Online Appendix

"Fixed, Proportional, or Menu-based? A Study of Managed Security Service Provider Contracts"

Appendix A: Proofs of Statements

This appendix provides the detailed proofs for all lemmas, and propositions presented in the main text.

Before deriving the equilibrium, we first establish the foundational game-theoretic framework and specify the key parameters. Given the complex interactions between the MSSP and heterogeneous clients, we model this scenario as a static game where both parties make a single set of decisions (Hui et al. 2019). Within this static framework, all functions and parameters reflect the rational expectations of the MSSP and clients in this single period (Guan et al. 2025).

To elucidate the formulation of these rational expectations, we use client negligence as a detailed illustrative example. In practice, clients may exhibit heterogeneous levels of negligence; however, the MSSP can form a rational expectation of β based on observable historical client data, which is denoted by $\tilde{\beta}$. Therefore, we model client negligence as a random variable $\tilde{\beta}$. Specifically, we express it as $\tilde{\beta} = \beta + \epsilon$, where ϵ is a mean-zero random shock capturing the uncertainty. Let $\epsilon \sim U[-\delta, \delta]$, with δ representing the degree of uncertainty. The expected client negligence is

$$E[\tilde{\beta}] = \int_{\beta-\delta}^{\beta+\delta} \tilde{\beta} \cdot \frac{1}{2\delta} \; d\tilde{\beta} = \beta.$$

In a rational expectations equilibrium, the MSSP's expectation of client negligence must be consistent with actual negligence; that is, $\tilde{\beta} = \beta$. For ease of exposition, we use β to represent the equilibrium negligence level in all subsequent analyses of equilibrium paths. Furthermore, we assume that β_F denotes client negligence under the fixed compensation contract (F-Contract), and β_R denotes client negligence under the proportional compensation contract (R-Contract). Note that β_F can be higher or lower than β_R, reflecting that clients' post-contractual negligence behaviors may differ depending on their chosen contract type.

While non-decision parameters such as the security loss risk factor (α) are treated as expected values in our static game, they are firmly grounded in estimable industry metrics rather than arbitrary assumptions. To reliably assess the security loss risk factor (α) and security environments, clients and MSSPs often utilize advanced methodologies such as Bayesian belief networks, decision trees (e.g., Andoh-Baidoo and Osei-Bryson 2007), and historical abnormal return analyses. Following information systems literature (e.g., Sen and Borle 2015), the likelihood and financial impact of data breaches can be rigorously estimated using contextual firm data and empirical models. Moreover, many organizations today keep detailed audit trails that can facilitate post-breach investigation using computer forensic techniques (Hui et al. 2019). These methodologies allow MSSPs to approximate loss parameters with a high degree of statistical confidence.

Proof of Lemma 1

When the MSSP offers only the fixed compensation contract (F-Contract), let θ_F denote the valuation of the marginal client who is indifferent between accepting the F-Contract and rejecting it. Given the heterogeneity in clients' valuations of the MSSP services, clients with perceived service value exceeding θ_F will purchase the security service under the F-Contract. From the definition of θ_F, we have $E[u_F(\theta_F)] = \theta_F - p_F - b\beta_F N_F \alpha\theta_F + b\beta_F N_F d = 0$, and $\theta_F = 1 - N_F$. We then derive the service fee as

$$p_F = (1 - N_F)(1 - \alpha b\beta_F N_F) + b\beta_F N_F d. \quad \text{(A1)}$$

The MSSP's profit under the F-Contract is

$$\pi_F(p_F) = p_F N_F - b\beta_F N_F d N_F - \Gamma(s). \quad \text{(A2)}$$

Taking Equation (A1) into Equation (A2), we have $\max_{N_F} \pi_F(N_F) = N_F(1 - N_F)(1 - \alpha b\beta_F N_F) - \Gamma(s)$.

Then, deriving the first-order condition with respect to N_F, we have

$$\frac{\partial \pi_F}{\partial N_F} = 3\alpha b\beta_F {N_F}^2 - 2\alpha b\beta_F N_F - 2N_F + 1. \quad \text{(A3)}$$

Setting the first-order condition to zero and solving for the optimal client base size, we obtain the equilibrium solutions: $N_F^* = \frac{1-A+\alpha b\beta_F}{3\alpha b\beta_F}$ and $\theta_F^* = 1 - N_F^* = \frac{2\alpha b\beta_F + A - 1}{3\alpha b\beta_F}$, where $A = \sqrt{(\alpha b\beta_F)^2 - \alpha b\beta_F + 1}$. We can easily verify that $N_F^* - \frac{1}{2} = \frac{2-\alpha b\beta_F - 2A}{6\alpha b\beta_F} < 0$ since $(2 - \alpha b\beta_F)^2 - (2A)^2 = -3(\alpha b\beta_F)^2 < 0$. Furthermore, the second-order derivative is $\frac{\partial^2 \pi_F}{\partial {N_F}^2} = 2\alpha b\beta_F(2N_F - 1) + 2(\alpha b\beta_F N_F - 1)$. In practice, the probability of being attacked in a given time period, $q = b\beta N$, is low for the vast majority of clients. Therefore, we safely assume that q is sufficiently small such that $\alpha q < 1$ holds (where $\alpha q = \alpha b \beta_F N_F$). Under these conditions, we have $\frac{\partial^2 \pi_F}{\partial {N_F}^2} < 0$ since $N_F^* < \frac{1}{2}$ and $\alpha q < 1$. The second-order condition is satisfied, ensuring that the critical point N_F^* constitutes a profit maximum for the MSSP.

Under the assumptions of $d > 0$, $b \in (0,1)$, $\theta \in [0,1]$, $\beta_F > 1$, and $\varphi \in [0,1]$, we now prove $0 < \theta_F^* < 1$, which ensures that a strictly positive fraction of clients will accept the F-Contract. First, to prove $\theta_F^* < 1$, we need to prove $\frac{2\alpha b \beta_F + A - 1}{3\alpha b \beta_F} < 1$, which simplifies to $A = \sqrt{(\alpha b \beta_F)^2 - \alpha b \beta_F + 1} < \alpha b \beta_F + 1$. Since $A^2 < (\alpha b \beta_F + 1)^2$ always holds, the inequality $\theta_F^* < 1$ holds. Next, to prove $\theta_F^* > 0$, we need to prove $\frac{2\alpha b \beta_F + A - 1}{3\alpha b \beta_F} > 0$, which simplifies to $A > 1 - 2\alpha b \beta_F$. Consider two cases based on the value of $\alpha b \beta_F$: (i) If $1 - 2\alpha b \beta_F < 0$, the inequality holds trivially since $A > 0$; (ii) If $1 - 2\alpha b \beta_F \geq 0$ (which implies $\alpha b \beta_F \leq \frac{1}{2}$), both sides are non-negative. We can square both sides to obtain $A^2 > (1 - 2\alpha b \beta_F)^2$. Substituting A^2 yields $(\alpha b \beta_F)^2 - \alpha b \beta_F + 1 > 1 - 4\alpha b \beta_F + 4(\alpha b \beta_F)^2$. This expression simplifies to $3\alpha b \beta_F (1 - \alpha b \beta_F) > 0$. Since $\alpha b \beta_F \leq \frac{1}{2} < 1$ in this case, the condition is strictly satisfied. Therefore, $A > 1 - 2\alpha b \beta_F$ holds universally, establishing that $\theta_F^* > 0$. In conclusion, inequality $0 < \theta_F^* < 1$ holds. Note that the other root of the first-order condition falls outside the feasible range and is therefore discarded.

Finally, based on the solution $N_F^* = \frac{1 - \sqrt{(\alpha b \beta_F)^2 - \alpha b \beta_F + 1} + \alpha b \beta_F}{3\alpha b \beta_F}$, we can derive the expressions for the equilibrium service fee $p_F^* = \frac{2}{3} + \frac{\left(\alpha b \beta_F + 1 - \sqrt{(\alpha b \beta_F)^2 - \alpha b \beta_F + 1}\right)(3b\beta_F d - \alpha b \beta_F - 1)}{9\alpha b \beta_F}$, the total expected compensation $T_F^* = b\beta_F N_F d N_F = \frac{(-1 + \sqrt{(\alpha b \beta_F)^2 - \alpha b \beta_F + 1} - \alpha b \beta_F)^2 d}{9\alpha^2 b \beta_F}$, and the MSSP's profit $\pi_F^* = \frac{(1 - A + \alpha b \beta_F)(2\alpha b \beta_F + A - 1)(2 + A - \alpha b \beta_F)}{27(\alpha b \beta_F)^2} - \Gamma(s)$. ■

