Appendices

A Proof of Lemma 1

In equilibrium, the platform ensures the following constraint holds:

$$d(\frac{\bar{v}-p_N}{\bar{v}}) = s\frac{w_N}{\bar{c}}. \tag{40}$$

Suppose that this constraint does not hold, then 1) when $d(\frac{\bar{v}-p_N}{\bar{v}}) < s\frac{w_N}{\bar{c}}$, the platform can reduce the cost by decreasing the wage slightly without affecting the revenue, which increases its profit; or 2) when $d(\frac{\bar{v}-p_N}{\bar{v}}) > s\frac{w_N}{\bar{c}}$, the platform can increase the revenue by increasing the price slightly without affecting the cost, which also increases its profit. Therefore, the optimal wage should satisfy

$$w_N(p_N) = \frac{\bar{c}(\bar{v}-p_N)d}{\bar{v}s} \tag{41}$$

Substituting (40) and (41) into (5), the platform's profit is

$$\pi_N = d\frac{\bar{v}-p_N}{\bar{v}}(p_N - w_N(p_N)) \tag{42}$$

Maximizing the profit by choosing p_N, the first-order-condition is

$$\frac{\partial \pi_N}{\partial p_N} = \frac{d(2d\bar{c}(\bar{v}-p_N) + s\bar{v}(\bar{v}-2p_N))}{s\bar{v}^2} = 0 \tag{43}$$

and the optimal price p_N^* is given in (7). Substituting (7) into (41), we obtain the optimal wage w_N^* in (6). We can check that the second-order derivative is always negative.

Based on the equilibrium outcomes, we can calculate the expected profit as:

$$\begin{aligned}\mathbb{E}(\pi_N^*) &= \sum_s \sum_d \frac{1}{4}(d\frac{\bar{v}-p_N^*}{\bar{v}}(p_N^* - w_N^*)) \\ &= \frac{\bar{v}^2(m\bar{c}(n-\delta)(\delta+n) + n\bar{v}(m-\theta)(\theta+m))(2mn\bar{c}\bar{v} + \bar{c}^2(n-\delta)(\delta+n) + \bar{v}^2(m-\theta)(\theta+m))}{4(\bar{c}(n-\delta)+\bar{v}(m-\theta))(\bar{c}(\delta+n)+\bar{v}(m-\theta))(\bar{c}(n-\delta)+\bar{v}(\theta+m))(\bar{c}(\delta+n)+\bar{v}(\theta+m))}\end{aligned} \tag{44}$$

□

B Proof of Lemma 2

In equilibrium, the platform ensures the following constraint holds:

$$\alpha n + (d-\alpha n)\frac{\bar{v}-p_C}{\bar{v}} = s\frac{w_C}{\bar{c}}. \tag{45}$$

Again, suppose that this constraint (45) does not hold, then following the same argument in the proof of Lemma 1, the platform can either reduce the wage or increase the price to increase its profit. Therefore, the optimal wage satisfies

$$w_C(p_C) = \frac{\bar{c}(d(\bar{v}-p_C) + \alpha n p_C)}{s\bar{v}} \tag{46}$$

Substituting (45) and (46) into (8), the platform's profit is

$$\pi_C = \alpha n(-w_C) + (d-\alpha n)\frac{\bar{v}-p_C}{\bar{v}}(p_C - w_C(p_C)) \tag{47}$$

Maximizing the profit by choosing p_C, the first-order-condition is

$$\frac{\partial \pi_C}{\partial p_C} = \frac{(d-\alpha n)(2\bar{c}(d\bar{v}+p_C(\alpha n-d))+s\bar{v}(\bar{v}-2p_C))}{s\bar{v}^2} = 0 \tag{48}$$

and the optimal price p_C^* is given in (14). We can check that the second-order derivative is always negative. Substituting (14) into (46), we obtain the optimal wages w_C^* in (13).

In equilibrium, the platform charges a membership fee that is just below ΔU^r. Calculating $\Delta U^r(d,s)$ for any given d and s by substituting (14) into (11), we have

$$\Delta U^r(d,s) = p_C^*(d,s)(1-\frac{p_C^*(d,s)}{2\bar{v}}) = \frac{\bar{v}}{8}\frac{(2\bar{c}+\frac{s}{d}\bar{v})(2\bar{c}(1-\frac{\alpha n}{d})+3\frac{s}{d}\bar{v})}{(\bar{c}(1-\frac{\alpha n}{d})+\frac{s}{d}\bar{v})^2} \tag{49}$$

Now take expectation of (49) for all possible d and s, we derive the equilibrium membership fee in (12).

Then the expected profit is given as:

$$\begin{aligned}
\mathbb{E}(\pi_C^*) &= \sum_s \sum_d \frac{1}{4}[\alpha n(-w_C^*) + (d-\alpha n)\frac{\bar{v}-p_C^*}{\bar{v}}(p_C^*-w_C^*)] + \alpha n R_C^* \\
&= \frac{1}{32}\bar{v}(\alpha n \bar{c}^2(\frac{((\alpha+1)n-\delta)^2}{(\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta))^2} + \frac{((\alpha+1)n-\delta)^2}{(\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m))^2} \\
&+ \frac{(\delta+(\alpha+1)n)^2}{(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta))^2} + \frac{(\delta+(\alpha+1)n)^2}{(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m))^2}) \\
&- 2\bar{c}(\frac{(n-\delta)((\alpha+1)n-\delta)}{\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta)} + \frac{(n-\delta)((\alpha+1)n-\delta)}{\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m)} \\
&+ \frac{(\delta+n)(\delta+(\alpha+1)n)}{\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta)} + \frac{(\delta+n)(\delta+(\alpha+1)n)}{\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m)}) + 4(\alpha+2)n)
\end{aligned} \tag{50}$$

□

C Analysis for Scenario (C) in Section 4.2

In Section 4.2, we present the analysis for scenario (a) and (b). In this section, we perform the analysis for scenario (c) and show the condition for when it is more profitable for the platform to offer membership to only regular consumers, compared to providing membership to both regular and occasional consumers. Assume that in stage 2, $\gamma_r \in [0,1]$ fraction of the regular consumers choose to become subscribers, and $\gamma_o \in [0,1]$ fraction of the occasional consumers choose to become members. It is clear that, in the equilibrium, if $(\gamma_r,\gamma_o)=(1,1)$, all consumers join the membership; if $(\gamma_r,\gamma_o)=(1,0)$, only regular consumers join the membership. We use subscript T to denote this case. The platform's problem in the spot period is

$$\begin{aligned}
&\max_{p_{C,T},w_{C,T}} -w_{C,T}(\gamma_r\alpha n+\gamma_o(d-\alpha n)) \\
&+ (p_{C,T}-w_{C,T})\cdot \min\{((1-\gamma_r)\alpha n+(1-\gamma_o)(d-\alpha n))\frac{\bar{v}-p_{C,T}}{\bar{v}}, s\frac{w_{C,T}}{\bar{c}} - (\gamma_r\alpha n+\gamma_o(d-\alpha n))\}.
\end{aligned} \tag{51}$$

The first term represents the cost of serving all member consumers, and the second term denotes the profit from serving non-member consumers in the spot period. We denote the platform's optimal spot price and optimal spot wage under the consumer-side membership plan as $p_{C,T}^*(d,s)$ and $w_{C,T}^*(d,s)$, respectively.

In stage 2, a consumer joins membership if her expected surplus gain from membership is at least as high as the membership fee, $R_{C,T}$, given the consumer's belief about the size and composition of consumers who join the membership. In the equilibrium, this belief should be consistent with the platform's belief about the size and composition of consumers who join the membership, which impacts the platform's decision on the membership fee. Given (γ_r, γ_o), for a regular consumer, the expected purchase-related surplus during the spot period (excluding membership fee) if she joins the membership plan is

$$U^r_{C,T} = \sum_s \sum_d \frac{1}{4}[\int_0^{\bar{v}} \frac{z}{\bar{v}} dz] = \frac{\bar{v}}{2}, \tag{52}$$

and the expected surplus if she does not join the membership plan is

$$\begin{aligned} U^r_{N,T}(\gamma_r, \gamma_o) &= \sum_s \sum_d \frac{1}{4}[\int_{p^*_{C,T}(d,s,\gamma_r,\gamma_o)}^{\bar{v}} \frac{z - p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{\bar{v}} dz] \\ &= \frac{\bar{v}}{2} - \mathbb{E}[p^*_{C,T}(d, s, \gamma_r, \gamma_o) \cdot (1 - \frac{p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{2\bar{v}})]. \end{aligned} \tag{53}$$

Then, a regular consumer's expected surplus gain from membership is:

$$\Delta U^r(\gamma_r, \gamma_o) = U^r_{C,T} - U^r_{N,T}(\gamma_r, \gamma_o) = \mathbb{E}[p^*_{C,T}(d, s, \gamma_r, \gamma_o) \cdot (1 - \frac{p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{2\bar{v}})]. \tag{54}$$

For an occasional consumer, given (γ_r, γ_o), the expected purchase-related surplus during the spot period (excluding membership fee) if she joins the membership plan is

$$U^o_{C,T} = \sum_s \sum_d \frac{1}{4} \frac{d - \alpha n}{\frac{n_o}{\hat{n}_o}(1-\alpha)n}[\int_0^{\bar{v}} \frac{z}{\bar{v}} dz] = \frac{\hat{n}_o}{n_o}\frac{\bar{v}}{2}, \tag{55}$$

and the expected surplus if she does not join the membership plan is

$$\begin{aligned} U^o_{N,T}(\gamma_r, \gamma_o) &= \sum_s \sum_d \frac{1}{4} \frac{d - \alpha n}{\frac{n_o}{\hat{n}_o}(1-\alpha)n}[\int_{p^*_{C,T}(d,s,\gamma_r,\gamma_o)}^{\bar{v}} \frac{z - p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{\bar{v}} dz] \\ &= \frac{\hat{n}_o}{n_o}(\frac{\bar{v}}{2} - \mathbb{E}[p^*_{C,T}(d, s, \gamma_r, \gamma_o) \cdot (1 - \frac{p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{2\bar{v}})]). \end{aligned} \tag{56}$$

