Online Appendix A. Selected Recent Gamification Literature

Table A.1 Selected Recent Literature of Gamification

Reference	Gamification Elements	Interested Outcomes	Methodology	Design Improvement
This study	Badge: Volume, Variety, & Valence	Contribution Quantity & Quality	Structural HMM with Copula Endogeneity Correlation	Counterfactual Strategies
Chen et al. (2018)	Badge: Volume	Contribution Quantity	Structural HMM	
Cavusoglu et al. (2015)	Badge: Variety or Valence	Contribution Quantity	Regressions	
Ho et al. (2022)	Badge: Variety Leaderboard	Shopping Engagement	Field Experiment & Survey	
Leung et al. (2023)	Badge: Variety Leaderboard	Learning Engagement and Test Score	Field Experiment	
Hydari et al. (2023)	Leaderboard	Daily Steps Count	Field Experiment & Survey	
Bojd et al. (2022)	Leaderboard	Weight Loss	Regressions	

Online Appendix B. Correlation of Key Variables

Table B.1. Correlations of Key Variables

	$Volume$	$Variety$	$Valence$	$Tenure$	$AvgAnswer$	$AvgQuestion$
$Volume$	1.00	-0.058	0.504	0.423	0.608	0.293
$Variety$		1.000	-0.063	0.034	-0.044	0.420
$Valence$			1.000	0.017	0.523	0.018
$Tenure$				1.000	-0.071	-0.028
$AvgAnswer$					1.000	-0.032
$AvgQuestion$						1.000

Online Appendix C. Prior Distributions and Model Estimation

For all parameters in our model, we use vague priors to minimize the impact of prior distributions. For the parameters η, α_s, β, β^a, $\zeta_{s,}$, ξ_s and ξ_s^a, we assume a normal prior with a mean of 0 and a variance of 1000:

$$\eta_s \sim N(0,1000), \alpha_s \sim N(0,1000), \beta_s \sim N(0,1000),\ \beta_s^a \sim N(0,1000),$$

$$\zeta_s \sim N(0,1000), \xi_s \sim N(0,1000), \xi_s^a \sim N(0,1000),\ s = 1,2,\cdots S.$$

For the individual parameters β_{i0} and β_{i0}^*, the following prior distributions are used:

$$\beta_{i01} = \Delta_{i1} \sim N\left(\overline{\Delta}_1, \sigma_{\Delta_1}^2\right)$$

$$\beta_{i01}^a = \Delta_{i1}^a \sim N\left(\overline{\Delta}_1^a, \sigma_{\Delta_1^a}^2\right)$$

For $s = 2, \cdots, S$,

$$\beta_{i0s} = \beta_{i0s-1} + \exp(\Delta_{is}), \qquad \Delta_{is} \sim N\left(\overline{\Delta}_s, \sigma_{\Delta_s}^2\right),$$

$$\beta_{i0s}^a = \beta_{i0s-1}^a + \exp(\Delta_{is}^a), \qquad \Delta_{is}^a \sim N\left(\overline{\Delta}_s^a, \sigma_{\Delta_s^a}^2\right),$$

$$\overline{\Delta}_s \sim N(0,1000), \qquad \sigma_{\Delta_s}^2 \sim InvGamma(0.01,0.01), \qquad s = 1, \cdots, S$$

$$\overline{\Delta}_s^a \sim N(0,1000), \qquad \sigma_{\Delta_s^a}^2 \sim InvGamma(0.01,0.01), \qquad s = 1, \cdots, S$$

For the individual transition threshold μ_{irj}, the following prior distributions are used:

$$\mu_{i11} = \delta_{i11} \sim N\left(\bar{\delta}_{11}, \sigma_{\delta_{11}}^2\right)$$

$$\mu_{iS2} = \delta_{iS2} \sim N\left(\bar{\delta}_{S2}, \sigma_{\delta_{S2}}^2\right)$$

For $r = 2, \cdots, S-1$,

$$\mu_{ir1} = \delta_{ir1} \sim N\left(\bar{\delta}_{r1}, \sigma^2_{\delta_{r1}}\right)$$

$$\mu_{ir2} = \delta_{ir1} + \exp(\delta_{ir2})\,, \qquad \delta_{ir2} \sim N\left(\bar{\delta}_{r2}, \sigma^2_{\delta_{r2}}\right),$$

$$\bar{\delta}_{rj} \sim N(0{,}1000), \qquad \sigma^2_{\delta_{rj}} \sim InvGamma(0.01{,}0.01), \qquad r = 1{,}2, \cdots, S, \qquad j = 1{,}2.$$

Lastly, for the prior distribution on correlation matrix Γ_r in Gaussian copula model, we follow Liechty et al. (2004) and assume a normal prior $N(0{,}1)$ on correlation coefficients:

$$\pi(\Gamma_1, \Gamma_2, \cdots, \Gamma_S) = c^* \prod_{i<j} \exp\left\{-\frac{\gamma_{ij}^2}{2}\right\} \prod_{r=1}^{S} \prod_{j=1}^{J} \exp\left\{-\frac{p_{rj}^2}{2}\right\} \cdot \prod_{r=1}^{S} I_{\{\Gamma_r \in \mathcal{R}\}}$$

where $\mathcal{R}$ is the space of all correlation matrices of dimension $J+1$, and

$$c^{*-1} = \int_{\Gamma_1, \Gamma_2, \cdots, \Gamma_S \in \mathcal{R}} \prod_{i<j} \exp\left\{-\frac{\gamma_{ij}^2}{2}\right\} d\gamma_{ij} \cdot \prod_{r=1}^{S} \prod_{j=1}^{J} \exp\left\{-\frac{p_{rj}^2}{2}\right\} dp_{rj},$$