Proof of Lemma 2

When the MSSP offers only a proportional compensation contract (R-Contract), let θ_R denote the valuation of the marginal client who is indifferent between accepting the R-Contract and rejecting it. Analogous to Case F, θ_R divides the potential market into two segments. From the definition of θ_R, we have $E[u_R(\theta_R)] = \theta_R - p_R - b\beta_R N_R(1-\varphi)\alpha\theta_R = 0$, and $\theta_R = 1 - N_R$. Similar to Case F, we derive the service fee as

$$p_R = (1 - N_R)[1 - \alpha b\beta_R(1-\varphi)N_R]. \tag{A4}$$

The MSSP's expected profit under R-Contract is:

$$\pi_R(p_R) = p_R N_R - \int_{\theta_R}^{1}(b\beta_R N_R\varphi\alpha\theta)d\theta - \Gamma(s). \tag{A5}$$

Taking Equation (A4) into Equation (A5), we have $\max_{N_R} \pi_R(N_R) = N_R(1-N_R) - \alpha b\beta_R {N_R}^2(2 - 2N_R + \varphi N_R)/2 - \Gamma(s)$. By taking the first-order derivative with respect to N_R, we obtain:

$$\frac{\partial \pi_R}{\partial N_R} = 1 - \frac{3\alpha b\beta_R(\varphi-2){N_R}^2}{2} - \frac{(4\alpha b\beta_R+4)N_R}{2} \quad . \tag{A6}$$

Setting the first-order condition to zero, we obtain the equilibrium solutions $N_R^* = \frac{C}{3\alpha b\beta_R(\varphi-2)}$ and $\theta_R^* = 1 - N_R^* = \frac{3\alpha b\beta_R\varphi - 4\alpha b\beta_R - B + 2}{3\alpha b\beta_R(\varphi-2)}$, where $B = \sqrt{(2\alpha b\beta_R + 2)^2 + 6\alpha b\beta_R(\varphi-2)}$ and $C = -2\alpha b\beta_R + B - 2$. Since $N_R^* - \frac{1}{2} = \frac{\alpha b\beta_R(2-3\varphi) - 4 + 2\sqrt{(2\alpha b\beta_R+2)^2 + 6\alpha b\beta_R(\varphi-2)}}{6\alpha b\beta_R(\varphi-2)}$, $(\varphi - 2) < 0$ and $(\alpha b\beta_R(2-3\varphi)-4)^2 - \left(2\sqrt{(2\alpha b\beta_R+2)^2 + 6\alpha b\beta_R(\varphi-2)}\right)^2 < 0$, we have $N_R^* < \frac{1}{2}$. In addition, $\frac{\partial^2 \pi_R}{\partial {N_R}^2} = 2\alpha b\beta_R(2N_R - 1) + 2(\alpha b\beta_R N_R - 1) - 3\alpha b\beta_R\varphi N_R < 0$ given $N_R^* < \frac{1}{2}$ and $\alpha q < 1$. Thus, the second-order condition is satisfied, ensuring that the critical point N_R^* constitutes a profit maximum for the MSSP.

Next, we show that $0 < \theta_R^* = \frac{3\alpha b\beta_R\varphi - 4\alpha b\beta_R - B + 2}{3\alpha b\beta_R(\varphi-2)} < 1$ holds, ensuring that a positive fraction of clients will accept the R-Contract. First, we can prove that the position of an indifferent client under R-Contract always increases with α, that is $\frac{\partial \theta_R^*}{\partial \alpha} = \frac{-2B + 4 + b\beta_R(3\varphi-2)\alpha}{3\alpha^2 b\beta_R(\varphi-2)B} > 0$. Note that the denominator is strictly negative, since $\varphi < 1$ implies $\varphi - 2 < 0$. Thus, to prove $\frac{\partial \theta_R^*}{\partial \alpha} > 0$, we only need to show that the numerator is negative. When $4 + b\beta_R(3\varphi-2)\alpha < 0$, we have $-2B + 4 + b\beta_R(3\varphi-2)\alpha < 0$ and $\frac{\partial \theta_R^*}{\partial \alpha} > 0$. When $4 + b\beta_R(3\varphi-2)\alpha > 0$, we have $(4 + b\beta_R(3\varphi-2)\alpha)^2 - (-2B)^2 = 3(\alpha b\beta_R)^2\left((3\varphi-4)\varphi - 4\right) < 0$. Therefore, $|2B| > 4 + b\beta_R(3\varphi-2)\alpha$ and $\frac{\partial \theta_R^*}{\partial \alpha} > 0$. Next, since $\theta_R^*(\alpha)$ is monotonically increasing in α, with $\lim_{\alpha \to 0^+} \theta_R^*(\alpha) = \frac{1}{2}$, and $\lim_{\alpha \to +\infty} \theta_R^*(\alpha) = 1$, we can conclude that $0 < \theta_R^* < 1$ holds for all $\alpha > 0$. Note that the other mathematical root of the first-order condition falls outside the feasible domain and is thus excluded.

Finally, based on the equilibrium client base $N_R^* = \frac{-2\alpha b\beta_R + \sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)}-2}{3\alpha b\beta_R(\varphi-2)}$, we can derive the equilibrium service fee $p_R^* = \frac{C(\alpha b\beta_R(3\varphi-2)(\varphi-1)+\varphi+2)}{9\alpha b\beta_R(\varphi-2)^2} + \frac{\varphi-4}{3(\varphi-2)}$, the total expected compensation $T_R^* = \frac{\varphi C^2(6\alpha b\beta_R(2-\varphi)+C)}{54(\alpha b\beta_R)^2(2-\varphi)^3}$, and the MSSP's profit $\pi_R^* = \frac{C(-C(\alpha b\beta_R+1)-6\alpha b\beta_R(2-\varphi))}{27(\alpha b\beta_R)^2(2-\varphi)^2} - \Gamma(s)$. ■

Proof of Lemma 3

When the MSSP offers a menu-based contract regime (M-Contract), let θ_{MF} denote the valuation of the marginal client who is indifferent between accepting the F-Contract and not purchasing any security service, and θ_{MR} be the valuation of the marginal client who is indifferent between adopting the F-Contract and the R-Contract.

Following a similar logic to Case F and Case R, the MSSP sets a uniform service fee p_M to determine the market boundaries. At equilibrium, the marginal client with valuation θ_{MF} receives zero expected utility ($E[u_M(\theta_{MF})] = 0$), and the marginal client with valuation θ_{MR} is indifferent between accepting the F- and R-Contract. Thus, we have the following two equations:

$$\begin{cases} \theta_{MF} - p_M - b\beta_F N_M(\alpha\theta_{MF} - d) = 0, \\ \theta_{MR} - p_M - b\beta_R N_M(1-\varphi)\alpha\theta_{MR} = \theta_{MR} - p_M - b\beta_F N_M(\alpha\theta_{MR} - d). \end{cases} \tag{A7}$$

By solving the Equations in (A7) and utilizing the relationships $N_{MR} = 1 - \theta_{MR}$ and $N_M = N_{MF} + N_{MR}$, we obtain N_{MR}^* and p_M.

$$\begin{cases} N_{MR}^* = \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)} \\ p_M = \left(1 - N_{MF} - \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}\right)\left(1 - \alpha b\beta_F\left(N_{MF} + \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}\right)\right) \\ \quad + b\beta_F d\left(N_{MF} + \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}\right) \end{cases} \tag{A8}$$

The MSSP's expected profit under the M-Contract is:

$$\pi_M = p_M N_M - b\beta_F N_M d N_{MF} - \int_{\theta_{MR}}^{1} (b\beta_R N_M \varphi\alpha\theta)\ d\theta - \Gamma(s) \tag{A9}$$

Equation (A8) reveals that within the menu-based contract framework, the size of the R-Contract sub-market, N_{MR}, is independent of the service fee p_M, whereas the size of the F-Contract sub-market,

N_{MF}, is endogenously determined by p_M. According to Lemma 3, N_{MR} represents the size of the high-valuation client base, N_{MF} represents the size of the middle-valuation client base, and $1 - N_{MF} - N_{MR}$ denotes the low-valuation client base. Therefore, under the M-Contract, the MSSP has no direct control over the contract choice of high-valuation clients through pricing. However, it can effectively influence middle- and low-valuation clients' adoption of F-Contract by adjusting the service fee p_M.