Thus, an occasional consumer's expected surplus gain from membership is:

$$\Delta U^o(\gamma_r, \gamma_o) = U^o_{C,T} - U^o_{N,T}(\gamma_r, \gamma_o) = \frac{\hat{n}_o}{n_o}\mathbb{E}[p^*_{C,T}(d, s, \gamma_r, \gamma_o) \cdot (1 - \frac{p^*_{C,T}(d, s, \gamma_r, \gamma_o)}{2\bar{v}})]. \tag{57}$$

Note that the term $\frac{d-\alpha n}{\frac{n_o}{\hat{n}_o}(1-\alpha)n}$ represents an occasional consumer's probability of needing a service in a period when the demand potential is d, where $d - \alpha n$ is the number of occasional consumers who need a service in the spot period, and $\frac{n_o}{\hat{n}_o}(1 - \alpha)n$ is the total number of occasional consumers. Recall that $d \in \{n + \delta, n - \delta\}$ with equal probabilities, and an occasional consumer's expected probability of needing a service is $\frac{1}{2}\frac{(1-\alpha)n+\delta}{\frac{n_o}{\hat{n}_o}(1-\alpha)n} + \frac{1}{2}\frac{(1-\alpha)n-\delta}{\frac{n_o}{\hat{n}_o}(1-\alpha)n} = \frac{(1-\alpha)n}{\frac{n_o}{\hat{n}_o}(1-\alpha)n} = \frac{\hat{n}_o}{n_o}$, $\frac{\hat{n}_o}{n_o} \in (0, 1)$, as specified in Section 3. It is clear that $\Delta U^o(\gamma_r, \gamma_o) < \Delta U^r(\gamma_r, \gamma_o)$ since $\frac{\hat{n}_o}{n_o} < 1$. Therefore, given any $(d, s, \gamma_r, \gamma_o)$, a regular consumer's expected surplus gain from membership is greater than that of an occasional consumer. Given this, the platform can never induce only the occasional consumers to become members.

Under this belief, in stage 1, the platform would set the membership fee. Assume that the platform set membership fee $R_{C,T}(\gamma_r, \gamma_o) > \Delta U^r(\gamma_r, \gamma_o)$, then both the regular consumers and occasional consumers would find that the membership fee is more than the expected surplus gain from becoming members. Therefore, none of the consumers would choose to become members, then in the equilibrium, $(\gamma_r, \gamma_o) = (0, 0)$. Now assume that the platform set membership fee $\Delta U^r(\gamma_r, \gamma_o) \geq R_{C,T}(\gamma_r, \gamma_o) > \Delta U^o(\gamma_r, \gamma_o)$, a regular consumer joins the membership as her expected surplus gain is not less than the membership fee.[13] However, an occasional consumer does not join since her expected surplus gain is smaller than the subscription fee. Thus, in the equilibrium, all the regular consumers choose to become members, but none of the occasional consumers join, which means $(\gamma_r, \gamma_o) = (1, 0)$. Then the platform's optimal choice of membership fee is $R_{C,T}(\gamma_r = 1, \gamma_o = 0) = \Delta U^r(\gamma_r = 1, \gamma_o = 0)$, since any membership fee lower than that does not increase the number of members, but reduces the membership fee revenue. Finally, assume that $\Delta U^o(\gamma_r, \gamma_o) \geq R_{C,T}(\gamma_r, \gamma_o)$, apparently, a regular consumer is strictly better off to become a member, and an occasional consumer is not worse off if she becomes a member. Hence, the equilibrium is that all consumers choose to join the membership, i.e., $(\gamma_r, \gamma_o) = (1, 1)$, and the platform's optimal membership fee is $R_{C,T}(\gamma_r = 1, \gamma_o = 1) = \Delta U^o(\gamma_r = 1, \gamma_o = 1)$. It can be shown that $\Delta U^r(\gamma_r = 1, \gamma_o = 0) > \Delta U^o(\gamma_r = 1, \gamma_o = 1)$ so that when the platform set $R_{C,T}(\gamma_r, \gamma_o) = \Delta U^r(\gamma_r = 1, \gamma_o = 0)$, no occasional consumer is better off joining the membership.

Lemma 7. *When the platform offers the membership plan to both types of consumers, in equilibrium, the platform sets the membership fee, spot price and spot wage as follows:*

$$R^*_{C,T} = \sum_s \sum_d \frac{d - \alpha n}{\frac{n_o}{\hat{n}_o}(1-\alpha)n} \frac{1}{32\bar{v}} [\frac{(2d\bar{c} + s\bar{v})(3s\bar{v} - 2d\bar{c})}{s^2\bar{v}}] \tag{58}$$

$$w^*_{C,T}(d, s) = \frac{\bar{c}s}{d} \tag{59}$$

$$p^*_{C,T}(d, s) = w^*_{C,T}(d, s) + \frac{\bar{v}}{2} \tag{60}$$

When the platform does not offer membership or if it offers the membership only to regular consumers, the platform's equilibrium decisions are provided in Lemma 1 and 2, respectively. To derive Lemma 7, we find that the platform ensures the the following condition:

$$(\gamma_r \alpha n + \gamma_o(d - \alpha n)) + ((1-\gamma_r)\alpha n + (1-\gamma_o)(d - \alpha n))\frac{\bar{v} - p_{C,T}}{\bar{v}} = s\frac{w_{C,T}}{\bar{c}} \tag{61}$$

If this constraint does not hold, then following the same argument in the proof of Lemma 1, the platform can either reduce the wage or increase the price to increase its profit. Therefore, the optimal wage satisfies

$$w_{C,T}(p_{C,T}) = \frac{\bar{c}(d\bar{v} + p_{C,T}(\gamma_o(d - \alpha n) - d + \alpha n\gamma_r))}{s\bar{v}}. \tag{62}$$

[13] For ease of illustration, we assume that a consumer always joins the membership if her expected surplus gain is not less than the membership fee.

Substituting (61) and (62) into (51), the platform's profit is

$$\begin{aligned}\pi_{C,T} &= -w_{C,T}(p_{C,T})(\gamma_r \alpha n + \gamma_o(d-\alpha n)) \\ &\quad + ((1-\gamma_r)\alpha n + (1-\gamma_o)(d-\alpha n))\frac{\bar{v}-p_{C,T}}{\bar{v}}(p_{C,T} - w_{C,T}(p_{C,T})).\end{aligned} \tag{63}$$

Maximizing the profit by choosing $p_{C,T}$, the first-order-condition is

$$\frac{\partial \pi_{C,T}}{\partial p_{C,T}} = -\frac{(\gamma_o(d-\alpha n) - d + \alpha n\gamma_r)(2\bar{c}(d\bar{v} + p_{C,T}(\gamma_o(d-\alpha n) - d + \alpha n \gamma_r)) + s\bar{v}(\bar{v} - 2p_{C,T}))}{s\bar{v}^2} = 0, \tag{64}$$

then we can derive

$$p^*_{C,T} = \frac{\bar{v}(2d\bar{c} + s\bar{v})}{2s\bar{v} - 2\bar{c}(\gamma_o(d-\alpha n) - d + \alpha n\gamma_r)} \tag{65}$$

Substituting $\gamma_r = 1$ and $\gamma_o = 1$ in to the expression above, the optimal price $p^*_{C,T}$ is given in (60). We can check that the second-order derivative is always negative. Substituting (60), $\gamma_r = 1$, and $\gamma_o = 1$ into (62), we obtain the optimal wages $w^*_{C,T}$ in (59). Substituting (60), $\gamma_r = 1$, and $\gamma_o = 1$ into (57), we obtain the optimal membership fee $R^*_{C,T}$ in (58). And the platform's expected profit when it offers the membership plan to both types of consumers is

$$\begin{aligned}\mathbb{E}(\pi^*_{C,T}) &= \sum_s \sum_d \frac{1}{4}[-w^*_{C,T}d] + (\alpha n + \frac{n_o}{\hat{n}_o}(1-\alpha)n)R^*_{C,T} \\ &= \frac{1}{8(\alpha-1)\bar{v}(m-\theta)^2(\theta+m)^2}(-4n\bar{c}^2(\theta^2+m^2)(\alpha(\frac{\hat{n}_o}{n_o}-1)+1)((\alpha-3)\delta^2 + (\alpha-1)n^2) \\ &\quad + 4m\bar{c}\bar{v}(m^2-\theta^2)((\alpha-1)n^2(\alpha(\frac{\hat{n}_o}{n_o}-1)-1) - \delta^2(\alpha + \alpha\frac{\hat{n}_o}{n_o} - 1)) \\ &\quad + 3(\alpha-1)n\bar{v}^2(m^2-\theta^2)^2(\alpha(\frac{\hat{n}_o}{n_o}-1)+1)).\end{aligned} \tag{66}$$

We now compare $\mathbb{E}(\pi^*_{C,T})$ to $\mathbb{E}(\pi^*_C)$ in Equation (50). Subtract $\mathbb{E}(\pi^*_C)$ by $\mathbb{E}(\pi^*_{C,T})$, we have

$$\begin{aligned}\mathbb{E}(\pi^*_C) - \mathbb{E}(\pi^*_{C,T}) = \sum_s \sum_d \frac{1}{4}[&\frac{\bar{v}(s\bar{v} - 2\alpha n\bar{c})(2d^2\bar{c} + s\bar{v}(2d+\alpha n))}{8(\bar{c}(d-\alpha n) + s\bar{v})^2} + \frac{d^2\bar{c}}{s} \\ &- \frac{(\alpha(\frac{\hat{n}_o}{n_o}-1)+1)(d-\alpha n)\left(4d^2\bar{c}^2 - 4ds\bar{c}\bar{v} - 3s^2\bar{v}^2\right)}{8(\alpha-1)s^2\bar{v}}]\end{aligned} \tag{67}$$

