Online Supplement to: "Extortionality" in Ransomware Attacks: A Microeconomic Study of Extortion and Externality

Debabrata Dey
(deb.dey@ku.edu)
University of Kansas, Lawrence, KS

Atanu Lahiri
(atanu.lahiri@utdallas.edu)
University of Texas, Dallas, TX

Appendices

A Broader Literature

Our work draws from a diverse stream of literature on externality, extortion, and repeated games. Here, we provide a quick overview of this literature.

A.1 Externality

Economic models are particularly helpful in understanding negative externality that has long been recognized as typical of the cybersecurity context. Negative externality arises primarily because users are connected through telecommunication networks and a breach in one user's security can quickly make others susceptible. Further, the actions of a user are often driven by self-interest and not the collective interest of the entire user population. For example, August and Tunca (2006) explain why users who find patching inconvenient may leave their systems unpatched even when doing so can put others in harm's way. Likewise, Dey et al. (2012) argue that users' inertia in adopting anti-virus software is in part attributable to their desire to free-ride on the adopters. In our context as well, ransom payments involve a similar negative externality. However, this externality has nothing to do with users being connected through the same communication network; rather, it stems from the fact that a victim's decision to comply with an attacker's ransom demand raises the future threat level for all users, an issue not considered in prior research.

The negative externality considered in this work is also reminiscent of pollution. Pollution is a *technological externality* and, unlike a pecuniary externality, it does jeopardize the Pareto optimality of competitive equilibria (Laffont 1989). This is because a polluter simply ignores any costs that pollution imposes on the society and considers only its own costs when making production decisions. A way to restore Pareto optimality in such a scenario could be creating an efficient market that allows the pollutees to charge the polluters (Cooter 1989). Alternatively, governments can impose pollution quotas or levy penalties/taxes to mitigate the externality. We are not considering such fiscal interventions alone; rather, the primary intervention of our interest is a complete ban on ransom payments. First, a blanket ban on pollution is neither feasible nor practical as that would require completely shutting down the production process itself. Moreover, the externality in our context arises from extortion, the presence of another strategic agent—the extortionist—introducing a unique dimension absent in prior literature.

A.2 Extortion

The literature on the economics of extortion examines strategic interactions between an extortionist and his victims (Sandler 2000, Shavell 1993). In the case of extortion, a threatener gets itself into a *position* to make a ransom demand of a victim in exchange for something that is rightfully the victim's—a demand that the victim may accept or reject—and if the victim rejects, the threatener has the choice of carrying out the threat. It is a one-shot game and, irrespective of whether the victim accedes or the threatener carries out its threat, the threatener loses its position, making the game end right there. Blackmail is similar, except that it is essentially an offer to trade silence for money, and the blackmailer can maintain its position over a prolonged period, allowing it to make repeated demands from the same victim (Block and Anderson 2000, Shavell 1993). Conceptually, ransomware is a form of extortion, and many of the ideas from the traditional literature extend. At the same time, however, our context is different in a significant way. First, we view the interactions between extortionists and their victims through the lens of a negative externality. As a result, unlike a typical single-period extortion game, ours becomes a repeated game. Similar to an extortionist, a ransomware attacker also loses its threatening position after the focal period, but because of the negative externality that permeates from one period to the next, it may regain its position in the future through a new breach. This way, the negative externality influences victims' future breach probability, connecting their current decisions to future pay-offs and making ours a Markovian repeated game. Second, although traditional extortion literature provides ample economic arguments in support of criminalizing extortion, it remains silent about sanctions on victims themselves. In our context, though, there is no easy way to bring the extortionist to justice, so all policy interventions are invariably geared towards restraining the victims and mitigating the externality that way. Third, we consider strategic interactions among victims themselves, an issue that, to the best of our knowledge, has not been a priority of the extortion literature.

A.3 Repeated Games

Methodologically, our work is related to games that are repeated over an infinite horizon (Rasmusen 2007, chap.5). To be sure, our game is also a *repeated game*, and ours also spans an infinite horizon, but there is an important distinction. Unlike a typical infinite-period repeated game—such as the repeated versions of "battle of sexes" or "prisoner's dilemma," where the payoff matrix remains the same from one period to another (Fudenberg and Tirole 1991, chap.5)—our cost matrix changes from one state to another, the states themselves evolving according to a Markov decision process (Ross 1992, chap.6).

As far as repeated games are concerned, there is also a version that runs over a finite number of periods. A well-known example is that of the "chain store paradox" due to Selten (1978), where backward induction guarantees a Pareto optimal outcome in all periods. In other cases, though—say, in prisoner's dilemma—it is not possible to reach the Pareto optimal outcome within the confines of a finite-period setting, but certainly possible when played over an infinite horizon, for instance, by adopting a "grim" strategy (Rasmusen 2007, chap.5), one that we also employ in our analysis in Section 8.2.

References

August, T., T.I. Tunca. 2006. Network software security and user incentives. *Management Science* **52**(11) 1703–1720.

Block, W., G.M. Anderson. 2000. Blackmail, extortion, and exchange. *New York Law Review* **44**(3 and 4) 541–562.

Cavusoglu, H., S. Raghunathan, W.T. Yue. 2008. Decision-theoretic and game-theoretic approaches to IT security investment. *Journal of Management Information Systems* **25**(2) 281–304.

Cooter, R.D. 1989. The Coase theorem. J. Eatwell, M. Millgate, P. Newman, eds., *Allocation, Information and Markets*. Palgrave Macmillan, London, UK, 64–70.

Dey, D., A. Lahiri, G. Zhang. 2012. Hacker behavior, network effects, and the security software market. *Journal of Management Information Systems* **29**(2) 77–108.

Fudenberg, D., J. Tirole. 1991. *Game Theory*. MIT Press, Cambridge, MA.

Laffont, J.-J. 1989. Externalities. J. Eatwell, M. Millgate, P. Newman, eds., *Allocation, Information and Markets*. Palgrave Macmillan, London, UK, 112–116.

Rasmusen, E. 2007. *Games and Information: An Introduction to Game Theory*. 4th ed. Blackwell, Malden, MA.

Ross, S.M. 1992. *Applied Probability Models with Optimization Applications*. Dover, New York, NY.

Sandler, T. 2000. Economic analysis of conflict. *The Journal of Conflict Resolution* **44**(6) 723–729.

Selten, R. 1978. The chain store paradox. *Theory and Decision* **9**(2) 127–159.

Shavell, S. 1993. An economic analysis of threats and illegality: Blackmail, extortion, and robbery. *University of Pennsylvania Law Review* **141**(5) 1877–1903.

B Model Extensions

In this appendix, we conduct robustness checks by relaxing some of the assumptions of the original setup to verify under what conditions our results remain applicable and when they do not.

B.1 Targeted Attacks

In the original setup, we had ignored targeted attacks, a possibility that we consider now. Specifically, if a firm pays ransom in period t, attackers will target it disproportionately more in the next period, that is, period $t+1$. In other words, the breach probability for a firm does not only depend on how many firms paid ransom in the previous period, but it also depends on whether the firm itself paid ransom.

To model this scenario, we need to relax Assumption 1 and redefine the breach probability for a firm as:

$$\beta_m(z) = \beta(1 + \alpha r m) + \nu z,$$

where $z = 1$ if the firm itself paid ransom in the previous period and $z = 0$ otherwise; ν captures the effect of targeted attacks, and $m \geq z$ is the number of firms that paid ransom in the previous period. As before, α is the externality parameter, and β is the inherent vulnerability. We do not exclude the possibility that ν can be negative—if paying ransom were to reduce the chance of a breach, such a scenario can be accommodated as well.

Note that, just as z reflects whether the focal firm paid in the previous period, $m-z$ represents the decision of the non-focal firm. Specifically, if the non-focal firm paid in the previous period, $m-z$ would be one, and it would be zero otherwise. Based on this, we can adjust the expression for $V(m,s)$ for the two-firm case (Lemma 1) as follows:

$$\begin{aligned} V(m,z,s) = &(1-\beta_m(z))(1-\beta_m(m-z))v_{00}(s) + \beta_m(z)(1-\beta_m(m-z))v_{10}(s) \\ &+ \beta_m(m-z)(1-\beta_m(z))v_{01}(s) + \beta_m(z)\beta_m(m-z)v_{11}(s). \end{aligned}$$

We can now solve the two-firm case as before by solving (1), (2), and (3), while using $V(m,z,s)$ in place of $V(m,s)$. The new solution is given by:

$$v_{ij}\left(\mathrm{N}^2\right) = r(1+\gamma)\left(\frac{\eta}{\alpha(1-\delta)}+i\right), \qquad v_{ij}\left(\mathrm{P}^2\right) = r\left(\frac{\frac{\eta}{\alpha(1-\delta)}+\frac{\eta r(j-i)}{1-\delta\nu}+i}{1-2\eta r-\delta\nu}\right),$$

$$v_{ij}(\mathrm{PN}) = \frac{r\left(\frac{\eta}{\alpha(1-\delta)}+i\right)}{1-\eta r-\delta\nu}, \text{ and } \qquad v_{ij}(\mathrm{NP}) = r(1+\gamma)\left(\frac{\left(\frac{\eta}{\alpha(1-\delta)}+i\right)(1-\delta\nu)+\eta r(j-i)}{1-\eta r-\delta\nu}\right),$$

where $\eta = \alpha\beta\delta$ as in our original setup. Note that if $\nu = 0$, this new solution reduces to the one mentioned in Lemma 2.

We are now ready adjust the thresholds χ_1 and χ_2 defined in Proposition 2. Proceeding exactly as we did in the original analysis, we obtain:

$$\chi_1 = \frac{\eta r+\delta\nu}{1-\eta r-\delta\nu} \text{ and } \chi_2 = \frac{\eta r\left(\frac{1-2\delta\nu}{1-\delta\nu}\right)+\delta\nu}{1-2\eta r-\delta\nu}.$$

It is clear that these thresholds coincide with their original values when $\nu = 0$. In fact, the n-firm case can also be solved. Specifically, Theorem 1 can be generalized by incorporating the impact of ν on χ_k, $1 \le k \le n$, while leaving $\chi_0 = 0$ and $\chi_{n+1} = +\infty$ as originally defined:

$$\chi_k = \frac{\eta r\left(\frac{1-k\delta\nu}{1-\delta\nu}\right)+\delta\nu}{1-k\eta r-\delta\nu}.$$

Since $\frac{\partial\chi_k}{\partial\nu} = \frac{\delta}{k}\left(\frac{k-1}{(1-\delta\nu)^2}+\frac{1}{(1-k\eta r-\delta\nu)^2}\right) > 0$, it is clear that χ_k is increasing in ν. We can also generalize the welfare analysis in Theorem 2 by recomputing the ξ_1 threshold:

$$\xi_1 = \frac{n\eta r+\delta\nu}{1-n\eta r-\delta\nu},$$

which is also increasing in ν.

Overall, ν has little impact on our analyses except that its introduction shifts the χ_k and ξ_1 thresholds. An important point to note here is that we do not put any restrictions on ν; it could be either positive or negative. When ν is negative, it represents a situation where ransom payment by a firm after a security breach may reduce the firm's future breach probability. Economically, a negative ν enhances a firm's willingness to pay ransom, so all thresholds shift downward; the opposite happens for a positive ν, which works as a damper to the firm's willingness to pay. Notwithstanding these shifts, the thresholds' relative order remains the same, and $\xi_1 > \chi_n$ still holds. Thus, ν has no structural impact on our analyses or results and, qualitatively, all findings remains intact.