Notice that because N_{MR}^* is determined solely by exogenous parameters, optimizing the service fee p_M is analytically equivalent to optimizing only the F-Contract client base size N_{MF}. Taking Equation (A8) into Equation (A9), and deriving the first-order condition with respect to N_{MF}, we obtain the equilibrium solutions of the client base as: $N_{MF}^* = \frac{6b{\beta_F}^2 d-(4\alpha b\beta_F-2)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1}}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$. Since $N_{MR}^* = \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}$, we can derive the total equilibrium client base $N_M^* = N_{MR}^* + N_{MF}^* = \frac{2(\alpha b\beta_F+1)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1}}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$. Therefore, $\theta_{MF}^* = 1 - N_M^* = 1 - \frac{2(\alpha b\beta_F+1)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1}}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$ and $\theta_{MR}^* = 1 - N_{MR}^* = \frac{\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}$, where $Y1 = 4b^2{\beta_F}^4(\alpha^2 - 3\alpha d + 3d^2) + 4(\beta_F + \varphi\beta_R - \beta_R)^2 + 2b{\beta_F}^3(\alpha^2 b\beta_R(7\varphi - 4) + 3b\beta_R d^2(\varphi - 2) - 2\alpha) - 2\alpha b\beta_F(\varphi\beta_R - \beta_R)^2(1 - 3\alpha b\varphi\beta_R + 6b\beta_F d) - 4\alpha b\beta_R{\beta_F}^2(\varphi - 1)(2 + \alpha b\beta_R(1 - 4\varphi) + 6b\beta_F d)$. Since $\theta_{MR}^* \in (0,1)$, the condition $\beta_F + \varphi\beta_R - \beta_R > 0$ needs to hold for an interior solution.

Accordingly, we can derive the equilibrium service fee $p_M^* = \frac{((1+\alpha b\beta_F-3b\beta_F d)(\beta_F+\varphi\beta_R-\beta_R)\sqrt{Y1}+2(\beta_F+\varphi\beta_R-\beta_R)^2(b\beta_F(3d+4\alpha)-1)+Y2)}{18\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)^2}$, the expected total compensation $T_M^* = b\beta_F N_M^* d N_{MF}^* + \frac{\alpha b\beta_R\varphi N_M^* N_{MR}^*(2-N_{MR}^*)}{2}$, and the equilibrium profit $\pi_M^* = p_M^* N_M^* - b\beta_F N_M^* d N_{MF}^* - \frac{\alpha b\beta_R\varphi N_M^* N_{MR}^*(2-N_{MR}^*)}{2} - \Gamma(s)$, where $Y2 = (-2\alpha^2 + 6d^2)b^2{\beta_F}^4 + \alpha^2 b^2\beta_F(\varphi - 1)\left((4\varphi + 2)\beta_F{\beta_R}^2 + 3\varphi{\beta_R}^3(\varphi - 1)\right) + b^2{\beta_F}^3\beta_R\left(3d^2(\varphi - 2) + \alpha^2(4 - \varphi)\right) + b{\beta_F}^3(6d + 8\alpha)$. In addition, $\frac{\partial^2\pi_M}{\partial {N_M}^2} = 2\alpha b\beta_F(2N_M - 1) + 2(\alpha b\beta_F N_M - 1) < 0$ given $N_M^* < \frac{1}{2}$ and $\alpha q < 1$. The second-order condition is satisfied, verifying that N_M^* constitutes a profit maximum for the MSSP. Note that the other mathematical root of the first-order condition is discarded as it falls outside the feasible domain.

We now explain the self-selection mechanism that drives high-valuation clients to prefer the R-Contract. Comparing $E[u_{MR}(\theta_{MR})] = \theta_{MR} - p_M - b\beta_R N_M(1-\varphi)\alpha\theta_{MR}$ with $E[u_{MF}(\theta_{MR})] = \theta_{MR} - p_M - b\beta_F N_M(\alpha\theta_{MR} - d)$, we find that when both contract types are offered to clients, high-valuation clients ($\theta > \theta_{MR} = \frac{\beta_F d}{\beta_F+\varphi\beta_R-\beta_R}$) will prefer the R-Contract, middle- and low-valuation clients ($\theta < \theta_{MR} = \frac{\beta_F d}{\beta_F+\varphi\beta_R-\beta_R}$) will select the F-Contract. Thus, the necessary and sufficient conditions for the equilibrium solutions above are $0 < \theta_{MF} < \theta_{MR} < 1$. However, as the values of the parameters vary, the condition $0 < \theta_{MF} < \theta_{MR} < 1$ may not always hold. As a result, three scenarios may appear:

(i) Reduction to Case F: when $\theta_{MR} \geq 1$, we have $\alpha \leq \alpha_F = \frac{\beta_F d}{\beta_F+\varphi\beta_R-\beta_R}$, indicating that θ_{MR} is outside of the defined range [0,1], then the equilibrium client strategy profile under M-Contract reduces to F-Contract. This implies that no clients select the R-Contract, even though the MSSP offers both contract types.

(ii) Reduction to Case R: when $\theta_{MR} < 1$, we have $\alpha > \alpha_F$ and heterogeneous choices of compensation contracts exist among clients. As MSSP's pricing does not impact clients' preference between F-Contract and R-Contract, its key decision is the client base purchasing its service by adjusting the fee p_M. When $\theta_{MF} \leq \theta_{MR}$ is violated (i.e., $\theta_{MF} \geq \theta_{MR}$), we have $\alpha \geq \alpha_R = \frac{3{\beta_F}^2 bd - b\beta_F d\beta_R(\varphi-1) + (\beta_F+\varphi\beta_R-\beta_R) - \sqrt{(\beta_F+\varphi\beta_R-\beta_R)^2 + b^2{\beta_F}^2 d^2(\beta_F+\beta_R)^2 + 2b\beta_F d(\varphi^2{\beta_R}^2 - (\beta-\beta_R)^2)}}{(2\beta_F-\varphi\beta_R)(\beta_F+\varphi\beta_R-\beta_R)}$ and the equilibrium client strategy profile reduces to that of Case R, indicating that no clients opt for the F-Contract, despite the simultaneous availability of both contract types.

(iii) The Menu-based Equilibrium: when $0 < \theta_{MF} < \theta_{MR} < 1$, that is, $\alpha_F < \alpha < \alpha_R$, MSSP will set a service fee p_M so that potential clients with $\theta \leq \theta_{MF}$ will opt out, those with $\theta_{MF} < \theta < \theta_{MR}$ will adopt F-Contract, and those with $\theta_{MR} \leq \theta \leq 1$ will adopt R-Contract. This represents a true menu-based market structure where heterogeneous choices coexist.

Based on the above analysis, we have equilibrium solutions in Case M:

(a) if $\alpha \leq \alpha_F$, leading to $\theta^*_{MR} \geq 1$, the equilibrium MSSP solution for the menu-based contract reduces to that for Case F in Lemma 1. Formally, $p^*_M = p^*_F$ and $N^*_M = N^*_F$, when $\alpha \leq \alpha_F$;

(b) if $\alpha_F < \alpha < \alpha_R$, corresponding to $0 < \theta^*_{MF} < \theta^*_{MR} < 1$, high-valuation ($N^*_{MR} =$

$\frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}$) clients located in $[\theta_{MR}^*, 1]$ will choose the R-Contract. The MSSP will set the service fee at $p_M^* = \frac{((1+\alpha b\beta_F-3b\beta_F d)(\beta_F+\varphi\beta_R-\beta_R)\sqrt{Y1}+2(\beta_F+\varphi\beta_R-\beta_R)^2(b\beta_F(3d+4\alpha)-1)+Y2)}{18\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)^2}$ to ensure that middle-valuation ($N_{MF}^* = \frac{(6b\beta_F{}^2 d-(4\alpha b\beta_F-2)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1})}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$) clients whose valuations lie in $(\theta_{MF}^*, \theta_{MR}^*)$ will choose the F-Contract, and the low-valuation clients whose valuations lie in $[0, \theta_{MF}^*]$ will not purchase the service;

(c) if $\alpha \geq \alpha_R$, resulting in $\theta_{MF}^* \geq \theta_{MR}^*$, the equilibrium MSSP solution for the menu-based contract reduces to that for Case R in Lemma 2. Formally, $p_M^* = p_R^*$ and $N_M^* = N_R^*$ when $\alpha \geq \alpha_R$. ■

Proof of Proposition 1

We first prove Proposition 1(a). Under the F-Contract, the first-order derivative of the client base with respect to α is $\frac{\partial N_F^*}{\partial\alpha} = \frac{2-\alpha b\beta_F-2\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1}}{6\alpha^2 b\beta_F\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1}}$. As $(2-\alpha b\beta_F)^2 - \left(2\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1}\right)^2 = -3(\alpha b\beta_F)^2 < 0$, we have $\frac{\partial N_F^*}{\partial\alpha} < 0$.

Similarly, under the R-Contract, the first-order derivative with respect to α is $\frac{\partial N_R^*}{\partial\alpha} = \frac{\alpha b\beta_R(2-3\varphi)-4+2\sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)}}{3\alpha^2 b\beta_R(\varphi-2)\sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)}}$. Since $(\varphi-2) < 0$, we have the denominator $3\alpha^2 b\beta_R(\varphi-2)\sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)} < 0$. Next, evaluating the squared terms in the numerator yields $(\alpha b\beta_R(2-3\varphi)-4)^2 - \left(2\sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)}\right)^2 = 3\alpha^2 b^2\beta_R{}^2(3\varphi^2-4\varphi-4) < 0$, since $(3\varphi^2-4\varphi-4) < 0$ given $0 < \varphi < 1$. Thus, $2\sqrt{(2\alpha b\beta_R+2)^2+6\alpha b\beta_R(\varphi-2)} > |\alpha b\beta_R(2-3\varphi)-4|$, which implies the numerator of $\frac{\partial N_R^*}{\partial\alpha}$ is positive, and consequently $\frac{\partial N_R^*}{\partial\alpha} < 0$.

