-home. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= \frac{r_I(V - T_6)}{V} - ck_I^2 \\ \pi_E^{SS} &= \frac{r_E(T_6 - T_2)}{V} - ck_E^2\end{aligned}$$

- (iii)  $G \leq r_I \leq k_I(1 + \beta)V$ , consumers with usage between  $(T \in (T_2, V))$  consume E-only. Others abstain from the market.

$$\begin{aligned}\pi_I^{SS} &= 0 \\ \pi_E^{SS} &= \frac{r_E(V - T_2)}{V} - ck_E^2\end{aligned}$$

**I-Usage, E-Usage:** When both the incumbent and the entrant adopt a usage-based strategy, the market segmentation derivation is as follows:

**I-R constraints:**

- (a) I-only  $k_I T(1 + \beta)(1 - p_I x) \geq 0 \Rightarrow p_I \leq \frac{1}{x}$  (which holds).  
 (b) E-only  $k_E T(1 - p_E x) \geq 0 \Rightarrow p_E \leq \frac{1}{x}$  (which holds).

$$(c) \text{ Multi-home } k_I T(1+\beta)(1-p_I x) + k_E T(1-p_E x) - \gamma T(1 - \frac{p_I x}{2} - \frac{p_E x}{2}) \geq 0 \Rightarrow k_I(1+\beta)(1-p_I x) + k_E(1-p_E x) - \gamma(1 - \frac{p_I x}{2} - \frac{p_E x}{2}) \geq 0 \Rightarrow p_E \leq \frac{2(k_I(1+\beta) + k_E - \gamma)}{(2k_E - \gamma)x} - \frac{(2k_I(1+\beta) - \gamma)p_I}{(2k_E - \gamma)}$$

Check if

$$\begin{aligned} & \frac{2(k_I(1+\beta) + k_E - \gamma)}{(2k_E - \gamma)x} - \frac{(2k_I(1+\beta) - \gamma)p_I}{(2k_E - \gamma)} \leq p_I \\ \Rightarrow & \frac{2(k_I(1+\beta) + k_E - \gamma)}{(2k_E - \gamma)x} \leq \frac{2(k_I(1+\beta) + k_E - \gamma)p_I}{(2k_E - \gamma)x} \\ \Rightarrow & p_I \geq \frac{1}{x} \text{ (which does not hold)} \end{aligned}$$

**I-C constraints: (a) Comparison of I-only with Multi-home**  $k_I T(1+\beta)(1-p_I x) \geq k_I T(1+\beta)(1-p_I x) + k_E T(1-p_E x) - \gamma T(1 - \frac{p_I x}{2} - \frac{p_E x}{2}) \Rightarrow T(k_E(1-p_E x) - \gamma(1 - \frac{p_I x}{2} - \frac{p_E x}{2})) \leq 0 \Rightarrow (k_E(1-p_E x) - \gamma(1 - \frac{p_I x}{2} - \frac{p_E x}{2})) \leq 0 \Rightarrow (k_E - \gamma) \leq x(p_E k_E - \frac{\gamma p_I}{2} - \frac{\gamma p_E}{2}) \Rightarrow \frac{2(k_E - \gamma)}{x} \leq p_E(2k_E - \gamma) - \gamma p_I \Rightarrow p_E \geq \frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)}$ .

Check if  $\frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \leq p_I \Rightarrow \frac{(2\gamma - 2k_E)p_I}{(2k_E - \gamma)} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \leq 0 \Rightarrow p_I \geq \frac{1}{x}$  (which does not hold.)  $\Rightarrow \frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \geq p_I$ .

Check if  $\frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \leq \frac{1}{x} \Rightarrow \frac{\gamma p_I}{2k_E - \gamma} \leq \frac{\gamma}{(2k_E - \gamma)x} \Rightarrow p_I \leq \frac{1}{x}$  (which holds.)

Check if  $\frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \geq 0 \Rightarrow p_I \leq \frac{2(k_I(1+\beta) + k_E - \gamma)}{x(2k_I(1+\beta) - \gamma)}$ . And Check if  $\frac{2(k_I(1+\beta) + k_E - \gamma)}{x(2k_I(1+\beta) - \gamma)} \leq \frac{1}{x} \Rightarrow (2k_E - \gamma) \leq 0$  (which does not hold.)  $\Rightarrow \frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)} \geq 0 \forall p_I \leq \frac{1}{x}$

**(b) Comparison of I-only with E-only**  $k_I T(1+\beta)(1-p_I x) \geq k_E T(1-p_E x) \Rightarrow k_I(1+\beta)(1-p_I x) \geq k_E(1-p_E x) \Rightarrow p_E \geq \frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E}$

Check if  $\frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E} \geq p_I \Rightarrow \frac{k_I(1+\beta) - k_E}{k_E}(p_I - \frac{1}{x}) \geq 0 \Rightarrow p_I \geq \frac{1}{x}$  (which does not hold.)  $\Rightarrow \frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E} \leq p_I$

Check if  $\frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E} \leq \frac{1}{x} \Rightarrow \frac{k_I(1+\beta) - k_E}{k_E}(p_I - \frac{1}{x}) \leq 0 \Rightarrow p_I \leq \frac{1}{x}$  (which holds.)

Check if  $\frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E} \geq 0 \Rightarrow p_I \geq \frac{k_I(1+\beta) - k_E}{k_I(1+\beta)x} (\leq \frac{1}{x})$

**(c) Comparison of E-only with Multi-home**  $k_E T - r_E \geq k_I T(1+\beta)(1-p_I x) + k_E T(1-p_E x) - \gamma T(1 - \frac{p_I x}{2} - \frac{p_E x}{2}) \Rightarrow 0 \geq 2k_I(1+\beta)(1-p_I x) - \gamma(2-p_I x - p_E x) \Rightarrow 2k_I(1+\beta)p_I x - \gamma x(p_I + p_E) \geq 2(k_I(1+\beta) - \gamma) \Rightarrow p_E \leq \frac{(2k_I(1+\beta) - \gamma)p_I}{2(k_I(1+\beta) - \gamma)} - \frac{x\gamma}{2(k_I(1+\beta) - \gamma)}$

Check if  $\frac{(2k_I(1+\beta) - \gamma)p_I}{2(k_I(1+\beta) - \gamma)} - \frac{x\gamma}{2(k_I(1+\beta) - \gamma)} \geq p_I \Rightarrow p_I \geq \frac{1}{x}$  (which does not hold.)

Check if  $\frac{(2k_I(1+\beta) - \gamma)p_I}{2(k_I(1+\beta) - \gamma)} - \frac{x\gamma}{2(k_I(1+\beta) - \gamma)} \leq \frac{1}{x} \Rightarrow p_I \leq \frac{1}{x}$  (which holds.)

Check if  $\frac{(2k_I(1+\beta) - \gamma)p_I}{2(k_I(1+\beta) - \gamma)} - \frac{x\gamma}{2(k_I(1+\beta) - \gamma)} \geq 0 \Rightarrow p_I \geq \frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)} (\leq \frac{1}{x})$

Check if  $\frac{k_I(1+\beta)p_I}{k_E} - \frac{k_E x}{k_I(1+\beta) - k_E} \geq \frac{(2k_I(1+\beta) - \gamma)p_I}{2(k_I(1+\beta) - \gamma)} - \frac{x\gamma}{2(k_I(1+\beta) - \gamma)} \Rightarrow p_I = \begin{cases} \leq \frac{1}{x}; & (2k_E - \gamma)k_I(1+\beta) - k_E \gamma \geq 0 \\ \geq \frac{1}{x}; & (2k_E - \gamma)k_I(1+\beta) - k_E \gamma \leq 0 \end{cases}$

Check if  $(2k_E - \gamma)k_I(1 + \beta) - k_E\gamma \geq 0 \Rightarrow k_E(2k_I(1 + \beta) - \gamma) \geq \gamma k_I(1 + \beta) \Rightarrow k_E \geq \frac{\gamma k_I(1 + \beta)}{2k_I(1 + \beta) - \gamma}$

Check if  $\frac{\gamma k_I(1 + \beta)}{2k_I(1 + \beta) - \gamma} \leq k_I \Rightarrow \gamma(1 + \beta) \leq 2k_I(1 + \beta) - \gamma \Rightarrow 2(k_I - \gamma) + \beta(2k_I - \gamma) \geq 0$  (It holds)

Let us consider  $A = \frac{k_I(1 + \beta)p_I}{k_E} - \frac{k_I(1 + \beta) - k_E}{k_E x}$ ,  $B = \frac{(2k_I(1 + \beta) - \gamma)p_I}{\gamma} - \frac{2(k_I(1 + \beta) - \gamma)}{x\gamma}$ ,  $C = \frac{\gamma p_I}{2k_E - \gamma} + \frac{2(k_E - \gamma)}{x(2k_E - \gamma)}$ . Market segmentation is as follows:

(a)  $0 \leq p_I \leq \frac{2(k_I(1 + \beta) - \gamma)}{x(2k_I(1 + \beta) - \gamma)}$  We have  $A \leq 0, B \leq 0$ , and  $C \geq 0$ .

(i)  $0 \leq p_E \leq C$ , All consumers with usage between 0 and V opt for multi-home.

(ii)  $C \leq p_E \leq \frac{1}{x}$ , consumers with usage between 0 and V opt for the incumbent's usage.

(b)  $\frac{2(k_I(1 + \beta) - \gamma)}{x(2k_I(1 + \beta) - \gamma)} \leq p_I \leq \frac{1}{x}$

(i)  $0 \leq p_E \leq B$ , All consumers with usage between 0 and V opt for the entrant's usage.

(ii)  $B \leq p_E \leq C$ , All consumers with usage between 0 and V opt for multi-home.

(iii)  $C \leq p_E \leq \frac{1}{x}$ , consumers with usage between 0 and V opt for the incumbent's usage.

(a)  $0 \leq p_I \leq \frac{2(k_I(1 + \beta) - \gamma)}{x(2k_I(1 + \beta) - \gamma)}$

(i)  $0 \leq p_E \leq C$ ,  $\pi_I^{UU} = p_I(k_I(1 + \beta) - \frac{\gamma}{2})x - ck_I^2$ ,  $\pi_E^{UU} = p_E(k_E - \frac{\gamma}{2})x - ck_E^2$

(ii)  $C \leq p_E \leq \frac{1}{x}$ ,  $\pi_I^{UU} = p_I k_I(1 + \beta)x - ck_I^2$ ,  $\pi_E^{UU} = 0$

(b)  $\frac{2(k_I(1 + \beta) - \gamma)}{x(2k_I(1 + \beta) - \gamma)} \leq p_I \leq \frac{1}{x}$

(i)  $0 \leq p_E \leq B$ ,  $\pi_I^{UU} = 0$ ,  $\pi_E^{UU} = p_E k_E x - ck_E^2$

(ii)  $B \leq p_E \leq C$ ,  $\pi_I^{UU} = p_I(k_I(1 + \beta) - \frac{\gamma}{2})x - ck_I^2$ ,  $\pi_E^{UU} = p_E(k_E - \frac{\gamma}{2})x - ck_E^2$

(iii)  $C \leq p_E \leq \frac{1}{x}$ ,  $\pi_I^{UU} = p_I k_I(1 + \beta)x - ck_I^2$ ,  $\pi_E^{UU} = 0$

**I-Subscription, E-Usage:** When the incumbent adopts a subscription-based strategy and the entrant adopts a usage-based strategy, the market segmentation is as follows:

**I-R constraints:**

(a) I-only  $k_I T(1 + \beta) - r_I \geq 0 \Rightarrow T \geq \frac{r_I}{k_I(1 + \beta)}$ . And  $\frac{r_I}{k_I(1 + \beta)} \leq V \Rightarrow r_I \leq k_I(1 + \beta)V$  (which holds).

(b) E-only  $k_E T(1 - p_E x) \geq 0 \Rightarrow p_E \leq \frac{1}{x}$  (which holds).

(c) Multi-home

$$\begin{aligned} & k_I T(1 + \beta) + T(k_E - \gamma)(1 - p_E x) - r_I \geq 0 \\ \Rightarrow & T(k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)) \geq r_I \\ \Rightarrow & T \geq \frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} \end{aligned}$$

Given the assumptions above,  $k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x) \geq 0$ . And  $\frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} \leq V \Rightarrow r_I \leq (k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x))V$ . Also,  $(k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x))V \geq V k_I(1 + \beta)$ . It implies  $\frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} \leq V$  for  $0 \leq r_I \leq k_I(1 + \beta)V$ . Also,  $\frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} \leq \frac{r_I}{k_I(1 + \beta)}$ .

**I-C constraints:**

(a) Comparison of I-only with E -only

$$\begin{aligned} k_I T(1 + \beta) - r_I &\geq k_E T(1 - p_E x) \\ T(k_I(1 + \beta) - k_E(1 - p_E x)) &\geq r_I \\ T &\geq \frac{r_I}{k_I(1 + \beta) - k_E(1 - p_E x)} \text{ if } k_I(1 + \beta) - k_E(1 - p_E x) \geq 0 \text{ (which holds.)} \end{aligned}$$

(b) Comparison of E-only with Multi-home

$$\begin{aligned} k_E T(1 - p_E x) &\geq k_I T(1 + \beta) - r_I + (k_E - \gamma)T(1 - p_E x) \\ 0 &\geq -\gamma T(1 - p_E x) + k_I(1 + \beta)T - r_I \\ T(k_I(1 + \beta) - \gamma(1 - p_E x)) &\leq r_I \\ T &\leq \frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)} \end{aligned}$$

And,  $(k_I(1 + \beta) - \gamma(1 - p_E x)) \geq 0$ . Also,  $\frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)} \leq V \Rightarrow r_I \leq (k_I(1 + \beta) - \gamma(1 - p_E x))V$ .

(c) Comparison of I-only with Multi-home

$$\begin{aligned} k_I T(1 + \beta) - r_I &\geq k_I T(1 + \beta) - r_I + (k_E - \gamma)T(1 - p_E x) \\ 0 &\geq T(k_E - \gamma)(1 - p_E x) \end{aligned}$$

This inequality does not hold as  $(k_E - \gamma)(1 - p_E x) \geq 0$ . It implies that the utility under multi-home is greater than the utility under I-only.

Let us consider  $T_1 = \frac{r_I}{k_I(1 + \beta)}$ ,  $T_2 = \frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)}$ ,  $T_3 = \frac{r_I}{k_I(1 + \beta) - k_E(1 - p_E x)}$ ,  $T_4 = \frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)}$ . Check if

$$\begin{aligned} \frac{r_I}{k_I(1 + \beta) - k_E(1 - p_E x)} &\leq \frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)} \\ \Rightarrow k_I(1 + \beta) - \gamma(1 - p_E x) &\leq k_I(1 + \beta) - k_E(1 - p_E x) \\ \Rightarrow -\gamma &\leq -k_E \\ \Rightarrow \gamma &\geq k_E \text{ (which does not hold.)} \end{aligned}$$

Check if

$$\begin{aligned} \frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} &\leq \frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)} \\ \Rightarrow k_I(1 + \beta) - \gamma(1 - p_E x) &\leq k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x) \\ \Rightarrow -\gamma &\leq (k_E - \gamma) \\ \Rightarrow k_E &\geq 0 \text{ (which holds.)} \end{aligned}$$

Check if

$$\begin{aligned} \frac{r_I}{k_I(1 + \beta) - k_E(1 - p_E x)} &\leq \frac{r_I}{k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x)} \\ \Rightarrow k_I(1 + \beta) + (k_E - \gamma)(1 - p_E x) &\leq k_I(1 + \beta) - k_E(1 - p_E x) \\ \Rightarrow (k_E - \gamma) &\leq -k_E \\ \Rightarrow k_E &\leq \frac{\gamma}{2} \text{ (which does not hold.)} \end{aligned}$$

Check if

$$\begin{aligned} \frac{r_I}{k_I(1+\beta) + (k_E - \gamma)(1 - p_E x)} &\leq \frac{r_I}{k_I(1+\beta)} \\ \Rightarrow k_I(1+\beta) &\leq k_I(1+\beta) + (k_E - \gamma)(1 - p_E x) \\ \Rightarrow 0 &\leq (k_E - \gamma)(1 - p_E x) \quad (\text{which holds.}) \end{aligned}$$

Also  $\frac{r_I}{k_I(1+\beta) - k_E(1 - p_E x)} \geq \frac{r_I}{k_I(1+\beta)}$ , and  $\frac{r_I}{k_I(1+\beta) - \gamma(1 - p_E x)} \geq \frac{r_I}{k_I(1+\beta)}$ . We have  $T_2 \leq t_1 \leq T_4 \leq T_3$ . Market segmentation is as follows:

- (a)  $0 \leq r_I \leq (k_I(1+\beta) - k_E(1 - p_E x))V$ , consumers with usage with  $(0, T_4)$  consume E-only, and others ( $T \in (T_4, V)$ ) do multi-home.
- (b)  $(k_I(1+\beta) - k_E(1 - p_E x))V \leq r_I \leq (k_I(1+\beta) - \gamma(1 - p_E x))V$ , consumers with usage with  $(0, T_4)$  consume E-only, and others ( $T \in (T_4, V)$ ) do multi-home.
- (c)  $(k_I(1+\beta) - \gamma(1 - p_E x))V \leq r_I \leq k_I(1+\beta)V$ , consumers with usage with  $(0, V)$  go with the entrant's usage plan.