Denote $pr^1 = \frac{(1-\alpha)(2s^2\bar{c}\bar{v}^2(d^2+2\alpha dn - 4\alpha^2n^2) + 2d\bar{c}^3(d-\alpha n)^3 + s\bar{c}^2\bar{v}(5d+3\alpha n)(d-\alpha n)^2 - s^3\bar{v}^3(d-4\alpha n))}{\alpha(d-\alpha n)(3s\bar{v}-2d\bar{c})(\bar{c}(d-\alpha n)+s\bar{v})^2}$, it is easy to verify that if $\frac{\hat{n}_o}{n_o} > pr^1$, the summand of Equation (67) is negative; if $\frac{\hat{n}_o}{n_o} < pr^1$, we can verify that the summand of Equation (67) is positive. We also show that the summand of Equation (67) is decreasing in the ratio $\frac{\hat{n}_o}{n_o}$ for any demand potential d and supply potential s, therefore, the profit difference $\mathbb{E}(\pi^*_C) - \mathbb{E}(\pi^*_{C,T})$ decreases in $\frac{\hat{n}_o}{n_o}$. Thus, there exists a threshold pr^{CT}, such that if $\frac{\hat{n}_o}{n_o} > pr^{CT}$, $\mathbb{E}(\pi^*_C) - \mathbb{E}(\pi^*_{C,T}) < 0$, indicating that providing membership to only regular consumers is less profitable than providing membership to both types of consumers; otherwise, if $\frac{\hat{n}_o}{n_o} < pr^{CT}$, $\mathbb{E}(\pi^*_C) - \mathbb{E}(\pi^*_{C,T}) > 0$, providing membership to only regular consumers is more profitable than providing membership to both types of consumers.

We now compare $\mathbb{E}(\pi^*_{C,T})$ to $\mathbb{E}(\pi^*_N)$ in Equation (44). Subtract $\mathbb{E}(\pi^*_N)$ by $\mathbb{E}(\pi^*_{C,T})$, we have

$$\mathbb{E}(\pi^*_N) - \mathbb{E}(\pi^*_{C,T}) = \sum_s \sum_d \frac{1}{4}[\frac{ds\bar{v}^2}{4d\bar{c} + 4s\bar{v}} + \frac{d^2\bar{c}}{s} - \frac{(\alpha(\frac{\hat{n}_o}{n_o} - 1) + 1)(d - \alpha n)\left(4d^2\bar{c}^2 - 4ds\bar{c}\bar{v} - 3s^2\bar{v}^2\right)}{8(\alpha - 1)s^2\bar{v}}] \tag{68}$$

Denote $pr^2 = \frac{(1-\alpha)\left(2d^2\bar{c}^2(d-\alpha n)+ds\bar{c}\bar{v}(3d+\alpha n)-s^2\bar{v}^2(d-3\alpha n)\right)}{\alpha(d-\alpha n)(3s\bar{v}-2d\bar{c})(d\bar{c}+s\bar{v})}$, we show that if $\frac{\hat{n}_o}{n_o} > pr^2$, the summand of Equation (68) is negative; if $\frac{\hat{n}_o}{n_o} < pr^2$, we can verify that the summand of Equation (68) is positive. We also show that the summand of Equation (68) is decreasing in the ratio $\frac{\hat{n}_o}{n_o}$, therefore, the profit difference $\mathbb{E}(\pi^*_N) - \mathbb{E}(\pi^*_{C,T})$ decreases in $\frac{\hat{n}_o}{n_o}$. Thus, there exists a threshold pr^{NT}, such that if $\frac{\hat{n}_o}{n_o} > pr^{NT}$, $\mathbb{E}(\pi^*_N) - \mathbb{E}(\pi^*_{C,T}) < 0$, indicating that not providing a membership is less profitable than providing membership to both types of consumers; otherwise, if $\frac{\hat{n}_o}{n_o} < pr^{NT}$, $\mathbb{E}(\pi^*_N) - \mathbb{E}(\pi^*_{C,T}) > 0$, indicating that not providing a membership is more profitable than providing membership to both types of consumers.

Consequently, if $\frac{\hat{n}_o}{n_o} < \min\{pr^{CT}, pr^{NT}\}$, offering either no-membership plan or a membership plan to regular consumers only leads to greater profit than a membership plan that targets both types of consumers.

C.1 Proof of Technical Condition in Equation (3)

To ensure that all spot prices are positive and remain under $\bar{v}$, and all spot wages are positive and remain under $\bar{c}$, we need to add the following two constraints

$$0 < p^*_N(d, s) < \bar{v}, 0 < p^*_C(d, s) < \bar{v}, 0 < p^*_{C,T}(d, s) < \bar{v}, \tag{69}$$

$$0 < w^*_N(d, s) < \bar{c}, 0 < w^*_C(d, s) < \bar{c}, 0 < w^*_{C,T}(d, s) < \bar{c}, \tag{70}$$

Substitute $w^*_N(d, s)$, $w^*_C(d, s)$, and $w^*_{C,T}(d, s)$ from Equations (6), (13), and (59) into constraint (69), and substitute $p^*_N(d, s)$, $p^*_C(d, s)$, and $p^*_{C,T}(d, s)$ from Equations (7), (14), and (60) into constraint (70). Combining the two constraints and simplifying, making sure that those two constraints are satisfied under all possible values of d and s, we have Equation (3).

□

D Proof of Proposition 1

We subtract $\mathbb{E}(\pi_C^*)$ in (50) by $\mathbb{E}(\pi_N^*)$ in (44), which yields

$$
\begin{aligned}
\mathbb{E}(\pi_C^*) - \mathbb{E}(\pi_N^*) = \alpha n \bar{v}[&\frac{1}{2} \\
& - \frac{1}{16}(\frac{(\bar{v}(m-\theta)-2\alpha n\bar{c})^2}{2(\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta))^2} - \frac{(2\bar{c}(n-\delta)+\bar{v}(m-\theta))^2}{(\bar{c}(n-\delta)+\bar{v}(m-\theta))(\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta))} \\
& + \frac{(\bar{v}(m-\theta)-2\alpha n\bar{c})^2}{2(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta))^2} - \frac{(2\bar{c}(\delta+n)+\bar{v}(m-\theta))^2}{(\bar{c}(\delta+n)+\bar{v}(m-\theta))(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta))} \\
& + \frac{(\bar{v}(\theta+m)-2\alpha n\bar{c})^2}{2(\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m))^2} - \frac{(2\bar{c}(n-\delta)+\bar{v}(\theta+m))^2}{(\bar{c}(n-\delta)+\bar{v}(\theta+m))(\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m))} \\
& + \frac{(\bar{v}(\theta+m)-2\alpha n\bar{c})^2}{2(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m))^2} - \frac{(2\bar{c}(\delta+n)+\bar{v}(\theta+m))^2}{(\bar{c}(\delta+n)+\bar{v}(\theta+m))(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m))})].
\end{aligned} \tag{71}
$$

Therefore, $\mathbb{E}(\pi_C^*) > \mathbb{E}(\pi_N^*)$ if and only if the RHS of equation (71) is positive. Remove the coefficient term $\alpha n \bar{v}$, the RHS of equation (71) becomes

$$
\begin{aligned}
& \frac{1}{2} - \frac{1}{16}(\frac{(\bar{v}(m-\theta)-2\alpha n\bar{c})^2}{2(\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta))^2} - \frac{(2\bar{c}(n-\delta)+\bar{v}(m-\theta))^2}{(\bar{c}(n-\delta)+\bar{v}(m-\theta))(\bar{c}((1-\alpha)n-\delta)+\bar{v}(m-\theta))} \\
& + \frac{(\bar{v}(m-\theta)-2\alpha n\bar{c})^2}{2(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta))^2} - \frac{(2\bar{c}(\delta+n)+\bar{v}(m-\theta))^2}{(\bar{c}(\delta+n)+\bar{v}(m-\theta))(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(m-\theta))} \\
& + \frac{(\bar{v}(\theta+m)-2\alpha n\bar{c})^2}{2(\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m))^2} - \frac{(2\bar{c}(n-\delta)+\bar{v}(\theta+m))^2}{(\bar{c}(n-\delta)+\bar{v}(\theta+m))(\bar{c}((1-\alpha)n-\delta)+\bar{v}(\theta+m))} \\
& + \frac{(\bar{v}(\theta+m)-2\alpha n\bar{c})^2}{2(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m))^2} - \frac{(2\bar{c}(\delta+n)+\bar{v}(\theta+m))^2}{(\bar{c}(\delta+n)+\bar{v}(\theta+m))(\bar{c}(\delta+(1-\alpha)n)+\bar{v}(\theta+m))}).
\end{aligned} \tag{72}
$$

Denote the expression in (72) as $\Delta\pi(m,n)$. We can show that $\Delta\pi(m,n) > 0$ for sure when $\frac{\bar{c}}{\bar{v}} \leq \frac{\sqrt{17}-3}{4}$. We also verify that $\frac{\partial \Delta\pi(m,n)}{\partial m} > 0$ and $\frac{\partial \Delta\pi(m,n)}{\partial n} < 0$. In addition, we can show that $\Delta\pi(m,n)|_{m\to\theta} < 0$, $\Delta\pi(m,n)|_{m\to\infty} > 0$, $\Delta\pi(m,n)|_{n\to\delta} > 0$, and $\Delta\pi(m,n)|_{n\to\infty} < 0$. Therefore, when $\frac{\bar{c}}{\bar{v}} > \frac{\sqrt{17}-3}{4}$, there exists thresholds in terms of m and n such that $\Delta\pi(m,n) = 0$. Define threshold of m given n, $m^C(n) > 0$, and threshold of n given m, $n^C(m) > 0$, such that when $m = m^C(n)$ or $n = n^C(m)$, $\Delta\pi(m^C(n)) = 0$ or $\Delta\pi(n^C(m)) = 0$ (the RHS of equation (71) equals to 0 as well). Then, for any $m \geq m^C(n)$, $\Delta\pi(m) \geq \Delta\pi(m^C(n)) = 0$, or for any $n \leq n^C(m)$, $\Delta\pi(n) \geq \Delta\pi(n^C(m)) = 0$, hence, the RHS of equation (71) is non-negative, and the platform offers consumer-side-only membership plan; otherwise, for any $m < m^C(n)$ or $n > n^C(m)$, it must be $\Delta\pi(m) < \Delta\pi(m^C(n)) = 0$ or $\Delta\pi(n) < \Delta\pi(n^C(m)) = 0$, respectively, then the RHS of equation (71) is negative, and the platform does not offer consumer-side-only membership plan.