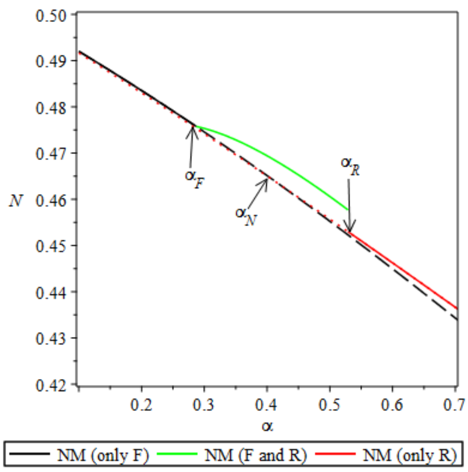
Next, we prove the statement in Proposition 1(b) regarding how N_M^* varies with α. From Lemma 3, we obtained $N_M^* = \frac{(6b\beta_F{}^2 d-(4\alpha b\beta_F-2)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1})}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$. By taking the first-order derivative of N_M^* with respect to α and setting it equal to zero, i.e., $\frac{\partial N_M^*}{\partial\alpha} = 0$, we can obtain a unique critical point, denoted as $\alpha_M = \frac{\beta_F d(-2\sqrt{H}+(2\beta_F+\varphi\beta_R-2\beta_R)bd\beta_F{}^2(3bd\beta_F+1))}{(\beta_F+\varphi\beta_R-\beta_R)^2(3b^2\beta_F{}^3 d^2+2bd\beta_F{}^2-\beta_F-2\varphi\beta_R)}$, where $H = \left(\beta_F + \frac{\varphi\beta_R}{2} - \beta_R\right)(b^2\beta_F{}^3 d^2(\beta_F{}^2+2\varphi\beta_F\beta_R+2\beta_F\beta_R-3\beta_R{}^2) - (2bd\beta_F{}^2-\beta_F-2\varphi\beta_R)(\beta_F+\varphi\beta_R-\beta_R)^2)$.

We now focus on the case where $\beta_F > \beta_R$ because it is a necessary condition for the existence of a positive critical point $\alpha_M > 0$. To determine whether α_M is a maximum point, we examine the behavior of $\frac{\partial N_M^*}{\partial \alpha}$ at the boundaries of the domain. As $\alpha \to 0^+$, we have the following asymptotic expansion: $\frac{\partial N_M^*}{\partial \alpha} = \frac{\sqrt{4(\beta_F+\varphi\beta_R-\beta_R)^2+6b^2{\beta_F}^3 d^2(2\beta_F-2\beta_R+\varphi\beta_R)}-2(\beta_F+\varphi\beta_R-\beta_R)}{6b\beta_F(\beta_F+\varphi\beta_R-\beta_R)\alpha^2} + o\left(\frac{1}{\alpha^3}\right) > 0$. Conversely, as $\alpha \to +\infty$, the expansion yields: $\frac{\partial N_M^*}{\partial \alpha} = -\frac{\left(2\sqrt{b^2\beta_F(2\beta_F+3\varphi\beta_R)(\beta_F+\varphi\beta_R-\beta_R)^2}+\sqrt{2}b\beta_F(3b\beta_F d+1)(\beta_F+\varphi\beta_R-\beta_R)\right)}{6b\beta_F\sqrt{b^2\beta_F(2\beta_F+3\varphi\beta_R)(\beta_F+\varphi\beta_R-\beta_R)^2}\alpha^2} + o\left(\frac{1}{\alpha^3}\right) < 0$. Since $\frac{\partial N_M^*}{\partial \alpha}$ is continuous and has only one root (α_M), it is certain that the transition from positive to negative occurs at α_M. Furthermore, the uniqueness of α_M guarantees that it is the global maximizer. Therefore, we conclude that the function N_M^* is unimodal, and it is strictly increasing on the interval $(0, \alpha_M)$ and decreasing on the interval $(\alpha_M, +\infty)$.

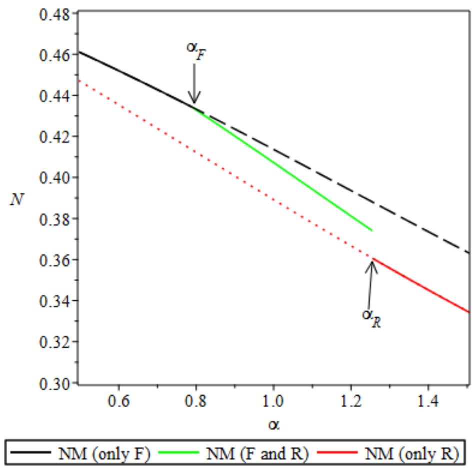
Moreover, to ensure that N_M^* exhibits an increasing trend within the relevant M-Contract market interval (α_F, α_R), it is required that the critical point $\alpha_M > \alpha_F$. Here we introduce a threshold client negligence level for the F-Contract, denoted as β_1, above which we have $\alpha_M > \alpha_F$. That is, $\alpha_M > \alpha_F$ holds when $\beta_F > \beta_1$, where β_1 satisfies the equation $3bd{\beta_1}^2(\beta_1 - \beta_R) = (\beta_1 + \varphi\beta_R - \beta_R)(\beta_1 + 2\sqrt{3\beta_1\beta_R - 2\beta_1^2})$ subject to $\beta_1 > \beta_R$. Therefore, when the condition $\beta_F > \beta_1 > \beta_R$ is satisfied, $\frac{\partial N_M^*}{\partial \alpha} > 0$ holds for $\alpha \in (\alpha_F, \alpha_M)$, as stated in Proposition 1(b). As a numerical illustration, please refer to Figure 6(b) in the main text. We set $\beta_F = 1.8, \beta_R = 1.05$ such that $\beta_F > \beta_1 = 1.44 > \beta_R$.

Specifically, by comparing the relative position of the peak α_M against the market boundaries α_F and α_R, we can analytically categorize the market dynamics into four detailed scenarios: (1) If $\alpha_F < \alpha_M < \alpha_R$, (i.e., $\beta_F > \beta_1 > \beta_R$ holds), the client base first increases ($\frac{\partial N_M^*}{\partial \alpha} > 0$) for $\alpha \in (\alpha_F, \alpha_M)$, and subsequently decreases ($\frac{\partial N_M^*}{\partial \alpha} < 0$) for $\alpha \in (\alpha_M, \alpha_R)$. (2) If $\alpha_M > \alpha_R$, (i.e., $\beta_F > \beta_1 > \beta_R$ holds), the peak lies beyond the feasible upper bound. Due to the unimodality of N_M^*, we have $\frac{\partial N_M^*}{\partial \alpha} > 0$ globally across $\alpha \in (\alpha_F, \alpha_R)$. (3) However, when $\beta_1 > \beta_F > \beta_R$, we have $\alpha_M < \alpha_F$ and $\frac{\partial N_M^*}{\partial \alpha} < 0$ for all $\alpha \in (\alpha_F, \alpha_R)$. Figure A1(a) depicts the scenario where $\alpha_M < \alpha_F$ (i.e., $\beta_1 > \beta_F > \beta_R$). (4)

Moreover, when $\beta_R > \beta_F$, the structural properties of the market further bifurcate into two sub-cases based on the threshold $\beta_R\left(1-\frac{\varphi}{2}\right)$. Specifically, if $\beta_R > \beta_F > \beta_R\left(1-\frac{\varphi}{2}\right)$, the peak falls below the lower bound ($\alpha_M < \alpha_F$), yielding $\frac{\partial N_M^*}{\partial \alpha} < 0$ for all $\alpha \in (\alpha_F, \alpha_R)$. Conversely, if $\beta_R > \beta_R\left(1-\frac{\varphi}{2}\right) > \beta_F$, the positive critical point α_M ceases to exist. Despite these distinct mathematical underlying conditions, both sub-cases lead to the same monotonic outcome: the client base N_M^* is monotonically decreasing throughout the interval $\alpha \in (\alpha_F, \alpha_R)$, which is numerically illustrated in Figure A1(b) for the broader condition $\beta_R > \beta_F$. In addition, we have $\frac{\partial N_{MR}^*}{\partial \alpha} = \frac{\beta_F d}{\alpha^2(\beta_F+\varphi\beta_R-\beta_R)} > 0$, indicating that the market share of the R-Contract strictly expands with α.■



(a) $\beta_1 > \beta_F > \beta_R$, ($\beta_1 = 1.38, \beta_F = 1.24, \beta_R = 1.01$)



(b) $\beta_R > \beta_F$, ($\beta_R = 1.2, \beta_F = 1.1$)

$d = 0.1, \varphi = 0.2, b = 0.5$

Figure A1. Variation of N_M^* with α under low and high β_F.