### EC.6.2. Stage 4 Decision: Incumbent's Price and Content Acquisition

Given the market segment outlined in Stage 5, we now solve for prices and content acquisition index of the incumbent. The incumbent simultaneously decides content acquisition index ( $k_I$ ) and prices ( $r_I$  and  $p_I$  under subscription and usage-based, respectively) by maximizing the profits obtained under each pricing strategy.

**I-Usage, E-Subscription:** Here, incumbent decides  $p_I$ , and  $k_I$  simultaneously.

- (a)  $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$

$$\begin{aligned} \pi_I^{S^U} &= p_I [k_I(1+\beta)x \frac{T_4}{V} + (k_I(1+\beta) - \gamma)x \frac{(V - T_4)}{V}] - ck_I^2 \\ &= p_I k_I(1+\beta)x - \frac{p_I \gamma x (V - T_4)}{V} - ck_I^2 \\ &= p_I (k_I(1+\beta) - \gamma)x + \frac{p_I \gamma x T_4}{V} - ck_I^2 \\ &= p_I (k_I(1+\beta) - \gamma)x + \frac{p_I \gamma x r_E}{(k_E - \gamma(1 - p_I x))V} - ck_I^2 \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \pi_I^{S^U}}{\partial p_I} &= (k_I(1+\beta) - \gamma)x + \frac{\gamma x r_E (k_E - \gamma(1 - p_I x) - p_I \gamma x)}{V(k_E - \gamma(1 - p_I x))^2} \\ &= (k_I(1+\beta) - \gamma)x + \frac{\gamma x r_E (k_E - \gamma)}{V(k_E - \gamma(1 - p_I x))^2} \\ \frac{\partial^2 \pi_I^{S^U}}{\partial p_I^2} &= \frac{-2\gamma x r_E (k_E - \gamma) \gamma x}{V(k_E - \gamma(1 - p_I x))^3} \\ &< 0 \\ \frac{\partial \pi_I^{S^U}}{\partial k_I} &= p_I x(1+\beta) - 2ck_I \\ \frac{\partial^2 \pi_I^{S^U}}{\partial k_I \partial p_I} &= x(1+\beta) \\ \frac{\partial^2 \pi_I^{S^U}}{\partial k_I^2} &= -2c < 0 \end{aligned}$$

Hessian of the above function is positive semi-definite for  $0 \leq c \leq \frac{V(1+\beta)^2(k_E - \gamma(1-p_I x))^3}{4\gamma^2 r_E(k_E - \gamma)}$ . Solving for  $\frac{\partial \pi_I^{S^U}}{\partial p_I} = 0$ , we get two complex roots. And evaluating  $\pi_I$  at  $p_I = \frac{1}{x}$ , is positive. We solve for  $k_I$ , and the solution is  $k_I = \frac{(1+\beta)}{2c}$ . The solution is  $p_I = \frac{1}{x}$ ,  $k_I = \frac{(1+\beta)}{2c}$ ,  $\pi_I^{S^U} = \frac{(1+\beta)^2}{4c} - \gamma + \frac{r_E \gamma}{k_E V}$ , and  $\pi_E^{S^U} = r_E(1 - \frac{r_E}{k_E V}) - ck_E^2$ .

- (b)  $(k_E - \gamma(1-p_I x))V \leq r_E \leq k_E V$ ,  $\pi_I^{S^U} = p_I k_I (1+\beta)x - ck_I^2$ , linear in  $p_I$ . Solution is  $p_I = \frac{1}{x}$ ,  $k_I = \frac{(1+\beta)}{2c}$ , and  $\pi_I^{S^U} = \frac{(1+\beta)^2}{4c}$ .

**I-Subscription, E-Subscription:** Here, incumbent decides  $r_I$  and  $k_I$  simultaneously. We solve for the incumbent's optimal prices ( $r_I$ ) and the content amount ( $k_I$ ).

- (a) For  $0 \leq r_E \leq (k_E - \gamma)V$

- (i) For  $0 \leq r_I \leq B$ , the first ordered conditions are:

$$\begin{aligned} \frac{\partial \pi_I^{S^S}}{\partial r_I} &= 1 - \frac{2r_I}{k_I V(1+\beta)} \\ \frac{\partial \pi_I^{S^S}}{\partial k_I} &= \frac{r_I^2}{V(1+\beta)k_I^2} - 2ck_I \\ \frac{\partial^2 \pi_I^{S^S}}{\partial r_I^2} &= \frac{-2}{k_I(1+\beta)V} < 0 \\ \frac{\partial^2 \pi_I^{S^S}}{\partial k_I^2} &= \frac{-2r_I^2}{k_I^3(1+\beta)V} - 2c < 0 \\ \frac{\partial^2 \pi_I^{S^S}}{\partial r_I \partial k_I} &= \frac{2r_I}{k_I^2(1+\beta)V} \end{aligned}$$

By solving the above first ordered equations, we have  $k_I = \frac{(1+\beta)V}{8c}$ ,  $r_I = \frac{(1+\beta)^2 V^2}{16c}$ , and  $\pi_I^{S^S} = \frac{(1+\beta)^2 V^2}{64c}$ . The Hessian matrix of the above profit function is negative semi-definite. Check if  $r_I \leq B \Rightarrow r_E \geq \frac{k_E V}{2}$ . And  $\frac{k_E V}{2} \leq (k_E - \gamma)V \Rightarrow k_E \geq 2\gamma$ . Solution is  $r_E =$

$$\left\{ \left\{ \begin{array}{l} B; \\ B; \\ \frac{(1+\beta)^2 V^2}{16c}; \end{array} \right. \left\{ \begin{array}{l} 0 \leq r_E \leq \frac{k_E V}{2} \\ \frac{k_E V}{2} \leq r_E \leq (k_E - \gamma)V \end{array} \right. \right\}; \left. \begin{array}{l} \gamma \leq k_E \leq 2\gamma \\ k_E \geq 2\gamma \end{array} \right\}$$

When  $r_I = B$ , solving for  $k_I$ . The solution is  $k_I = \frac{r_E(1+\beta)(k_E V - r_E)}{2ck_E^2 V}$ ,  $r_I = \frac{r_E^2(1+\beta)^2(k_E V - r_E)}{2ck_E^3 V}$ , and  $\pi_I^{S^S} = \frac{r_E^2(1+\beta)^2(k_E V - r_E)^2}{4ck_E^4 V^2}$

- (ii) For  $B \leq r_I \leq D$ , the first-ordered equations are:

$$\begin{aligned} \frac{\partial \pi_I^{S^S}}{\partial r_I} &= \frac{1}{V} \left( V - \frac{2r_I - r_E}{k_I(1+\beta) - k_E} \right) \\ \frac{\partial \pi_I^{S^S}}{\partial k_I} &= \frac{r_I(r_I - r_E)(1+\beta)}{V(k_I(1+\beta) - k_E)^2} - 2ck_I \\ \frac{\partial^2 \pi_I^{S^S}}{\partial r_I^2} &= \frac{-2}{V(k_I(1+\beta) - k_E)} < 0 \\ \frac{\partial^2 \pi_I^{S^S}}{\partial k_I^2} &= \frac{-2r_I(r_I - r_E)(1+\beta)^2}{V(k_I(1+\beta) - k_E)^3} - 2c < 0 \end{aligned}$$

$$\frac{\partial^2 \pi_I^{SS}}{\partial r_I \partial k_I} = \frac{(2r_I - r_E)(1 + \beta)}{V(k_I(1 + \beta) - k_E)^2}$$

The hessian matrix is negative semi-definite for  $c \geq \frac{r_E^2(1 + \beta)^2}{4V(k_I(1 + \beta) - k_E)^3}$ . Solving first-ordered equations for  $r_I$  and  $k_I$ , we get negative value for  $r_I$  and  $\pi_I^{SS}$ . This case is not to be considered for further analysis.

(iii) For  $D \leq r_I \leq H$ , the first-ordered conditions are:

$$\begin{aligned} \frac{\partial \pi_I^{SS}}{\partial r_I} &= \frac{1}{V} \left( V - \frac{2r_I}{k_I(1 + \beta) - \gamma} \right) \\ \frac{\partial \pi_I^{SS}}{\partial k_I} &= \frac{r_I^2(1 + \beta)}{V(k_I(1 + \beta) - k_E)^2} - 2ck_I \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I^2} &= \frac{-2}{V(k_I(1 + \beta) - k_E)} < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial k_I^2} &= \frac{-2r_I^2(1 + \beta)^2}{V(k_I(1 + \beta) - \gamma)^2} - 2c < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I \partial k_I} &= \frac{2r_I(1 + \beta)}{V(k_I(1 + \beta) - k_E)^2} \end{aligned}$$

The Hessian matrix is negative semi-definite. By solving the first ordered equations, solution is  $r_I = \frac{(1 + \beta)^2 V^2}{16c} - \frac{\gamma V}{2}$ ,  $k_I = \frac{(1 + \beta)V}{8c}$ , and  $\pi_I^{SS} = \frac{(1 + \beta)^2 V^2}{64C} - \frac{\gamma V}{4}$ . Also  $r_I \leq H$ . And  $r_I \geq D \Rightarrow r_E \leq \frac{(k_E - \gamma)V}{2}$ . When  $r_I = D$ , solution is  $k_I = \frac{(1 + \beta)r_E((k_E - \gamma)V - r_E)}{2c(k_E - \gamma)^2 V}$ , and  $\pi_I^{SS} = \frac{r_E^2(1 + \beta)^2((k_E - \gamma)V - r_E)^2}{4c(k_E - \gamma)^4 V^2} - \frac{\gamma r_E((k_E - \gamma)V - r_E)}{V(k_E - \gamma)^2}$ .

(iv) For  $H \leq r_I \leq k_I(1 + \beta)V$ ,  $\pi_I^{SS} = 0$ .

(b)  $(k_E - \gamma)V \leq r_E \leq k_E V$

(i) For  $0 \leq r_I \leq B$ , the first-ordered equations are:

$$\begin{aligned} \frac{\partial \pi_I^{SS}}{\partial r_I} &= \frac{1}{V} \left( V - \frac{2r_I}{k_I(1 + \beta)} \right) \\ \frac{\partial \pi_I^{SS}}{\partial k_I} &= \frac{r_I^2}{V k_I^2(1 + \beta)} - 2ck_I \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I^2} &= \frac{-2}{k_I(1 + \beta)V} < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial^2 k_I} &= \frac{-2r_I^2}{V(1 + \beta)k_I^3} - 2c < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I \partial k_I} &= \frac{2r_I}{V k_I^2(1 + \beta)} \end{aligned}$$

By solving the above first ordered equations, we have  $k_I = \frac{(1 + \beta)V}{8c}$ ,  $r_I = \frac{(1 + \beta)^2 V^2}{16c}$ , and  $\pi_I^{SS} = \frac{(1 + \beta)^2 V^2}{64c}$ . The Hessian matrix of the above profit function is negative semi-definite. Check if  $r_I \leq B \Rightarrow r_E \geq \frac{k_E V}{2}$ . And  $\frac{k_E V}{2} \leq (k_E - \gamma)V \Rightarrow k_E \geq 2\gamma$ . Solution is  $r_E =$

$$\left\{ \begin{array}{l} B ; \\ \left\{ \begin{array}{l} B ; \\ \frac{(1 + \beta)^2 V^2}{16c} ; \end{array} \right. \left. \begin{array}{l} 0 \leq r_E \leq \frac{k_E V}{2} \\ \frac{k_E V}{2} \leq r_E \leq (k_E - \gamma)V \end{array} \right\} ; \left. \begin{array}{l} \gamma \leq k_E \leq 2\gamma \\ k_E \geq 2\gamma \end{array} \right\}$$

When  $r_I = B$ , solving for  $k_I$ . The solution is  $k_I = \frac{r_E(1+\beta)(k_EV - r_E)}{2ck_E^2V}$ ,  $r_I = \frac{r_E^2(1+\beta)^2(k_EV - r_E)}{2ck_E^3V}$ , and  $\pi_I^{SS} = \frac{r_E^2(1+\beta)^2(k_EV - r_E)^2}{4ck_E^4V^2}$

(ii) For  $B \leq r_I \leq G$ , the first-ordered equations are:

$$\begin{aligned}\frac{\partial \pi_I^{SS}}{\partial r_I} &= \frac{1}{V} \left( V - \frac{2r_I - r_E}{k_I(1+\beta) - k_E} \right) \\ \frac{\partial \pi_I^{SS}}{\partial k_I} &= \frac{r_I(r_I - r_E)(1+\beta)}{V(k_I(1+\beta) - k_E)^2} - 2ck_I \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I^2} &= \frac{-2}{V(k_I(1+\beta) - k_E)} < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial k_I^2} &= \frac{-2r_I(r_I - r_E)(1+\beta)^2}{V(k_I(1+\beta) - k_E)^3} - 2c < 0 \\ \frac{\partial^2 \pi_I^{SS}}{\partial r_I \partial k_I} &= \frac{(2r_I - r_E)(1+\beta)}{V(k_I(1+\beta) - k_E)^2}\end{aligned}$$

The hessian matrix is negative semi-definite for  $c \geq \frac{r_E^2(1+\beta)^2}{4V(k_I(1+\beta) - k_E)^3}$ . Solving first-ordered equations for  $r_I$  and  $k_I$ , we get negative value for  $r_I$  and  $\pi_I^{SS}$ . This case is not to be considered for further analysis.

(iii)  $G \leq r_I \leq k_I(1+\beta)V$ ,  $\pi_I^{SS} = 0$ .