□

E Proof of Proposition 2

(i) In the no-membership equilibrium, consumer surplus of regular consumers and occasional consumers are given as

$$CS_N^r = \sum_s \sum_d \frac{1}{4}[\int_{p_N^*}^{\bar{v}} \alpha n \frac{z - p_N^*}{\bar{v}} dz] \tag{73}$$

$$CS_N^o = \sum_s \sum_d \frac{1}{4}[\int_{p_N^*}^{\bar{v}} (d - \alpha n) \frac{z - p_N^*}{\bar{v}} dz] \tag{74}$$

In the consumer-side-only membership equilibrium, consumer surplus of regular consumers and occasional consumers are given as

$$CS_C^r = \alpha n(U_C^r - \Delta U^r) = \alpha n(U_N^r) = \sum_s \sum_d \frac{1}{4}[\int_{p_C^*}^{\bar{v}} \alpha n \frac{z - p_C^*}{\bar{v}} dz] \tag{75}$$

$$CS_C^o = \sum_s \sum_d \frac{1}{4}[\int_{p_C^*}^{\bar{v}} (d - \alpha n) \frac{z - p_C^*}{\bar{v}} dz] \tag{76}$$

It can be shown that $CS_N^r > CS_C^r$ and $CS_N^o > CS_C^o$ because $p_C^* > p_N^*$. Therefore, the total consumer surplus is lower in the membership equilibrium, compared to that in the no-membership equilibrium.

(ii) The provider surplus in the no-membership equilibrium is

$$PS_N = \sum_s \sum_d \frac{1}{4}[\int_0^{w_N^*} s \frac{w_N^* - z}{\bar{c}} dz] \tag{77}$$

The provider surplus in the membership equilibrium is

$$PS_C = \sum_s \sum_d \frac{1}{4}[\int_0^{w_C^*} s \frac{w_C^* - z}{\bar{c}} dz] \tag{78}$$

It can be shown that $PS_N < PS_C$ because $w_C^* > w_N^*$. Therefore, the provider surplus is higher in the consumer-side-only membership equilibrium, compared to that in no-membership equilibrium.

(iii) The social welfare is the total valuation of all served consumers minus the total cost of all serving providers. Under the no-membership equilibrium, the social welfare can be expressed with

$$SW_N = \sum_s \sum_d \frac{1}{4}[\int_{p_N^*}^{\bar{v}} d \frac{z}{\bar{v}} dz - \int_0^{w_N^*} s \frac{z}{\bar{c}} dz] \tag{79}$$

Under the consumer-side-only membership equilibrium, the social welfare can be expressed with

$$SW_C = \sum_s \sum_d \frac{1}{4}[\int_{p_C^*}^{\bar{v}} (d - \alpha n) \frac{z}{\bar{v}} dz + \int_0^{\bar{v}} \alpha n \frac{z}{\bar{v}} dz - \int_0^{w_C^*} s \frac{z}{\bar{c}} dz] \tag{80}$$

For any d and s, when the technique condition in constraint (3) holds, the summand in (79) is less than the summand in (80), i.e., $\int_{p_N^*}^{\bar{v}} d\frac{z}{\bar{v}}dz - \int_0^{w_N^*} s\frac{z}{\bar{c}}dz < \int_{p_C^*}^{\bar{v}} (d-\alpha n)\frac{z}{\bar{v}}dz + \int_0^{\bar{v}} \alpha n\frac{z}{\bar{v}}dz - \int_0^{w_C^*} s\frac{z}{\bar{c}}dz$, which indicates $SW_N < SW_M$.

□

F Proof of Lemma 3

In the equilibrium, the platform ensures the following constraint holds:

$$d(\frac{\bar{v}-p_P}{\bar{v}})-\beta m=(s-\beta m)\frac{w_P}{\bar{c}}. \tag{81}$$

Again, suppose that this constraint (81) does not hold, then following the same argument in the proof of Lemma 1, the platform can either reduce the wage or increase the price to increase its profit. Therefore, the optimal wage satisfies

$$w_P(p_P)=\frac{\bar{c}(\bar{v}(d-\beta m)-dp_P)}{\bar{v}(s-\beta m)} \tag{82}$$

Substituting (81) and (82) into (16), the platform's profit is

$$\pi_P=\beta m p_P+(p_P-w_P(p_P))\cdot(d(\frac{\bar{v}-p_P}{\bar{v}})-\beta m) \tag{83}$$

Maximizing the profit by choosing p_P, the first-order-condition is

$$\frac{\partial \pi_P}{\partial p_P}=\frac{d}{\bar{v}^2}(\bar{v}(\bar{v}-2p_P)-\frac{2\bar{c}(\bar{v}(\beta m-d)+dp_P)}{s-\beta m})=0 \tag{84}$$

and the optimal price p_P^* is given in (22). We can check that the second-order derivative is always negative. Substituting (22) into (82), we obtain the optimal wages w_P^* in (21).

In equilibrium, the platform offers a fixed membership wage that is just above $-\Delta U^{provider}$. Calculating $-\Delta U^{provider}(d,s)$ for any given d and s by substituting (17) and (18) into (19), we have

$$-\Delta U^{provider}(d,s)=\mathbb{E}[\frac{w_P^{*2}}{2\bar{c}}]+\frac{\bar{c}}{2}=\frac{\bar{c}}{2}+\frac{\bar{c}}{8}[\frac{\bar{v}(d-2\beta m)}{(\bar{v}(s-\beta m)+\bar{c}d}]^2 \tag{85}$$

Now take expectation of (85) for all possible d and s, we derive the equilibrium fixed membership wage in (20).

Then the expected profit is given as:

$$\begin{aligned}
&\mathbb{E}(\pi_P^*)=\sum_s\sum_d\frac{1}{4}[\beta m p_P^*+(p_P^*-w_P^*)\cdot(d(\frac{\bar{v}-p_P^*}{\bar{v}})-\beta m)]-\beta m R_P^*\\
&=\frac{1}{32}(2\bar{v}(-\frac{4\beta m\bar{c}(\delta+\beta m-n)+\bar{v}(n-\delta)((\beta-1)m-\theta)}{\bar{c}(n-\delta)+\bar{v}(\theta-\beta m+m)}\\
&+\frac{4\beta m\bar{c}(\delta-\beta m+n)-\bar{v}(\delta+n)((\beta-1)m-\theta)}{\bar{c}(\delta+n)+\bar{v}(\theta-\beta m+m)}+\frac{4\beta m\bar{c}(\delta-\beta m+n)-\bar{v}(\delta+n)(\theta+(\beta-1)m)}{\bar{c}(\delta+n)-\bar{v}(\theta+(\beta-1)m)}\\
&-\frac{4\beta m\bar{c}(\delta+\beta m-n)+\bar{v}(n-\delta)(\theta+(\beta-1)m)}{\bar{c}(n-\delta)-\bar{v}(\theta+(\beta-1)m)})-\frac{\beta m\bar{c}\bar{v}^2(\delta+2\beta m-n)^2}{(\bar{c}(\delta-n)+\bar{v}(\theta+(\beta-1)m))^2}-16\\
&-\frac{\beta m\bar{c}\bar{v}^2(\delta+2\beta m-n)^2}{(\bar{c}(n-\delta)+\bar{v}(\theta-\beta m+m))^2}-\frac{\beta m\bar{c}\bar{v}^2(\delta-2\beta m+n)^2}{(\bar{c}(\delta+n)+\bar{v}(\theta-\beta m+m))^2}-\frac{\beta m\bar{c}\bar{v}^2(\delta-2\beta m+n)^2}{(\bar{c}(\delta+n)-\bar{v}(\theta+(\beta-1)m))^2})
\end{aligned} \tag{86}$$