Proof of Proposition 2

Proof of Proposition 2(a)

We now first establish a benchmark case without negative security externalities. This benchmark allows for a rigorous examination of the role that security externalities play in the MSSP's decision-making process.

In the main model, security externalities are captured through a multiplier in the expression for the likelihood of a successful security breach, i.e., $b\beta_F N_F$ under the F-Contract and $b\beta_R N_R$ under the R-Contract. Without these externalities, the probability of security exploitation for an individual client should be independent of the total client base size. Thus, we replace the externality with an exogenous constant G (yielding $b\beta_F G$ under the F-Contract and $b\beta_R G$ under the R-Contract). It is important to emphasize that our objective here is to assess how the equilibrium dynamics fundamentally shift in the presence versus the absence of externalities, rather than to compare the absolute magnitudes of the solutions. Hence, the specific magnitude of G relative to the client base size is immaterial to our comparative analysis.

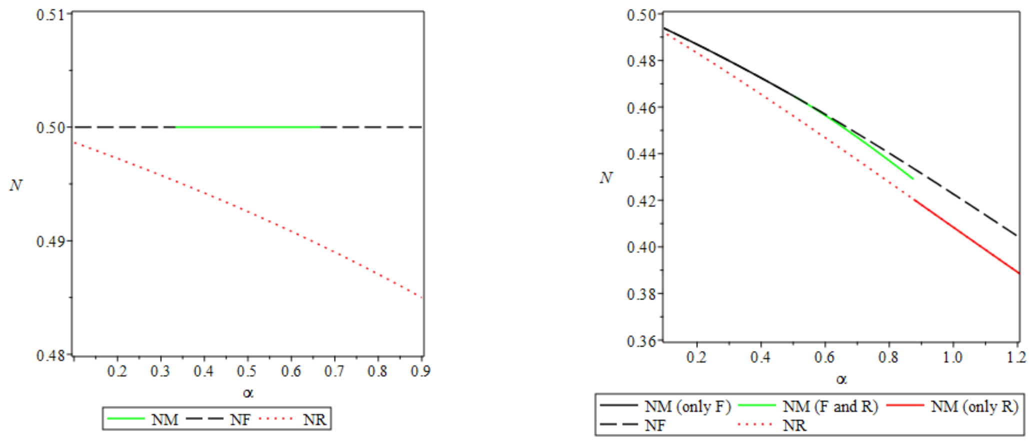
Assuming the security externality is independent of the client base size N_F, then the MSSP profit under the F-Contract can be expressed as $\pi_F = N_F(p_F - b\beta_F Gd) - \Gamma(s)$. To maximize its own profit, the MSSP will set a service fee p_F so that the MSSP can determine the client base N_F by impacting the client's utility. Given $\theta_F = 1 - N_F$, we have $p_F = (1 - N_F)(1 - \alpha b\beta_F G) + b\beta_F Gd$. Thus, the MSSP's expected profit becomes $\max_{N_F} \pi_F(N_F) = N_F(1 - N_F)(1 - \alpha b\beta_F G) - \Gamma(s)$. Taking the first-order and second-order conditions with respect to N_F, we have $\frac{\partial \pi_F}{\partial N_F} = (1 - 2N_F)(1 - \alpha b\beta_F G) = 0$ and $\frac{\partial^2 \pi_F}{\partial {N_F}^2} = 2(\alpha b\beta_F G - 1) < 0$. As a result, the MSSP will choose to serve exactly half of the potential market (that is, $\widehat{N}_F^* = 1/2$). Remarkably, this optimal market coverage is independent of the total security risk faced by the client pool, i.e., $\alpha b\beta_F G$.

Similarly, in the absence of security externalities, the expected breach probability under the R-Contract becomes independent of the client base. Consequently, the MSSP's expected profit can be expressed as: $\pi_R = N_R(p_R - \alpha b\beta_R G\varphi(2 - N_R)/2) - \Gamma(s)$. To maximize its profit, the MSSP optimally sets the service fee p_R to induce the desired equilibrium client base. Utilizing the marginal client condition $\theta_R = 1 - N_R$, the service fee is given by $p_R = \theta_R(1 - \alpha b\beta_R G(1 - \varphi)) = (1 - N_R)(1 - \alpha b\beta_R G(1 - \varphi))$. Thus, the MSSP's expected profit becomes $\pi_R = N_R(1 - N_R) - \alpha b\beta_R G N_R(2 - 2N_R + \varphi N_R)/2 - \Gamma(s)$. Taking the first-order and second-order derivatives with respect to N_R, we can obtain $\frac{\partial \pi_R}{\partial N_R} = -(1 + (\varphi - 2)N_R)\alpha b\beta_R G - 2N_R + 1 = 0$ and $\frac{\partial^2 \pi_R}{\partial {N_R}^2} = 2(\alpha b\beta_R G - 1) - \alpha b\beta_R G\varphi < 0$. As a result, we obtain the R-Contract's equilibrium client base $\widehat{N}_R^* = \frac{1}{2} - \frac{\alpha b\beta_R \varphi G}{4 + 2\alpha b\beta_R G(\varphi - 2)}$ in the absence of externalities.

Similar to Case F and Case R, in the absence of security externalities, the MSSP's expected profit under the M-Contract can be expressed as $\pi_M = p_M N_M - \ b\beta_F G d N_{MF} - \int_{\theta_{MR}}^{1} (b\beta_R G\varphi\alpha\theta)\ d\theta - \Gamma(s)$. Since the indifference condition between the F-Contract and R-Contract relies solely on the relative expected utilities—where the uniform service fee p_M cancels out—the marginal client type θ_{MR} and the corresponding high-valuation client base remain unaffected by the absence of externalities. Thus, we have $\widehat{N}_{MR}^* = N_{MR}^* = 1 - \frac{\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}$. This demonstrates that regardless of whether security externalities are present, the number of high-valuation clients who self-select the R-Contract under the menu-based regime remains unchanged. Therefore, the MSSP only needs to determine the location of the marginal client who is indifferent between purchasing F-Contract and not purchasing the security service.

To maximize its own profit, the MSSP will set the service fee p_M to adjust the client base to its desired level. Since $\theta_{MF} = 1 - N_{MR} - N_{MF}$, we have $p_M = ((N_{MR} + N_{MF} - 1)\alpha + d)b\beta_F G - N_{MR} - N_{MF} + 1$. Thus, the MSSP's expected profit is $\pi_M = N_{MF}(N_{MR} + N_{MF} - 1)(\alpha b\beta_F G - 1) + \big(((N_{MR} + N_{MF} - 1)\alpha + d)b\beta G - N_{MR} - N_{MF} + 1\big)N_{MR} + \frac{\alpha b\beta_R G\varphi N_{MR}(2-N_{MR})}{2} - \Gamma(s)$. Taking the first-order and second-order derivatives with respect to N_{MF}, we have $\frac{\partial\pi_M}{\partial N_{MF}} = (\alpha b\beta_F G - 1)\left(2N_{MF} - 1 + 2\frac{\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}\right) = 0$ and $\frac{\partial^2\pi_M}{\partial {N_{MF}}^2} = 2(\alpha b\beta_F G - 1) < 0$. Therefore, we obtain the equilibrium client base $\widehat{N}_{MF}^* = \frac{-\alpha(\beta_F+\varphi\beta_R-\beta_R)+2\beta_F d}{2\alpha(\beta_F+\varphi\beta_R-\beta_R)}$ without externality. Finally, the equilibrium client base for the MSSP is $\widehat{N}_M^* = \widehat{N}_{MF}^* + \widehat{N}_{MR}^* = \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)} + \frac{-\alpha(\beta_F+\varphi\beta_R-\beta_R)+2\beta_F d}{2\alpha(\beta_F+\varphi\beta_R-\beta_R)} = \frac{1}{2}$ in the absence of externality.

In conclusion, in the absence of security externalities, the total equilibrium client bases under the three contract scenarios are explicitly given by $\widehat{N}_F^* = \frac{1}{2}$, $\widehat{N}_R^* = \frac{1}{2} - \frac{\alpha b\beta_R\varphi G}{4+2\alpha b\beta_R G(\varphi-2)}$, and $\widehat{N}_M^* = \frac{1}{2}$. These analytical solutions satisfy the condition $\widehat{N}_F^* = \widehat{N}_M^* > \widehat{N}_R^*$, as shown in Figure A2(a). That is, $\widehat{N}_F^* \geq max\{\widehat{N}_M^*, \widehat{N}_R^*\}$ holds.