We compare the profits for different scenarios and the incumbent's profit under this case is as follows:

$$\pi_I^{SS} = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \frac{(1+\beta)^2V^2}{64c} - \frac{\gamma V}{4ck_E^4V^2}; \quad 0 \leq c \leq c_1 \\ \frac{r_E^2(1+\beta)^2(k_EV - r_E)^2}{4ck_E^4V^2}; \quad c \geq c_1 \end{array} \right\}; \quad 0 \leq r_E \leq \frac{(k_E - \gamma)V}{2} \\ \left\{ \begin{array}{l} \frac{r_E^2(1+\beta)^2((k_E - \gamma)V - r_E)^2}{4c(k_E - \gamma)^4V^2} - \frac{\gamma r_E((k_E - \gamma)V - r_E)}{V(k_E - \gamma)^2}; \quad 0 \leq c \leq c_2 \\ \frac{r_E^2(1+\beta)^2(k_EV - r_E)^2}{4ck_E^4V^2}; \quad c \geq c_2 \end{array} \right\}; \quad \frac{(k_E - \gamma)V}{2} \leq r_E \leq \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \\ \frac{r_E^2(1+\beta)^2(k_EV - r_E)^2}{4ck_E^4V^2}; \quad \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \leq r_E \leq \frac{k_EV}{2} \\ \frac{(1+\beta)^2V^2}{64c}; \quad \frac{k_EV}{2} \leq r_E \leq k_EV \end{array} \right\}$$

Where

$$\begin{aligned}c_1 &= \frac{(k_EV - 2r_E)^2(1+\beta)^2(k_E^2V^2 - 4r_E^2 + 4r_Ek_EV)}{16k_E^4V^3\gamma} \\ c_2 &= \frac{r_E(1+\beta)^2(k_E^2V + r_E\gamma - k_E(2r_E + V\gamma))(2k_E^3V - r_E\gamma^2 + k_E\gamma(2r_E + V\gamma) - k_E^2(2r_E + 3V\gamma))}{4k_E^4V((k_E - \gamma)V - r_E)(k_E - \gamma)^2}\end{aligned}$$

**I-Usage and E-Usage:** We solve for the optimal prices ( $p_I$ ) and the content amount ( $k_I$ ) of the incumbent.

$$\begin{aligned}\text{(a)} \quad 0 \leq p_I &\leq \frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)} \\ \text{(i)} \quad 0 \leq p_E &\leq C,\end{aligned}$$

$$\pi_I^{UU} = p_I(k_I(1+\beta) - \frac{\gamma}{2})x - ck_I^2$$

Solving for  $p_I$  and  $k_I$ , we get  $k_I = \frac{\gamma}{2(1+\beta)}$ ,  $p_I = \frac{c\gamma}{(1+\beta)^2x}$ . At this point,  $\frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)}$  is  $-\infty$ . This case is not possible.

$$(ii) C \leq p_E \leq \frac{1}{x},$$

$$\pi_I^{UU} = p_I k_I (1 + \beta)x - ck_I^2$$

This is the case of a monopolist. By solving for the prices and the content amount, we get  $p_I = \frac{1}{x}$ ,

$$k_I = \frac{(1 + \beta)}{2c}, \text{ and } \pi_I^{UU} = \frac{(1 + \beta)^2}{4c}.$$

$$(b) \frac{2(k_I(1 + \beta) - \gamma)}{x(2k_I(1 + \beta) - \gamma)} \leq p_I \leq \frac{1}{x}$$

$$(i) 0 \leq p_E \leq B,$$

$$\pi_I^{UU} = 0$$

$$\pi_E^{UU} = p_E k_E x - ck_E^2$$

$$(ii) B \leq p_E \leq C,$$

$$\pi_I^{UU} = p_I(k_I(1 + \beta) - \frac{\gamma}{2})x - ck_I^2$$

The profit function is positive semi-definite in  $p_I$  and  $k_I$ . By solving sequentially, we get  $k_I = \frac{\gamma}{2(1 + \beta)}$ ,  $p_I = \frac{c\gamma}{x(1 + \beta)^2}$ , and  $\pi_I^{UU} = \frac{-c\gamma^2}{4(1 + \beta)^2} < 0$ . So, the optimal solution is  $p_I = \frac{1}{x}$ ,  $k_I = \frac{(1 + \beta)}{2c}$ , and  $\pi_I^{UU} = \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}$ .  $\pi_I^{UU} > 0$  for  $0 \leq c \leq \frac{(1 + \beta)^2}{2\gamma}$ . For  $c > \frac{(1 + \beta)^2}{2\gamma}$ , we consider the monopoly case (for the entrant) under the usage-based strategy.

$$(iii) C \leq p_E \leq \frac{1}{x},$$

$$\pi_I^{UU} = p_I k_I (1 + \beta)x - ck_I^2$$

This is the case of a monopolist. By solving for the prices and the content amount, we get  $p_I = \frac{1}{x}$ ,

$$k_I = \frac{(1 + \beta)}{2c}, \text{ and } \pi_I^{UU} = \frac{(1 + \beta)^2}{4c}.$$

**I-Subscription, E-Usage:** Here, incumbent decides  $r_I$  and  $k_I$  simultaneously.

$$(a) 0 \leq r_I \leq (k_I(1 + \beta) - \gamma(1 - p_E x))V$$

$$\pi_I^{US} = r_I \frac{(V - T_4)}{V} - ck_I^2$$

$$= \frac{r_I}{V} \left( V - \frac{r_I}{k_I(1 + \beta) - \gamma(1 - p_E x)} \right) - ck_I^2$$

The first-ordered conditions are:

$$\frac{\partial \pi_I^{US}}{\partial r_I} = 1 - \frac{2r_I}{(k_I(1 + \beta) - \gamma(1 - p_E x))V}$$

$$\frac{\partial \pi_I^{US}}{\partial k_I} = \frac{r_I^2(1 + \beta)}{V(k_I(1 + \beta) - \gamma(1 - p_E x))^2} - 2ck_I$$

$$\frac{\partial^2 \pi_I^{US}}{\partial r_I^2} = \frac{-2}{(k_I(1 + \beta) - \gamma(1 - p_E x))V} < 0$$

$$\frac{\partial^2 \pi_I^{US}}{\partial k_I^2} = \frac{-2r_I^2(1 + \beta)^3}{V(k_I(1 + \beta) - \gamma(1 - p_E x))^3} - 2c < 0$$

$$\frac{\partial^2 \pi_I^{US}}{\partial r_I \partial k_I} = \frac{2r_I(1 + \beta)}{V(k_I(1 + \beta) - \gamma(1 - p_E x))^2} < 0$$

The Hessian matrix is negative semi-definite, implying that the function is jointly concave in  $r_I$  and  $k_I$ . Solving for  $r_I$  and  $k_I$ , we get  $r_I = \frac{V}{2} \left( \frac{(1 + \beta)^2 V}{8c} - \gamma(1 - p_E x) \right)$ , and  $k_I = \frac{(1 + \beta)V}{8c}$ .

$$(b) (k_I(1 + \beta) - \gamma(1 - p_E x))V \leq r_I \leq (k_I(1 + \beta)V, \pi_I^{US} = 0.$$

### EC.6.3. Stage 3 Decision: Entrant's Price and Content Acquisition

In this section, we solve for the entrant's prices and content acquisition index simultaneously conditional on the incumbent's decision in Stage 4.

**I-Usage, E-Subscription:** Here, the entrant chooses  $r_E$  and  $k_E$  simultaneously by maximizing profits. Given the incumbent's optimal decisions, we solve for the entrant's optimal prices ( $r_E$ ) and content amount ( $k_E$ ).

$$\begin{aligned}\frac{\partial \pi_E^{SU}}{\partial r_E} &= 1 - \frac{2r_E}{k_E V} \\ \frac{\partial \pi_E^{SU}}{\partial k_E} &= \frac{r_E^2}{V k_E^2} - 2ck_E \\ \frac{\partial^2 \pi_E^{SU}}{\partial r_E^2} &= \frac{-2}{V k_E} < 0 \\ \frac{\partial^2 \pi_E^{SU}}{\partial k_E^2} &= \frac{r_E^2}{V k_E^3} - 2c < 0 \\ \frac{\partial^2 \pi_E^{SU}}{\partial r_E \partial k_E} &= \frac{2r_E}{V k_E^2}\end{aligned}$$

The Hessian matrix of the above profit function is negative semi-definite. The solution is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{V^2}{16c}$ ,  $\pi_E^{SU} = \frac{V^2}{64c}$ , and  $\pi_I^{SU} = \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ .  $\pi_I^{SU} > 0 \Rightarrow c \leq \frac{(1+\beta)^2}{2\gamma}$ . For  $c > \frac{(1+\beta)^2}{2\gamma}$ , the entrant will have monopoly profits, which is equivalent to  $\frac{V^2}{64c}$  under subscription.

**I-Subscription, E-Subscription:** Here, the entrant chooses  $r_E$  and  $k_E$  simultaneously by maximizing profits. Given the incumbent's optimal decisions, we solve for the entrant's optimal prices ( $r_E$ ) and content amount ( $k_E$ ).

$$(a) \ 0 \leq r_E \leq \frac{(k_E - \gamma)V}{2}$$

(i) For  $0 \leq c \leq c_1$ , the first-ordered equations are:

$$\begin{aligned}\frac{\partial \pi_E^{SS}}{\partial r_E} &= 1 - \frac{2r_E}{k_E V} \\ \frac{\partial \pi_E^{SS}}{\partial k_E} &= \frac{r_E^2}{V k_E^2} - 2ck_E \\ \frac{\partial^2 \pi_E^{SS}}{\partial r_E^2} &= \frac{-2}{V k_E} < 0 \\ \frac{\partial^2 \pi_E^{SS}}{\partial k_E^2} &= \frac{-2r_E^2}{V k_E^3} - 2c < 0 \\ \frac{\partial^2 \pi_E^{SS}}{\partial r_E \partial k_E} &= \frac{2r_E}{V k_E^2}\end{aligned}$$

And the Hessian matrix of the above profit function is negative semi-definite. The solution is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{V^2}{16c}$ , and  $\pi_E^{SS} = \frac{V^2}{64c}$ . Now  $\frac{V^2}{16c} > \frac{(V - 8c\gamma)V}{16c}$ . The solution is  $r_E = \frac{(V - 8c\gamma)V}{16c}$ .

(ii) For  $c > c_1$ , the first-ordered equations are:

$$\frac{\partial \pi_E^{SS}}{\partial r_E} = 1 - \frac{2r_E}{(k_E - \gamma)V}$$

$$\begin{aligned}\frac{\partial \pi_E^{SS}}{\partial k_E} &= \frac{r_E^2}{V(k_E - \gamma)^2} - 2ck_E \\ \frac{\partial^2 \pi_E^{SS}}{\partial r_E^2} &= \frac{-2}{V(k_E - \gamma)} < 0 \\ \frac{\partial^2 \pi_E^{SS}}{\partial k_E^2} &= \frac{-2r_E^2}{V(k_E - \gamma)^3} - 2c < 0 \\ \frac{\partial^2 \pi_E^{SS}}{\partial r_E \partial k_E} &= \frac{2r_E}{V(k_E - \gamma)^2}\end{aligned}$$

And the Hessian matrix of the above profit function is negative semi-definite. The solution is

$$k_E = \frac{V}{8c}, r_E = \frac{(V - 8c\gamma)V}{16c}, \text{ and } \pi_E^{SS} = \frac{V^2}{64c} - \frac{\gamma V}{4}.$$

$$(b) \frac{(k_E - \gamma)V}{2} \leq r_E \leq \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)}$$

(i) For  $0 \leq c \leq c_2$ , solution is  $r_E = \frac{k_E V}{2} > \frac{(k_E - \gamma)k_E V}{(2k_E - \gamma)}$ . The solution is  $r_E = \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)}$ , and  $k_E = \frac{V}{8c}$ .

(ii) For  $c > c_2$ , the solution is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{(V - 8c\gamma)V}{16C}$ , and  $\pi_E^{SS} = \frac{V^2}{64c} - \frac{\gamma V}{4}$ .

(c)  $\frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)} \leq r_E \leq \frac{k_E V}{2}$ , Solution is  $r_E = \frac{(k_E - \gamma)V}{2} < \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)}$ . The solution is  $r_E = \frac{k_E(k_E - \gamma)V}{(2k_E - \gamma)}$ , we then solve for  $k_E$ .  $\frac{k_E V}{2} \leq r_E \leq k_E V$ , the solution is  $r_E = \frac{(k_E - \gamma)V}{2} < \frac{k_E V}{2}$ . The solution is  $r_E = \frac{k_E V}{2}$ , we then solve for  $k_E$ .

We compare profits and derive the following outcome:

$$\begin{aligned}r_E &= \begin{cases} \frac{(V - 8c\gamma)V}{16c}; & 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; & c \geq \frac{V}{16\gamma} \end{cases} \\ k_E &= \begin{cases} \frac{V}{8c}; & 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; & c \geq \frac{V}{16\gamma} \end{cases} \\ \pi_E^{SS} &= \begin{cases} \frac{V^2}{64c} - \frac{\gamma V}{4}; & 0 \leq c \leq \frac{V}{16\gamma} \\ 0; & c \geq \frac{V}{16\gamma} \end{cases}\end{aligned}$$

**I-Usage, E-Usage:** Here, the entrant chooses  $p_E$  and  $k_E$  simultaneously by maximizing profits. Given the incumbent's optimal decisions, we solve for the entrant's optimal prices ( $p_E$ ) and the content amount ( $k_E$ ).

When  $\pi_I^{UU} = \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}$ , we have  $\pi_E^{UU} = p_E(k_E - \frac{\gamma}{2})x - ck_E^2$ . Solving for  $p_E$  and  $k_E$ , we have

$$\pi_E^{UU} = \left\{ \left\{ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1 \right\} ; \left\{ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \right\} ; \left\{ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq c_L \\ 0; \quad c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ \frac{1}{4c}; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \right\} ; \quad \beta > \frac{2}{\sqrt{3}} - 1 \right\}$$

Where  $\pi_{E1} = \frac{(1+\beta)^2 - 4cx\gamma(1+\beta)^2 + 2c^2x\gamma^2}{4cx^2(1+\beta)^2}$ ,  $c_L = \frac{(1+\beta)(2x(1+\beta) - \sqrt{4x^2(1+\beta)^2 - 2x})}{2x\gamma}$ . Note that we consider the monopoly profits of the entrant for  $c > \frac{(1+\beta)^2}{2\gamma}$ .

**I-Subscription, E-Usage:** Here, the entrant chooses  $p_E$  and  $k_E$  simultaneously by maximizing profits. Given the incumbent's optimal decisions, we solve for the entrant's optimal prices ( $p_E$ ) and the content amount ( $k_E$ ). Entrant's profit function is  $\pi_E^{US} = \frac{p_E(2k_E - \gamma)x}{2} - ck_E^2$ . The profit function is jointly convex in  $p_E$  and  $k_E$ . By solving it sequentially, we get  $p_E = \frac{1}{x}$ ,  $k_E = \frac{1}{2c}$ , and  $\pi_E^{US} = \frac{1}{4c} - \frac{\gamma}{2}$ .  $\pi_E > 0$  for  $0 \leq c \leq \frac{1}{2\gamma}$ . For  $c > \frac{1}{2\gamma}$ , we consider the monopolist profit for the incumbent under subscription.

#### EC.6.4. Stage 2: Incumbents' Pricing Strategy Decision

In this stage, we solve for the incumbent's optimal strategy given entrant's strategy. Incumbent has two options to choose from: *i* Subscription-based strategy or *ii* usage-based strategy. Given the optimal decisions by the incumbent (in Stage 4) and entrant (in Stage 4), we compare the incumbents' optimal profits when the entrant is usage-based and subscription-based. When the entrant adopts a usage-based strategy, profits of the incumbent under a usage-based strategy is :

$$\pi_I^{UU} = \left\{ \left\{ \left\{ \begin{array}{l} \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}; \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ 0; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \right\} ; \left\{ \left\{ \begin{array}{l} \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}; \quad 0 \leq c \leq c_L \\ \frac{(1+\beta)^2}{4c}; \quad c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ 0; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \right\} ; \quad 0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1 \right\}$$

The incumbent's profits under subscription, when the entrant is a usage-based service provider, is:  $\pi_I^{US} = \frac{(1+\beta)^2 V^2}{64c}$ .

We now compare the above profits when the entrant is a usage-based service provider. Check if,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} < \frac{(1+\beta)^2 V^2}{64c} \Rightarrow c > \frac{(16-V^2)(1+\beta)^2}{32\gamma}$  if  $(16-V^2) > 0 \Rightarrow V < 4$ .