□

G Proof of Proposition 3

We subtract $\mathbb{E}(\pi_P^*)$ in (86) by $\mathbb{E}(\pi_N^*)$ in (44), which yields

$$\mathbb{E}(\pi_P^*) - \mathbb{E}(\pi_N^*) = \sum_s \sum_d \frac{1}{4}[\frac{\beta m\bar{c}(2d\bar{c} - \bar{v}(d-2s))(-2d^2\bar{c}^2 + d\bar{c}\bar{v}(3d-4s) + \bar{v}^2(-2\beta dm + 3ds - 2s^2))}{8(d\bar{c} + s\bar{v})(d\bar{c} + \bar{v}(s-\beta m))^2}]$$

$$= \frac{1}{32}\bar{c}(\frac{2\bar{v}(\delta + 2\beta m - n)^2}{\bar{c}(\delta - n) + \bar{v}(\theta + (\beta - 1)m)} - \frac{\beta m\bar{v}^2(\delta + 2\beta m - n)^2}{(\bar{c}(\delta - n) + \bar{v}(\theta + (\beta - 1)m))^2} - 16\beta m$$
$$+ \frac{2\bar{v}(\delta - 2\beta m + n)^2}{\bar{v}(\theta + (\beta - 1)m) - \bar{c}(\delta + n)} - \frac{\beta m\bar{v}^2(\delta + 2\beta m - n)^2}{(\bar{c}(n - \delta) + \bar{v}(\theta - \beta m + m))^2} - \frac{\beta m\bar{v}^2(\delta - 2\beta m + n)^2}{(\bar{c}(\delta + n) + \bar{v}(\theta - \beta m + m))^2}$$
$$+ \frac{2\bar{v}(\delta + 2\beta m - n)^2}{\bar{c}(\delta - n) + (\beta - 1)m\bar{v} - \theta\bar{v}} + \frac{2\bar{v}(\delta - 2\beta m + n)^2}{-(\bar{c}(\delta + n)) + (\beta - 1)m\bar{v} - \theta\bar{v}} - \frac{\beta m\bar{v}^2(\delta - 2\beta m + n)^2}{(\bar{c}(\delta + n) - \bar{v}(\theta + (\beta - 1)m))^2}$$
$$+ \frac{2\bar{v}(n - \delta)^2}{\bar{c}(n - \delta) + \bar{v}(m - \theta)} + \frac{2\bar{v}(n - \delta)^2}{\bar{c}(n - \delta) + \bar{v}(\theta + m)} + \frac{2\bar{v}(\delta + n)^2}{\bar{c}(\delta + n) + \bar{v}(m - \theta)} + \frac{2\bar{v}(\delta + n)^2}{\bar{c}(\delta + n) + \bar{v}(\theta + m)}) \tag{87}$$

Remove coefficient term $\beta m\bar{c}$ in the summand of (87) and denote the rest of the summand as $\Delta\pi_{N,P}(s,d)$. We verify $\frac{\partial\Delta\pi_{N,P}(s,d)}{\partial m} < 0$ and $\frac{\partial\Delta\pi_{N,P}(s,d)}{\partial n} > 0$. Also, we show that $\Delta\pi_{N,P}(s,d) < 0$ for sure when $\frac{\bar{c}}{\bar{v}} > \frac{1}{2}$ or when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta > \frac{s(\bar{v}^2 - \bar{c}\bar{v} - 2\bar{c}^2)}{2\bar{v}^2 m}$. Thus, it is only possible for $\frac{\partial\Delta\pi_{N,P}(s,d)}{\partial m} > 0$ if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta < \frac{s(\bar{v}^2 - \bar{c}\bar{v} - 2\bar{c}^2)}{2\bar{v}^2 m}$. We further show that $\frac{s(\bar{v}^2 - \bar{c}\bar{v} - 2\bar{c}^2)}{2\bar{v}^2 m} > 0$ for sure. Thus, there exists a threshold of β, $\beta^P > 0$, such that when $\frac{\bar{c}}{\bar{v}} > \frac{1}{2}$ or when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta > \beta^P$, $\mathbb{E}(\pi_P^*) - \mathbb{E}(\pi_N^*) < 0$. If $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta < \beta^P$, the platform's membership strategy depends on n and m. In addition, we show that $\Delta\pi_{N,P}(s,d)|_{m\to\infty} < 0$ and $\Delta\pi_{N,P}(s,d)|_{d\to 0} < 0$ for sure, and if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta < \frac{s(\bar{v}^2 - \bar{c}\bar{v} - 2\bar{c}^2)}{2\bar{v}^2 m}$, we show that $\Delta\pi_{N,P}(s,d)|_{s\to d} > 0$ and $\Delta\pi_{N,P}(s,d)|_{d\to s} > 0$. Note that by the technique condition (3), we need $s > d$.

Therefore, when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $\beta < \beta^P$, there exists threshold of m given n, $m^P(n) > 0$, and threshold of n given m, $n^P(m) > 0$, such that when $m = m^P(n)$ (or $n = n^P(m)$), $\mathbb{E}(\pi_P^*) - \mathbb{E}(\pi_N^*) = 0$. Then, for any $m < m^P(n)$ (or $n > n^P(m)$), $\mathbb{E}(\pi_P^*) - \mathbb{E}(\pi_N^*) > 0$, and the platform offers provider-side-only membership plan; otherwise, for any $m > m^P(n)$ (or $n < n^P(m)$), it must be $\mathbb{E}(\pi_P^*) - \mathbb{E}(\pi_N^*) < 0$, and the platform offers no membership plan.

□

H Proof of Proposition 4

(i) In the no-membership equilibrium, consumer surplus is given as

$$CS_N = \sum_s \sum_d \frac{1}{4}[\int_{p_N^*}^{\bar{v}} d\frac{z - p_N^*}{\bar{v}} dz] \tag{88}$$

In the consumer-side-only membership equilibrium, consumer surplus of regular consumers and occasional consumers are given as

$$CS_P = \sum_s \sum_d \frac{1}{4}[\int_{p_P^*}^{\bar{v}} d\frac{z - p_P^*}{\bar{v}} dz] \tag{89}$$

It can be shown that $CS_N < CS_P$ because $p_P^* < p_N^*$. Therefore, the total consumer surplus is higher in the provider-side-only membership equilibrium, compared to that in the no-membership equilibrium.

(ii) In the no-membership equilibrium, the provider surplus of regular providers and occasional providers are

$$PS_N^r = \sum_s \sum_d \frac{1}{4}[\int_0^{w_N^*} \beta m \frac{w_N^* - z}{\bar{c}} dz] \tag{90}$$

$$PS_N^o = \sum_s \sum_d \frac{1}{4}[\int_0^{w_N^*} (s - \beta m) \frac{w_N^* - z}{\bar{c}} dz] \tag{91}$$

The provider surplus of regular providers and occasional providers in the provider-side-only membership equilibrium are

$$PS_P^r = \beta m (U_P^{provider} - \Delta U_P^{provider}) = \sum_s \sum_d \frac{1}{4}[\int_0^{w_P^*} \beta m \frac{w_P^* - z}{\bar{c}} dz] \tag{92}$$

$$PS_P^o = \sum_s \sum_d \frac{1}{4}[\int_0^{w_P^*} (s - \beta m) \frac{w_P^* - z}{\bar{c}} dz] \tag{93}$$

It can be shown that $PS_N^r > PS_P^r$ and $PS_N^o > PS_P^o$ because $w_P^* < w_N^*$. Therefore, the provider surplus is lower in the provider-side-only membership equilibrium, compared to that in no-membership equilibrium.

(iii) The social welfare is the total valuation of all served consumers minus the total cost of all serving providers. Under the no-membership equilibrium, the social welfare is given in (79). Under the provider-side-only membership equilibrium, the social welfare can be expressed with

$$SW_P = \sum_s \sum_d \frac{1}{4}[\int_{p_P^*}^{\bar{v}} d\frac{z}{\bar{v}} dz - \int_0^{\bar{c}} \beta m \frac{z}{\bar{c}} dz - \int_0^{w_P^*} (s - \beta m) \frac{z}{\bar{c}} dz] \tag{94}$$

For any d and s, the summand in (79) is less than the summand in (94), i.e., $\int_{p_N^*}^{\bar{v}} d\frac{z}{\bar{v}}dz - \int_0^{w_N^*} s\frac{z}{\bar{c}}dz < \int_{p_P^*}^{\bar{v}} d\frac{z}{\bar{v}}dz - \int_0^{\bar{c}} \beta m\frac{z}{\bar{c}}dz - \int_0^{w_P^*}(s - \beta m)\frac{z}{\bar{c}}dz$, if $s < \frac{(3v-2c)d}{2v}$. Therefore, there exists $m^{PSW}(n) > 0$, such that if $m < m^{PSW}(n)$, $SW_N < SW_P$, and if $m > m^{PSW}(n)$, $SW_N > SW_P$.

□

I Proof of Lemma 4

In the equilibrium, the platform ensures the following constraint holds:

$$(d - \alpha n)(\frac{\bar{v} - p_B}{\bar{v}}) = (s - \beta m)\frac{w_B}{\bar{c}} - (\alpha n - \beta m). \tag{95}$$

Following the same argument in the proofs of Lemma 1 and 3. Therefore, the optimal wage satisfies

$$w_B(p_B) = \frac{\bar{c}(\bar{v}(d - \beta m) + p_B(\alpha n - d))}{\bar{v}(s - \beta m)} \tag{96}$$

Substituting (95) and (96) into (24) or (25), the platform's profit is

$$\pi_B = \frac{1}{\bar{v}^2}(\bar{v}p_B(d - \alpha n)(\bar{v} - p_B) - \frac{\bar{c}(\bar{v}(d - \beta m) + p_B(\alpha n - d))^2}{s - \beta m}) \tag{97}$$

Maximizing the profit by choosing p_B, the first-order-condition is

$$\frac{\partial \pi_B}{\partial p_B} = \frac{(d-\alpha n)(2\bar{c}(\bar{v}(d-\beta m)+p_B(\alpha n-d))+\bar{v}(s-\beta m)(\bar{v}-2p_B))}{\bar{v}^2(s-\beta m)} = 0 \tag{98}$$

and the optimal price p_B^* is given in (29). We can check that the second-order derivative is always negative. Substituting (29) into (96), we obtain the optimal wages w_B^* in (28).