B.2 Heterogeneous Firms

We now extend our analysis to a setting where the firms are not identical. In particular, when not acceding to ransom demands, firms could end up incurring different losses. Accordingly, we denote the loss incurred by firm i as c_i. Two cases are possible: (i) attackers are strategic, and (ii) they are not.

If attackers are strategic, they choose the ransom demand r_i for firm i in such a way that every firm ends up facing the same loss ratio, that is, $\frac{c_i - r_i}{r_i}$ becomes the same for all firms. This is because if the attacker wants k out of n firms to pay in the equilibrium, it would choose r_i's in a manner that makes $\gamma_i = \chi_k$ for all i; see Section 6.2. Viewed this way, the apparent heterogeneity disappears, our earlier analysis applies verbatim, and there is little else to do. So, we now consider the case where attackers are not strategic and r is fixed. In that case, different firms would face different loss ratios, and that for firm i is now given by $\gamma_i = \frac{c_i - r}{r}$. Now, if all those loss ratios fall in the same region, that is, if there exists a $k \in \{0, 1, \ldots, n\}$ such that, for all $i \in \{1, \ldots, n\}$, $\chi_k \leq \gamma_i \leq \chi_{k+1}$, then the equilibrium outcome would be $\mathrm{P}^k\mathrm{N}^{n-k}$ or $\mathrm{N}^{n-k}\mathrm{P}^k$, depending on whether the focal firm plays P or N. And, again, all our earlier analyses apply verbatim. In other words, when the heterogeneity is not large and γ_i's are all narrowly bounded, our earlier insights continue to apply.

B.2.1 General Analysis

We next consider the most general case where the heterogeneity is large, that is, γ_i's are all scattered across different regions. Even in this situation, much of our earlier analysis goes through. Specifically:

Remark 1 *Let $\chi_k < \gamma_i < \chi_{k+1}$ represent firm i's relative position, for some fixed k. Then, if fewer than k other firms are playing* P*, firm i would play* P*; it would play* N *otherwise.*

To better understand this strategy and its associated outcomes, we use the illustration in Table B1. Consider Case 1 in the table. In that case, $\gamma_1, \gamma_2 > \chi_5$, so firms 1 and 2 would always pay. Similarly, since $\gamma_5 < \chi_1$, firm 5 would never pay. Firms 3 and 4 both have loss ratios less than χ_3, and they want to pay only if fewer than 2 firms are paying; see Remark 1. Since we already have two other firms (1 and 2) paying, firms 3 and 4 would not pay, resulting in the equilibrium $\mathrm{P}^2\mathrm{N}^3$. Other cases in the table can be interpreted in an analogous manner. It is clear that the heterogeneity in γ is likely to result in an asymmetric outcome—unless all the γ_i's are above χ_n or all below χ_1, we will get an equilibrium where some firms pay and others do not.

Table B1: Examples of Firms' Strategies and Their Associated Outcomes for $n = 5$

No.	**Case**	**Strategy**	**Equilibrium**
1	$\gamma_1, \gamma_2 > \chi_5$; $\chi_2 < \gamma_3, \gamma_4 < \chi_3$; $\gamma_5 < \chi_1$	Firms 1 and 2 will pay; Firms 3, 4, and 5 will not	$\mathrm{P}^2\mathrm{N}^3$
2	$\gamma_1, \gamma_2 > \chi_5$; $\chi_3 < \gamma_3 < \chi_4$; $\gamma_5 < \chi_1$; $\chi_2 < \gamma_4 < \chi_3$	Firms 1, 2, and 3 will pay; Firms 4 and 5 will not	$\mathrm{P}^3\mathrm{N}^2$
3	$\chi_4 < \gamma_1, \gamma_2, \gamma_3 < \chi_5$; $\chi_3 < \gamma_4 < \chi_4$; $\chi_1 < \gamma_5 < \chi_2$	Firms 1, 2, and 3 will pay; Firms 4 and 5 will not	$\mathrm{P}^3\mathrm{N}^2$
4	$\chi_4 < \gamma_1, \gamma_2, \gamma_3, \gamma_4 < \chi_5$; $\chi_3 < \gamma_5 < \chi_4$	Firms 1, 2, 3, and 4 will pay; Firm 5 will not	$\mathrm{P}^4\mathrm{N}$
5	$\gamma_1 > \chi_5$; $\chi_4 < \gamma_2 < \chi_5$; $\chi_3 < \gamma_3 < \chi_4$; $\chi_2 < \gamma_4 < \chi_3$; $\gamma_5 < \chi_1$	Firms 1, 2, and 3 will pay; Firms 4 and 5 will not	$\mathrm{P}^3\mathrm{N}^2$

Now, although not much changes conceptually, fully characterizing the equilibrium for an n-firm game becomes difficult because of combinatorial complexities. To get insights, therefore, we first focus on the case of two firms. Let χ_1 and χ_2 be as defined in Proposition 4. Consistent with our earlier findings, if $\gamma_i \leq \chi_1$ for both $i \in \{1, 2\}$, then N^2 is the equilibrium; if $\gamma_i \geq \chi_2$ for both, then P^2 is the equilibrium; in all other cases, one firm playing P and the other opting for N is the equilibrium. Figure B1 depicts these equilibria. To understand this figure properly, we can refer back to Figures 3 and 5. There, the firms were identical, so each could be represented by a single point on the $(\eta r, \gamma)$ plane. However, now that they have different loss ratios, γ_1 and γ_2, the same representation does not work. Instead, we now need to layout possible real-life contexts on the (γ_1, γ_2) plane. To understand how this new representation relates back to the original one, we need to trace the yellow diagonal in Figure B1, which represents the case of $\gamma_1 = \gamma_2$. As we go from the left-bottom corner to the top-right one, we first encounter the green region where neither firm pays. As the loss ratio increases, tracing the diagonal takes us to the asymmetric outcome region in dark blue, and then

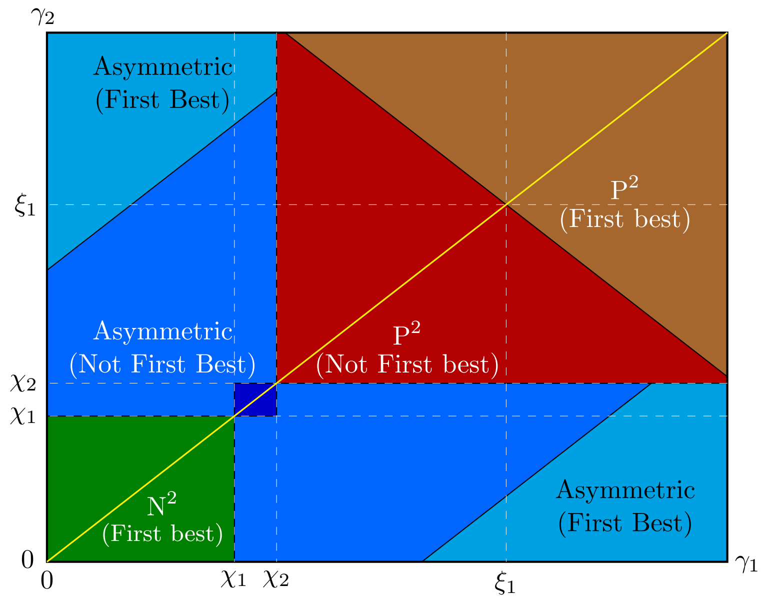

Figure B1: Impact of Prohibition with Asymmetric Firms

eventually to the red and brown regions where both firms pay. This is exactly the same trend observed in both Figures 3 and 5.

Not surprisingly, new possibilities could arise when we are off the diagonal. As before, when both firms (that is, both loss ratios) are below χ_1, or both are above χ_2, we get a symmetric outcome. Moreover, we get an asymmetric equilibrium when both firms are located between χ_1 and χ_2; this is the case corresponding to the tiny rectangular region shown in the darkest shade of blue. However, the overall asymmetric region now spans additional cases: (i) firm 1 is below χ_2 and firm 2 above χ_1, and (ii) firm 1 is above χ_1 and firm 2 below χ_2. These additions are illustrated in light blue and cyan in Figure B1.

Using Figure B1, we can also revisit the role of ξ_1 discussed earlier in Theorem 2 and Figure 5. When $\gamma_i \geq \chi_2$ for both $i \in \{1, 2\}$ and the equilibrium is P^2, the total welfare deviates from the first best if and only if $\sum_{k=1}^{2}(\gamma_k - \xi_1) < 0$ (red region in Figure B1), and it coincides with the first best otherwise (brown region). Therefore, exactly as we saw in Figure 5, when P^2 is the equilibrium, a ban improves welfare if and only if the equilibrium falls in the red region and not in the brown. Regarding the asymmetric equilibrium regions, if $\gamma_i > \chi_1$ and $\gamma_j < \chi_2$ ($j \neq i$), and firm i plays P in the asymmetric equilibrium, a ban is effective if and only if $\gamma_i < \chi_1(2 + \gamma_j)$. Thus, similar to the brown region, in the cyan as well, a ban is counterproductive, while it remains useful in both deep and light blue regions. In the green region where $\gamma_i < \chi_1$ for both i, the equilibrium is N^2, and as before, imposing a ban has no material effect. All in all, in this extended setup, the asymmetric region is the only one that is a bit different from what we observed earlier. In the original setup, the entire asymmetric region deviated from the first best. Interestingly, now a portion of it—specifically the two parts in cyan—coincides with the first best. What is the intuition behind this new finding? When one firm faces a very high loss ratio, notwithstanding the externality, it becomes socially optimal for it to pay the attacker and avoid excessive losses.

B.2.2 A Special Case

Extending Figure B1 to $n > 2$ firms is not realistic because of obvious visualization difficulties. We, therefore, examine a special case of the n-firm setting in which some firms, say an n_H number of them, face a very high loss ratio, so they always pay upon a breach. Also, let there be n_L other firms who face a very low loss ratio and never pay. The remaining $\hat{n} = n - n_L - n_H$ firms are assumed to be identical as in the original model, and they all face the same (or very similar) loss ratio of γ. Once again, our earlier results carry over to such a setting, albeit with minor adjustments. In particular, the χ_k thresholds defined in Theorem 1 still apply, except that the relevant range for k becomes $n_H + 1 \le k \le n - n_L$. When γ is above χ_{n-n_L}, all but n_L firms pay, and when γ is below χ_{n_H+1} only n_H firms pay. In between, the number of firms playing P is as given in Theorem 1. To make this point visually, we can revise Figure 4 to Figure B2 for $n = 5$, $n_L = 1$, and $n_H = 1$. As shown in this figure, all $\hat{n} = n - n_L - n_H = 3$ firms would pay if breached as long as $\gamma > \chi_{n-n_L} = \chi_4$. Likewise, none of these $\hat{n}$ firms would pay after a breach if $\gamma < \chi_{n_H+1} = \chi_2$. Between χ_2 and χ_4, only some of these $\hat{n}$ firms would pay.