For  $V > 4$ , the above inequality holds for all  $c > 0$ , implying that  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} < \frac{(1+\beta)^2 V^2}{64c}$ . Also  $\frac{(16-V^2)(1+\beta)^2}{32\gamma} < \frac{(1+\beta)^2}{2\gamma}$ . Check if  $\frac{(1+\beta)^2}{4c} < \frac{(1+\beta)^2 V^2}{64c} \Rightarrow V > 4$ . Check if  $\frac{(16-V^2)(1+\beta)^2}{32\gamma} < c_L$ .

$$c_L = \begin{cases} > \frac{(16-V^2)(1+\beta)^2}{32\gamma}; & \frac{2}{\sqrt{3}} - 1 \leq \beta \leq \frac{32}{\sqrt{(48+V^2)(16-V^2)}} - 1 \\ \left\{ \begin{array}{l} > \frac{(16-V^2)(1+\beta)^2}{32\gamma}; \\ < \frac{(16-V^2)(1+\beta)^2}{32\gamma}; \end{array} \right. & \frac{2}{3(1+\beta)^2} < x \leq \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}; \beta > \frac{32}{\sqrt{(48+V^2)(16-V^2)}} - 1 \\ & \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)} < x \leq \frac{1}{2} \end{cases}$$

When the entrant adopts a usage-based strategy, the optimal pricing strategy for the incumbent is

$$j^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \left(\frac{2}{\sqrt{3}} - 1\right) \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \frac{(1+\beta)^2(16-V^2)}{32\gamma} < c < c_L \end{array} \right\}; \quad \left(\frac{2}{\sqrt{3}} - 1\right) < \beta \leq \beta_1 \\ \left\{ \begin{array}{l} U; c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x \leq \frac{1}{2} \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq x \leq \frac{2}{3(1+\beta)^2} \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \frac{(1+\beta)^2(16-V^2)}{32\gamma} < c < c_L \end{array} \right\}; \quad \frac{2}{3(1+\beta)^2} < x < x_1 \\ \left\{ \begin{array}{l} U; c_L < c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad \beta > \beta_1 \\ \left\{ \begin{array}{l} U; 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ S; c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}; \quad x_1 < x \leq \frac{1}{2} \\ S; \end{array} \right\}; \quad 0 \leq V \leq 4 \quad \left. \vphantom{j^*} \right\} \quad V > 4$$

When the entrant adopts a subscription-based strategy, the incumbent's profits under a usage-based strategy, is  $\pi_I^{S^U} = \left\{ \begin{array}{l} \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}; \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ 0; \quad c > \frac{(1+\beta)^2}{2\gamma} \end{array} \right\}$

The incumbent's profit under a subscription-based strategy, when the entrant adopts a subscription-based strategy, is  $\pi_I^{S^S} = \left\{ \begin{array}{l} \frac{(1+\beta)^2(V^2 - 64c^2\gamma^2)}{64cV^2}; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ \frac{(1+\beta)^2 V^2}{64c}; \quad c \geq \frac{V}{16\gamma} \end{array} \right\}$ .

$$j^* = \left\{ \left\{ \left\{ \begin{array}{l} S; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ U; \quad \frac{V}{16\gamma} \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\} ; \quad 0 \leq \beta \leq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \right\} ; \quad 0 \leq V \leq 4 \right. \\ \left. \left\{ \left\{ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ S; \quad c \geq \frac{(1+\beta)^2(16-V^2)}{(1+\beta)^2} \end{array} \right\} ; \quad \beta \geq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \right\} \right. \\ \left. \left\{ \begin{array}{l} S; \quad 0 \leq \beta \leq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1 \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ S; \quad c \geq \frac{V}{16\gamma} \end{array} \right\} ; \quad \beta \geq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1 \end{array} \right\} ; \quad 4 \leq V \leq 8(1+\beta)^2 \right. \\ \left. \left. \left. S; \quad V \geq 8(1+\beta)^2 \right. \right. \right. \end{array} \right.$$

### EC.6.5. Stage 1: Entrant's Optimal Pricing Strategy

Given the optimal pricing strategy in Stage 2, we now compare the profits of the entrant for the optimal

pricing strategy. When the entrant chooses a subscription-based strategy, the profit of the entrant is

$$\pi_E^S = \left\{ \left\{ \left\{ \left\{ \begin{array}{l} \frac{V^2}{64c} - \frac{\gamma V}{4}; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ \frac{V^2}{64c}; \quad \frac{V}{16\gamma} \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ 0; \quad c > \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\} ; \quad 0 \leq \beta \leq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \right\} ; \quad 0 \leq V \leq 4 \right. \\ \left. \left\{ \left\{ \left\{ \begin{array}{l} \frac{V^2}{64c}; \quad 0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ 0; \quad c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\} ; \quad \beta \geq -1 + \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} \right\} \right. \\ \left. \left\{ \left\{ \begin{array}{l} \frac{V^2}{64c} - \frac{\gamma V}{4}; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ 0; \quad c \geq \frac{V}{16\gamma} \end{array} \right\} ; \quad 0 \leq \beta \leq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1 \right\} ; \quad 4 \leq V \leq 8(1+\beta)^2 \right. \\ \left. \left\{ \left\{ \begin{array}{l} \frac{V^2}{64c}; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ 0; \quad c \geq \frac{V}{16\gamma} \end{array} \right\} ; \quad \beta \geq \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1 \right\} \right. \\ \left. \left\{ \left\{ \begin{array}{l} \frac{V^2}{64c} - \frac{\gamma V}{4}; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ 0; \quad c \geq \frac{V}{16\gamma} \end{array} \right\} ; \quad V \geq 8(1+\beta)^2 \right. \right. \end{array} \right.$$

. Entrant's profits under a usage-based strategy are

$$\pi_E^U = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq \frac{(16-V^2)(1+\beta)^2}{32\gamma} \\ 0; \quad c > \frac{(16-V^2)(1+\beta)^2}{32\gamma} \end{array} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \\ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq \frac{(16-V^2)(1+\beta)^2}{32\gamma} \\ \frac{1}{4c} - \frac{\gamma}{2}; \quad \frac{(16-V^2)(1+\beta)^2}{32\gamma} < c \leq \frac{1}{2\gamma} \\ 0; \quad c > \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq \frac{(16-V^2)(1+\beta)^2}{32\gamma} \\ \frac{1}{4c} - \frac{\gamma}{2}; \quad \frac{(16-V^2)(1+\beta)^2}{32\gamma} < c \leq \frac{1}{2\gamma} \\ 0; \quad c > \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq x \leq x_1 \\ \left\{ \begin{array}{l} \pi_{E1}; \quad 0 \leq c \leq c_L \\ 0; \quad c > c_L \end{array} \right\}; \quad \beta > \beta_1 \\ \left\{ \begin{array}{l} \frac{1}{4c} - \frac{\gamma}{2}; \quad 0 \leq c \leq \frac{1}{2\gamma} \\ 0; \quad c > \frac{1}{2\gamma} \end{array} \right\}; \quad x_1 < x \leq \frac{1}{2} \\ \left\{ \begin{array}{l} \frac{1}{4c} - \frac{\gamma}{2}; \quad 0 \leq c \leq \frac{1}{2\gamma} \\ 0; \quad c > \frac{1}{2\gamma} \end{array} \right\}; \quad V > 4 \end{array} \right\}; \quad \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} < V \leq 4$$

We now compare the above profits for the optimal strategy of the entrant. The entrant's optimal pricing

$$\text{strategy is } l^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} U; \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} U; \quad 0 \leq x \leq \bar{x}_1 \\ S; \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ \text{n.a.}; \quad c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} U; \quad 0 \leq x \leq \bar{x}_1 \\ S; \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ U; \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ \text{n.a.}; \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_2 \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq c_u \\ \left\{ \begin{array}{l} U; \quad 0 \leq x \leq \bar{x}_1 \\ S; \quad \bar{x}_1 \leq x \leq \frac{1}{2} \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ U; \quad 0 \leq x \leq x_1 \\ \text{n.a.}; \quad x_1 \leq x \leq \frac{1}{2} \\ \text{n.a.}; \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \beta \geq \beta_2 \\ \left\{ \begin{array}{l} U; \quad 0 \leq c \leq \frac{1}{2\gamma} \\ \text{n.a.}; \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left\{ \begin{array}{l} S; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ U; \quad \frac{V}{16\gamma} \leq c \leq \frac{1}{2\gamma} \\ \text{n.a.}; \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \beta \geq \beta_1 \\ \left\{ \begin{array}{l} S; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad 4 \leq V \leq 8 \\ \left\{ \begin{array}{l} S; \quad 0 \leq c \leq \frac{V}{16\gamma} \\ \text{n.a.}; \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad V \geq 8 \end{array} \right\}; \quad \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \leq V \leq 4$$

$$\text{where } c_u = \frac{(1+\beta)^2}{\gamma} - \frac{(1+\beta)\sqrt{V^2+64\beta(2+\beta)}}{8\gamma}, \quad \bar{x}_1 = \frac{4(4c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{V^2(1+\beta)^4+16c^2\gamma(-2(1+\beta)^2+c\gamma)^2})}{V^2(1+\beta)^2},$$

$$x_1 = \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}, \quad \beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}} \text{ and } \beta_1 = \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma)^2}} - 1.$$

**Proof of Lemma EC.1 (a):** The incumbent's optimal optimal pricing strategy is derived in the Stage 2 of the game (refer Section EC.6.4 of the e-companion) when the entrant is a subscription-based service provider.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example. at  $\{V = 1, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , incumbent chooses a usage-based strategy.

**Proof of Lemma EC.1(b):** The optimal pricing strategy of the incumbent is derived in Stage 2 of the game (refer Section EC.6.4 of the e-companion) when the entrant adopts a usage-based pricing strategy.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example. at  $\{V = 1, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , incumbent chooses a usage-based strategy.

**Proof of Lemma EC.2:** The optimal pricing strategy of the entrant is derived in Stage 1 of the game (refer Section EC.6.5 of the e-companion) .

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example. at  $\{V = 1, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , entrant chooses a usage-based strategy.

**Proof of Lemma EC.3:** In this, we combine the optimal pricing strategy derived in Lemma EC.1 and Lemma EC.2. The complete Nash equilibrium (combined pricing strategy) is



**Proof of Proposition 1(c)** Given the optimal strategy in Lemma EC.3, we get this result.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example. at  $\{V = 6, c = 0.5, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , the incumbent firm prefers a subscription-based pricing strategy, while the entrant optimally adopts a usage-based strategy.

**Proof of Proposition 2** Given the optimal strategy in Lemma EC.3, we get this result.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example. at  $\{V = 2, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , the incumbent firm prefers a usage-based pricing strategy, while the entrant does not enter the market.

**Proof of Proposition 3 (i):** We compare the profits of the incumbent and the entrant for this. The optimal profit for the incumbent and entrant is

$$(\pi_I^*, \pi_E^*) = \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq c \leq c_u \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq x \leq \bar{x}_1 \right\} \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2 (V^2 - 64c^2 \gamma^2)}{64cV^2}, \frac{V^2}{64c} - \frac{\gamma V}{4} \right); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \right\} \right. \\ \left. \left( \frac{(1+\beta)^2}{4c}, 0 \right); \quad c \geq \frac{(1+\beta)^2 (16 - V^2)}{32\gamma} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \end{array} \right\}; \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq c \leq c_u \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq x \leq \bar{x}_1 \right\} \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \right\} \right. \\ \left. \left( \frac{(1+\beta)^2 V^2}{64c}, \frac{1}{4c} - \frac{\gamma}{2} \right); \quad \frac{(1+\beta)^2 (16 - V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \right\}; \quad 0 \leq \beta \leq \beta_2 \\ \left( \frac{(1+\beta)^2}{4c}, 0 \right); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq c \leq c_u \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \pi_{E1} \right); \quad 0 \leq x \leq \bar{x}_1 \right\} \right. \\ \left. \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad \bar{x}_1 \leq x \leq \frac{1}{2} \right\} \right. \\ \left. \left( \frac{(1+\beta)^2 V^2}{64c}, \frac{1}{4c} - \frac{\gamma}{2} \right); \quad 0 \leq x \leq x_1 \right\}; \quad \frac{(1+\beta)^2 (16 - V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ \left( \frac{(1+\beta)^2}{4c}, 0 \right); \quad x_1 \leq x \leq \frac{1}{2} \\ \left( \frac{(1+\beta)^2}{4c}, 0 \right); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \beta \geq \beta_2 \end{array} \right\}; \quad \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \leq V \leq 4 \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2 V^2}{64c}, \frac{1}{4c} - \frac{\gamma}{2} \right); \quad 0 \leq c \leq \frac{1}{2\gamma} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad 0 \leq c \leq \frac{V}{16\gamma} \right\}; \\ \left\{ \left( \frac{(1+\beta)^2 V^2}{64c}, \frac{1}{4c} - \frac{\gamma}{2} \right); \quad \frac{V}{16\gamma} \leq c \leq \frac{1}{2\gamma} \right\}; \quad \beta \geq \beta_1 \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2 (V^2 - 64c^2 \gamma^2)}{64cV^2}, \frac{V^2}{64c} - \frac{\gamma V}{4} \right); \quad 0 \leq c \leq \frac{V}{16\gamma} \right\}; \quad 0 \leq \beta \leq \beta_1 \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad 0 \leq c \leq \frac{V}{16\gamma} \right\}; \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad \beta \geq \beta_1 \end{array} \right\}; \quad 4 \leq V \leq 8 \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2 (V^2 - 64c^2 \gamma^2)}{64cV^2}, \frac{V^2}{64c} - \frac{\gamma V}{4} \right); \quad 0 \leq c \leq \frac{V}{16\gamma} \right\}; \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad 8 \leq V \leq 8(1+\beta)^2 \\
 \left\{ \left. \begin{array}{l} \left\{ \left( \frac{(1+\beta)^2 (V^2 - 64c^2 \gamma^2)}{64cV^2}, \frac{V^2}{64c} - \frac{\gamma V}{4} \right); \quad 0 \leq c \leq \frac{V}{16\gamma} \right\}; \\ \left( \frac{(1+\beta)^2 V^2}{64c}, 0 \right); \quad c \geq \frac{V}{16\gamma} \end{array} \right\}; \quad V \geq 8(1+\beta)^2 \end{array} \right\}$$

where  $c_u = \frac{(1+\beta)^2}{\gamma} - \frac{(1+\beta)\sqrt{V^2+64\beta(2+\beta)}}{8\gamma}$ ,  $\bar{x}_1 = \frac{4(4c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{V^2(1+\beta)^4+16c^2\gamma(-2(1+\beta)^2+c\gamma)^2})}{V^2(1+\beta)^2}$ ,  
 $x_1 = \frac{512}{(1+\beta)^2(48+V^2)(16-V^2)}$ ,  $\beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}}$  and  $\beta_1 = \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1$ .

(a)  $0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)}$

(i)  $0 \leq c \leq c_u$ , we find that  $\pi_I^* = \left\{ \begin{array}{l} \leq \pi_E^* ; \\ \left\{ \begin{array}{l} \leq \pi_E^* ; \quad 0 \leq x \leq \hat{x} \\ \geq \pi_E^* ; \quad \hat{x} \leq x \leq \frac{1}{2} \end{array} \right\} ; \quad \beta \geq -1 + \frac{2}{\sqrt{3}} \end{array} \right\}$ , where  $\hat{x} = \frac{c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{(1+\beta)^6 - 2c(1+\beta)^4\gamma + 4c^2(1+\beta)^4\gamma^2 - 4c^3(1+\beta)^2\gamma^3 + c^4\gamma^4}}{(1+\beta)^2((1+\beta)^2 - 2c\gamma)}$

(ii)  $c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$

(1)  $0 \leq x \leq \bar{x}_1$ , we find that  $\pi_I^* = \left\{ \begin{array}{l} \leq \pi_E^* ; \\ \left\{ \begin{array}{l} \leq \pi_E^* ; \quad 0 \leq x \leq \hat{x} \\ \geq \pi_E^* ; \quad \hat{x} \leq x \leq \bar{x}_1 \end{array} \right\} ; \quad \beta \geq -1 + \frac{2}{\sqrt{3}} \end{array} \right\}$ , where  $\hat{x} = \frac{c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{(1+\beta)^6 - 2c(1+\beta)^4\gamma + 4c^2(1+\beta)^4\gamma^2 - 4c^3(1+\beta)^2\gamma^3 + c^4\gamma^4}}{(1+\beta)^2((1+\beta)^2 - 2c\gamma)}$

(2)  $\bar{x}_1 \leq x \leq \frac{1}{2}$ ,  $\pi_I^* \geq \pi_E^*$ .