In equilibrium, the platform charges a membership fee that is just below ΔU_B^r and offers a fixed membership wage that is just above $-\Delta U_B^{provider}$. For any given d and s, we have

$$\Delta U_B^r(d,s) = \mathbb{E}[p_B^*(1-\frac{p_B^*}{2\bar{v}})] = \frac{\bar{v}}{8}[4-\frac{\bar{v}}{32}[\frac{(\bar{c}(2\beta m-2\alpha n)+\bar{v}(s-\beta m))}{(\bar{c}(d-\alpha n)+\bar{v}(s-\beta m))}]^2] \tag{99}$$

$$-\Delta U_B^{provider}(d,s) = \mathbb{E}[\frac{w_B^{*2}}{2\bar{c}}]+\frac{\bar{c}}{2} = \frac{\bar{c}}{2}+[\frac{\bar{v}(d-2\beta m+\alpha n)}{(\bar{c}(d-\alpha n)+\bar{v}(s-\beta m))}]^2 \tag{100}$$

Now take expectation of (99) and (100) for all possible d and s, we derive the equilibrium membership fee and fixed membership wage in (26) and (27), respectively.

□

J Proof of Proposition 5

When the platform offers both-sides membership plan, the expected profit is given as:

$$\mathbb{E}(\pi_B^*) = \sum_s \sum_d \frac{1}{4}[\frac{1}{\bar{v}^2}(\bar{v}p_B^*(d-\alpha n)(\bar{v}-p_B^*) - \frac{\bar{c}(\bar{v}(d-\beta m)+p_B^*(\alpha n-d))^2}{s-\beta m})] + \alpha n R_{C,B}^* - \beta m R_{P,B}^* \tag{101}$$

Replace $\beta = \frac{\alpha n}{m}$ into (101), we can rearrange the expected profit as

$$\begin{aligned} E^{SC}(\pi_B^*) = \sum_s \sum_d \frac{1}{4}[\frac{1}{8}(\frac{2\bar{v}^2(d-\alpha n)(s-\alpha n)}{\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n)} + \alpha n\bar{v}(4-\frac{\bar{v}^2(s-\alpha n)^2}{(\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n))^2}) \\ -\alpha n\bar{c}(\frac{\bar{v}^2(d-\alpha n)^2}{(\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n))^2}+4))] \end{aligned} \tag{102}$$

We subtract $E^{SC}(\pi_B^*)$ in (102) by $\mathbb{E}(\pi_N^*)$ in (44), which yields

$$\begin{aligned} E^{SC}(\pi_B^*) - \mathbb{E}(\pi_N^*) = \sum_s \sum_d \frac{1}{4}[\frac{1}{8}(\frac{2\bar{v}^2(d-\alpha n)(s-\alpha n)}{\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n)} + \alpha n\bar{v}(4-\frac{\bar{v}^2(s-\alpha n)^2}{(\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n))^2}) \\ -\alpha n\bar{c}(\frac{\bar{v}^2(d-\alpha n)^2}{(\bar{c}(d-\alpha n)+\bar{v}(s-\alpha n))^2}+4) - \frac{2ds\bar{v}^2}{d\bar{c}+s\bar{v}})] \end{aligned} \tag{103}$$

Denote the summand of the equation (103) as $\Delta\pi_{BN}$. It can be shown that $\Delta\pi_{BN} \geq 0$ when $\frac{\bar{c}}{\bar{v}} \leq \frac{1}{4}$ and $\Delta\pi_{BN} < 0$ when $\frac{\bar{c}}{\bar{v}} > \frac{1}{2}$. When $\frac{1}{4} < \frac{\bar{c}}{\bar{v}} \leq \frac{1}{2}$, we show that the $\Delta\pi_{BN}$ always decreases in s, indicating that it is only optimal to offer membership plans to both sides when the supply potential is small enough. Also, we show that $\Delta\pi_{BN}|_{s\to d} < 0$ and $\Delta\pi_{BN}|_{s\to\infty} > 0$ for any s and d. Denote a threshold s^{SC} such that when $s = s^{SC}$, $\Delta\pi_{BN} = 0$. Therefore, exists threshold of m given n, $m^{SC}(n) > 0$, such that for any $m < m^{SC}(n)$, $E^{SC}(\pi_B^*) - \mathbb{E}(\pi_N^*) > 0$, and the platform offers membership plans to both sides; otherwise, for any $m > m^{SC}(n)$, we have $E^{SC}(\pi_B^*) - \mathbb{E}(\pi_N^*) < 0$, and the platform offers no-membership plan.

□

K Proof of Proposition 6

(i) In the no-membership equilibrium, surplus of regular consumers, CS_N^r, and surplus of occasional consumers, CS_N^o, are given in (73) and (74), respectively. In the both-sides membership equilibrium, surplus of regular consumers and occasional consumers are given as

$$CS_B^r = \alpha n(U_B^r - \Delta U^r) = \alpha n(U_N^r) = \sum_s \sum_d \frac{1}{4}[\int_{p_B^*}^{\bar{v}} \alpha n \frac{z - p_B^*}{\bar{v}} dz] \tag{104}$$

$$CS_B^o = \sum_s \sum_d \frac{1}{4}[\int_{p_B^*}^{\bar{v}} (d - \alpha n) \frac{z - p_B^*}{\bar{v}} dz] \tag{105}$$

In the no-membership equilibrium, the provider surplus of regular providers and occasional providers are given in (90) and (91), respectively. The provider surplus of regular providers and occasional providers in the both-sides membership equilibrium are

$$PS_B^r = \beta m(U_B^{provider} - \Delta U_B^{provider}) = \sum_s \sum_d \frac{1}{4}[\int_0^{w_B^*} \beta m \frac{w_B^* - z}{\bar{c}} dz] \tag{106}$$

$$PS_B^o = \sum_s \sum_d \frac{1}{4}[\int_0^{w_B^*} (s - \beta m) \frac{w_B^* - z}{\bar{c}} dz] \tag{107}$$

Apparently, $CS_N^r < CS_B^r$, $CS_N^o < CS_B^o$, $PS_N^r > PS_B^r$, and $PS_N^o > PS_B^o$ if and only if $p_B^* < p_N^* \Rightarrow w_B^* < w_N^*$, where w_N^* is given in (6) and w_B^* is given in (28).

We show that $w_B^* < w_N^*$ if and only if $\beta > \frac{\alpha n(2d\bar{c}+s\bar{v})}{m(2d\bar{c}-\bar{v}(d-2s))}$ and $w_B^* > w_N^*$ if and only if $\beta < \frac{\alpha n(2d\bar{c}+s\bar{v})}{m(2d\bar{c}-\bar{v}(d-2s))}$, for any d and s. Also, we show that $0 < \frac{\alpha n(2d\bar{c}+s\bar{v})}{m(2d\bar{c}-\bar{v}(d-2s))} < 1$ for any d and s. Therefore, exists $\beta^{BCPS} \in (0, 1)$ such that when $\beta > \beta^{BCPS}$ the total consumer (provider) surplus is higher (lower) in the both-sides membership equilibrium, compared to that in the no-membership equilibrium.

(ii) The social welfare is the total valuation of all served consumers minus the total cost of all serving providers. Under the no-membership equilibrium, the social welfare is given in (79). Under the both-sides membership equilibrium, the social welfare can be expressed with

$$SW_B = \sum_s \sum_d \frac{1}{4}[\int_{p_B^*}^{\bar{v}} (d - \alpha n) \frac{z}{\bar{v}} dz + \int_0^{\bar{v}} \alpha n \frac{z}{\bar{v}} dz - \int_0^{\bar{c}} \beta m \frac{z}{\bar{c}} dz - \int_0^{w_B^*} (s - \beta m) \frac{z}{\bar{c}} dz] \tag{108}$$

Let $\beta^{BN} = \frac{\alpha n\bar{v}(s^2\bar{v}^2-4d^2\bar{c}^2)}{m(\bar{c}\bar{v}^2(3d^2+\alpha dn-8ds+4s^2)+4d\bar{c}^3(d-\alpha n)-4\bar{c}^2\bar{v}(2d(d-s)+\alpha ns)+\alpha ns\bar{v}^3)}$, when $\beta = \beta^{BN}$, the summand in (79) equals to that in (108). Denote $s^{BN} = \frac{1}{8}\sqrt{-\frac{16\alpha dn\bar{v}}{\bar{c}} + \frac{64\alpha dn\bar{c}}{\bar{v}} + \frac{\alpha^2 n^2\bar{v}^2}{\bar{c}^2} + \frac{16\alpha^2 n^2\bar{c}^2}{\bar{v}^2} + 16d^2 - 8\alpha^2 n^2} + \frac{\bar{c}(\alpha n-2d)}{2\bar{v}} - \frac{\alpha n\bar{v}}{8\bar{c}} + d$. We find that when $\beta^{BN} < 0$, $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $s \leq s^{BN}$ and the summand in (79) is always less than the summand in (108). Therefore, there exists a threshold $m^{BSW}(n)$, such that if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m \leq m^{BSW}(n)$, $SW_N < SW_B$. When $\beta^{BN} > 0$, for and d and s, the summand in (79) is less than the summand in (108) if and only if $\beta < \beta^{BN}$. Therefore, there exists a threshold of $\beta^{BSW} > 0$, such that if

$\beta < \beta^{BSW}$, $SW_N < SW_B$.

□

L Proof of Proposition 7

(i) When $\alpha n = \beta m$, the platform's expected profit when it offers membership on both sides is given in (102). Replace $\beta = \frac{\alpha n}{m}$ into (50), we can rearrange the platform's expected profit when it offers consumer-side-only membership as

$$E^{SC}(\pi_C^*) = \sum_s \sum_d \frac{1}{4}[\frac{\bar{v}(s\bar{v} - 2\alpha n\bar{c})(2d^2\bar{c} + s\bar{v}(2d + \alpha n))}{8(\bar{c}(d - \alpha n) + s\bar{v})^2}]. \tag{109}$$

Replace $\beta = \frac{\alpha n}{m}$ into (86), we can rearrange the platform's expected profit when it offers provider-side-only membership as

$$E^{SC}(\pi_P^*) = \sum_s \sum_d \frac{1}{4}[\frac{\bar{v}(4\alpha n\bar{c}(d - \alpha n) + d\bar{v}(s - \alpha n))}{4(d\bar{c} + \bar{v}(s - \alpha n))} - \frac{1}{8}\alpha n\bar{c}(\frac{\bar{v}^2(d - 2\alpha n)^2}{(d\bar{c} + \bar{v}(s - \alpha n))^2} + 4)]. \tag{110}$$

It can be shown that, for any d and s, the summand of $E^{SC}(\pi_P^*)$ in (110) is always smaller than the summand of $E^{SC}(\pi_B^*)$ in (102). Therefore, $E^{SC}(\pi_B^*) > E^{SC}(\pi_P^*)$ for sure, it is never optimal for the platform to offer provider-side-only membership when $\alpha n = \beta m$.