(a) $\beta_F = 1.2, \beta_R = 1.05,\ G = 0.5$
(without security externality)

(b) $\beta_F = \beta_R = 1$
(without client negligence)

$d = 0.1, \varphi = 0.2, b = 0.5$

Figure A2. Comparison of client bases without externality or negligence

We now examine the benchmark scenario that isolates the effect of client negligence by assuming homogeneous baseline negligence, normalized to $\beta_F = \beta_R = 1$. Based on Lemmas 1, 2, and 3, substituting these parameters yields the respective equilibrium client bases $\widehat{N}_F^* = \frac{1-\sqrt{(\alpha b)^2-\alpha b+1}+\alpha b}{3\alpha b}$, $\widehat{N}_R^* = \frac{-2\alpha b+\sqrt{(2\alpha b+2)^2+6\alpha b(\varphi-2)}-2}{3\alpha b(\varphi-2)}$, and $\widehat{N}_M^* = \frac{2\alpha b-\sqrt{2(\frac{3(\alpha b\varphi)^2+(2+2\alpha^2b^2-6\alpha b^2d-2\alpha b)\varphi+3d^2b^2}{\varphi})+2}}{6\alpha b}$. Taking the difference $\widehat{N}_F^* - \widehat{N}_R^* = \frac{-\varphi(\alpha b+1)+(\varphi-2)\sqrt{(\alpha b)^2-\alpha b+1}+\sqrt{(2\alpha b+2)^2+6\alpha b(\varphi-2)}}{3\alpha b(2-\varphi)}$, we have $(\sqrt{(2\alpha b+2)^2+6\alpha b(\varphi-2)})^2 - \left(-\varphi(\alpha b+1)+(\varphi-2)\sqrt{(\alpha b)^2-\alpha b+1}\right)^2 = -2(\varphi-2)\varphi\left((-1-\alpha b)\sqrt{(\alpha b)^2-\alpha b+1}+(\alpha b)^2+\frac{\alpha b}{2}+1\right) > 0$. Therefore, $\widehat{N}_F^* > \widehat{N}_R^*$ when $\beta_F = \beta_R = 1$. Taking the difference $\widehat{N}_F^* - \widehat{N}_M^* = \frac{\sqrt{2(\frac{3(\alpha b\varphi)^2+(2+2\alpha^2b^2-6\alpha b^2d-2\alpha b)\varphi+3d^2b^2}{\varphi})}-2\sqrt{(\alpha b)^2-\alpha b+1}}{6\alpha b}$, we have

$$\left(\sqrt{2\left(\frac{3(\alpha b\varphi)^2+(2+2\alpha^2b^2-6\alpha b^2d-2\alpha b)\varphi+3d^2b^2}{\varphi}\right)}\right)^2 - \left(2\sqrt{(\alpha b)^2-\alpha b+1}\right)^2 = \frac{6b^2(d-\alpha\varphi)^2}{\varphi} > 0 \quad .$$

Therefore, $\widehat{N}_F^* > \widehat{N}_M^*$ when $\beta_F = \beta_R = 1$. To be concluded, $\widehat{N}_F^* \geq max\{\widehat{N}_M^*, \widehat{N}_R^*\}$ holds when $\beta_F = \beta_R = 1$, as shown in Figure A2(b). Please refer to Figure A2 for scenarios without security externalities or client negligence.

Proof of Proposition 2(b)

We first present a comprehensive comparison of the equilibrium client bases under the three contract schemes, which is formally summarized in the following proposition.

Proposition EC.1. *For* $\alpha \in (\alpha_F, \alpha_F^+)$*, the equilibrium client bases satisfy the following ordering: (a)* $N_F^* > N_M^* > N_R^*$ *when* $\beta_F < \beta_R$*; (b)* $N_M^* > N_F^* > N_R^*$ *when* $\beta_R < \beta_F < min\{\beta_2, (\frac{3\varphi}{2}+1)\beta_R\}$*; (c)* $N_R^* > N_M^* > N_F^*$ *when* $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$*. Moreover,* $N_M^* > N_R^* > N_F^*$ *holds when* $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$ *and* $\alpha \in (\alpha_E, \alpha_E^+)$*.*

Note that Proposition 2(b) in the main text corresponds directly to Proposition EC.1(b). Collectively, the complete results of Proposition EC.1 demonstrate that any of the three contract types can achieve the largest client base, depending on the specific parameter regions. We now provide the detailed proof for Proposition EC.1.

We first compare the relationships among the pure-strategy contracts (N_F^*, N_R^*), and then, on that basis, compare all three contract types. Specifically, we aim to establish the following conditions: (a) when $\beta_F < \beta_R$, $N_R^* < N_F^*$ holds (Scenario FR-1); (b) when $\beta_R < \beta_F < \left(\frac{3\varphi}{2}+1\right)\beta_R$, $\begin{cases} \alpha > \alpha_N, N_R^* > N_F^* \ \text{(Scenario FR} - 3) \\ \alpha \le \alpha_N, N_R^* \le N_F^* \ \text{(Scenario FR} - 2) \end{cases}$ holds; and (c) when $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$, $N_R^* > N_F^*$ holds (Scenario FR-4).

We begin by analyzing the scenario where $\beta_F > \beta_R$, including Scenarios FR-2, FR-3, and FR-4. Let $\Delta N = N_F^* - N_R^* = \frac{(\varphi-2)(1-A)\beta_R + \beta_F(\alpha b \beta_R \varphi + 2 - B)}{3\alpha b \beta_F \beta_R(\varphi-2)}$. By setting $\Delta N = 0$, we derive a critical point $\alpha_N = \frac{(2\beta_F + \varphi\beta_R - 2\beta_R)(3\varphi\beta_R + 2\beta_R - 2\beta_F)}{8b\beta_F\varphi\beta_R(\beta_F - \beta_R)}$. It is straightforward to verify that a unique positive root $\alpha_N > 0$ exists if and only if $\beta_R < \beta_F \le (\frac{3\varphi}{2}+1)\beta_R$.

First, we prove that when $\beta_R < \beta_F \le (\frac{3\varphi}{2}+1)\beta_R$, $\begin{cases} \alpha > \alpha_N, N_R^* > N_F^* \\ \alpha \le \alpha_N, N_R^* \le N_F^* \end{cases}$ (Scenarios FR-2 and 3) holds. In this region, $\alpha_N > 0$ uniquely exists. We can obtain $\frac{\partial \Delta N}{\partial \alpha} = \frac{\left((-4\beta_F - 2\beta_R\varphi + 4\beta_R)B + \beta_F(6\alpha b \beta_R \varphi - 4\alpha b \beta_R + 8)\right)A - (\varphi-2)(\alpha b \beta_F - 2)\beta_R B}{6\alpha^2 b \beta_F \beta_R(\varphi-2)AB}$. When $\beta_R < \beta_F \le (\frac{3\varphi}{2}+1)\beta_R$, if $\alpha \to 0^+$, we have $\lim_{\alpha\to 0^+} \Delta N = N_F^* - N_R^* = 0$ and $\lim_{\alpha\to 0^+} \frac{\partial \Delta N}{\partial \alpha} = \frac{\partial N_F^*}{\partial \alpha} - \frac{\partial N_R^*}{\partial \alpha} = -\frac{b(2\beta_F - 3\varphi\beta_R - 2\beta_R)}{16} > 0$. If $\alpha \to +\infty$, we can obtain $\Delta N = N_F^* - N_R^* = \frac{\beta_R - \beta_F}{2\alpha b \beta_F \beta_R} + o\left(\frac{1}{\alpha^2}\right) < 0$. Furthermore, they intersect at a unique point $\alpha_N > 0$, with $\frac{\partial N_F^*}{\partial \alpha} < 0$ and $\frac{\partial N_R^*}{\partial \alpha} < 0$, as proved in Proposition 1. Thus, as α increases, N_F^* is initially greater than N_R^*, equals N_R^* at α_N, and becomes smaller than N_R^* when α is sufficiently large. That is, $N_F^* > N_R^*$ when $\alpha < \alpha_N$, and $N_F^* < N_R^*$ when $\alpha > \alpha_N$. Please refer to Figure A3(a) for the scenario when $\beta_R < \beta_F \leq (\frac{3\varphi}{2} + 1)\beta_R$.