(iii)  $c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$ ,  $\pi_I^* \geq \pi_E^*$ .

(b)  $\frac{4\sqrt{\beta(1+\beta)}}{(1+\beta)} \leq V \leq 4$

(A)  $0 \leq \beta \leq \beta_2$

(i)  $0 \leq c \leq c_u$ , we find that  $\pi_I^* = \left\{ \begin{array}{l} \leq \pi_E^* ; \\ \left\{ \begin{array}{l} \leq \pi_E^* ; \quad 0 \leq x \leq \hat{x} \\ \geq \pi_E^* ; \quad \hat{x} \leq x \leq \frac{1}{2} \end{array} \right\} ; \quad -1 + \frac{2}{\sqrt{3}} \leq \beta \leq \beta_2 \end{array} \right\}$ , where  $\hat{x} = \frac{c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{(1+\beta)^6 - 2c(1+\beta)^4\gamma + 4c^2(1+\beta)^4\gamma^2 - 4c^3(1+\beta)^2\gamma^3 + c^4\gamma^4}}{(1+\beta)^2((1+\beta)^2 - 2c\gamma)}$

(ii)  $c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$

(1)  $0 \leq x \leq \bar{x}_1$ , we find that  $\pi_I^* = \left\{ \begin{array}{l} \leq \pi_E^* ; \\ \left\{ \begin{array}{l} \leq \pi_E^* ; \quad 0 \leq x \leq \hat{x} \\ \geq \pi_E^* ; \quad \hat{x} \leq x \leq \bar{x}_1 \end{array} \right\} ; \quad -1 + \frac{2}{\sqrt{3}} \leq \beta \leq \beta_2 \end{array} \right\}$ , where  $\hat{x} = \frac{c\gamma(-2(1+\beta)^2+c\gamma) + \sqrt{(1+\beta)^6 - 2c(1+\beta)^4\gamma + 4c^2(1+\beta)^4\gamma^2 - 4c^3(1+\beta)^2\gamma^3 + c^4\gamma^4}}{(1+\beta)^2((1+\beta)^2 - 2c\gamma)}$

(2)  $\bar{x}_1 \leq x \leq \frac{1}{2}$ ,  $\pi_I^* \geq \pi_E^*$ .

(iii)  $\frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma}$ ,  $\pi_I^* \geq \pi_E^*$ .

(iv)  $c \geq \frac{1}{2\gamma}$ ,  $\pi_I^* \geq \pi_E^*$ .

(B)  $\beta \geq \beta_2$

(i)  $0 \leq c \leq c_u$ ,  $\pi_I^* \geq \pi_E^*$ .

(ii)  $c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$ ,

- (i)  $0 \leq x \leq \bar{x}_1, \pi_I^* \geq \pi_E^*$ .
  - (ii)  $\bar{x}_1 \leq x \leq \frac{1}{2}, \pi_I^* \geq \pi_E^*$ .
  - (iii)  $\frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma}$ 
    - (1)  $0 \leq x \leq x_1, \pi_I^* \geq \pi_E^*$ .
    - (2)  $x_1 \leq x \leq \frac{1}{2}, \pi_I^* \geq \pi_E^*$ .
  - (iv)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
- (c)  $4 \leq V \leq 8$
- (A)  $0 \leq \beta \leq \beta_1$ 
    - (i)  $0 \leq c \leq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
    - (ii)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
  - (B)  $\beta \geq \beta_1$ 
    - (i)  $0 \leq c \leq \frac{V}{16\gamma}$ , we have  $\pi_I^* = \begin{cases} \leq \pi_E^*; & \beta_1 \leq \beta \leq \frac{\sqrt{(V^2+32c\gamma)}-4}{4} \\ \geq \pi_E^*; & \beta \geq \frac{\sqrt{(V^2+32c\gamma)}-4}{4} \end{cases}$
    - (ii)  $\frac{v}{16\gamma} \leq c \leq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
    - (iii)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
- (d)  $8 \leq V \leq 8(1+\beta)^2$
- (A)  $0 \leq \beta \leq \beta_1$ 
    - (i)  $0 \leq c \leq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
    - (ii)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
  - (B)  $\beta \geq \beta_1$ 
    - (i)  $0 \leq c \leq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
    - (ii)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
- (e)  $V \geq 8(1+\beta)^2$
- (i)  $0 \leq c \leq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .
  - (ii)  $c \geq \frac{1}{2\gamma}, \pi_I^* \geq \pi_E^*$ .

**Part (a) of Proposition 3(i)** When the valuation is low (i.e.,  $0 \leq V \leq 4$ ), the entrant gets a higher profit when the cost of content acquisition ( $c$ ) and consumer stickiness are low enough (i.e.,  $0 \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$  and  $0 \leq \beta \leq \frac{2}{\sqrt{3}} - 1$ ). In that setting, both incumbent and entrant choose a usage-based strategy.

We verify that the model parameter space ( $V \geq 0, c > 0, 0 \leq x \leq \frac{1}{2}, \gamma > 0, \beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 1, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , both incumbent and entrant choose a usage-based strategy. And  $\pi_E > \pi_I$ .

**Part (b) of Proposition 3(ii)** When the valuation is low (i.e.,  $0 \leq V \leq 4$ ), incumbent gets a higher profit when the cost of content acquisition is high enough (i.e.,  $c \geq \frac{(16-V^2)(1+\beta)^2}{32\gamma}$ ). In that scenario, the incumbent differentiates its strategy from that of the entrant.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 2, c = 1.8, x = 0.4, \gamma = 0.5, \beta = 0.5\}$ , the entrant chooses a subscription-based strategy whereas the incumbent chooses a usage-based strategy. And  $\pi_E < \pi_I$ .

**Proof of Proposition 3 (ii):** When the valuation is high (i.e.,  $V \geq 4$ ), the incumbent gets a higher profit.

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $0 \leq x \leq \frac{1}{2}$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 4.5, c = 0.5, x = 0.4, \gamma = 0.5, \beta = 0.5\}$ ,  $\pi_E < \pi_I$ .

**Proof of Proposition 4:** Given the optimal strategy outlined in Lemma EC.3, the following are the content attractive index chosen by the entrant and incumbent.

$$(k_I^*, k_E^*) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)(V^2 - 64c^2\gamma^2)}{8cV}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)}{2c}, \text{n.a.} \right); \end{array} \right. \left. \begin{array}{l} 0 \leq c \leq c_u \\ 0 \leq x \leq \bar{x}_1 \\ \bar{x}_1 \leq x \leq \frac{1}{2} \\ c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \end{array} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \\ \left\{ \begin{array}{l} \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \frac{1}{2c} \right); \\ \left( \frac{(1+\beta)}{2c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{1}{2cx} \right); \\ \left( \frac{(1+\beta)V}{8c}, \frac{1}{2c} \right); \\ \left( \frac{(1+\beta)}{2c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)}{2c}, \text{n.a.} \right); \end{array} \right\}; \quad \begin{array}{l} 0 \leq c \leq c_u \\ 0 \leq x \leq \bar{x}_1 \\ \bar{x}_1 \leq x \leq \frac{1}{2} \\ \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \\ 0 \leq c \leq c_u \\ c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma} \\ \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \begin{array}{l} 0 \leq \beta \leq \beta_2 \\ \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \leq V \leq 4 \\ \beta \geq \beta_2 \end{array} \\ \left\{ \begin{array}{l} \left( \frac{(1+\beta)V}{8c}, \frac{1}{2c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \frac{1}{2c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)V}{8c}, \frac{1}{2c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \end{array} \right\}; \quad \begin{array}{l} 0 \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \\ 0 \leq c \leq \frac{V}{16\gamma} \\ \frac{V}{16\gamma} \leq c \leq \frac{1}{2\gamma} \\ c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad \begin{array}{l} 0 \leq \beta \leq \beta_1 \\ 4 \leq V \leq 8 \\ \beta \geq \beta_1 \end{array} \\ \left\{ \begin{array}{l} \left( \frac{(1+\beta)(V^2 - 64c^2\gamma^2)}{8cV}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)}{2c}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \\ \left( \frac{(1+\beta)(V^2 - 64c^2\gamma^2)}{8cV}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \end{array} \right\}; \quad \begin{array}{l} 0 \leq c \leq \frac{V}{16\gamma} \\ c \geq \frac{V}{16\gamma} \\ 0 \leq \beta \leq \beta_1 \\ \beta \geq \beta_1 \end{array} \\ \left\{ \begin{array}{l} \left( \frac{(1+\beta)(V^2 - 64c^2\gamma^2)}{8cV}, \frac{V}{8c} \right); \\ \left( \frac{(1+\beta)V}{8c}, \text{n.a.} \right); \end{array} \right\}; \quad \begin{array}{l} 0 \leq c \leq \frac{V}{16\gamma} \\ c \geq \frac{V}{16\gamma} \end{array} \end{array} \right\}; \quad \begin{array}{l} 0 \leq \beta \leq \beta_1 \\ 8 \leq V \leq 8(1+\beta)^2 \\ V \geq 8(1+\beta)^2 \end{array} \\ \text{(a) } 0 \leq V \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \end{array} \right\}$$

- (i)  $0 \leq c \leq c_u$ , we find that  $k_I = \left\{ \begin{array}{l} \leq k_E; \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; \quad \frac{1}{(1+\beta)} \leq x \leq \frac{1}{2} \end{array} \right\}; \quad \beta \geq 1 \end{array} \right\}$
- (ii)  $c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$
- (1)  $0 \leq x \leq \bar{x}_1$ , we find that  $k_I = \left\{ \begin{array}{l} \leq k_E; \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; \quad \frac{1}{(1+\beta)} \leq x \leq \bar{x}_1 \end{array} \right\}; \quad \beta \geq 1 \end{array} \right\}$
- (2)  $\bar{x}_1 \leq x \leq \frac{1}{2}$ , we find that  $\frac{(1+\beta)(V^2 - 64c^2\gamma^2)^2}{8cV} \geq \frac{V}{8c}$  ( $k_I \geq k_E$ ).
- (iii)  $c \geq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$ ,  $k_I \geq k_E$ .
- (b)  $\frac{4\sqrt{\beta(1+\beta)}}{(1+\beta)} \leq V \leq 4$
- (A)  $0 \leq \beta \leq \beta_2$
- (i)  $0 \leq c \leq c_u$ , we have  $k_I = \left\{ \begin{array}{l} \leq k_E; \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq \beta \leq 1 \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; \quad \frac{1}{(1+\beta)} \leq x \leq \frac{1}{2} \end{array} \right\}; \quad 1 \leq \beta \leq \beta_2 \end{array} \right\}; \quad \frac{4\sqrt{\beta(1+\beta)}}{(1+\beta)} \leq V \leq 4\sqrt{\sqrt{3}-1} \\ 4\sqrt{\sqrt{3}-1} \leq V \leq 4 \end{array} \right\}$
- (ii)  $c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma}$
- (1)  $0 \leq x \leq \bar{x}_1$ , we have  $k_I = \left\{ \begin{array}{l} \leq k_E; \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq \beta \leq 1 \\ \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; \quad \frac{1}{(1+\beta)} \leq x \leq \bar{x}_1 \end{array} \right\}; \quad 1 \leq \beta \leq \beta_2 \end{array} \right\}; \quad \frac{4\sqrt{\beta(1+\beta)}}{(1+\beta)} \leq V \leq 4\sqrt{\sqrt{3}-1} \\ 4\sqrt{\sqrt{3}-1} \leq V \leq 4 \end{array} \right\}$
- (2)  $\bar{x}_1 \leq x \leq \frac{1}{2}$ , check if  $\frac{(1+\beta)}{2c} \leq \frac{V}{8c} \Rightarrow V \geq 4(1+\beta) \geq 4$ . It implies that  $\frac{(1+\beta)}{2c} \geq \frac{V}{8c}$  ( $k_I \geq k_E$ ).
- (iii)  $\frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma}$ , check if  $\frac{(1+\beta)V}{8c} \leq \frac{1}{2c} \Rightarrow V \leq \frac{4}{(1+\beta)} \leq 4 \forall \beta \geq 0$ . Further, check if  $\frac{4}{(1+\beta)} \geq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \Rightarrow 1 \geq \beta(2+\beta) \Rightarrow \beta \leq -1 + \sqrt{2}$ . Further, check if  $-1 + \frac{4}{\sqrt{(48+V^2)(16-V^2)}} \leq -1 + \sqrt{2} \Rightarrow 512 \leq 768 - 32V^2 - V^4$  (which does not hold). It implies that  $k_I = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \leq k_E; \quad \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \leq V \leq \frac{4}{(1+\beta)} \\ \geq k_E; \quad \frac{4}{(1+\beta)} \leq V \leq 4 \end{array} \right\}; \quad 0 \leq \beta \leq \sqrt{2}-1 \\ \geq k_E; \quad \sqrt{2}-1 \leq \beta \leq \beta_1 \end{array} \right\}$ .
- (iv)  $c \geq \frac{1}{2\gamma}$ ,  $k_I \geq k_E$ .
- (B)  $\beta \geq \beta_2$
- (i)  $0 \leq c \leq c_u$ , we find that  $k_I = \left\{ \begin{array}{l} \leq k_E; \quad 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; \quad \frac{1}{(1+\beta)} \leq x \leq \frac{1}{2} \end{array} \right\}$

$$(ii) \quad c_u \leq c \leq \frac{(1+\beta)^2(16-V^2)}{32\gamma},$$

$$(1) \quad 0 \leq x \leq \bar{x}_1, \text{ we find that } k_I = \begin{cases} \leq k_E; & 0 \leq x \leq \frac{1}{(1+\beta)} \\ \geq k_E; & \frac{1}{(1+\beta)} \leq x \leq \frac{1}{2} \end{cases}$$

$$(2) \quad \bar{x}_1 \leq x \leq \frac{1}{2}, \text{ we find that } k_I \geq k_E.$$

$$(iii) \quad \frac{(1+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma}$$

$$(1) \quad 0 \leq x \leq x_1, \text{ check if } \frac{(1+\beta)V}{8c} \leq \frac{1}{2c} \Rightarrow V \leq \frac{4}{(1+\beta)} \leq 4. \text{ Also, } \frac{4}{(1+\beta)} \leq \frac{4\sqrt{\beta(2+\beta)}}{(1+\beta)} \Rightarrow \beta \geq (-1+\sqrt{2}), \text{ and } \beta_2 \geq (-1+\sqrt{2}). \text{ It implies that } k_I \geq k_E.$$

$$(2) \quad x_1 \leq x \leq \frac{1}{2}, k_I \geq k_E.$$

$$(iv) \quad c \geq \frac{1}{2\gamma}, k_I \geq k_E.$$

$$(c) \quad 4 \leq V \leq 8$$

$$(A) \quad 0 \leq \beta \leq \beta_1$$

$$(i) \quad 0 \leq c \leq \frac{1}{2\gamma}, k_I \geq k_E.$$

$$(ii) \quad c \geq \frac{1}{2\gamma}, k_I \geq k_E.$$

$$(B) \quad \beta \geq \beta_1$$

$$(i) \quad 0 \leq c \leq \frac{V}{16\gamma}, \text{ we have } k_I = \begin{cases} \left\{ \begin{array}{l} \geq k_E; \quad 4 \leq V \leq 4(1+\beta) \\ \leq k_E; \quad 4(1+\beta) \leq V \leq 8 \end{array} \right\}; & 0 \leq \beta \leq 1 \\ \geq k_E; & \beta \geq 1 \end{cases}$$

$$(ii) \quad c \geq \frac{V}{16\gamma}, \text{ we have } k_I \geq k_E.$$

$$(d) \quad 8 \leq V \leq 8(1+\beta)^2$$

$$(A) \quad 0 \leq \beta \leq \beta_1$$

$$(i) \quad 0 \leq c \leq \frac{V}{16\gamma}, \text{ we have we have } k_I = \begin{cases} \left\{ \begin{array}{l} \geq k_E; \quad 0 \leq c \leq \frac{V\sqrt{\beta}}{8\gamma\sqrt{(1+\beta)}} \\ \leq k_E; \quad \frac{V\sqrt{\beta}}{8\gamma\sqrt{(1+\beta)}} \leq c \leq \frac{V}{16\gamma} \end{array} \right\}; & 0 \leq \beta \leq \frac{1}{3} \\ \geq k_E; & \beta \geq \frac{1}{3} \end{cases}$$

$$(ii) \quad c \geq \frac{V}{16\gamma}, \text{ we find that } k_I \geq k_E.$$

$$(B) \quad \beta \geq \beta_1$$

$$(i) \quad 0 \leq c \leq \frac{V}{16\gamma}, \text{ we have } k_I = \begin{cases} \left\{ \begin{array}{l} \leq k_E; & 0 \leq \beta \leq 1 \\ \geq k_E; \quad 8 \leq V \leq 4(1+\beta) \\ \leq k_E; \quad 4(1+\beta) \leq V \leq 8(1+\beta)^2 \end{array} \right\}; & \beta \geq 1 \end{cases}$$

$$(ii) \quad c \geq \frac{V}{16\gamma}, \text{ we find that } k_I \geq k_E.$$

$$(e) \quad V \geq 8(1+\beta)^2$$

$$(i) \quad 0 \leq c \leq \frac{V}{16\gamma}, \text{ we find that } k_I = \begin{cases} \left\{ \begin{array}{l} \geq k_E; \quad 0 \leq c \leq \frac{V\sqrt{\beta}}{8\gamma\sqrt{(1+\beta)}} \\ \leq k_E; \quad c \geq \frac{V\sqrt{\beta}}{8\gamma\sqrt{(1+\beta)}} \end{array} \right\}; & 0 \leq \beta \leq \frac{1}{3} \\ \geq k_E; & \beta \geq \frac{1}{3} \end{cases}$$

$$(ii) \ c \geq \frac{V}{16\gamma}, \ k_I \geq k_E.$$

We verify that the model parameter space ( $V \geq 0, c > 0, 0 \leq x \leq \frac{1}{2}, \gamma > 0, \beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 1, c = 0.6, x = 0.4, \gamma = 0.9, \beta = 0.1\}$ , we have  $k_E > k_I$ .