Denote a threshold in terms of $\frac{\bar{c}}{\bar{v}}$: $t^{NC} = \frac{(\sqrt{9d^2+4\alpha dn+4\alpha^2n^2}-d-2\alpha n)}{4d}$, for any d and s, we show that $t^{NC} \in (\frac{1}{4}, \frac{1}{2})$ and when $\frac{\bar{c}}{\bar{v}} \leq t^{NC}$, the summand of $E^{SC}(\pi_C^*)$ in equation (109) is greater than the summand of $\mathbb{E}(\pi_N^*)$ in equation (44). Therefore, there exists a threshold in terms of $\frac{\bar{c}}{\bar{v}}$, c^{NC}, such that when $\frac{\bar{c}}{\bar{v}} \leq c^{NC}$, the platform's profit is higher under the consumer-side-only membership plan than the no-membership plan. Thus, when $\frac{\bar{c}}{\bar{v}} \leq c^{NC}$, only need to compare consumer-side-only and both-sides membership plans. Subtract the summand of $E^{SC}(\pi_C^*)$ from the summand of $E^{SC}(\pi_B^*)$ and denote the result as $\Delta\pi_{CB}^{SC}$, we can show that $\Delta\pi_{CB}^{SC}$ always decreases in s for any s and d, and $\Delta\pi_{CB}^{SC}|_{s\to d} > 0$ and $\Delta\pi_{CB}^{SC}|_{s\to\infty} < 0$ when $\frac{\bar{c}}{\bar{v}} \leq t^{NC}$. Denote a threshold s^{CB} such that when $s = s^{CB}$, $\Delta\pi_{CB}^{SC} = 0$. Therefore, there exists a threshold of m given n, $m^{CB}(n)$, such that for any $m \leq m^{CB}(n)$, $E^{SC}(\pi_B^*) \geq E^{SC}(\pi_C^*)$, and the platform offers both-sides membership plan; otherwise, when $m > m^{CB}(n)$, the platform offers consumer-side-only membership plan.

(ii) Following the Proof of Proposition 5, when $\frac{\bar{c}}{\bar{v}} > \frac{1}{2}$, we only need to compare profits $E^{SC}(\pi_C^*)$ and $\mathbb{E}(\pi_N^*)$, since $E^{SC}(\pi_B^*) < \mathbb{E}(\pi_N^*)$ for sure. Subtract the summand of $\mathbb{E}(\pi_N^*)$ from the summand of $E^{SC}(\pi_C^*)$ and denote the result as $\Delta\pi_{CN}^{SC}$, we can show that $\Delta\pi_{CN}^{SC}$ always increases in s for any s and d, and $\Delta\pi_{CN}^{SC}|_{s\to\frac{2\bar{c}}{\bar{v}}d} < 0$ and $\Delta\pi_{CN}^{SC}|_{s\to\infty} > 0$ when $\frac{\bar{c}}{\bar{v}} > \frac{1}{2}$. Denote a threshold s^{CN} such that when $s = s^{CN}$, $\Delta\pi_{CN}^{SC} = 0$. Therefore, there exists a threshold of m given n, $m^{CN}(n)$, such that for any $m \leq m^{CN}(n)$, $E^{SC}(\pi_C^*) \leq \mathbb{E}(\pi_N^*)$, and the platform offers no-membership plan; otherwise, when $m > m^{CN}(n)$, the platform offers consumer-side-only membership plan.

(iii) When $c^{NC} < \frac{\bar{c}}{\bar{v}} \leq \frac{1}{2}$, we need to compare profits $E^{SC}(\pi_C^*)$, $E^{SC}(\pi_B^*)$, and $\mathbb{E}(\pi_N^*)$. We show that $E^{SC}(\pi_C^*) > E^{SC}(\pi_B^*)$ and $E^{SC}(\pi_C^*) > \mathbb{E}(\pi_N^*)$ when $m > m^{CN}(n)$, $\mathbb{E}(\pi_N^*) > E^{SC}(\pi_B^*)$ and $\mathbb{E}(\pi_N^*) \geq E^{SC}(\pi_C^*)$ when $m^{SC}(n) < m \leq m^{CN}(n)$, and $E^{SC}(\pi_B^*) > E^{SC}(\pi_C^*)$ and $E^{SC}(\pi_B^*) \geq \mathbb{E}(\pi_N^*)$ when $m \leq m^{SC}(n)$.

□

M Proof of Proposition 8

The social welfare under no-membership plan, consumer-side-only membership plan, provider-side-only membership plan, and both-sides membership plan are provided in Equation (79), Equation (80), Equation (94), and Equation (108), respectively.

In the Proof of Proposition 2 part (iii), we have shown that the social welfare is always lower under the no-membership plan compared to the consumer-side-only membership plan. When the technique conditions in constraints (3) and (4) hold, for any d and s, we show that the summand of Equation (108) is greater than that in Equation (94). Therefore, no-membership plan or provider-side-only membership plan can never lead to the highest social welfare.

Next, we compare social welfare under consumer-side-only membership plan and both-sides membership plan. We find that the summand in Equation (108) is greater than the summand in Equation (80) if and only if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $s < d + (\frac{1}{2} - \frac{\bar{c}}{\bar{v}})(d - \alpha n)$; otherwise, if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $s > d + (\frac{1}{2} - \frac{\bar{c}}{\bar{v}})(d - \alpha n)$, or if $\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$, the summand in Equation (108) is smaller than the summand in Equation (80). Also, denote the difference between the summand in Equation (108) and the summand in Equation (80), i.e., $(\int_{p_B^*}^{\bar{v}} (d - \alpha n)\frac{z}{\bar{v}}dz + \int_0^{\bar{v}} \alpha n \frac{z}{\bar{v}}dz - \int_0^{\bar{c}} \beta m \frac{z}{\bar{c}}dz - \int_0^{w_B^*}(s - \beta m)\frac{z}{\bar{c}}dz) - (\int_{p_C^*}^{\bar{v}} (d - \alpha n)\frac{z}{\bar{v}}dz + \int_0^{\bar{v}} \alpha n \frac{z}{\bar{v}}dz - \int_0^{w_C^*} s\frac{z}{\bar{c}}dz)$, as ΔSW^{CB}, we show that ΔSW^{CB} is decreasing in s, and when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$, we have $\Delta SW^{CB}|_{s \to d} > 0$ and $\Delta SW^{CB}|_{s \to \infty} < 0$. Thus, there exists a threshold $m^{ASW}(n)$, such that if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m < m^{ASW}(n)$, the social welfare is the highest under the both-sides membership plan; otherwise, if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m > m^{ASW}(n)$, or if $\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$, the social welfare is the highest under the consumer-side-only membership plan.

□

N Proof of Corollary 1

(i) According to Proposition 7(a), it is only possible for the platform to offer membership plans to both consumers and providers if $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$. When $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and the summand in Equation (102) is greater than the summand in Equation (109), we show that the summand in Equation (108) is greater than the summand in Equation (80) if $s < d + (\frac{1}{2} - \frac{\bar{c}}{\bar{v}})(d - \alpha n)$, for any d and s; otherwise, if $d + (\frac{1}{2} - \frac{\bar{c}}{\bar{v}})(d - \alpha n) < s < s^{CB}$, the summand in Equation (108) is smaller than the summand in Equation (80) for any d and s. Therefore, when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and the platform optimally offers the both-sides membership plan, the social welfare is also the highest under the both-sides membership plan if $m \leq m^{ASW}(n)$.

(ii) When $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and the summand in Equation (102) is smaller than the summand in Equation (109), the summand in Equation (108) is always smaller than the summand in Equation (80). Thus, when $\frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and the platform optimally offers the consumer-side-only membership plan, the social welfare is also the highest under the consumer-side-only membership plan.

In addition, according to Proposition 7(b) and (c), when $\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$, the platform offers either no-membership plan or consumer-side-only membership plan. Also, According to the Proof of Proposition 8, when $\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$, the social welfare is maximized under the consumer-side-only membership plan. Therefore, whenever the platform's profit is maximized under the consumer-side-only membership plan, the social welfare is also maximized under the consumer-side-only membership plan.

□

O Proof of Proposition 9

(i) According to the the Proof of Proposition 7, we have shown that, under the balanced-membership, it is never optimal for the platform to offer a provider-side-only membership plan. Thus, when a provider-side membership is mandated, the platform is forced to offer a both-sides membership plan. Consequently, under the conditions of Proposition 7(a) (i.e., either (i)$\frac{\bar{c}}{\bar{v}} \leq c^{NC}$ and $m \leq m^{CB}(n)$ or (ii)$c^{NC} < \frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m \leq m^{SC}(n)$), the platform optimally offers a both-sides membership plan without the mandate, so the mandate has no effects on the consumer surplus, provider surplus, or social welfare.