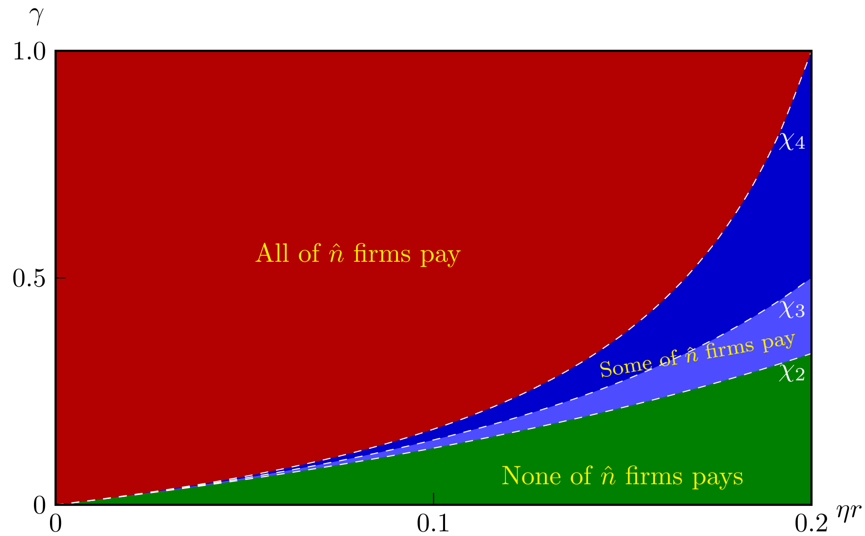

Figure B2: Equilibrium with Heterogeneous firms; $n = 5$, $n_L = 1$, $n_H = 1$, $\hat{n} = n - n_L - n_H = 3$

Since the problem with the three types of firms is structurally the same as the homogeneous case in the main paper, all our earlier findings qualitatively extend. The only necessary caveat is that a blanket prohibition on ransom payments would excessively hurt all n_H firms who face a large loss ratio, so a complete ban might no longer be socially optimal. This is akin to what happens in the cyan regions in Figure B1. A partial ban that exempts firms with high loss ratios could be a possible solution.

B.3 Security Investment Endogenized

Consider a case where a firm can influence its own breach probability through investments in cybersecurity. The breach probability for firm i can be expressed as:

$$\beta_{mi} = b_i(1 + \alpha r m),$$

where b_i, the intrinsic vulnerability of firm i, is a strategic lever for the firm. Firm i can set its own b_i by choosing its security investment to be $\frac{g}{b_i}$, where $g > 0$ is a constant. We start with the two-firm case. Without any loss of generality, we consider an initial state of $(0, 0)$. The overall cost to firm 1 can then be found as:

$$v_{00}^{(1)}(\mathrm{N}^2) = \frac{b_1 \delta r(1+\gamma)}{1-\delta} + \frac{g}{b_1}, \qquad v_{00}^{(1)}(\mathrm{P}^2) = \frac{b_1 \delta r}{(1-\delta)(1-\alpha\delta r(b_1+b_2))} + \frac{g}{b_1},$$

$$v_{00}^{(1)}(\mathrm{PN}) = \frac{b_1 \delta r}{(1-\delta)(1-\alpha b_1 \delta r)} + \frac{g}{b_1}, \text{ and} \qquad v_{00}^{(1)}(\mathrm{NP}) = \frac{b_1 \delta r(1+\gamma)}{(1-\delta)(1-\alpha b_2 \delta r)} + \frac{g}{b_1}.$$

The corresponding cost to firm 2 can be written as:

$$v_{00}^{(2)}(\mathrm{N}^2) = \frac{b_2\delta r(1+\gamma)}{1-\delta} + \frac{g}{b_2}, \qquad v_{00}^{(2)}(\mathrm{P}^2) = \frac{b_2\delta r}{(1-\delta)(1-\alpha\delta r(b_1+b_2))} + \frac{g}{b_2},$$

$$v_{00}^{(2)}(\mathrm{NP}) = \frac{b_2\delta r}{(1-\delta)(1-\alpha b_2\delta r)} + \frac{g}{b_2}, \text{ and} \qquad v_{00}^{(2)}(\mathrm{PN}) = \frac{b_2\delta r(1+\gamma)}{(1-\delta)(1-\alpha b_1\delta r)} + \frac{g}{b_2}.$$

It is clear from the above expressions that, when firm j invests in cybersecurity, b_j goes down, forcing in turn both $v_{00}^{(i)}(\mathrm{P}^2)$ and $v_{00}^{(i)}(\mathrm{NP})$, $i \neq j$, to go down as well, effectively allowing firm i to get away with a higher b_i of its own, that is, with a lower investment in cybersecurity. In other words, another externality—akin to the one in (Cavusoglu et al. 2008)—is baked into this extended model.

To endogenize b_i, we will start with the N^2-strategy. In that case, firms' investment decisions are identical but independent. Minimizing $v_{00}^{(1)}(\mathrm{N}^2)$, we find the optimal investment level for either firm as $b^*(\mathrm{N}^2) = \sqrt{\frac{g(1-\delta)}{\delta r(1+\gamma)}}$. Plugging this back in to the expression for $v_{00}^{(i)}(\mathrm{N}^2)$, we get $v_{00}^{(i)*}(\mathrm{N}^2) = 2\sqrt{\frac{g\delta r(1+\gamma)}{1-\delta}}$ in equilibrium, for $i = 1, 2$. Next, when the strategy is PN, firm 1's total cost is independent of firm 2's investment, but not the other way. Accordingly, firm 1's optimal investment can be found by minimizing $v_{00}^{(1)}(\mathrm{PN})$ over b_1, which leads to $b_1^*(\mathrm{PN}) = \frac{1}{\alpha\delta r+\sqrt{\frac{\delta r}{g(1-\delta)}}}$ and $v_{00}^{(1)*}(\mathrm{PN}) = g\alpha\delta r + 2\sqrt{\frac{g\delta r}{1-\delta}}$. Substituting this $b_1^*(\mathrm{PN})$ into the expression for $v_{00}^{(2)}(\mathrm{PN})$ and minimizing it, we get $b_2^*(\mathrm{PN}) = \frac{1}{\sqrt{\delta r(1+\gamma)\left(\frac{1}{g(1-\delta)}+\alpha\sqrt{\frac{\delta r}{g(1-\delta)}}\right)}}$ and $v_{00}^{(2)*}(\mathrm{PN}) = 2\sqrt{\delta r(1+\gamma)\left(\frac{1}{1-\delta}+\alpha\sqrt{\frac{g\delta r}{1-\delta}}\right)}$. Comparing $v_{00}^{(1)*}(\mathrm{N}^2)$ and $v_{00}^{(1)*}(\mathrm{PN})$, we find that N^2 has lower cost than PN if and only if $\gamma < \chi_1 \triangleq \alpha\sqrt{g\delta r(1-\delta)} + \frac{\alpha^2 g\delta r(1-\delta)}{4}$, implying that N^2 is a valid Nash equilibrium if and only if $\gamma < \chi_1$.

We now consider strategy P^2. To find the optimal investment level in this case, we solve $\frac{\partial v_{00}^{(1)}(\mathrm{P}^2)}{\partial b_1} = 0$ and $b_1 = b_2$ simultaneously. The optimal investment level can then be obtained as $b^*(\mathrm{P}^2)$, which is the smallest positive real root of the cubic equation $\frac{\delta r(1-\alpha b\delta r)}{(1-\delta)(1-2\alpha b\delta r)^2} - \frac{g}{b^2} = 0$.[†] Substituting $b^*(\mathrm{P}^2)$ for b_1 and b_2, we can get the equilibrium cost to either firm under the P^2 strategy, $v_{00}^{(i)*}(\mathrm{P}^2)$. Next, to verify when P^2 is a valid Nash equilibrium, we consider when firm 1 would switch from P^2 to NP. Now, $v_{00}^{(1)*}(\mathrm{NP})$ must be equal to $v_{00}^{(2)*}(\mathrm{PN})$, so $v_{00}^{(1)*}(\mathrm{NP}) = 2\sqrt{\delta r(1+\gamma)\left(\frac{1}{1-\delta}+\alpha\sqrt{\frac{g\delta r}{1-\delta}}\right)}$. Let χ_2 be the solution of $v_{00}^{(1)*}(\mathrm{P}^2) = v_{00}^{(1)*}(\mathrm{NP})$, then firm 1 would not move from P^2 to NP—or firm 2 from P^2 to PN—as long as $\gamma > \chi_2$. Therefore, P^2 is a valid Nash equilibrium if $\gamma > \chi_2$.

Now, in our original setup, χ_2 was always greater than χ_1, giving rise to a window (χ_1, χ_2) where neither symmetric solution—that is, neither N^2 nor P^2—was valid. However, now in this extended setup, it can be verified that χ_1 is larger than χ_2, ensuring that at least one symmetric outcome exists for the entire region. Since a symmetric solution always exists, we need not look for asymmetric ones anymore. In fact, the asymmetric solutions are also dominated by one or both of the symmetric outcomes, implying that PN and NP can be eliminated from further considerations.

When we extend this analysis to n-firms, all the above observations extend seamlessly. In particular, χ_1, the threshold between N^n and PN^{n-1}, remains exactly the same as above. We can also find χ_n, the threshold between P^n and NP^{n-1}; it is easy to verify that χ_n is also less than χ_1, implying that at least one of the symmetric solutions (N^n or P^n) is valid throughout. All in all, the n-firm equilibrium can be expressed as:

Remark 2 *Let* $\chi_1 = \alpha\sqrt{g\delta r(1-\delta)} + \frac{\alpha^2 g\delta r(1-\delta)}{4}$. *All firms play* N *if* $\gamma < \chi_1$ *and play* P *otherwise.*

The interaction of the two externality effects—extortinality and investment externality—creates interesting dynamics. When a firm plays P, it creates a negative externality on all others by increasing their breach probability. When it invests in cybersecurity, on the other hand, it creates a positive externality by decreasing the risk level of everyone else. It turns out that, when a firm adopts P in equilibrium, it also increases its

[†]Note that, although a closed-form expression for $b^*(\mathrm{P}^2)$ exists, it is too large to include even in an appendix.

investment to compensate for the marginal increase in its own risk, the extra investment now creating a counterweight. The end result is that, in the entire region that used to be asymmetric in Figure 5, all firms now prefer to play P and then compensate by making extra investments in cyber-defense.

Even though the asymmetric region disappears now, the overall externality effect does not. It manifests itself in a substantial portion of the parameter space where the outcome is not first best:

Remark 3 *Let ξ_1 be the solution of $v_{00}^{(i)*}(\mathrm{P}^n) = v_{00}^{(i)*}(\mathrm{N}^n)$ for any i, $i = 1, 2, \ldots, n$, and let χ_1 be as above. If $\gamma > \xi_1$ or if $\gamma < \chi_1$, the outcome is first best. If γ is between χ_1 and ξ_1, the outcome is not first best.*

This is illustrated in Figure B3. It would appear that structurally everything remains the same and earlier insights prevail. A ban could help if $\gamma \in (\chi_1, \xi_1)$ but would hurt social welfare if $\gamma > \xi_1$; a ban is irrelevant below χ_1.