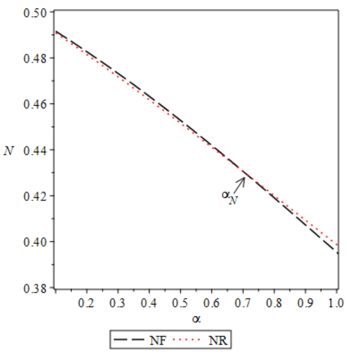
Second, we prove that when $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$, $N_F^* < N_R^*$ holds (Scenario FR-4). Under this extreme condition, the threshold α_N becomes negative and falls outside the feasible domain. Here, we prove that $\Delta N = N_F^* - N_R^* < 0$ by exploiting its monotonicity with respect to the compensation ratio φ. By taking the derivative of ΔN with respect to φ, we find $\frac{\partial \Delta N}{\partial \varphi} = \frac{(-2\alpha b\beta_R - 2)B + 4 + 4\alpha^2 b^2 {\beta_R}^2 + \alpha b\beta_R(3\varphi+2)}{3\alpha b\beta_R(\varphi-2)^2 B} > 0$ because $(4 + 4\alpha^2 b^2 {\beta_R}^2 + \alpha b\beta_R(3\varphi + 2))^2 - \left((-2\alpha b\beta_R - 2)B\right)^2 = 9\alpha^2 b^2 {\beta_R}^2 (\varphi - 2)^2 > 0$. Hence, $\frac{\partial \Delta N}{\partial \varphi} > 0$ holds. Moreover, given $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$, it implies $\varphi < \frac{2\beta_F - 2\beta_R}{3\beta_R}$. That is, $\frac{2\beta_F - 2\beta_R}{3\beta_R}$ is the maximum value of φ in this situation. Next, we prove that when φ takes its maximum value $\frac{2\beta_F - 2\beta_R}{3\beta_R}$, ΔN is still less than 0. Thus, we will have $\Delta N < 0$ always holds true under $\varphi < \frac{2\beta_F - 2\beta_R}{3\beta_R}$ (that is $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$).

By setting $\Delta N = 0$, we obtain the critical point $\varphi_1 = \frac{2(\beta_F - \beta_R)(\alpha b\beta_F + 2A - 2)}{\beta_R\left((-2\alpha b\beta_F - 2)A + 2(\alpha b\beta_F)^2 + \alpha b\beta_F + 2\right)}$. We can find $\varphi_1 - \frac{2\beta_F - 2\beta_R}{3\beta_R} = \frac{4(\beta_F - \beta_R)\left(-(\alpha b\beta_F + 4)A + (\alpha b\beta_F)^2 - \alpha b\beta_F + 4\right)}{\beta_R(6(\alpha b\beta_F)^2 - 6(\alpha b\beta_F + 1)A + 3\alpha b\beta_F + 6)} > 0$. Since $\Delta N(\varphi_1) = 0$ and $\frac{\partial \Delta N}{\partial \varphi} > 0$, it mathematically follows that $\Delta N < 0$ holds universally for all feasible $\varphi < \frac{2\beta_F - 2\beta_R}{3\beta_R}$. Thus, we can conclude that $\Delta N < \Delta N_{\varphi = \frac{2\beta_F - 2\beta_R}{3\beta_R}} < 0$. That is, when $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$, $\Delta N < 0$ always holds ($N_F^* < N_R^*$). Therefore, the client base under the R-Contract consistently exceeds that under the F-Contract when $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$. Please refer to Figure A3(b) for the scenario when $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$.

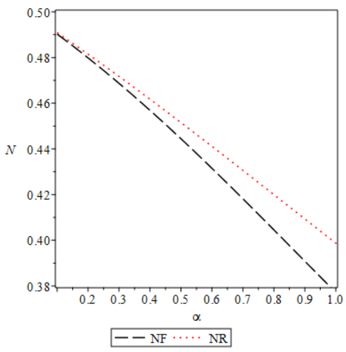
We now turn to the scenario where $\beta_F < \beta_R$, $N_R^* < N_F^*$ holds (Scenario FR-1). At this point, the threshold value α_N does not exist in the positive domain. When α tends to 0, we find that $\lim_{\alpha \to 0^+} \Delta N = N_F^* - N_R^* = 0$ and $\lim_{\alpha \to 0^+} \frac{\partial \Delta N}{\partial \alpha} = -\frac{b(2\beta_F - 3\varphi\beta_R - 2\beta_R)}{16} > 0$. Furthermore, as α tends to infinity, $\Delta N = \frac{\beta_R - \beta_F}{2\alpha b \beta_F \beta_R} + o(\frac{1}{\alpha^2}) > 0$ since $\beta_F < \beta_R$. Moreover, we have proven that both N_F^* and N_R^* are monotonically decreasing in α, and that the equation $N_F^* = N_R^*$ has no real roots in the domain $\alpha > 0$. Integrating these findings, we conclude that $N_R^* < N_F^*$ holds when $\beta_F < \beta_R$. Please refer to Figure A3(c) for the scenario when $\beta_F < \beta_R$.

In conclusion, the sizes of the equilibrium client bases under the F-Contract and R-Contract satisfy: (a) when $\beta_F < \beta_R$, $N_R^* < N_F^*$ holds (Scenario FR-1); (b) when $\beta_R < \beta_F < \left(\frac{3\varphi}{2} + 1\right)\beta_R$, $\begin{cases} \alpha > \alpha_N, N_R^* > N_F^* & \text{(Scenario FR} - 3) \\ \alpha \le \alpha_N, N_R^* \le N_F^* & \text{(Scenario FR} - 2) \end{cases}$ holds; and (c) when $\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$, $N_R^* > N_F^*$ holds (Scenario FR-4).

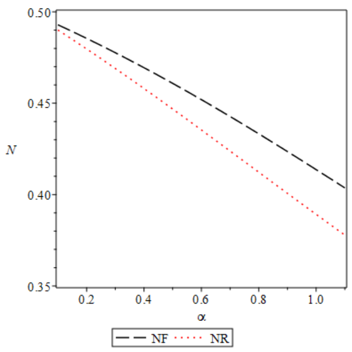
Here is the reason. While Proposition 2(a) establishes that the F-Contract holds a coverage advantage ($N_F^* \ge N_R^*$) when $\beta_F = \beta_R = 1$, incorporating heterogeneous client negligence introduces a critical trade-off. Specifically, when clients under the F-Contract exhibit higher negligence ($\beta_F > \beta_R$), compared to the R-Contract, the F-Contract faces an intensified externality-driven contraction effect due to the higher overall breach probability ($q = bN\beta$). Consequently, the R-Contract surpasses the F-Contract's coverage ($N_R^* > N_F^*$) either (i) when the F-Contract client base is sufficiently negligent ($\beta_F > (\frac{3\varphi}{2} + 1)\beta_R$), compelling the MSSP to restrict F-Contract's coverage to mitigate this sharply amplified breach probability, or (ii) when it is moderately negligent ($\beta_R < \beta_F < (\frac{3\varphi}{2} + 1)\beta_R$) but the resulting mild externality-driven contraction effect is intensified by high security loss risks ($\alpha > \alpha_N$). Under all other conditions ($\beta_F < \beta_R$, or when $\beta_R < \beta_F \le \left(\frac{3\varphi}{2} + 1\right)\beta_R$ and $\alpha < \alpha_N$ both hold), the F-Contract still maintains its dominance ($N_F^* > N_R^*$).



(a) $\beta_R < \beta_F \le \left(\frac{3\varphi}{2} + 1\right)\beta_R$,



(b) $\beta_F > \left(\frac{3\varphi}{2} + 1\right)\beta_R > \beta_R$,



(c) $\beta_F < \beta_R$,
$(\beta_F = 1.1, \beta_R = 1.2)$

$(\beta_F = 1.3, \beta_R = 1.1)$ $(\beta_F = 1.5, \beta_R = 1.1)$

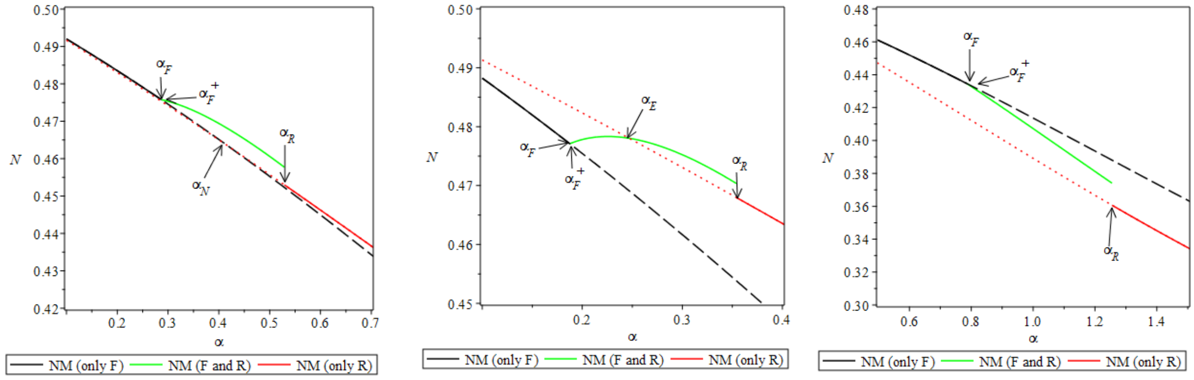
$d = 0.1, \varphi = 0.2, b = 0.5$

Figure A3. Comparison between N_F^* and N_R^*.