We devise a Markov Chain Monte Carlo (MCMC) algorithm to estimate the proposed HMM. We sample all parameters iteratively using either the random-walk Metropolis-Hasting algorithm (Chibs and Greenburg 1995) or the Gibbs sampling (Gelfand 2000). For Gibbs update, we generate the random variable from its full conditional distribution. For Metropolis-Hasting update, we generate the proposal draw either from a normal distribution or uniform distribution and then we accept the new draw with probability $\min\left\{1, \frac{f\left(y_{it}, y_{it}^a | \theta_j^{new}, \theta_{-j}\right)\pi\left(\theta_j^{new}\right)}{f\left(y_{it}, y_{it}^a | \theta_j^{(t-1)}, \theta_{-j}\right)\pi\left(\theta_j^{(t-1)}\right)}\right\}$, where $f(.)$ is the likelihood function of the proposed model.

To increase the efficiency of estimation, we use collapsed Gibbs sampler to first sample correlation coefficients P_r $(r = 1{,}2 \cdots, S)$ before sampling the latent states. Then, we sample latent states and update the rest of the parameters accordingly. Our MCMC algorithm is implemented as follows:

1) Sample $P_r, r = 1{,}2, \cdots, S$ using the Metropolis-Hasting update.
2) Sample $S_{it}, i = 1{,}2, \cdots, n, s = 1{,}2 \cdots, S$, using forward-backward recursions (Scott, 2002).
3) Sample Γ using the Metropolis-Hasting update.
4) Sample $\alpha_s, \zeta_{s,} \tau_{ts}$ $s = 1{,}2, \cdots, S$ using the Metropolis-Hasting update.
5) Sample $\beta_s, \xi_s, s = 1{,}2, \cdots, S$ using the Metropolis-Hasting update.

6) Sample β_s^*, ξ_s^* $s = 1,2,\cdots,S$ using the Metropolis-Hasting update.

7) Sample η_s, $s = 1,2,\cdots,S$ using the Metropolis-Hasting update.

8) Sample $\delta_{irj}, i = 1,2,\cdots n$, $r = 1,2,\cdots,S$, $j = 1,2$ using the Metropolis-Hasting update.

9) Sample $\Delta_{is}, i = 1,2,\cdots n$, $s = 1,2,\cdots,S$ using using the Metropolis-Hasting update.

10) Sample $\Delta_{is}^a, i = 1,2,\cdots n$, $s = 1,2,\cdots,S$ using the Metropolis-Hasting update.

11) Sample $\bar{\delta}_{rj}$ and $\sigma_{\delta_{rj}}^2$, $r = 1,2,\cdots,S$, $j = 1,2$ using the Gibbs update:

$$\sigma_{\delta_{rj}}^2 \sim InvGamma\left(0.01 + \frac{n}{2}, 0.01 + \frac{\sum_{i=1}^n \left(\delta_{irj} - \bar{\delta}_{rj}\right)^2}{2}\right)$$

$$\bar{\delta}_{rj} \sim N\left(\frac{1000 * \sum_{i=1}^n \delta_{irj}}{\sigma_{\delta_{rj}}^2 + n * 1000}, \frac{1000 * \sigma_{\delta_{rj}}^2}{\sigma_{\delta_{rj}}^2 + n * 1000}\right)$$

12) Sample $\overline{\Delta}_s$ and $\sigma_{\Delta_s}^2$, $s = 1,2,\cdots,S$ using the Gibbs update:

$$\sigma_{\Delta_s}^2 \sim InvGamma\left(0.01 + \frac{n}{2}, 0.01 + \frac{\sum_{i=1}^n \left(\Delta_{is} - \overline{\Delta}_s\right)^2}{2}\right)$$

$$\overline{\Delta}_s \sim N\left(\frac{1000 * \sum_{i=1}^n \Delta_{is}}{\sigma_{\Delta_s}^2 + n * 1000}, \frac{1000 * \sigma_{\Delta_s}^2}{\sigma_{\Delta_s}^2 + n * 1000}\right)$$

13) Sample $\overline{\Delta}_s^a$ and $\sigma_{\Delta_s^a}^2$, $s = 1,2,\cdots,S$ using Gibbs update:

$$\sigma_{\Delta_s^a}^2 \sim InvGamma\left(0.01 + \frac{n}{2}, 0.01 + \frac{\sum_{i=1}^n \left(\Delta_{is}^a - \overline{\Delta}_s^a\right)^2}{2}\right)$$

$$\overline{\Delta}_s^a \sim N\left(\frac{1000 * \sum_{i=1}^n \Delta_{is}^a}{\sigma_{\Delta_s^a}^2 + n * 1000}, \frac{1000 * \sigma_{\Delta_s^a}^2}{\sigma_{\Delta_s^a}^2 + n * 1000}\right)$$

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