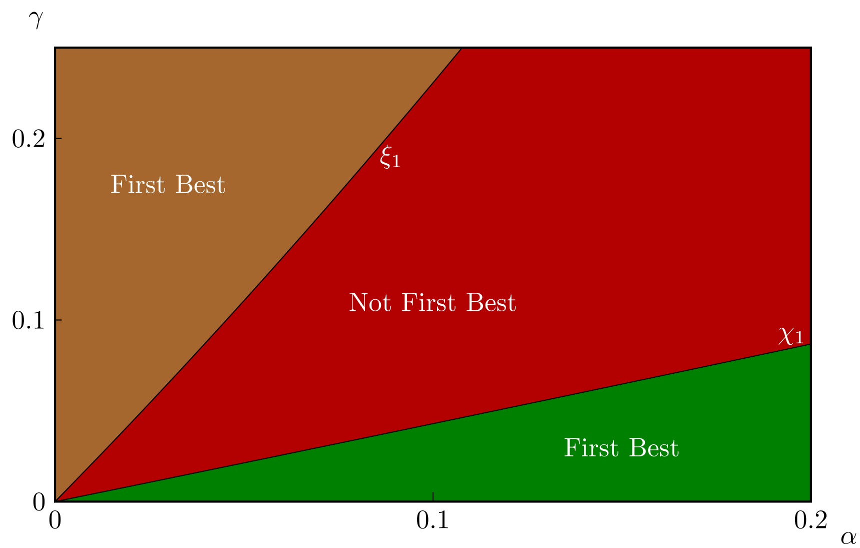

Figure B3: Equilibrium and First-Best Regions with Endogenized Security Investment; $n = 5$

B.4 A Two-Period Setup

We used an infinite-horizon model in the main paper. Such models are common in the literature, because the discount factor, $\delta \in (0, 1)$, can be reduced to effectively shorten the horizon of a firm's decision problem. The cost incurred in the k-th period into the future gets multiplied by δ^k, which declines sharply with δ, significantly diminishing the contribution of that period towards the net present cost. The costs incurred in periods further into the future would get discounted even more heavily, so they too should become irrelevant, especially as δ becomes small. Notwithstanding this discounting angle, it is important to check if using a finite-horizon setup would qualitatively change our results.

To this end, we now consider a simple setup with n firms but only two periods, Periods 1 and 2. In Period 2, which is also the last period in this revised setup, all breached firms pay in optimality, because doing so results in an immediate saving of $c - r$ and there is also no future to worry about. However, paying may not be optimal in the first period, as doing so increases the probability of a breach in the second.

To see when paying in the first period is optimal, we need to compare the costs of paying and not paying. The total cost incurred by a paying firm, assuming $m - 1$ other firms are paying as well, can be written as:

$$v_p = r + \delta \beta_m r,$$

where r is the cost incurred in Period 1 and $\beta_m r$ is the expected cost incurred in Period 2; recall that $\beta_m \triangleq \beta(1 + \alpha m r)$. Now, if the firm does not pay, the expected cost in Period 2 decreases to $\beta_{m-1} r$ but the cost incurred in the first period rises to c. Accordingly, the total cost associated with not paying in Period 1 becomes:

$$v_n = c + \delta\beta_{m-1}r.$$

Setting $v_p = v_n$, we obtain:

$$\gamma = \frac{c-r}{r} = \alpha\beta\delta r = \eta r.$$

Thus, it is optimal for a firm to accede to a ransom demand if $\gamma > \eta r$, and it is optimal to not pay otherwise. Since the threshold ηr does not depend on m, playing this way is also the dominant strategy for each firm. The implication is that there are only two regions in equilibrium: For $\gamma \leq \eta r$, the outcome is N^n, that is, no breached firm pays; for $\gamma > \eta r$, we end up with P^n, that is, all breached firms pay. In other words, there are no asymmetric outcomes anymore.

Now, is the outcome always the first best? If $\gamma \leq \eta r$, whether or not breached, no one pays in the first period, so we trivially reach the first best. Accordingly, we now consider the other possibility where $m > 0$ firms are breached in the first period and $\gamma > \eta r$. In this case, all m firms will pay in period 1, and the total cost faced by the firms in equilibrium will simply be the total ransom paid in Period 1 plus the expected payments in Period 2, that is, $mr + n\delta\beta_m r$. Now, if no one had paid, this cost would have been $mc + n\delta\beta_0 r$. Comparing the two, we get another threshold for γ, equivalent to ξ_1 in our original model:

$$mr + n\delta\beta_m r \leq mc + n\delta\beta_0 r \Leftrightarrow \gamma = \frac{c-r}{r} \geq n\alpha\beta\delta r = n\eta r.$$

Evidently, regardless of m, the strategy of paying ransom is socially optimal only if $\gamma \geq n\eta r$. In other words, Prisonner's Dilemma takes hold when $\eta r < \gamma < n\eta r$, similar to what we observed in the red region in Figure 5. Thus, although the asymmetric equilibrium disappears, the issue of externality remains relevant in this two-period setup much as it did in our original model, and policy interventions including prohibition also do as well.

What would happen if we increase the number of periods? The analysis becomes somewhat cumbersome and tedious beyond the second period, so we refrain from that exercise. In general terms, newer thresholds seem to emerge when we go beyond two or more periods. Theoretically, as the number of periods increases, these thresholds should all converge to the ones found for the infinite-horizon game.

C Proofs

Proof of Lemma 1

Since β_m is the probability of a breach for a single firm, the probability of both of them getting breached is β_m^2, only one of them getting breached is $\beta_m(1-\beta_m)$, and no one getting breached is $(1-\beta_m)^2$. The result follows. ■

Proof of Proposition 1

To show that `pn` or `np` is not played by either firm in equilibrium, we first use use (1)–(3) to determine $v_{ij}(s_1, s_2)$, $\forall\ s_1, s_2 \in \{$`pp`,`pn`,`np`,`nn`$\}$. This allows us to compare the four possible strategies. The 4×4 payoff matrix for this case is shown in Table 3. For the sake of clarity, we analyze one column at a time.

Let us first consider what firm 1 should play if firm 2 plays pp. According to (1)–(3),

$$
\begin{aligned}
v_{ij}(\texttt{pp},\texttt{pp}) &= \frac{r(\beta\delta + (1-\delta)(i(1-\alpha r\beta\delta) + j\alpha r\beta\delta))}{(1-\delta)(1-2\alpha r\beta\delta)}, \\
v_{ij}(\texttt{pn},\texttt{pp}) &= \frac{\beta\delta\left(r(1-\beta) + c\beta + \alpha r\beta^2\delta(2c-r)(1+\alpha r)\right)}{(1-\delta)(1-\alpha r\beta\delta(2(1-\beta)-\alpha r\beta))} + \frac{i(r+\alpha r\beta\delta(c\beta(2+\alpha r)-r))}{1-\alpha r\beta\delta(2(1-\beta)-\alpha r\beta)} \\
&\quad + \frac{j\alpha r\beta\delta(r+\beta(2+\alpha r)(c-r))}{1-\alpha r\beta\delta(2(1-\beta)-\alpha r\beta)} + \frac{ij(c-r-\alpha r\beta\delta(2c-r))}{1-\alpha r\beta\delta(2(1-\beta)-\alpha r\beta)}, \\
v_{ij}(\texttt{np},\texttt{pp}) &= \frac{\beta\delta\left(c - c\beta + r\beta - \alpha r\beta^2\delta(2c(1+2\alpha r) - r(1+\alpha r))\right)}{(1-\delta)(1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta))))} + ic \\
&\quad + \frac{j\alpha r\beta\delta(c(1-\beta(2+\alpha r(1+2\alpha r\beta\delta))) + r\beta(2+\alpha r))}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))} \\
&\quad - \frac{ij(c(1-2\alpha r\beta\delta) - r(1-\alpha r\beta\delta))}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))}, \\
v_{ij}(\texttt{nn},\texttt{pp}) &= \frac{c(\beta\delta + (1-\delta)(i(1-\alpha r\beta\delta) + j\alpha r\beta\delta))}{(1-\delta)(1-\alpha r\beta\delta)}.
\end{aligned}
$$

It follows that $v_{ij}(\texttt{pp},\texttt{pp}) \le v_{ij}(\texttt{pn},\texttt{pp})$ if $\gamma \ge \frac{\alpha r\beta\delta}{1-2\alpha r\beta\delta}$, and $v_{ij}(\texttt{nn},\texttt{pp}) < v_{ij}(\texttt{pn},\texttt{pp})$ if $\gamma < \frac{\alpha r\beta\delta}{1-2\alpha r\beta\delta}$. Thus, from firm 1's perspective, either pp or nn dominates pn when firm 2 plays pp. Similarly, $v_{ij}(\texttt{pp},\texttt{pp}) \le v_{ij}(\texttt{np},\texttt{pp})$ if $\gamma \ge \frac{\alpha r\beta\delta}{1-2\alpha r\beta\delta}$, and $v_{ij}(\texttt{nn},\texttt{pp}) < v_{ij}(\texttt{np},\texttt{pp})$ if $\gamma < \frac{\alpha r\beta\delta}{1-2\alpha r\beta\delta}$, implying that either pp or nn dominates np as well. Clearly, firm 1 will play neither pn nor np if firm 2 plays pp. Using a similar argument, we can prove that firm 2 will not play pn or np if firm 1 plays pp. Altogether, (pn,pp), (np,pp), (pp,pn), and (pp,np) are not possible in equilibrium.

Let us now consider what happens if firm 2 plays nn. In this case,

$$
\begin{aligned}
v_{ij}(\texttt{pp},\texttt{nn}) &= \frac{r(i(1-\delta)+\beta\delta)}{(1-\delta)(1-\alpha r\beta\delta)}, \\
v_{ij}(\texttt{pn},\texttt{nn}) &= \frac{\beta\delta(r(1-\beta) + c\beta(1+\alpha r(1+\alpha r)\beta\delta))}{(1-\delta)(1-\alpha r\beta\delta(1-\beta(2+\alpha r)))} + \frac{i(c\alpha r\beta^2\delta(2+\alpha r)+r)}{1-\alpha r\beta\delta(1-\beta(2+\alpha r))} + \frac{ij(c(1-\alpha r\beta\delta)-r)}{1-\alpha r\beta\delta(1-\beta(2+\alpha r))}, \\
v_{ij}(\texttt{np},\texttt{nn}) &= \frac{\beta\delta\left(r\beta + c\left(1-\beta-\alpha r\beta^2\delta(1+\alpha r)\right)\right)}{(1-\delta)\left(1-\alpha r\beta^2\delta(2+\alpha r)\right)} + ic - \frac{ij(c(1-\alpha r\beta\delta)-r)}{1-\alpha r\beta^2\delta(2+\alpha r)}, \\
v_{ij}(\texttt{nn},\texttt{nn}) &= \frac{c(i(1-\delta)+\beta\delta)}{1-\delta}.
\end{aligned}
$$

It follows that $v_{ij}(\texttt{pp},\texttt{nn}) \le v_{ij}(\texttt{pn},\texttt{nn})$ if $\gamma \ge \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta}$, and $v_{ij}(\texttt{nn},\texttt{nn}) < v_{ij}(\texttt{pn},\texttt{nn})$ if $\gamma < \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta}$. Thus, from firm 1's perspective, either pp or nn dominates pn when firm 2 plays nn. Similarly, $v_{ij}(\texttt{pp},\texttt{nn}) \le v_{ij}(\texttt{np},\texttt{nn})$ if $\gamma \ge \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta}$, and $v_{ij}(\texttt{nn},\texttt{nn}) < v_{ij}(\texttt{np},\texttt{nn})$ if $\gamma < \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta}$, implying that either pp or nn dominates np as well. Clearly, firm 1 will play neither pn nor np if firm 2 plays nn. Using a similar argument, firm 1 will not play pn or np if firm 1 plays nn. Altogether, (pn,nn), (np,nn), (nn,pn), and (nn,np) are not possible in equilibrium.