## EC.7. Solution and Results of the Extension Model in Section 5.1

In this section, we solve the game, illustrated in Figure EC.1 through backward induction. The decision tree in Figure EC.1 mirrors the decision sequences in Figure 1 except Stage 0. We carry forward the results derived under Stages 1, 2, 3, 4, and 5, and solve for Stage 0.

### EC.7.1. Stage 0: Strategic Exclusivity Decision

We will compare the incumbent profits when there is an investment on the strategic exclusivity index ( $\alpha$ ) to the case of no investment on strategic exclusivity index. The profits under strategic exclusivity index is as follows:

$$\pi_I(\alpha) = -F + \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq c \leq c_u^\alpha \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq x \leq \bar{x}_1^\alpha \\ \left( \frac{64cV^2}{(1+\alpha+\beta)^2(V^2 - 64c^2\gamma^2)^2}; \quad \bar{x}_1^\alpha \leq x \leq \frac{1}{2} \right) \end{array} \right\}; \quad c_u \leq c \leq \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \\ \left( \frac{(1+\alpha+\beta)^2}{4c}; \quad c \geq \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \right) \end{array} \right\}; \quad 0 \leq V \leq \frac{4\sqrt{(\alpha+\beta)(2+\alpha+\beta)}}{(1+\alpha+\beta)} \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq c \leq \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \right); \quad 0 \leq \beta \leq \beta_2 - \alpha \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \right); \\ \left( \frac{(1+\alpha+\beta)^2}{4c}; \quad c \geq \frac{1}{2\gamma} \right) \end{array} \right\}; \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq c \leq \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \right); \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad 0 \leq x \leq \bar{x}_1^\alpha \\ \left( \frac{64c}{(1+\alpha+\beta)^2}; \quad \bar{x}_1^\alpha \leq x \leq \frac{1}{2} \right) \end{array} \right\}; \quad \frac{(1+\alpha+\beta)^2(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ \left( \frac{(1+\alpha+\beta)^2}{4c}; \quad c \geq \frac{1}{2\gamma} \right) \end{array} \right\}; \quad \beta \geq \beta_2 - \alpha \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad 0 \leq \beta \leq \beta_1 - \alpha \right) \end{array} \right\}; \quad \frac{4\sqrt{(\alpha+\beta)(2+\alpha+\beta)}}{(1+\alpha+\beta)} \leq V \leq 4 \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq c \leq \frac{V}{16\gamma} \right); \quad \beta \geq \beta_1 - \alpha \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad c \geq \frac{V}{16\gamma} \right) \end{array} \right\}; \quad 4 \leq V \leq 8 \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2(V^2 - 64c^2\gamma^2)^2}{64cV^2}; \quad 0 \leq c \leq \frac{V}{16\gamma} \right); \quad 0 \leq \beta \leq \beta_1 - \alpha \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad c \geq \frac{V}{16\gamma} \right) \end{array} \right\}; \quad 8 \leq V \leq 8(1+\alpha+\beta)^2 \\ \left\{ \begin{array}{l} \left( \frac{(1+\alpha+\beta)^2 - \frac{\gamma}{2}}{4c}; \quad 0 \leq c \leq \frac{V}{16\gamma} \right); \quad \beta \geq \beta_1 - \alpha \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad c \geq \frac{V}{16\gamma} \right) \end{array} \right\}; \\ \left( \frac{(1+\alpha+\beta)^2(V^2 - 64c^2\gamma^2)^2}{64cV^2}; \quad 0 \leq c \leq \frac{V}{16\gamma} \right); \quad V \geq 8(1+\alpha+\beta)^2 \\ \left( \frac{(1+\alpha+\beta)^2 V^2}{64c}; \quad c \geq \frac{V}{16\gamma} \right); \end{array} \right\}$$

$$\text{where } \bar{x}_1^\alpha = \frac{4(4c\gamma(-2(1+\alpha+\beta)^2 + c\gamma) + \sqrt{V^2(1+\alpha+\beta)^4 + 16c^2\gamma(-2(1+\alpha+\beta)^2 + c\gamma)^2})}{V^2(1+\alpha+\beta)^2}, \quad x_1^\alpha = \frac{512}{(1+\alpha+\beta)^2(48+V^2)(16-V^2)},$$

$$c_u^\alpha = \frac{(1+\alpha+\beta)^2}{\gamma} - \frac{(1+\alpha+\beta)\sqrt{V^2 + 64(\alpha+\beta)(2+\alpha+\beta)}}{8\gamma}, \quad \beta_2 = -1 + \frac{32}{\sqrt{(48+V^2)(16-V^2)}} \text{ and } \beta_1 = \sqrt{\frac{32cV^2\gamma}{16V^2 - (V^2 - 64c^2\gamma^2)^2}} - 1.$$

While comparing profits, we have the following results:

$$\begin{aligned}
\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} &= \begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & 0 \leq \alpha \leq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & \alpha \geq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF} \end{cases}, \\
\frac{(1+\beta)^2}{4c} &= \begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & 0 \leq \alpha \leq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF + 2c\gamma} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & \alpha \geq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF + 2c\gamma} \end{cases}, \\
\frac{(1+\beta)^2}{4c} &= \begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - F; & 0 \leq \alpha \leq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - F; & \alpha \geq -(1+\beta) + \sqrt{(1+\beta)^2 + 4cF} \end{cases}, \quad \frac{(1+\beta)^2 V^2}{64c} = \\
\begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & 0 \leq \alpha \leq \frac{-4(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF + 32c\gamma}}{4} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & \alpha \geq \frac{-4(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF + 32c\gamma}}{4} \end{cases}, \quad (1+\beta)^2 A = \\
\begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & 0 \leq \alpha \leq -(1+\beta) + \sqrt{4Ac(1+\beta)^2 + 2c\gamma + 4cF} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - \frac{\gamma}{2} - F; & \alpha \geq -(1+\beta) + \sqrt{4Ac(1+\beta)^2 + 2c\gamma + 4cF} \end{cases}, \quad \text{where } A = \frac{(V^2 - 64c^2\gamma)^2}{64cV^2}, \\
\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} &= \begin{cases} \geq (1+\beta+\alpha)^2 A - F; & 0 \leq \alpha \leq -(1+\beta) + \sqrt{\frac{(1+\beta)^2 - 2c\gamma + 4cF}{4Ac}} \\ \leq (1+\beta+\alpha)^2 A - F; & \alpha \geq -(1+\beta) + \sqrt{\frac{(1+\beta)^2 - 2c\gamma + 4cF}{4Ac}} \end{cases}, \quad (1+\beta)^2 A = \\
\beta)^2 A &= \begin{cases} \geq (1+\beta+\alpha)^2 A - F; & 0 \leq \alpha \leq -(1+\beta)A + \sqrt{(1+\beta)^2 A^2 + AF} \\ \leq (1+\beta+\alpha)^2 A - F; & \alpha \geq -(1+\beta)A + \sqrt{(1+\beta)^2 A^2 + AF} \end{cases}, \quad \frac{(1+\beta)^2 V^2}{64c} = \\
\begin{cases} \geq \frac{(1+\beta+\alpha)^2}{4c} - F; & 0 \leq \alpha \leq \frac{-4(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF}}{4} \\ \leq \frac{(1+\beta+\alpha)^2}{4c} - F; & \alpha \geq \frac{-4(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF}}{4} \end{cases}, \quad \frac{(1+\beta)^2 V^2}{64c} = \\
\begin{cases} \geq \frac{(1+\beta+\alpha)^2 V^2}{64c} - F; & 0 \leq \alpha \leq \frac{-V(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF}}{V} \\ \leq \frac{(1+\beta+\alpha)^2 V^2}{64c} - F; & \alpha \geq \frac{-V(1+\beta) + \sqrt{V^2(1+\beta)^2 + 64cF}}{V} \end{cases}
\end{aligned}$$





**Proof of Proposition 5:** The results can be derived directly from the above analysis.

We verify that the model parameter space ( $V \geq 0, c > 0, \gamma > 0, \beta > 0, x > 0, F > 0$ ) is non-empty and all of our results are valid in this parameter space.

## EC.8. Solution and Results of the Extension Model in Section 5.2.1

In this section, we present the numerical analysis of the game structure depicted in Figure EC.2. The game is evaluated using backward induction.

We consider the following parameter values for the analysis:  $\beta = 0.1, c = 0.6, V = 1, x = 0.1,$  and  $\gamma = 0.9$ . In Stage 3, consumers' purchase decisions follow the same structure as in our main model, described in EC 3.3.1. In Stage 2, we use the closed-form solutions for the incumbent's pricing and content appeal decisions presented in Table EC.2. Based on these, we numerically solve for the incumbent's optimal pricing strategy. For the given parameter values, the incumbent chooses a usage-based pricing strategy regardless of the entrant's decision. Proceeding to Stage 1, and considering the incumbent's equilibrium choices in Stage 2, the entrant also opts for a usage-based pricing strategy under the same set of parameters. This explains that the part (a) of Proposition 1 (from our main model) holds when the consumer valuation and content acquisition costs are low. Further, for the same set of parameters, the entrant earns a higher profit than the incumbent, indicating that the incumbent's positional advantage is not guaranteed even under this alternative game structure (part (a) of Proposition 3(i) from our main model). Furthermore, the entrant's content attractiveness index is higher than that of the incumbent, qualitatively in line with Proposition 4.

For part (b) of Proposition 3(i), we evaluate the results for  $\beta = 0.1, c = 1.6, V = 1, x = 0.1,$  and  $\gamma = 0.9$ , we see that the incumbent and entrant differentiate their strategy, with the incumbent choosing a subscription-based and entrant opting for a usage-based strategy. Under this scenario, the positional advantage of the incumbent is retained in line with part (b) of Proposition 3(i).

We now consider another set of parameters for the analysis:  $\beta = 0.1, c = 0.6, V = 18, x = 0.1,$  and  $\gamma = 0.9$ , incumbent and entrant match up with subscription-based strategy indicating that the result is in line with part (b) of Proposition 1. Moreover, the incumbent's positional advantage is retained for this set of parameters, in line with part (ii) of Proposition 3.

## EC.9. Solution and Results of the Extension Model in Section 5.2.2

In this section, we solve the game, illustrated in Figure EC.3 through backward induction.

### EC.9.1. Stage 5: Consumer decision

As the utility structure remains unchanged, so there will not be any changes in the consumption plan for the consumers. We utilize the customer segmentation directly from Section EC.6.1.

### EC.9.2. Stage 4 Decision: Entrant's Price and Content Acquisition

Given the segmentation in stage 5 (refer Section EC.9.1), we will now solve the entrant's prices and content attractiveness. The entrant simultaneously decides content acquisition index ( $k_E$ ) and prices ( $r_E$  and  $p_E$  under subscription and usage-based, respectively) by maximizing the profits obtained under each pricing strategy,

**I-Subscription, E-Subscription:** Here, entrant decides  $r_E$  and  $k_E$  simultaneously.

(a)  $0 \leq r_E \leq (k_E - \gamma)V$

(i)  $0 \leq r_I \leq B$ , the first and second ordered partials are:  $\frac{\partial \pi_E^{SS}}{\partial r_E} = 1 - \frac{2r_E}{(k_E - \gamma)V}$ ,  $\frac{\partial \pi_E^{SS}}{\partial k_E} = \frac{r_E^2}{(k_E - \gamma)^2 V} - 2ck_E$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E^2} = \frac{-2r_E^2}{(k_E - \gamma)^3 V} - 2c < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial r_E^2} = \frac{-2}{(k_E - \gamma)V} < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E \partial r_E} = \frac{2r_E}{(k_E - \gamma)^2 V} > 0$ . The Hessian matrix of the above profit function is negative semi-definite. Solving these first ordered equations, the solution is  $r_E = \frac{V^2}{16c} - \frac{\gamma V}{2}$ ,  $k_E = \frac{V}{8c}$ ,  $\pi_E^{SS} = \frac{V^2}{64c} - \frac{\gamma V}{4}$ .

(ii)  $B \leq r_I \leq D$ , the entrant's profit function is convex within the defined parameter range and yields negative values

(iii)  $D \leq r_I \leq H$ , and  $H \leq r_I \leq k_I(1 + \beta)V$ , The first and second ordered partials are:  $\frac{\partial \pi_E^{SS}}{\partial r_E} = 1 - \frac{2r_E}{k_E V}$ ,  $\frac{\partial \pi_E^{SS}}{\partial k_E} = \frac{r_E^2}{k_E^2 V} - 2ck_E$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E^2} = \frac{-2r_E^2}{k_E^3 V} - 2c < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial r_E^2} = \frac{-2}{k_E V} < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E \partial r_E} = \frac{2r_E}{k_E^2 V} > 0$ . The Hessian matrix of the above profit function is negative semi-definite. Solving these first-ordered equations, the solution is  $r_E = \frac{V^2}{16c}$ ,  $k_E = \frac{V}{8c}$ , and  $\pi_E^{SS} = \frac{V^2}{64c}$ .

(b)  $(k_E - \gamma)V \leq r_E \leq k_E V$

(i)  $0 \leq r_I \leq B$ ,  $\pi_E^{SS} = 0$ .

(ii)  $B \leq r_I \leq G$ , the entrant's profit function is convex within the defined parameter range and yields negative values

(iii)  $G \leq r_I \leq k_I(1 + \beta)V$ , The first and second ordered partials are:  $\frac{\partial \pi_E^{SS}}{\partial r_E} = 1 - \frac{2r_E}{k_E V}$ ,  $\frac{\partial \pi_E^{SS}}{\partial k_E} = \frac{r_E^2}{k_E^2 V} - 2ck_E$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E^2} = \frac{-2r_E^2}{k_E^3 V} - 2c < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial r_E^2} = \frac{-2}{k_E V} < 0$ ,  $\frac{\partial^2 \pi_E^{SS}}{\partial k_E \partial r_E} = \frac{2r_E}{k_E^2 V} > 0$ . The Hessian matrix of the above profit function is negative semi-definite. The solution is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{V^2}{16c}$ , and  $\pi_E^{SS} = \frac{V^2}{64c}$ .