(ii) Under the conditions of Proposition 7(b) and (c) (i.e., (i)$\frac{\bar{c}}{\bar{v}} \leq c^{NC}$ and $m > m^{CB}(n)$, or (ii)$c^{NC} < \frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m > m^{SC}(n)$, or (iii)$\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$), the platform either offers a no-membership plan or a consumer-side-only membership plan. Comparing consumer surplus under the consumer-side-only membership plan given in Equation (75) and (76) to consumer surplus under the both-sides membership plan given in Equation (104) and (105), we show that consumer surplus is always higher under the both-sides membership plan than the consumer-side-only membership plan because $p_C^* > p_B^*$ for sure. Comparing provider surplus under the consumer-side-only membership plan given in Equation (78) to provider surplus under the both-sides membership plan given in Equation (106) and (107), we show that provider surplus is always lower under the both-sides membership plan than the consumer-side-only membership plan because $w_C^* > w_B^*$ for sure. We further show that the total surplus of consumers and providers is lower under the both-sides membership plan than the consumer-side-only membership plan. Therefore, when the platform's optimal strategy is consumer-side-only membership plan, and is forced to offer a both-sides membership plan instead, the social welfare is always reduced.

(iii) When $\alpha n = \beta m$, comparing consumer surplus under the no-membership plan given in Equation (73) and (74) to consumer surplus under the both-sides membership plan given in Equation (104) and (105) after plugging in $\beta = \frac{\alpha n}{m}$, we show that consumer surplus is always higher under the both-sides membership plan than the no-membership plan because $p_N^* > p_B^*$. Comparing provider surplus under the no-membership plan given in Equation (77) to provider surplus under the both-sides membership plan given in Equation (106) and (107) after plugging in $\beta = \frac{\alpha n}{m}$, we show that provider surplus is always lower under the both-sides membership plan than the no-membership plan because $w_N^* > w_B^*$. We further show that the total surplus of consumers and providers is higher under the both-sides membership plan than the no-membership plan.

(iv) Now, we compare social welfare under the no-membership plan and the both-sides membership plan under the balanced-membership. Note that this is different from Proposition 6 in which we compare those two plans in the general setting. First, we show that when $\frac{\bar{c}}{\bar{v}} \geq \frac{1}{2}$, the summand in Equation (79) is greater than the summand in Equation (108) for any d and s, indicating that the social welfare is always lower under the both-sides membership plan than the no-membership plan. Second, when $c^{NC} < \frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m^{SC}(n) < m \leq m^{CN}(n)$, the platform offers no-membership plan without the mandate. Denote $s^{MSW} = \frac{1}{8}(4\sqrt{\frac{(\bar{v}^2-4\bar{c}^2)(12d\bar{c}\bar{v}(d-\alpha n)-4\alpha^2 n^2\bar{c}^2+\alpha^2 n^2\bar{v}^2)}{\bar{v}^2(\bar{v}-4\bar{c})^2}} + \frac{3\bar{v}(2d-\alpha n)}{4\bar{c}-\bar{v}} + \frac{4\bar{c}(\alpha n-2d)}{\bar{v}} + 6d + \alpha n)$. We show that for any d and s, the summand in Equation (79) is greater than the summand in Equation (108) if and only if $s^{MSW} < s < s^{CN}$; otherwise, if $s^{SC} < s < s^{MSW}$, the summand in Equation (79) is smaller than the summand in Equation (108). Therefore, there exists a threshold m^{MSW}, such that when $c^{NC} < \frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m^{MSW}(n) < m < m^{CN}(n)$, the social welfare is lower under the both-sides membership plan than the no-membership plan; when $c^{NC} < \frac{\bar{c}}{\bar{v}} < \frac{1}{2}$ and $m^{SC}(n) < m < m^{MSW}(n)$, the social welfare is higher under the both-sides membership plan than the no-membership plan.

□

P Proof of Lemma 5

In the equilibrium, the platform ensures the following constraint holds:

$$(d-\alpha n)\frac{\bar{v}-p_M}{\bar{v}} = s\frac{w_M}{\bar{c}} - \alpha n\frac{\bar{v}-(1-\mu)p_M}{\bar{v}}. \tag{111}$$

The reasoning is provided in the proofs of previous Lemmas. Thus, the optimal wage should satisfy:

$$w_M(p_M) = \frac{\bar{c}(d\bar{v}-p_M(d-\mu\alpha n))}{s\bar{v}} \tag{112}$$

Substituting (111) and (112) into (30), the platform's profit is

$$\pi_M = \alpha n\frac{\bar{v}-(1-\mu)p_M}{\bar{v}}((1-\mu)p_M - w_M(p_M)) + (d-\alpha n)\frac{\bar{v}-p_M}{\bar{v}}(p_M - w_M(p_M)) \tag{113}$$

Maximizing the profit by choosing p_M, the first-order-condition is

$$\frac{\partial \pi_M}{\partial p_M} = \frac{\bar{v}(d-\alpha\mu n)(2d\bar{c}+s\bar{v}) - 2p_M(\bar{c}(d-\alpha\mu n)^2 + s\bar{v}(d+\alpha(\mu-2)\mu n))}{s\bar{v}^2} = 0 \tag{114}$$

and the optimal price p_M^* is given in (33). We can check that the second-order derivative is always negative. Substituting (33) into (112), we obtain the optimal wages w_M^* in (32).

In equilibrium, the platform charges a membership fee that is just below a regular consumer's expected surplus gain ΔU_M^r, which is

$$\Delta U_M^r = \sum_d \frac{1}{4}[\int_{(1-\mu)p_M^*(d,s)}^{\bar{v}} \frac{z-(1-\mu)p_M^*(d,s)}{\bar{v}} dz - \int_{p_M^*(d,s)}^{\bar{v}} \frac{z-p_M^*(d,s)}{\bar{v}} dz]. \tag{115}$$

Substituting (33) into (115), then, $\Delta U_M^r(d,s)$ for any given d and s is given by,

$$\Delta U_M^r(d,s) = \frac{\mu\bar{v}(d-\alpha\mu n)(2d\bar{c}+s\bar{v})(2\mu\bar{c}(d-2\alpha n)(d-\alpha\mu n) + s\bar{v}(d(\mu+2)+3\alpha(\mu-2)\mu n))}{8(\bar{c}(d-\alpha\mu n)^2 + s\bar{v}(d+\alpha(\mu-2)\mu n))^2} \tag{116}$$

Now take expectation of (116) for all possible d and s, we derive the equilibrium membership fee in (31).

□

Q Proof of Lemma 6

In the equilibrium, the platform ensures the following constraint holds:

$$(d-\alpha n)(\frac{\bar{v}-p_{B,M}}{\bar{v}}) = (s-\beta m)\frac{w_{B,M}}{\bar{c}} - (\alpha n\frac{\bar{v}-(1-\mu)p_{B,M}}{\bar{v}} - \beta m). \tag{117}$$

The reasoning is provided in the proofs of previous Lemmas. Thus, the optimal wage should satisfy:

$$w_{B,M}(p_{B,M}) = \frac{\bar{c}(\bar{v}(d-\beta m) - dp_{B,M} + \mu\alpha n p_{B,M})}{\bar{v}(s-\beta m)} \tag{118}$$

Substituting (117) and (118) into either (34) or (35), the platform's profit is obtained as

$$\begin{aligned}\pi_{B,M} &= (\alpha n\frac{\bar{v}-(1-\mu)p_{B,M}}{\bar{v}} - \beta m)((1-\mu)p_{B,M} - w_{B,M}(p_{B,M})) + \beta m(1-\mu)p_{B,M} \\ &+ (d-\alpha n)(\frac{\bar{v}-p_{B,M}}{\bar{v}})(p_{B,M} - w_{B,M}(p_{B,M}))\end{aligned} \tag{119}$$

Maximizing the profit by choosing $p_{B,M}$, the first-order-condition is

$$\frac{\partial \pi_{B,M}}{\partial p_{B,M}} = \frac{-\frac{2\bar{c}(d-\alpha\mu n)(\bar{v}(\beta m-d)+p_{B,M}(d-\alpha\mu n))}{s-\beta m} + \bar{v}(\bar{v}(d-\alpha\mu n) - 2p_{B,M}(d-\alpha(2-\mu)\mu n))}{\bar{v}^2} = 0 \tag{120}$$

and the optimal price $p_{B,M}^*$ is given in (39). We can check that the second-order derivative is always negative. Substituting (39) into (118), we obtain the optimal wages $w_{B,M}^*$ in (38).

In equilibrium, the platform charges a membership fee that is just below a regular consumer's expected surplus gain $\Delta U_{B,M}^r(d,s)$ and offers a fixed membership wage that is just above a regular provider's expected surplus loss $-\Delta U_B^{provider}(d,s)$. For any given d and s, we have

$$\begin{aligned}\Delta U_{B,M}^r(d,s) &= \frac{1}{4}[\int_{(1-\mu)p_{B,M}^*(d,s)}^{\bar{v}} \frac{z-(1-\mu)p_{B,M}^*(d,s)}{\bar{v}} dz - \int_{p_{B,M}^*(d,s)}^{\bar{v}} \frac{z-p_{B,M}^*(d,s)}{\bar{v}} dz] \\ &= \mu p_{B,M}^*(d,s)(1 - \frac{(2-\mu)p_{B,M}^*(d,s)}{2\bar{v}}).\end{aligned} \tag{121}$$

$$
\begin{aligned}
-\Delta U^{provider}_{B,M}(d,s) &= -\Big(\int_0^{\bar{c}} -\frac{z}{\bar{c}}dz - \int_0^{w^*_{B,M}(d,s)} \frac{w^*_{B,M}(d,s)-z}{\bar{c}}dz\Big) \\
&= \frac{1}{8}\bar{c}\Big(\frac{\bar{v}^2(d^2-2\beta dm-2\alpha d(1-\mu)\mu n-\alpha\mu n(\alpha\mu n-2\beta(2-\mu)m))^2}{(\bar{c}(d-\alpha\mu n)^2+\bar{v}(s-\beta m)(d-\alpha(2-\mu)\mu n))^2}+4\Big)
\end{aligned} \tag{122}
$$

Now take expectation of (121) and (122) for all possible d and s, we derive the equilibrium membership fee in (36) and (37), respectively.

□