Moving on to the case in which firm 2 plays pn, we find that:

$$v_{ij}(\texttt{pp,pn}) = \frac{r\beta\delta\left(1+\alpha r(1+\alpha r)\beta^2\delta\right)}{(1-\delta)(1-\alpha r\beta\delta(2-\beta(2+\alpha r)))} + \frac{ir(1-\alpha r\beta\delta(1-\beta(2+\alpha r)))}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))} + \frac{j(1-i)\alpha r^2\beta\delta}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))},$$

$$v_{ij}(\texttt{pn,pn}) = \frac{\beta\delta(r(1-\beta)+c\beta(1+2\alpha r\beta\delta(1+\alpha r)))}{(1-\delta)(1-2\alpha r\beta\delta(1-\beta(2+\alpha r)))} + \frac{i(r+\alpha r\beta\delta(\beta(2+\alpha r)(c+r)-r))}{1-2\alpha r\beta\delta(1-\beta(2+\alpha r))}$$
$$+ \frac{j\alpha r\beta\delta(\beta(2+\alpha r)(c-r)+r)}{1-2\alpha r\beta\delta(1-\beta(2+\alpha r))} + \frac{ij(c(1-2\alpha r\beta\delta)-r)}{1-2\alpha r\beta\delta(1-\beta(2+\alpha r))},$$

$$v_{ij}(\texttt{np,pn}) = \frac{\beta\delta\left(c(1-\beta)+r\beta-\alpha r\beta^2\delta(1+\alpha r)(c-r)\right)}{(1-\delta)(1-\alpha r\beta\delta)} + ic + \frac{j\alpha r\beta\delta(c-\beta(2+\alpha r)(c-r))}{1-\alpha r\beta\delta} - ij(c-r),$$

$$v_{ij}(\texttt{nn,pn}) = \frac{c\beta\delta\left(1+\alpha r\beta^2\delta(1+\alpha r)\right)}{(1-\delta)(1-\alpha r\beta\delta(1-\beta(2+\alpha r)))} + ic + \frac{j(1-i)c\alpha r\beta\delta}{1-\alpha r\beta\delta(1-\beta(2+\alpha r))}.$$

It follows that $v_{ij}(\texttt{pp,pn}) \leq v_{ij}(\texttt{pn,pn})$ if $\gamma \geq \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))}$, and $v_{ij}(\texttt{nn,pn}) < v_{ij}(\texttt{pn,pn})$ if $\gamma < \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))}$. Thus, from firm 1's perspective, either pp or nn dominates pn when firm 2 plays pn. Similarly, $v_{ij}(\texttt{pp,pn}) \leq v_{ij}(\texttt{np,pn})$ if $\gamma \geq \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))}$, and $v_{ij}(\texttt{nn,pn}) < v_{ij}(\texttt{np,pn})$ if $\gamma < \frac{\alpha r\beta\delta}{1-\alpha r\beta\delta(2-\beta(2+\alpha r))}$, implying that either pp or nn dominates np as well. Clearly, firm 1 will play neither pn nor np if firm 2 plays pn. Using a similar argument, firm 2 will not play pn or np, if firm 1 plays pn. Altogether, (pn,pn), (np,pn), and (pn,np) are not possible in equilibrium.

Finally, we consider firm 1's options when firm 2 plays np. Given what we have already ruled out, to complete the proof, we just need to show that (np,np) is also not possible in equilibrium. When firm 2 plays np, firm 1's payoffs are:

$$v_{ij}(\texttt{pp,np}) = \frac{r\beta\delta\left(1-\alpha\beta^2\delta r(1+3\alpha r)\right)}{(1-\delta)(1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta))))} + \frac{ir\left(1-\alpha r\beta^2\delta(2+3\alpha r)\right)}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))}$$
$$+ \frac{ij\alpha r^2\beta\delta}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))},$$

$$v_{ij}(\texttt{pn,np}) = \frac{\beta\delta\left(r(1-\beta)+c\beta+\alpha r\beta^2\delta(1+\alpha r)(c-r)\right)}{(1-\delta)(1-\alpha r\beta\delta)} + \frac{i\left(\alpha r\beta^2\delta(2+\alpha r)(c-r)+r\right)}{1-\alpha r\beta\delta} + ij(c-r),$$

$$v_{ij}(\texttt{np,np}) = \frac{\beta\delta\left(r\beta+c\left(1-\beta-2\alpha r\beta^2\delta(1+2\alpha r)\right)\right)}{(1-\delta)\left(1-4\alpha r\beta^2\delta(1+\alpha r)\right)} + ic - \frac{ij(c(1-2\alpha r\beta\delta)-r)}{1-4\alpha r\beta^2\delta(1+\alpha r)},$$

$$v_{ij}(\texttt{nn,np}) = \frac{c\beta\delta\left(1-\alpha r\beta^2\delta(1+\alpha r)\right)}{(1-\delta)\left(1-\alpha r\beta^2\delta(2+\alpha r)\right)} + ic + \frac{ijc\alpha r\beta\delta}{1-\alpha r\beta^2\delta(2+\alpha r)}.$$

It follows that $v_{ij}(\texttt{pp,np}) \leq v_{ij}(\texttt{np,np})$ if $\gamma \geq \frac{\alpha r\beta\delta\left(1-2\alpha^2r^2\beta^2\delta\right)}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))}$, and $v_{ij}(\texttt{nn,np}) < v_{ij}(\texttt{np,np})$ if $\gamma < \frac{\alpha r\beta\delta\left(1+2\alpha^2r^2\beta^2\delta\right)}{1-\alpha r\beta\delta(1+\beta(2+\alpha r+2\alpha^2r^2\beta\delta))}$. Now, since $\frac{\alpha r\beta\delta\left(1-2\alpha^2r^2\beta^2\delta\right)}{1-\alpha r\beta\delta(1+\beta(2+\alpha r(3-2\alpha r\beta\delta)))} < \frac{\alpha r\beta\delta\left(1+2\alpha^2r^2\beta^2\delta\right)}{1-\alpha r\beta\delta(1+\beta(2+\alpha r+2\alpha^2r^2\beta\delta))}$, either pp or nn dominates np, implying that (np,np) too is not possible in equilibrium.

In summary, even though firms have the option of playing differently in different states—that is, playing pn or np—it would end up responding the same way in all states, making pp and nn the only possible choices in the equilibrium. ∎

Proof of Lemma 2

When both firms play N, that is, the strategy combination is N^2, the simultaneous equations in (4) become:

$$v_{00}(\mathrm{N}^2) = v_{01}(\mathrm{N}^2) = \delta V(0,\mathrm{N}^2) \text{ and } v_{10}(\mathrm{N}^2) = v_{11}(\mathrm{N}^2) = c + \delta V(0,\mathrm{N}^2),$$

where $V(.,.)$ is as defined in Lemma 1. Solving these, we get:

$$v_{00}(\mathrm{N}^2) = v_{01}(\mathrm{N}^2) = \frac{c\beta\delta}{1-\delta} \quad \text{and} \quad v_{10}(\mathrm{N}^2) = v_{11}(\mathrm{N}^2) = c\left(\frac{\beta\delta}{1-\delta} + 1\right).$$

Since $c = r(1+\gamma)$ and $\eta = \alpha\beta\delta$, we can combine these four expressions into one and write:

$$v_{ij}(\mathrm{N}^2) = c\left(\frac{\beta\delta}{1-\delta} + i\right) = r(1+\gamma)\left(\frac{\eta}{\alpha(1-\delta)} + i\right).$$

The expressions for the other three strategy combinations, NP, PN, and P^2, can be obtained in a similar fashion by solving the following three sets of equations:

NP: $v_{00}(\mathrm{NP}) = \delta V(0, \mathrm{NP}), v_{01}(\mathrm{NP}) = \delta V(1, \mathrm{NP}), v_{10}(\mathrm{NP}) = c + \delta V(0, \mathrm{NP}),$ and $v_{11}(\mathrm{NP}) = c + \delta V(1, \mathrm{NP})$.

PN: $v_{00}(\mathrm{PN}) = v_{01}(\mathrm{PN}) = \delta V(0, \mathrm{PN})$ and $v_{10}(\mathrm{PN}) = v_{11}(\mathrm{PN}) = r + \delta V(1, \mathrm{PN})$.

P^2: $v_{00}(\mathrm{P}^2) = \delta V(0, \mathrm{P}^2), v_{01}(\mathrm{P}^2) = \delta V(1, \mathrm{P}^2), v_{10}(\mathrm{P}^2) = r + \delta V(1, \mathrm{P}^2),$ and $v_{11}(\mathrm{P}^2) = r + \delta V(2, \mathrm{P}^2)$.

The final expressions can then be obtained by rewriting the solutions as functions of i and j. ■

Proof of Proposition 2

Not paying a ransom is the dominant strategy if $v_{ij}(\mathrm{N}^2) < v_{ij}(\mathrm{P,N})$, which is equivalent to $\gamma < \chi_1$. On the other hand, paying is the dominant strategy if $v_{ij}(\mathrm{P}^2) \leq v_{ij}(\mathrm{N,P})$, that is, if $\gamma \geq \chi_2$.

In the remainder of our parameter space, each firm's best response is playing the opposite of what the other firm plays, because $v_{ij}(\mathrm{P,N}) \leq v_{ij}(\mathrm{N}^2)$ and $v_{ij}(\mathrm{N,P}) < v_{ij}(\mathrm{P}^2)$ when $\chi_1 \leq \gamma < \chi_2$.

The second part of the proposition follows from Theorem 2 by setting $n = 2$. ■

Proof of Proposition 3

We will prove this by induction. The proposition statement is trivially true for $n = 1$, becuase when there is only one firm, it must always adopt the same strategy. And, because of Proposition 1, the result holds for $n = 2$.