Comparing all the above profits derived above, the optimal outcome is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{V^2}{16c}$ , and  $\pi_E^{SS} = \frac{V^2}{64c}$ .

**I-Usage, E-Subscription:** Here, entrant decides  $r_E$ , and  $k_E$  simultaneously.

(a)  $0 \leq r_E \leq (k_E - \gamma(1 - p_I x))V$

$$\begin{aligned} \pi_E^{SU} &= \frac{r_E(V - T_A)}{V} - ck_E^2 \\ &= \frac{r_E}{V} \left( V - \frac{r_E}{k_E - \gamma(1 - p_I x)} \right) - ck_E^2 \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \pi_E^{SU}}{\partial r_E} &= \frac{1}{V} \left( V - \frac{2r_E}{k_E - \gamma(1 - p_I x)} \right) \\ \frac{\partial^2 \pi_E^{SU}}{\partial r_E^2} &= \frac{-2}{V(k_E - \gamma(1 - p_I x))} < 0 \\ \frac{\partial \pi_E^{SU}}{\partial k_E} &= \frac{r_E^2}{V(k_E - \gamma(1 - p_I x))^2} - 2ck_E \\ \frac{\partial^2 \pi_E^{SU}}{\partial k_E^2} &= \frac{-2r_E^2}{V(k_E - \gamma(1 - p_I x))^3} - 2c < 0 \\ \frac{\partial^2 \pi_E^{SU}}{\partial k_E \partial r_E} &= \frac{2r_E}{V(k_E - \gamma(1 - p_I x))^2} > 0 \end{aligned}$$

Hessian of the above profit function is negative semi-definite. Solution is  $k_E = \frac{V}{8c}$ ,  $r_E = \frac{V^2}{16c} - \frac{\gamma(1-p_I x)V}{2}$ , and  $\pi_E^{SU} = \frac{V^2}{64c} - \frac{\gamma(1-p_I x)V}{4}$ .

(b)  $(k_E - \gamma(1-p_I x))V \leq r_E \leq k_E V$ ,  $\pi_E^{SU} = 0$ .

**I-Subscription, E-Usage:** Here, entrant decides  $p_E$  and  $k_E$  simultaneously.

(a)  $0 \leq r_I \leq (k_I(1+\beta) - \gamma(1-p_E x))V$

$$\begin{aligned} \pi_E^{US} &= p_E k_E x \frac{T_A}{V} + p_E (k_E - \gamma)x \frac{(V - T_A)}{V} - ck_E^2 \\ &= p_E k_E x - p_E \gamma x \left(1 - \frac{r_I}{V(k_I(1+\beta) - \gamma(1-p_E x))}\right) - ck_E^2 \end{aligned}$$

The first-ordered conditions are:

$$\begin{aligned} \frac{\partial \pi_E^{US}}{\partial p_E} &= (k_E - \gamma)x + \frac{\gamma x r_I (k_I(1+\beta) - \gamma)}{(k_I(1+\beta) - \gamma(1-p_E x))^2 V} \\ \frac{\partial \pi_E^{US}}{\partial k_E} &= p_E x - 2ck_E \\ \frac{\partial^2 \pi_E^{US}}{\partial p_E^2} &= \frac{-2\gamma^2 x^2 r_I (k_I(1+\beta) - \gamma)}{(k_I(1+\beta) - \gamma(1-p_E x))^3 V} < 0 \\ \frac{\partial^2 \pi_E^{US}}{\partial k_E^2} &= -2c < 0 \\ \frac{\partial^2 \pi_E^{US}}{\partial p_E \partial k_E} &= x > 0 \end{aligned}$$

Solving for  $\frac{\partial \pi_E^{US}}{\partial p_E} = 0$ , we have two complex roots. Solution is  $p_E = \frac{1}{x}$ ,  $k_E = \frac{1}{2c}$ , and  $\pi_E^{US} = \frac{1}{4c} - \gamma + \frac{\gamma r_I}{k_I(1+\beta)V}$ .

(b)  $(k_I(1+\beta) - \gamma(1-p_E x))V \leq r_I \leq (k_I(1+\beta)V$ ,  $\pi_E^{US} = p_E k_E x - ck_E^2$ . Profit function is linear in  $p_E$ . Solution is  $p_E = \frac{1}{x}$ ,  $k_E = \frac{1}{2c}$ , and  $\pi_E^{US} = \frac{1}{4c}$ .

**I-Usage and E-Usage:** We solve for the optimal prices ( $p_E$ ) and the content amount ( $k_E$ ) of the entrant.

(a)  $0 \leq p_I \leq \frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)}$   
 (i)  $0 \leq p_E \leq C$ ,

$$\pi_E^{UU} = p_E (k_E - \frac{\gamma}{2})x - ck_E^2$$

The profit function is linear in  $p_E$ ; the solution is  $p_E = C$ . Evaluating the profit function at  $p_E = C$ , we have  $\pi_E^{UU} = k_E + \frac{(p_I x - 2)\gamma}{2} - ck_E^2$ . We now solve for  $k_E$ . Solution is  $k_E = \frac{1}{2c}$ , and  $\pi_E^{UU} = \frac{1}{4c} + \frac{(p_I x - 2)\gamma}{2}$ . Evaluating  $p_E$  at  $k_E$ , we have  $p_E = \frac{1 + c\gamma(p_I x - 2)}{x(1 - c\gamma)}$ .

(ii)  $C \leq p_E \leq \frac{1}{x}$ ,  $\pi_E^{UU} = 0$

(b)  $\frac{2(k_I(1+\beta) - \gamma)}{x(2k_I(1+\beta) - \gamma)} \leq p_I \leq \frac{1}{x}$   
 (i)  $0 \leq p_E \leq B$ ,

$$\pi_E^{UU} = p_E k_E x - ck_E^2$$

The profit function is linear in  $p_E$ ; the solution is  $p_E = B$ . Evaluating the profit function at  $p_E = B$ , we have  $\pi_E^{UU} = Bxk_E - ck_E^2$ . We now solve for  $k_E$ . Solution is  $k_E = \frac{Bx}{2c}$ , and  $\pi_E^{UU} = \frac{B^2 x^2}{4c}$ .

(ii)  $B \leq p_E \leq C$ ,

$$\pi_E^{UU} = p_E(k_E - \frac{\gamma}{2})x - ck_E^2$$

The profit function is linear in  $p_E$ ; the solution is  $p_E = C$ . Evaluating the profit function at  $p_E = C$ , we have  $\pi_E^{UU} = k_E + \frac{(p_I x - 2)\gamma}{2} - ck_E^2$ . We now solve for  $k_E$ . Solution is  $k_E = \frac{1}{2c}$ , and  $\pi_E^{UU} = \frac{1}{4c} + \frac{(p_I x - 2)\gamma}{2}$ . Evaluating  $p_E$  at  $k_E$ , we have  $p_E = \frac{1 + c\gamma(p_I x - 2)}{x(1 - c\gamma)}$ .

(iii)  $C \leq p_E \leq \frac{1}{x}$ ,  $\pi_E^{UU} = 0$

### EC.9.3. Stage 3 Decision: Incumbent's Price and Content Acquisition

In this section, we solve for the incumbent's prices and content acquisition index simultaneously conditional on the entrant's decision in Stage 4.

**I-Subscription, E-Subscription:** Here, the incumbent chooses  $r_I$  and  $k_I$  simultaneously by maximizing profits. Given the entrant's optimal decisions, we solve for the incumbent's optimal prices ( $r_I$ ) and content amount ( $k_I$ ). The first and second ordered partials are:  $\frac{\partial \pi_I^{SS}}{\partial r_I} = 1 - \frac{2r_I}{(k_I(1+\beta) - \gamma)V}$ ,  $\frac{\partial \pi_I^{SS}}{\partial k_I} = \frac{r_I^2}{V(k_I(1+\beta) - \gamma)^2} - 2ck_I$ ,  $\frac{\partial^2 \pi_I^{SS}}{\partial r_I^2} = \frac{-2}{V(k_I(1+\beta) - \gamma)} < 0$ ,  $\frac{\partial^2 \pi_I^{SS}}{\partial k_I^2} = \frac{r_I^2}{V(k_I(1+\beta) - \gamma)^3} - 2c < 0$ ,  $\frac{\partial^2 \pi_I^{SS}}{\partial r_I \partial k_I} = \frac{2r_I}{V(k_I(1+\beta) - \gamma)^2} > 0$ . The Hessian matrix of the above profit function is negative semi-definite. The solution is  $k_I = \frac{(1+\beta)V}{8c}$ ,  $r_I = \frac{(1+\beta)^2 V^2}{16c} - \frac{\gamma V}{2}$ , and  $\pi_I^{SS} = \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4} > 0 \Rightarrow c \leq \frac{(1+\beta)^2 V}{16\gamma}$ . For  $c > \frac{(1+\beta)^2 V}{16\gamma}$ , the entrant will have monopoly profits, which is equivalent to  $\frac{V^2}{64c}$  under subscription.

**I-Usage, E-Subscription:** Here, the incumbent chooses  $p_I$  and  $k_I$  simultaneously by maximizing profits. Given the entrant's optimal decisions, we solve for the incumbent's optimal prices ( $p_I$ ) and content amount ( $k_I$ ). Incumbent's profit is  $\pi_I^{SU} = \frac{p_I x(2k_I(1+\beta) - \gamma)}{2} - ck_I^2$ . The profit function is linear in  $p_I$ . Solution is  $p_E = \frac{1}{x}$ ,  $k_I = \frac{(1+\beta)}{2c}$ , and  $\pi_I^{SU} = \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . Also,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} > 0$  for  $c < \frac{(1+\beta)^2}{2\gamma}$ . For  $c > \frac{(1+\beta)^2}{2\gamma}$ , the entrant will have monopoly profits, which is equivalent to  $\frac{V^2}{64c}$  under subscription.

**I-Subscription, E-Usage:** Here, the incumbent chooses  $r_I$  and  $k_I$  simultaneously by maximizing profits. Given the entrant's optimal decisions, we solve for the incumbent's optimal prices ( $r_I$ ) and the content amount ( $k_I$ ). Incumbent's profit function is  $\pi_I^{US} = \frac{r_I}{V}(V - \frac{r_I}{k_I(1+\beta)}) - ck_I^2$ . The first-ordered conditions are:

$$\frac{\partial \pi_I^{US}}{\partial k_I} = \frac{r_I^2}{Vk_I^2(1+\beta)} - 2ck_I$$

$$\frac{\partial \pi_I^{US}}{\partial r_I} = \frac{1}{V}(V - \frac{2r_I}{k_I(1+\beta)})$$

The second ordered partial derivatives are:  $\frac{\partial^2 \pi_I^{US}}{\partial k_I^2} = \frac{-2r_I^2}{Vk_I^3(1+\beta)} - 2c < 0$ ,  $\frac{\partial^2 \pi_I^{US}}{\partial r_I^2} = \frac{-2}{Vk_I(1+\beta)} < 0$ , and

$\frac{\partial^2 \pi_I^{US}}{\partial k_I \partial r_I} = \frac{2r_I}{Vk_I^2(1+\beta)} > 0$ . The Hessian of the above profit function is negative semi-definite. By solving

the first-ordered conditions, solution is  $k_I = \frac{(1+\beta)V}{8c}$ ,  $r_I = \frac{(1+\beta)^2 V^2}{16c}$ , and  $\pi_I^{US} = \frac{(1+\beta)^2 V^2}{64c}$ .

**I-Usage, E-Usage:** Here, the incumbent chooses  $p_I$  and  $k_I$  simultaneously by maximizing profits. Given the entrant's optimal decisions, we solve for the incumbent's optimal prices ( $p_I$ ) and content amount ( $k_I$ ). Incumbent's profit is  $\pi_I^{S^U} = \frac{p_I x(2k_I(1+\beta) - \gamma)}{2} - ck_I^2$ . The profit function is linear in  $p_I$ . Solution is  $p_E = \frac{1}{x}$ ,  $k_I = \frac{(1+\beta)}{2c}$ , and  $\pi_I^{S^U} = \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . Also,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} > 0$  for  $c < \frac{(1+\beta)^2}{2\gamma}$ . For  $c > \frac{(1+\beta)^2}{2\gamma}$ , the entrant will have monopoly profits, which is equivalent to  $\frac{1}{4c}$  under a usage-based pricing strategy.

#### EC.9.4. Stage 2: Entrant's Pricing Strategy Decision

In this stage, we solve for the entrant's optimal strategy given incumbent's strategy. Entrant has two options to choose from: (i) Subscription-based strategy or (ii) usage-based strategy. Given the optimal decisions by the entrant (in Stage 4) and incumbent (in Stage 3), we compare the entrants' optimal profits when the incumbent is usage-based and subscription-based. When the incumbent adopts a usage-based strategy, profits of the entrant under a usage-based strategy is :

$$\pi_E^{U^U} = \begin{cases} \frac{1}{4c} - \frac{\gamma}{2}; & 0 \leq c \leq \frac{1}{2\gamma} \\ \frac{1}{4c}; & c > \frac{1}{2\gamma} \end{cases}$$

The entrant's profits under subscription, when the incumbent is a usage-based service provider, is:  $\pi_E^{U^S} = \frac{V^2}{64c}$ .

We now compare the above profits when the incumbent is a usage-based service provider. Check if,  $\frac{1}{4c} - \frac{\gamma}{2} < \frac{V^2}{64c} \Rightarrow c > \frac{(16-V^2)}{32\gamma}$  if  $(16-V^2) > 0 \Rightarrow V < 4$ .

For  $V > 4$ , the above inequality holds for all  $c > 0$ , implying that  $\frac{1}{4c} - \frac{\gamma}{2} < \frac{V^2}{64c}$ . Also,  $\frac{(16-V^2)}{32\gamma} < \frac{1}{2\gamma}$ .

For  $c \geq \frac{1}{2\gamma}$ , check if,  $\frac{1}{4c} \leq \frac{V^2}{64c} \Rightarrow V \geq 4$ . When the incumbent adopts a usage-based strategy, the optimal

$$\text{pricing strategy for the entrant is } l^* = \begin{cases} \begin{cases} U; & 0 \leq c \leq \frac{(16-V^2)}{32\gamma} \\ S; & \frac{(16-V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma}; & 0 \leq V \leq 4 \\ U; & c \geq \frac{1}{2\gamma} \end{cases} \\ S; & V > 4 \end{cases}$$

When the incumbent adopts a subscription-based strategy, the entrant's profits under a usage-based strategy is:  $\pi_E^{S^U} = \begin{cases} \frac{1}{4c} - \frac{\gamma}{2}; & 0 \leq c \leq \frac{1}{2\gamma} \\ 0; & c > \frac{1}{2\gamma} \end{cases}$

The entrant's profits under subscription, when the incumbent is a subscription-based service provider, is:  $\pi_E^{S^S} = \frac{V^2}{64c}$ .

We now compare the above profits when the incumbent is a subscription-based service provider. Check if,  $\frac{1}{4c} - \frac{\gamma}{2} < \frac{V^2}{64c} \Rightarrow c > \frac{(16-V^2)}{32\gamma}$  if  $(16-V^2) > 0 \Rightarrow V < 4$ .

For  $V > 4$ , the above inequality holds for all  $c > 0$ , implying that  $\frac{1}{4c} - \frac{\gamma}{2} < \frac{V^2}{64c}$ . Also,  $\frac{(16-V^2)}{32\gamma} < \frac{1}{2\gamma}$ .

When the incumbent adopts a subscription-based strategy, the optimal pricing strategy for the entrant is

$$l^* = \begin{cases} \begin{cases} U; & 0 \leq c \leq \frac{(16-V^2)}{32\gamma} \\ S; & c > \frac{(16-V^2)}{32\gamma} \end{cases} & ; \quad 0 \leq V \leq 4 \\ S; & V > 4 \end{cases}$$

### EC.9.5. Stage 1: Incumbent's Optimal Pricing Strategy

Given the optimal pricing strategy in Stage 2, we now compare the profits of the incumbent for the optimal pricing strategy. When the incumbent chooses a subscription-based strategy, the profit of the incumbent is

$$\pi_I^S = \begin{cases} \begin{cases} \frac{(1+\beta)^2 V^2}{64c}; & 0 \leq c \leq \frac{(16-V^2)}{32\gamma} \\ 0; & c \geq \frac{(16-V^2)}{32\gamma} \end{cases} & ; \quad 0 \leq V \leq -(1+\beta)^2 + \sqrt{16+(1+\beta)^4} \\ \begin{cases} \frac{(1+\beta)^2 V^2}{64c}; & 0 \leq c \leq \frac{(16-V^2)}{32\gamma} \\ \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4}; & \frac{(16-V^2)}{32\gamma} \leq c \leq \frac{(1+\beta)^2 V}{16\gamma} \\ 0; & c \geq \frac{(1+\beta)^2 V}{16\gamma} \end{cases} & ; \quad -(1+\beta)^2 + \sqrt{16+(1+\beta)^4} \leq V \leq 4. \\ \begin{cases} \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4}; & 0 \leq c \leq \frac{(1+\beta)^2 V}{16\gamma} \\ 0; & c \geq \frac{(1+\beta)^2 V}{16\gamma} \end{cases} & ; \quad V \geq 4 \end{cases}$$

$$\text{Incumbent's profits under a usage-based strategy are } \pi_I^U = \begin{cases} \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}; & 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ 0; & c > \frac{(1+\beta)^2}{2\gamma} \end{cases}$$

We now compare the above profits for the optimal strategy of the incumbent.

- (a)  $0 \leq V \leq -(1+\beta)^2 + \sqrt{16+(1+\beta)^4}$
- (i)  $0 \leq c \leq \frac{(16-V^2)}{32\gamma}$ , check if  $\frac{(1+\beta)^2 V^2}{64c} \leq \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . The condition holds for this region. The incumbent would choose usage-based pricing.
  - (ii)  $\frac{(16-V^2)}{32\gamma} \leq c \leq \frac{(1+\beta)^2}{2\gamma}$ ,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} > 0$ . The incumbent would choose usage-based pricing.
  - (iii)  $c \geq \frac{(1+\beta)^2}{2\gamma}$ , incumbent does not enter the market.
- (b)  $-(1+\beta)^2 + \sqrt{16+(1+\beta)^4} \leq V \leq 4$
- (i)  $0 \leq c \leq \frac{(16-V^2)}{32\gamma}$ , check if  $\frac{(1+\beta)^2 V^2}{64c} \leq \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . The condition holds for this region. The incumbent would choose usage-based pricing.
  - (ii)  $\frac{(16-V^2)}{32\gamma} \leq c \leq \frac{(1+\beta)^2 V}{16\gamma}$ ,  $\frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4} \leq \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . The condition holds for this region. The incumbent would choose usage-based pricing.
  - (iii)  $\frac{(1+\beta)^2 V}{16\gamma} \leq c \leq \frac{(1+\beta)^2}{2\gamma}$ ,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} > 0$ . The incumbent would choose usage-based pricing.
  - (iv)  $c \geq \frac{(1+\beta)^2}{2\gamma}$ , incumbent does not enter the market.
- (c)  $4 \leq V \leq 8$

- (i)  $0 \leq c \leq \frac{(1+\beta)^2 V}{16\gamma}$ , check if  $\frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4} \leq \frac{(1+\beta)^2}{4c} - \frac{\gamma}{2}$ . The condition holds for  $4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2}$ . The incumbent would choose  $j^* =$
- $$\begin{cases} \text{U ; } & 4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2} \\ \text{S ; } & \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2} \leq V \leq 8 \end{cases}.$$
- (ii)  $\frac{(1+\beta)^2 V}{16\gamma} \leq c \leq \frac{(1+\beta)^2}{2\gamma}$ ,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} > 0$ . The incumbent would choose usage-based pricing.
- (iii)  $c \geq \frac{(1+\beta)^2}{2\gamma}$ , incumbent does not enter the market.

(d)  $V \geq 8$

- (i)  $0 \leq c \leq \frac{(1+\beta)^2}{2\gamma}$ ,  $\frac{(1+\beta)^2}{4c} - \frac{\gamma}{2} < \frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4}$ . The incumbent would choose subscription-based pricing.

- (ii)  $\frac{(1+\beta)^2}{2\gamma} \leq c \leq \frac{(1+\beta)^2 V}{16\gamma}$ ,  $\frac{(1+\beta)^2 V^2}{64c} - \frac{\gamma V}{4} > 0$ . The incumbent would choose subscription-based pricing.

- (iii)  $c \geq \frac{(1+\beta)^2 V}{16\gamma}$ , incumbent does not enter the market.

The optimal pricing strategy for the incumbent is

$$j^* = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \text{U ; } \quad 0 \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \text{n.a. ; } \quad c \geq \frac{(1+\beta)^2}{2\gamma} \end{array} \right\} ; \quad 0 \leq V \leq \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2} \\ \left\{ \begin{array}{l} \text{S ; } \quad 0 \leq c \leq \frac{(1+\beta)^2 V}{16\gamma} \\ \text{U ; } \quad \frac{(1+\beta)^2 V}{16\gamma} \leq c \leq \frac{(1+\beta)^2}{2\gamma} \\ \text{n.a. ; } \quad c \geq \frac{(1+\beta)^2}{2\gamma} \end{array} \right\} ; \quad \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2} \leq V \leq 8 \\ \left\{ \begin{array}{l} \text{S ; } \quad 0 \leq c \leq \frac{(1+\beta)^2 V}{16\gamma} \\ \text{n.a. ; } \quad c \geq \frac{(1+\beta)^2 V}{16\gamma} \end{array} \right\} ; \quad V \geq 8 \end{array} \right\}$$

**Proof of Lemma EC.4:** We combine the Stage 1 and 2 decisions, and the optimal strategy of the game

$$(j^*, l^*) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (U, U); \quad 0 \leq c \leq \frac{(16 - V^2)}{32\gamma} \\ (U, S); \quad \frac{(16 - V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ (U, U); \quad \frac{1}{2\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., U); \quad c > \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq V \leq 4 \\ \left\{ \begin{array}{l} (U, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \\ \left\{ \begin{array}{l} (S, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ (U, S); \quad \frac{(1 + \beta)^2 V}{16\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \leq V \leq 8 \\ \left\{ \begin{array}{l} (S, S); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ (n.a., S); \quad c \geq \frac{(1 + \beta)^2 V}{16\gamma} \end{array} \right\}; \quad V \geq 8 \end{array} \right\}$$

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 1, c = 0.6, \gamma = 0.9, \beta = 0.1\}$ , both incumbent and entrant choose a usage-based strategy. The optimal profits corresponding to the optimal strategy are:

$$(\pi_I^*, \pi_E^*) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left( \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}, \frac{1}{4c} - \frac{1}{2} \right); \quad 0 \leq c \leq \frac{(16 - V^2)}{32\gamma} \\ \left( \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad \frac{(16 - V^2)}{32\gamma} \leq c \leq \frac{1}{2\gamma} \\ \left( \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}, \frac{1}{4c} \right); \quad \frac{1}{2\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ \left( 0, \frac{1}{4c} \right); \quad c > \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 0 \leq V \leq 4 \\ \left\{ \begin{array}{l} \left( \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad 0 \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ \left( 0, \frac{V^2}{64c} \right); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad 4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \\ \left\{ \begin{array}{l} \left( \frac{(1 + \beta)^2 V^2}{64c} - \frac{\gamma V}{4}, \frac{V^2}{64c} \right); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ \left( \frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2}, \frac{V^2}{64c} \right); \quad \frac{(1 + \beta)^2 V}{16\gamma} \leq c \leq \frac{(1 + \beta)^2}{2\gamma} \\ \left( 0, \frac{V^2}{64c} \right); \quad c \geq \frac{(1 + \beta)^2}{2\gamma} \end{array} \right\}; \quad \frac{4(2c\gamma + \sqrt{(1 + \beta)^4 - 2c(1 + \beta)^2\gamma + 4c^2\gamma^2})}{(1 + \beta)^2} \leq V \leq 8 \\ \left\{ \begin{array}{l} \left( \frac{(1 + \beta)^2 V^2}{64c} - \frac{\gamma V}{4}, \frac{V^2}{64c} \right); \quad 0 \leq c \leq \frac{(1 + \beta)^2 V}{16\gamma} \\ \left( 0, \frac{V^2}{64c} \right); \quad c \geq \frac{(1 + \beta)^2 V}{16\gamma} \end{array} \right\}; \quad V \geq 8 \end{array} \right\}$$

We compare the profits of the incumbent and the entrant to characterize conditions for the first-mover advantage. Check if,  $\frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2} > \frac{1}{4c} - \frac{\gamma}{2} \Rightarrow (1 + \beta)^2 > 1$  (which holds). Check if,  $\frac{(1 + \beta)^2}{4c} - \frac{\gamma}{2} > \frac{V^2}{64c} \Rightarrow c < \frac{16(1 + \beta)^2 - V^2}{32\gamma}$  if  $16(1 + \beta)^2 - V^2 > 0 \Rightarrow V < 4(1 + \beta)$ . Also,  $\frac{(16 - V^2)}{32\gamma} < \frac{16(1 + \beta)^2 - V^2}{32\gamma}$ . Check if,  $\frac{(1 + \beta)^2 V^2}{64c} - \frac{\gamma V}{4} > \frac{V^2}{64c} \Rightarrow c < \frac{\beta(2 + \beta)V}{16\gamma}$ . And  $\frac{\beta(2 + \beta)V}{16\gamma} < \frac{(1 + \beta)^2 V}{16\gamma}$ . For illustration purposes, we present the profit comparisons only for low consumer valuations; similar observations hold for higher valuations as well.

$$\pi_I = \left\{ \left\{ \left\{ \begin{array}{l} \geq \pi_E; \quad 0 \leq c \leq \frac{1}{2\gamma} \\ \leq \pi_E; \quad c \geq \frac{1}{2\gamma} \end{array} \right\}; \quad 0 \leq \beta \leq \sqrt{2} - 1 \right\}; \quad 0 \leq V \leq 4 \right. \\ \left. \left\{ \left\{ \begin{array}{l} \geq \pi_E; \quad 0 \leq c \leq \frac{\beta(2+\beta)}{2\gamma} \\ \leq \pi_E; \quad c \geq \frac{\beta(2+\beta)}{2\gamma} \end{array} \right\}; \quad \beta \geq \sqrt{2} - 1 \right\} \right. \\ \left. \left\{ \begin{array}{l} \geq \pi_I; \quad 0 \leq c \leq \frac{16(1+\beta)^2 - V^2}{32\gamma} \\ \leq \pi_E; \quad c \geq \frac{16(1+\beta)^2 - V^2}{32\gamma} \end{array} \right\}; \quad 4 \leq V \leq \frac{4(2c\gamma + \sqrt{(1+\beta)^4 - 2c(1+\beta)^2\gamma + 4c^2\gamma^2})}{(1+\beta)^2} \right\} \right\}$$

We verify that the model parameter space ( $V \geq 0$ ,  $c > 0$ ,  $\gamma > 0$ ,  $\beta > 0$ ) is non-empty and all of our results are valid in this parameter space. For example, at  $\{V = 1, c = 0.6, \gamma = 0.9, \beta = 0.1\}$ , entrant gets the higher profits.

### EC.10. Proof for the possibility of $\gamma$ being negative:

*Robustness to Mixed-Sign Similarity Preferences.* The special case below we analyze is intended to provide an analytically transparent demonstration of our point, rather than to exhaust all possible forms of heterogeneity in similarity preferences. The same qualitative conclusion should continue to hold under richer mixed-sign specifications, because the underlying mechanism in our model is driven by monotone threshold comparisons in a one-dimensional consumer index.

In the baseline model, let

$$\tau \equiv \frac{T}{V} \sim U[0, 1].$$

Suppose now that consumers are heterogeneous not only in  $\tau$ , but also in how they perceive content similarity. Let  $\gamma$  denote a consumer-specific similarity-preference term, independent of  $\tau$ , such that

$$\gamma \sim \text{DiscreteUniform}\{-a, 1 - a\}, \quad 0 < a < 1.$$

Hence, some consumers view similarity as harmful ( $\gamma = -a < 0$ ), whereas others view it as beneficial ( $\gamma = 1 - a > 0$ ).

Define the effective one-dimensional type index by

$$x \equiv \tau + \gamma.$$

Since  $\gamma \in \{-a, 1 - a\}$  with equal probability, it follows that  $x$  is uniformly distributed on  $[-a, 2 - a]$ . For notational convenience, define the shifted variable

$$z \equiv x + a = \tau + \gamma + a,$$

so that  $z \sim U[0, 2]$ . Let

$$\hat{\tau} \equiv \frac{z}{2}.$$

Then  $\hat{\tau} \sim U[0, 1]$ .

Accordingly, after a location shift and normalization, the model has exactly the same one-dimensional heterogeneity structure as the baseline model. Since all consumer-side cutoffs and comparative statics are driven by monotone threshold comparisons in that index, the equilibrium characterization and all qualitative results remain unchanged. The only difference is quantitative: because some consumers now assign positive value to similarity, the effective impact of  $\gamma$  is attenuated relative to the baseline specification in which similarity is always harmful.

*Proof.* Since  $\gamma \in \{-a, 1-a\}$  with equal probability, the random variable

$$x = \tau + \gamma$$

is a 50-50 mixture of a uniform random variable on  $[-a, 1-a]$  and a uniform random variable on  $[1-a, 2-a]$ .

Hence its density is

$$f_x(t) = \begin{cases} \frac{1}{2}, & t \in [-a, 1-a], \\ \frac{1}{2}, & t \in [1-a, 2-a], \\ 0, & \text{otherwise,} \end{cases}$$

which is simply the density of  $U[-a, 2-a]$ . Thus,

$$x \sim U[-a, 2-a].$$

Now define the shifted variable

$$z = x + a.$$

Then

$$z \sim U[0, 2].$$

Further define

$$\hat{\tau} = \frac{z}{2}.$$

By a standard change of variables,

$$\hat{\tau} \sim U[0, 1].$$

Next, all Stage-5 utility comparisons in the baseline model are affine in the one-dimensional consumer index. Thus, a generic utility difference can be written as

$$\Delta(\tau; \Theta) = A(\Theta) + B(\Theta)\tau,$$

where  $\Theta$  denotes the vector of prices and platform choices. Under the present extension, the corresponding comparison becomes

$$\tilde{\Delta}(x; \Theta) = \tilde{A}(\Theta) + \tilde{B}(\Theta)x.$$

Using  $x = z - a$ , we can rewrite this as

$$\tilde{\Delta}(z; \Theta) = \tilde{A}(\Theta) + \tilde{B}(\Theta)(z - a) = (\tilde{A}(\Theta) - a\tilde{B}(\Theta)) + \tilde{B}(\Theta)z.$$

Since  $z = 2\hat{\tau}$ , we have

$$\tilde{\Delta}(\hat{\tau}; \Theta) = (\tilde{A}(\Theta) - a\tilde{B}(\Theta)) + 2\tilde{B}(\Theta)\hat{\tau}.$$

Thus, after relabeling coefficients, the utility comparison has exactly the same functional form as in the baseline model with  $\hat{\tau} \sim U[0, 1]$ . Therefore, the cutoff structure, market partition, equilibrium logic, and comparative statics are qualitatively identical to those in the baseline model.

Finally, because some consumers now assign positive value to similarity, the net adverse effect of overlap is weaker than in the baseline case where similarity is uniformly harmful. Equivalently, the extension behaves like the baseline model with a smaller effective overlap parameter. This affects magnitudes, but not the signs or the qualitative structure of the results. Q.E.D.