Let us now assume that the proposition statement is true for $n = L$. We will show that it also holds for $n = L + 1$. Now, of the $L + 1$ firms, if any firm is not breached or if it does not pay in a period, its strategy has no impact on the cost-benefit calculus of the other L firms in that period. Therefore, we can drop this firm from further considerations without impacting others' strategies, thereby bringing us back to the case of L firms, which must abide by the induction hypothesis. Therefore, the only case where such a reduction is not possible is when all $L + 1$ firms are breached and they all play `p`. In that case, each firm finds it less costly to play `p` (over playing `n`) when L others play `p`. Naturally, if less than L other firms were to play `p`, playing `p` would still continue to be less costly than playing `n`, implying that the firm's strategy of playing `p` is valid in every conceivable state it could be in. This completes the proof. ■

Proof of Lemma 3

The probability that the focal firm gets breached is β_m. The probability that j of the other $n-1$ firms get breached is $\frac{(n-1)!}{j!(n-j-1)!}\beta_m^j (1-\beta_m)^{n-j-1}$. If firm 1 does get breached, it will be in state $(1, j)$ for some $j < n$. In contrast, if firm 1 does not get breached, it will be in state $(0, j)$ for some $j < n$. Putting all of these together and summing over all $j < n$, we get:

$$\begin{aligned} V(m, s) &= \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!}\beta_m^j (1-\beta_m)^{n-j-1} \left(\beta_m v_{1j}(s) + (1-\beta_m) v_{0j}(s)\right) \\ &= \sum_{i=0}^{1}\sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!}\beta_m^{i+j} (1-\beta_m)^{n-i-j} v_{ij}(s), \end{aligned}$$

the last step following from a simple rearrangement of terms. ■

Proof of Lemma 4

We start with the case where all firms play N. In that case, it is apparent from (5) that, for all $j < n$, $v_{0j}(\mathrm{N}^n) = \delta V(0, \mathrm{N}^n)$ and $v_{1j}(\mathrm{N}^n) = c + \delta V(0, \mathrm{N}^n)$. Substituting these into $V(0, \mathrm{N}^n)$ in Lemma 3, we get:

$$\begin{aligned}
V(0, \mathrm{N}^n) &= \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \beta^j (1-\beta)^{n-j-1} \left(\beta(c + \delta V(0, \mathrm{N}^n)) + (1-\beta)\, \delta V(0, \mathrm{N}^n)\right) \\
&= \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \beta^j (1-\beta)^{n-j-1} \left(\delta V(0, \mathrm{N}^n) + \beta c\right) \\
&= \left(\delta V(0, \mathrm{N}^n) + \beta c\right) \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \beta^j (1-\beta)^{n-j-1} \\
&= \left(\delta V(0, \mathrm{N}^n) + \beta c\right)(\beta + (1-\beta))^{n-1} = \delta V(0, \mathrm{N}^n) + \beta c.
\end{aligned}$$

Solving for $V(0, \mathrm{N}^n)$ now, we get $V(0, \mathrm{N}^n) = \frac{\beta c}{1-\delta}$, which can be plugged into (5) to obtain:

$$v_{ij}(\mathrm{N}^n) = ic + \delta V(0, \mathrm{N}^n) = c\left(\frac{\beta\delta}{1-\delta} + i\right), \quad \forall i \in \{0,1\},\ j \in \{0,1,2,\ldots,n-1\}.$$

Next, we consider the situation where firm 1 plays P, but all others play N. In that case, from (6), we can write, for all $j < n$, $v_{0j}(\mathrm{PN}^{n-1}) = \delta V(0, \mathrm{PN}^{n-1})$ and $v_{1j}(\mathrm{PN}^{n-1}) = r + \delta V(1, \mathrm{PN}^{n-1})$. Substituting these into $V(m, \mathrm{PN}^{n-1})$ in Lemma 3, for $m = 0, 1$, and using the previous trick for the summation term, we get:

$$\begin{aligned}
&V(0, \mathrm{PN}^{n-1}) = \beta(r + \delta V(1, \mathrm{PN}^{n-1})) + (1-\beta)\, \delta V(0, \mathrm{PN}^{n-1}), \text{ and} \\
&V(1, \mathrm{PN}^{n-1}) = \beta(1+\alpha r)(r + \delta V(1, \mathrm{PN}^{n-1})) + (1 - \beta(1+\alpha r))\, \delta V(0, \mathrm{PN}^{n-1}).
\end{aligned}$$

Solving them we get:

$$V(0, \mathrm{PN}^{n-1}) = \frac{r\beta}{(1-\delta)(1-\alpha r\beta\delta)} \quad \text{and} \quad V(1, \mathrm{PN}^{n-1}) = \frac{r\beta(1+\alpha r(1-\delta))}{(1-\delta)(1-\alpha r\beta\delta)}.$$

We can substitute these into (6) to get:

$$\begin{aligned}
v_{ij}(\mathrm{PN}^{n-1}) &= ir + \delta((1-i)V(0, \mathrm{PN}^{n-1}) + iV(1, \mathrm{PN}^{n-1})) \\
&= \frac{r}{1-\alpha r\beta\delta}\left(\frac{\beta\delta}{1-\delta} + i\right), \quad \forall i \in \{0,1\},\ j \in \{0,1,2,\ldots,n-1\}.
\end{aligned}$$

Next, we consider the strategy combination $s = \mathrm{P}^n$. In this case, from (7), we can write, for all $j < n$, $v_{0j}(\mathrm{P}^n) = \delta V(j, \mathrm{P}^n)$ and $v_{1j}(\mathrm{P}^n) = r + \delta V(j+1, \mathrm{P}^n)$. Substituting these into $V(m, \mathrm{P}^n)$ in Lemma 3, we get for all $m \le n$:

$$V(m, \mathrm{P}^n) = \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \beta_m^j (1-\beta_m)^{n-j-1} \left(\beta_m(r + \delta V(j+1, \mathrm{P}^n)) + (1-\beta_m)\, \delta V(j, \mathrm{P}^n)\right).$$

Although tedious, these $n+1$ linear equations can be solved for a unique solution $V(m, \mathrm{P}^n) = \frac{r\beta(1+m\alpha r(1-\delta))}{(1-\delta)(1-n\alpha r\beta\delta)}$. To ascertain that this is indeed the correct solution, we simply substitute for $V(j, \mathrm{P}^n)$ and $V(j+1, \mathrm{P}^n)$ in the above expression for $V(m,\mathrm{P}^n)$ and check whether it is satisfied. First, as per the proposed solution:

$$\beta_m(r+\delta V(j+1,\mathrm{P}^n))+(1-\beta_m)\,\delta V(j,\mathrm{P}^n)=r\beta_m+\frac{r\beta\delta}{1-n\alpha r\beta\delta}\left(\frac{1}{1-\delta}+\alpha r\beta_m+\alpha rj\right).$$

Substituting this into the expression for $V(m,\mathrm{P}^n)$ above and taking the terms independent of j outside of the summation, we get:

$$\begin{aligned}
V(m,\mathrm{P}^n)&=\sum_{j=0}^{n-1}\frac{(n-1)!}{j!(n-j-1)!}\beta_m^j\,(1-\beta_m)^{n-j-1}\left(r\beta_m+\frac{r\beta\delta}{1-n\alpha r\beta\delta}\left(\frac{1}{1-\delta}+\alpha r\beta_m+\alpha rj\right)\right)\\
&=r\beta_m+\frac{r\beta\delta}{1-n\alpha r\beta\delta}\left(\frac{1}{1-\delta}+\alpha r\beta_m\right)+\frac{\alpha r^2\beta\delta}{1-n\alpha r\beta\delta}\sum_{j=1}^{n-1}\frac{(n-1)!}{(j-1)!(n-j-1)!}\beta_m^j\,(1-\beta_m)^{n-j-1}\\
&=r\beta_m+\frac{r\beta\delta}{1-n\alpha r\beta\delta}\left(\frac{1}{1-\delta}+\alpha r\beta_m\right)+\frac{\alpha r^2\beta\delta\beta_m(n-1)}{1-n\alpha r\beta\delta}\sum_{k=0}^{n-2}\frac{(n-2)!}{k!(n-k-2)!}\beta_m^k\,(1-\beta_m)^{n-k-2}\\
&=r\beta_m+\frac{r\beta\delta}{1-n\alpha r\beta\delta}\left(\frac{1}{1-\delta}+\alpha r\beta_m\right)+\frac{\alpha r^2\beta\delta\beta_m(n-1)}{1-n\alpha r\beta\delta}\\
&=\frac{r\beta(1+m\alpha r(1-\delta))}{(1-\delta)(1-n\alpha r\beta\delta)}.\qquad\text{[after substituting }\beta_m=\beta(1+\alpha rm)]
\end{aligned}$$

Since the above is satisfied for all $m\le n$, $V(m,\mathrm{P}^n)=\frac{r\beta(1+m\alpha r(1-\delta))}{(1-\delta)(1-n\alpha r\beta\delta)}$ is indeed the unique solution. Substituting this $V(m,\mathrm{P}^n)$ into (7) gives us:

$$v_{ij}(\mathrm{P}^n)=r\left(\frac{\beta\delta(1+\alpha r(i+j)(1-\delta))}{(1-\delta)(1-n\alpha r\beta\delta)}+i\right)\quad\forall i\in\{0,1\},\ j\in\{0,1,2,\ldots,n-1\}.$$

Finally, we consider the situation where firm 1 plays N, but all others play P. In that case, from (8), we can write, for all $j<n$, $v_{0j}(\mathrm{NP}^{n-1})=\delta V(j,\mathrm{NP}^{n-1})$ and $v_{1j}=c+\delta V(j,\mathrm{NP}^{n-1})$. Substituting these into $V(m,\mathrm{NP}^{n-1})$ in Lemma 3, for $m\le n$, we get:

$$\begin{aligned}
V(m,\mathrm{NP}^{n-1})&=\sum_{j=0}^{n-1}\frac{(n-1)!}{j!(n-j-1)!}\beta_m^j\,(1-\beta_m)^{n-j-1}\left(\beta_m(c+\delta V(j,\mathrm{NP}^{n-1}))+(1-\beta_m)\,\delta V(j,\mathrm{NP}^{n-1})\right)\\
&=c\beta_m+\delta\sum_{j=0}^{n-1}\frac{(n-1)!}{j!(n-j-1)!}\beta_m^j\,(1-\beta_m)^{n-j-1}\,V(j,\mathrm{NP}^{n-1}).
\end{aligned}$$

As before, these $n+1$ linear equations can be solved to get $V(m,\mathrm{NP}^{n-1})=\frac{\beta c(1+\alpha rm(1-\delta))}{(1-\delta)(1-\alpha r\beta\delta(n-1))}$, for all $m\le n$. To confirm that this is indeed the solution, we plug this into the earlier expression for $V(m,\mathrm{NP}^{n-1})$:

$$\begin{aligned}
V(m,\mathrm{NP}^{n-1})&=c\beta_m+\delta\sum_{j=0}^{n-1}\frac{(n-1)!}{j!(n-j-1)!}\beta_m^j\,(1-\beta_m)^{n-j-1}\,\frac{\beta c(1+\alpha rj(1-\delta))}{(1-\delta)(1-\alpha r\beta\delta(n-1))}\\
&=\beta_m c+\frac{\beta\delta c}{(1-\delta)(1-\alpha r\beta\delta(n-1))}+\frac{\alpha r\beta\delta c\beta_m(n-1)}{1-\alpha r\beta\delta(n-1)}\sum_{k=0}^{n-2}\frac{(n-2)!}{k!(n-k-2)!}\beta_m^k\,(1-\beta_m)^{n-k-2}\\
&=\beta_m c+\frac{\beta\delta c}{(1-\delta)(1-\alpha r\beta\delta(n-1))}+\frac{\alpha r\beta\delta c\beta_m(n-1)}{1-\alpha r\beta\delta(n-1)}\\
&=\frac{\beta c(1+\alpha rm(1-\delta))}{(1-\delta)(1-\alpha r\beta\delta(n-1))},
\end{aligned}$$

which completes the proof. ■

Proof of Proposition 4

Not paying a ransom is the dominant strategy if $v_{ij}(\mathrm{N}^n) < v_{ij}(\mathrm{PN}^{n-1})$, which is equivalent to $\gamma < \chi_1$. On the other hand, paying is the dominant strategy if $v_{ij}(\mathrm{P}^n) \le v_{ij}(\mathrm{NP}^{n-1})$, that is, if $\gamma \ge \chi_n$. In between χ_1 and χ_n, there can only by asymmetric equilibria. ■

Proof of Theorem 1

Firm 1 would have no incentive to move from $\mathrm{P}^k\mathrm{N}^{n-k}$ to $\mathrm{N}^{n-k+1}\mathrm{P}^{k-1}$ if $v_{ij}(\mathrm{P}^k\mathrm{N}^{n-k}) \le v_{ij}(\mathrm{N}^{n-k+1}\mathrm{P}^{k-1})$, or equivalently, if $\gamma \ge \frac{\alpha r\beta\delta}{1-k\alpha r\beta\delta} \triangleq \chi_k$. The result follows. ■

Proof of Theorem 2

When ransom payments are not banned, w_k the total cost incurred by the n firms in region $\chi_k \le \gamma < \chi_{k+1}$, $0 \le k \le n$, is as specified in Equation (9). When a ban is in effect, the total cost becomes $W = \frac{n\eta r(1+\gamma)}{\alpha(1-\delta)}$. Since $W = w_0$, the first best is achieved no firm pays, that is, when $\gamma < \chi_1$, so prohibition has no effect there. When $\gamma \ge \chi_1$,

$$w_k - W = \frac{k\chi_k(1-n\eta r)\left(\frac{n\eta r}{1-n\eta r} - \gamma\right)}{\alpha(1-\delta)},$$

which is positive if $\gamma < \frac{n\eta r}{1-n\eta r} = n\chi_n \triangleq \xi_1$. Since $\xi_1 = n\chi_n > \chi_n > \chi_{n-1} \ldots > \chi_1$, the first best cannot be achieved in the asymmetric region $\chi_1 \le \gamma < \chi_n$, as well as in the part of the all-pay region where $\chi_n \le \gamma < \xi_1$; clearly, prohibition can be beneficial in these regions. Finally, when $\gamma \ge \xi_1$, the first best is already achieved and prohibition is counterproductive in that region because $w_k - W$ is negative there. ■

Proof of Proposition 5

The attacker's problem is to maximize $\frac{k}{n}\left(\frac{1+kc\eta-\sqrt{(1-kc\eta)^2+4c\eta}}{2\eta(k-1)}\right)$ over k, subject to $k \le n$. The first order condition leads to $k^* = 2 + \frac{1}{2c\eta}$; the second order condition is satisfied as well, because the second derivative at k^* is $-\frac{16c^3\eta^2(3c\eta+1)}{(2c\eta+1)^3(4c\eta+1)^2} < 0$.

Since $k \le n$, the optimal value is simply n when $2 + \frac{1}{2c\eta} \ge n \Leftrightarrow c \ge \frac{1}{2\eta(n-2)}$. The optimal ransom amount is obtained by substituting k^* into $r = \frac{1+kc\eta-\sqrt{(1-kc\eta)^2+4c\eta}}{2\eta(k-1)}$. ■

Proof of Corollary 1

Recall that $\eta = \alpha\beta\gamma$. Therefore, when $k^* < n$, $\frac{\partial k^*}{\partial c} = -\frac{1}{2c^2\eta} < 0$ and $\frac{\partial k^*}{\partial \beta} = -\frac{\alpha\delta}{2c\eta^2} < 0$. Further, $\frac{\partial r^*}{\partial c} = \frac{1}{(1+2c\eta)^2} > 0$ but $\frac{\partial r^*}{\partial \beta} = -\frac{2\alpha\delta c^2}{(1+2c\eta)^2} < 0$. ■

Proof of Theorem 3

From Equation (10), we know the total social cost:

$$w^* = \begin{cases} \frac{2c\eta(1+(n-2)(1+2c\eta))}{\alpha(1-\delta)}, & \text{if } c \ge \frac{1}{2\eta(n-2)}, \\ \frac{n\left(nc\eta-1+\sqrt{(1-nc\eta)^2+4c\eta}\right)}{2\alpha(1-\delta)}, & \text{otherwise.} \end{cases}$$

Under prohibition, the total cost becomes $W = \frac{nc\eta}{\alpha(1-\delta)}$. We first consider the case of $n = 2$. In that case,

$$w^* - W = \frac{2\left(2c\eta - 1 + \sqrt{(1-2c\eta)^2 + 4c\eta}\right)}{2\alpha(1-\delta)} - \frac{2c\eta}{\alpha(1-\delta)} = \frac{\sqrt{4c^2\eta^2+1} - 1}{\alpha(1-\delta)} > 0.$$

Turning our attention to the case where $n > 2$, we consider two possibilities. First, if $c \geq \frac{1}{2\eta(n-2)}$, we get:

$$w^* - W = \frac{2c\eta(1 + (n-2)(1+2c\eta))}{\alpha(1-\delta)} - \frac{nc\eta}{\alpha(1-\delta)} = \frac{c\eta(n-2)(4c\eta+1)}{\alpha(1-\delta)} > 0.$$

Next, we consider the second possibility, that is, $c < \frac{1}{2\eta(n-2)}$. In that case,

$$w^* - W = \frac{n\left(nc\eta - 1 + \sqrt{(1-nc\eta)^2 + 4c\eta}\right)}{2\alpha(1-\delta)} - \frac{nc\eta}{\alpha(1-\delta)}, \text{ and}$$

$$\frac{\partial^2 (w^* - W)}{\partial c^2} = \frac{2n\eta^2(n-1)}{\alpha(1-\delta)\left(\sqrt{(1-nc\eta)^2+4c\eta}\right)^3} > 0,$$

making $w^* - W$ a convex function of c, which attains its minimum at $c = 0$, since $\frac{\partial(w^*-W)}{\partial c}|_{c=0} = 0$. In other words, $w^* - W$ is strictly positive for all $c > 0$. This completes the proof. ■

Proof of Theorem 4

When attackers are not strategic, that is, when r is fixed, it is easy to verify that $\frac{\partial w_k}{\partial k} = \frac{\eta r(1-n\eta r)(\xi_1-\gamma)}{\alpha(1-\delta)(1-k\eta r)^2}$, which is positive only for $\gamma < \xi_1$. In other words, the social cost goes down as k decreases if $\gamma < \xi_1$, but increases for $\gamma > \xi_1$. Now, when a policymaker fiscally intervenes, either with a tax or a subsidy, γ decreases, since $\frac{\partial \gamma}{\partial \phi} = -\frac{c}{r} < 0$. Furthermore, since $\frac{\partial \chi_k}{\partial \tau} = \frac{\chi_k}{(1+\tau)(1-kr\eta(1+\tau))} > 0$, if the intervention involves a tax, χ_k increases at the same time. The net result is that the equilibrium shifts downward and rightward, resulting in a lower k and, hence, a lower (higher) social cost for $\gamma < \xi_1$ (for $\gamma > \xi_1$); see Figure 5. If the intervention is sufficiently strong, γ becomes smaller than χ_1, yielding the first best situation in which no one pays.

When attackers choose k and r strategically, k^* is always greater than or equal to two (Proposition 5), and the associated social cost must then be greater than that for $k = 0$ (prohibition). This makes fiscal intervention less effective than prohibition. Furthermore, for any interior solution ($2 < k^* < n$), we find that $\frac{\partial k^*}{\partial \phi} = \frac{1}{2c\eta(1-\phi)^2} > 0$, $\frac{\partial r^*}{\partial \sigma} = -\frac{c}{(1+\tau+2c\eta(1-\sigma))^2} < 0$, and $\frac{\partial r^*}{\partial \tau} = -\frac{2c\eta(1-\sigma)(1+\tau+c\eta(1-\sigma))}{\eta(1+\tau)^2(1+\tau+2c\eta(1-\sigma))^2} < 0$. The result follows. ■

Proof of Theorem 1′

Recall that v_{ij} can be obtained by solving a set of $2n$ simultaneous equations given by (11):

$$v_{ij}(s) = \sum_{k=0}^{j} \frac{j!}{k!(j-k)!} y^k (1-y)^{j-k} ((1-x)(ic + \delta V(k,s)) + x(ir + \delta V(i+k,s))),$$

where

$$V(m,s) = \sum_{i=0}^{1} \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \beta_m^{i+j} (1-\beta_m)^{n-i-j} v_{ij}(s). \tag{C1}$$

Now, consider the situation where firm 1 plays `p` with probability x and all other firms play `p` with y; let us represent this strategy combination as $s = (x, y)$. For this s, the solution of (11) is given by (12):

$$v_{ij}(x,y) = \left(i(1-n\eta ry) + (i+j)\eta ry + \frac{\beta\delta}{1-\delta}\right)\left(\frac{c(1-x)+rx}{1-\eta r(x+y(n-1))}\right).$$

To ascertain that (12) is indeed the unique solution, we first show that (12) satisfies (11) and (C1). To do so, we substitute (12) into (C1) to obtain:

$$V(m,s) = \left(\frac{\eta}{\alpha(1-\delta)} + \beta + \alpha\beta m r\right)\left(\frac{c(1-x)+rx}{1-\eta r(x+y(n-1))}\right).$$

Substituting this $V(m,s)$ back into (11), we obtain:

$$v_{ij}(x,y) = \left(i(1-n\eta r y) + (i+j)\eta r y + \frac{\beta\delta}{1-\delta}\right)\left(\frac{c(1-x)+rx}{1-\eta r(x+y(n-1))}\right),$$

which is exactly what is provided in (12) as the expected cost to firm 1. Therefore, (12) satisfies (11) and (C1). Furthermore, (11) and (C1) are linear equations, so this solution must be unique.

Now, firm 1 will mix `p` with `n` only if firm 1 is indifferent between playing `p` and `n`, that is, if the above payoff does not depend on x. This implies that $\frac{\partial v_{ij}(x,y)}{\partial x}$ must be 0. Therefore, to compel firm 1 to play a mixed strategy, the other firms must choose a y that solves $\frac{\partial v_{ij}(x,y)}{\partial x} = 0$:

$$y(\gamma) = \frac{1}{n-1}\left(\frac{1-\eta r}{\eta r} - \frac{1}{\gamma}\right).$$

Since the choice of firm 1 as the focal firm is arbitrary, it is clear that a unique, symmetric mixed-strategy equilibrium exists at $x = y = \frac{1}{n-1}\left(\frac{1-\eta r}{\eta r} - \frac{1}{\gamma}\right)$.

Finally, $y(\gamma)$ becomes 0 at $\gamma = \frac{\eta r}{1-\eta r} = \chi_1$, implying that firms will play N with a probability of one for all $\gamma < \chi_1$. Similarly, $y(\gamma)$ becomes 1 at $\gamma = \frac{\eta r}{1-n\eta r} = \chi_n$, which implies that firms will play `p` with certainty when $\gamma \geq \chi_n$. Finally, in the middle region where $\chi_1 \leq \gamma < \chi_n$, firms will play `p` with $y(\gamma)$. Putting together, in equilibrium, firms play `p` with probability $\mu^*(\gamma)$ where:

$$\mu^*(\gamma) = \begin{cases} 0, & \text{if } \gamma < \chi_1, \\ \frac{1}{n-1}\left(\frac{1-\eta r}{\eta r} - \frac{1}{\gamma}\right), & \text{if } \chi_1 \leq \gamma < \chi_n, \\ 1, & \text{if } \gamma \geq \chi_n. \end{cases}$$

This completes the proof. ∎

Proof of Theorem 2′

It is clear from Theorem 1′ that a ban is of no consequence when $\gamma < \chi_1$. According to Equation (13), when $\chi_1 \leq \gamma < \chi_n$, prohibition leads to a cost reduction of

$$\frac{n\gamma}{\alpha(1-\delta)} - \frac{n\eta r(1+\gamma)}{\alpha(1-\delta)} = \frac{n(1-\eta r)\left(\gamma - \frac{\eta r}{1-\eta r}\right)}{\alpha(1-\delta)} = \frac{n(1-\eta r)(\gamma - \chi_1)}{\alpha(1-\delta)},$$

which is positive since $\gamma \geq \chi_1$. Now, when $\gamma \geq \chi_n$, prohibition leads to a cost saving of

$$\frac{n\chi_n}{\alpha(1-\delta)} - \frac{n\eta r(1+\gamma)}{\alpha(1-\delta)} = \frac{n\left(\frac{\eta r}{1-n\eta r} - \eta r(1+\gamma)\right)}{\alpha(1-\delta)} = \frac{n\eta r\left(\frac{n\eta r}{1-n\eta r} - \gamma\right)}{\alpha(1-\delta)},$$

which is positive if and only if $\gamma < \frac{n\eta r}{1-n\eta r} = n\chi_n = \xi_1$. Thus, when γ is above ξ_1, the ban actually increases the total social cost. ∎

Proof of Proposition 5′

According to Theorem 1′, the probability of getting paid is $\mu^*(\gamma)$, so the attacker's problem would be to maximize the expected payoff $r\mu^*(\gamma)$. However, $\frac{\partial(r\mu^*(\gamma))}{\partial r} = -\frac{c^2}{(n-1)(c-r)^2} < 0$, so an attacker would reduce r

as long as doing so results in a higher probability of getting paid. This would result in a boundary solution of $\mu^*(\gamma) = 1$, at which point the attacker would relent from reducing r further, implying that γ would be equal to χ_n in equilibrium. Substituting $\gamma = \frac{c-r}{r}$ and $\chi_n = \frac{\eta r}{1-n\eta r}$ into $\gamma = \chi_n$ and then solving the equation, we get the desired r^*. ∎

Proof of Theorem 3′

We know from Proposition 5′ that attackers choose r^* in such a way that γ becomes χ_n. From the proof of Theorem 2′, we know that, when $\gamma = \chi_n$, prohibition reduces the total social cost by:

$$\frac{n\eta r\left(\frac{n\eta r}{1-n\eta r} - \gamma\right)}{\alpha(1-\delta)} = \frac{n\eta r\left(n\chi_n - \gamma\right)}{\alpha(1-\delta)}.$$

When $r = r^*$ and $\gamma = \chi_n$, this reduction becomes $\frac{n(n-1)\eta r^* \chi_n}{\alpha(1-\delta)} > 0$. ∎

Proof of Theorem 4′

We start with the case where r is fixed. When $\gamma \geq \xi_1$ (everyone pays) or $\gamma < \chi_1$ (no one pays), the situation is the same as the pure-strategy equilibrium, and the proof of Theorem 4 applies. So, we only consider the region $\chi_1 \leq \gamma < \xi_1$. As shown in the proof of Theorem 4, fiscal intervention puts a downward pressure on γ. Since $\frac{\partial \mu^*}{\partial \gamma} = \frac{1}{\gamma^2(1-n)} > 0$ and $\frac{\partial w_M}{\partial \gamma} = \frac{\eta}{\alpha(1-\delta)} > 0$, both the propensity to pay and social cost go down with fiscal intervention. When the intervention is sufficiently large, we reach $\mu^* = 0$, resulting in a situation that is equivalent to prohibition.

When attackers choose r strategically, we again reach a boundary solution of $\gamma = \chi_n$ or $\frac{c(1-\sigma)-r(1+\tau)}{r(1+\tau)} = \frac{\eta r(1+\tau)}{1-n\eta r(1+\tau)}$, resulting in $r^* = \frac{1+nc\eta(1-\sigma)+\sqrt{(1-nc\eta(1-\sigma))^2+4c\eta(1-\sigma)}}{2\eta(n-1)(1+\tau)}$ and $\mu^* = 1$. Therefore, despite fiscal intervention, all victims continue acceding to the ransom demand, although the ransom amount and the overall social cost decrease with fiscal intervention. ∎

Proof of Theorem 1″

As discussed in the paper, even when actions are fully observable, firms' strategies would remain the same for both $\gamma \geq \xi_1$ and $\gamma < \chi_1$. Therefore, we consider here what happens when $\chi_1 \leq \gamma < \xi_1$.

We start with the situation where $\chi_1 \leq \gamma < \chi_n$. Suppose all other firms are playing the grim strategy to push firm 1 towards playing `n`. In other words, if firm 1 ever plays `p`, they will retaliate by playing `p` going forward. If firm 1 is breached but sticks to `n`, then it will incur a cost of c in the current period and will also incur $\delta V(0, \mathrm{N}^n)$ in the future. If firm 1 deviates in the current period and plays `p`, it incurs only r now, but it faces $\delta V(1, \mathrm{NP}^{n-1})$ in the future because, when $\gamma < \chi_n$, firm 1's best response to others playing P is simply N. Firm 1 cooperates (that is, sticks to N) if the cost associated with cooperating is less than that of deviating, or if

$$c + \delta V(0, \mathrm{N}^n) < r + \delta V(1, \mathrm{NP}^{n-1}). \tag{C2}$$

Now, from the proof of Lemma 4, we know that

$$V(0, \mathrm{N}^n) = \frac{\beta c}{1-\delta} \quad \text{and} \quad V(m, \mathrm{NP}^{n-1}) = \frac{\beta c(1 + \alpha r m(1-\delta))}{(1-\delta)(1-\alpha r\beta\delta(n-1))}.$$

Using these values and the fact $c = r(1+\gamma)$, we can reduce this condition for cooperation to

$$\frac{\gamma}{1+\gamma} < \frac{\eta r}{1-(n-1)\eta r}\left(1 + \frac{(n-1)\beta\delta}{1-\delta}\right). \tag{C3}$$

The left hand side of (C3) is less than one for all possible values of γ. If the right hand side is greater than or equal to one, (C3) is automatically satisfied, so there is nothing more to do. Let us then consider the case where the right hand side is also less than one. In that case, (C3) is the algebraically equivalent to:

$$\gamma < \frac{\frac{\eta r}{1-(n-1)\eta r}\left(1+\frac{(n-1)\beta\delta}{1-\delta}\right)}{1-\frac{\eta r}{1-(n-1)\eta r}\left(1+\frac{(n-1)\beta\delta}{1-\delta}\right)} \triangleq \bar{\chi}.$$

Now,

$$\bar{\chi}-\chi_n = \frac{\frac{r\eta(1-(n-2)r\eta)}{(1-(n-1)r\eta)^2}\left(\frac{(n-1)\beta\delta}{1-\delta}+\frac{r\eta}{1-(n-2)r\eta}\right)}{1-\frac{\eta r}{1-(n-1)\eta r}\left(1+\frac{(n-1)\beta\delta}{1-\delta}\right)}. \tag{C4}$$

The numerator of the right hand side of (C4) is clearly positive, and so is the denominator because we assumed that the right hand side of (C3) is less than one. In other words, $\chi_n < \bar{\chi}$, and (C3) is automatically satisfied because $\chi_1 \le \gamma < \chi_n < \bar{\chi}$. Therefore, regardless of δ, firm 1 will cooperate if other firms play grim, and vice versa.

We now examine what happens when $\chi_n \le \gamma < \xi_1$. In this case, not cooperating still results in an immediate cost of r for firm 1, but doing so also prompts the other firms to retaliate and play $\mathtt{p}$ in all future periods. Since $\mathtt{p}$ is the best response to $\mathtt{p}$ when $\gamma \ge \chi_n$, such a retaliation would force the firms to get locked into P^n going forward, as opposed to the more desirable N^n. So, firm 1 cooperates if and only if:

$$c + \delta V(0, \mathrm{N}^n) < r + \delta V(1, \mathrm{P}^n).$$

Again, from the proof of Lemma 4, we know that

$$V(0,\mathrm{N}^n) = \frac{\beta c}{1-\delta} \quad \text{and} \quad V(m,\mathrm{P}^n) = \frac{r\beta(1+m\alpha r(1-\delta))}{(1-\delta)(1-n\alpha r\beta\delta)}.$$

Substituting these into the condition for cooperation immediately reduces it to

$$\gamma < \frac{\eta r}{1-n\eta r}\left(\frac{\alpha(1-\delta)+n\eta}{\alpha(1-\delta)+\eta}\right) \triangleq \xi_2.$$

Since $1 < \frac{\alpha(1-\delta)+n\eta}{\alpha(1-\delta)+\eta} < n$ for all $\delta \in (0,1)$, it is easy to see that ξ_2 is bounded by $\chi_n < \xi_2 < \xi_1$, implying that grim strategy leads to first best only in a part of the parameter space, specifically, where $\gamma < \xi_2$. ■

Proof of Theorem 2″

As mentioned in the proof of Theorem 1″, grim strategy expands the first-best region. Specifically, grim achieves first best also in $\chi_1 \le \gamma < \xi_2$, in addition to achieving it in the original first-best regions $\gamma \ge \xi_1$ and $\gamma < \chi_1$. Therefore, the only region where prohibition can still improve welfare is $\xi_2 \le \gamma < \xi_1$. Since $\xi_2 > \chi_n$, we can easily find the change in welfare from Equation (13):

$$\frac{n\chi_n}{\alpha(1-\delta)} - \frac{n\eta r(1+\gamma)}{\alpha(1-\delta)} = \frac{n\left(\frac{\eta r}{1-n\eta r}-\eta r(1+\gamma)\right)}{\alpha(1-\delta)} = \frac{n\eta r\left(\frac{n\eta r}{1-n\eta r}-\gamma\right)}{\alpha(1-\delta)} = \frac{n\eta r\left(\xi_1-\gamma\right)}{\alpha(1-\delta)}.$$

Since $\frac{n\eta r(\xi_1-\gamma)}{\alpha(1-\delta)} > 0$ when $\xi_2 \le \gamma < \xi_1$, we can infer that prohibition does improve welfare in this region. ■

Proof of Proposition 5″

With grim, all firms pay if $\gamma \ge \xi_2$ and no one pays otherwise. Therefore, attackers would set r such that γ is barely ξ_2. Substituting $\gamma = \frac{c-r}{r}$ into $\gamma = \xi_2$ and solving for r, we get the desired r^*. ■

Proof of Theorem 3$''$

From the proof of Theorem 2$''$, we know that, when $\gamma \geq \chi_n$, the impact of prohibition on welfare is simply $\frac{n\eta r(\xi_1-\gamma)}{\alpha(1-\delta)}$. Since $\gamma = \xi_2 > \chi_n$ in equilibrium, this change becomes $\frac{n\eta r(\xi_1-\xi_2)}{\alpha(1-\delta)}$. Now, as shown in the proof of Theorem 1$''$, $\xi_2 < \xi_1$. Therefore, $\frac{n\eta r(\xi_1-\xi_2)}{\alpha(1-\delta)} > 0$, implying that prohibition indeed improves welfare. ■

Proof of Theorem 4$''$

Consider the case where attackers are not strategic in their choice of r. When $\gamma \geq \xi_1$ (everyone pays) or $\gamma < \xi_2$ (no one pays), the result is obvious. So, we only consider the region $\xi_2 \leq \gamma < \xi_1$. Here as well, fiscal intervention puts a downward pressure on γ. When this pressure is sufficiently large, we reach a γ that is just below ξ_2, resulting in a situation that is equivalent to prohibition.

When attackers choose r strategically, we again reach a boundary solution of $\gamma = \xi_2$. Therefore, despite fiscal intervention, all victims continue acceding to the ransom demand, although the ransom amount and the overall social cost decreases with fiscal intervention. ■