Next, building on the preceding pairwise analyses, we now present the client base rankings across all three contracts within the interval $\alpha \in (\alpha_F, \alpha_F^+)$. Similarly, we begin by analyzing the scenario where $\beta_F \geq \beta_R$. According to Lemma 3(a), at the boundary $\alpha = \alpha_F$, the M-Contract converges to the F-Contract, yielding $N_M^*|_{\alpha=\alpha_F} = N_F^*|_{\alpha=\alpha_F}$. Evaluating the difference in their first-order derivatives with respect to α at this boundary, we obtain $\frac{\partial N_M^*}{\partial \alpha}|_{\alpha=\alpha_F} - \frac{\partial N_F^*}{\partial \alpha}|_{\alpha=\alpha_F} = \frac{(\beta_F+\varphi\beta_R-\beta_R)b(\beta_F-\beta_R)}{2\sqrt{X}} > 0$, where $X = b^2{\beta_F}^4d^2 - b{\beta_F}^3d + \left(1 - b\beta_R d(\varphi - 1)\right){\beta_F}^2 + 2\beta_F\beta_R(\varphi - 1) + (\varphi - 1)^2{\beta_R}^2$. This initial faster growth of N_M^* guarantees $N_M^* > N_F^*$ on the interval $\alpha \in (\alpha_F, \alpha_F^+)$. The reason is as follows. As α exceeds α_F, a re-segmentation process is triggered under the menu-based regime, prompting high-valuation clients to migrate to the R-Contract. This structural shift alters the network's security level, creating two distinct trajectories based on the relative level of client negligence, as stated in Proposition 1. If clients become less negligent upon upgrading ($\beta_F > \beta_R$), the shift triggers the segmentation-driven expansion effect. This improved network security encourages the MSSP to expand its market coverage, yielding $N_M^* > N_F^*$. Conversely, if upgrading clients are more negligent ($\beta_F < \beta_R$), the migration triggers the segmentation-driven contraction effect. To mitigate the overall breach probability, the MSSP must shrink its client base, resulting in $N_F^* > N_M^*$.

We now prove that, for $\alpha \in (\alpha_F, \alpha_F^+)$, the M-Contract yields the largest client base when $\beta_R < \beta_F < min\{\beta_2, (\frac{3\varphi}{2} + 1)\beta_R\}$; that is, the ranking $N_M^* > N_F^* > N_R^*$ holds, as depicted in Figure A4(a). According to the previous proof, the F-Contract serves more clients than R-Contract ($N_F^* > N_R^*$) if and only if $\beta_F \leq (\frac{3\varphi}{2} + 1)\beta_R$ and $\alpha \in (0, \alpha_N)$ (Scenario FR-2). Therefore, to ensure that $N_M^* > N_F^* > N_R^*$ holds for $\alpha \in (\alpha_F, \alpha_F^+)$, two conditions must be satisfied simultaneously: (i) $\beta_F \leq (\frac{3\varphi}{2} + 1)\beta_R$, and (ii) $\alpha_F^+ < \alpha_N$, which is guaranteed by $\beta_F < \beta_2$. Therefore, to satisfy both conditions, it is required that $\beta_R < \beta_F < min\{\beta_2, (\frac{3\varphi}{2} + 1)\beta_R\}$. Here, we introduce a threshold value β_2, below which $\alpha_F^+ < \alpha_N$ holds, and β_2 satisfies $8bd\varphi\beta_R{\beta_2}^2(\beta_2 - \beta_R) = (\beta_2 + \varphi\beta_R - \beta_R)(2\beta_2 + \varphi\beta_R - 2\beta_R)(-2\beta_2 + 3\varphi\beta_R + 2\beta_R)$. Consequently, we conclude that under middle client negligence for the F-Contract $\beta_R <$

$\beta_F < min\{\beta_2, (\frac{3\varphi}{2}+1)\beta_R\}$, the M-Contract achieves the largest client base ($N_M^* > N_F^* > N_R^*$) when $\alpha \in (\alpha_F, \alpha_F^+)$, corresponding to Proposition 2(b) (which is also Proposition EC.1(b)).



(a) $N_M^* \geq max\{N_F^*, N_R^*\}$ when $\beta_R < \beta_F < min\{\beta_2, (\frac{3\varphi}{2}+1)\beta_R\}, \beta_F = 1.24, \beta_R = 1.01$

(b) $N_R^* \geq max\{N_F^*, N_M^*\}$ when $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$, $\beta_F = 1.8, \beta_R = 1.05$

(c) $N_F^* \geq max\{N_M^*, N_R^*\}$ when $\beta_F < \beta_R$, $\beta_F = 1.1, \beta_R = 1.2$

$d = 0.1, \varphi = 0.2, b = 0.5$

Figure A4. Comparison of three contracts under low and high β_F.

We next proceed to interpret the phenomena under high client negligence for the F-Contract ($\beta_F > (\frac{3\varphi}{2}+1)\beta_R$), as depicted in Figure A4(b). Since N_R^* continuously decreases with α, while N_M^* first increases and then decreases (as proved in Proposition 1), solving $N_M^* = N_R^*$ yields two intersection points, α_E and α_L. Let α_E and α_L denote the left and right intersection points, respectively, such that $\alpha_E < \alpha_L$. Under $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$, we have established that $N_R^* > N_F^*$ (Scenario FR-4). Moreover, $N_M^* = N_F^*$ at $\alpha = \alpha_F$ (as shown in Lemma 3(a)) and $N_M^* > N_F^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$. Since N_M^* originates at N_F^* (which is below N_R^*), the first intersection point α_E where it overtakes N_R^* must be greater than α_F (i.e., $\alpha_F < \alpha_E$). This confirms that $N_R^* > N_M^* > N_F^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$, corresponding to Proposition EC.1(c). Furthermore, we can infer that when α exceeds the threshold α_E ($\alpha \in (\alpha_E, \alpha_E^+)$), the continuously increasing N_M^* overtakes N_R^*. As a result, there exists a threshold $\alpha_E \in (\alpha_F, \alpha_R)$ beyond which the M-Contract captures a larger client base than the pure contracts ($N_M^* > N_R^* > N_F^*$) for $\alpha \in (\alpha_E, \alpha_E^+)$ if $\beta_F > (\frac{3\varphi}{2}+1)\beta_R$, as stated in the last result of Proposition EC.1.

In greater detail, comparing the relative magnitudes of α_E, α_L, and α_R yields a precise scenario classification: (1) If $\alpha_E < \alpha_L < \alpha_R$, the menu-based contract N_M^* achieves the maximum client base within the interval (α_E, α_L). (2) If $\alpha_E < \alpha_R < \alpha_L$, N_M^* achieves the maximum client base within the interval (α_E, α_R), as shown in Figure A4(b). (3) If $\alpha_E > \alpha_R$, then within the range (α_F, α_R), we have $N_M^* < N_R^*$. In this case, the client base under M-Contract is consistently smaller than that under R-Contract. Synthesizing scenarios (1) and (2), provided the threshold $\alpha_E \in (\alpha_F, \alpha_R)$, the M-Contract achieves a larger total client base than either pure contract when $\alpha \in (\alpha_E, min\{\alpha_L, \alpha_R\})$. This region lies beyond the analytical boundaries established in Proposition 2(b) ($\alpha \in (\alpha_F, \alpha_F^+)$), and our analysis reveal that the M-Contract also maintains its dominance here. In summary, the preceding analysis demonstrates that under $\beta_F > \beta_R$, regardless of whether the negligence gap between the F-Contract and R-Contract is moderate or large, the menu-based regime can achieve a larger client base than either pure contract under moderate security loss risks ($N_M^* \geq max\{N_F^*, N_R^*\}$). This sharply contrasts with the baseline established in Proposition 2(a), where the pure F-Contract guarantees the maximum coverage ($\widehat{N}_F^* \geq max\{\widehat{N}_M^*, \widehat{N}_R^*\}$) without either externalities or negligence differences.

Finally, we focus on the case where $\beta_F < \beta_R$, as shown in Figure A4(c). Under this condition, at the boundary $\alpha = \alpha_F$, we have $N_M^*|_{\alpha=\alpha_F} = N_F^*|_{\alpha=\alpha_F}$ and $\frac{\partial N_M^*}{\partial \alpha}|_{\alpha=\alpha_F} - \frac{\partial N_F^*}{\partial \alpha}|_{\alpha=\alpha_F} = \frac{(\beta_F+\varphi\beta_R-\beta_R)b(\beta_F-\beta_R)}{2\sqrt{X}} < 0$. This implies that for $\alpha \in (\alpha_F, \alpha_F^+)$, $N_F^* > N_M^*$ holds. As previously established, if upgrading clients are more negligent ($\beta_F < \beta_R$), their migration triggers the segmentation-driven contraction effect. To mitigate the overall breach probability, the MSSP must proactively shrink its client base, resulting in $N_F^* > N_M^*$. Furthermore, we have established that $N_F^* > N_R^*$ always holds when $\beta_F < \beta_R$ (Scenario FR-1). Because N_M^* originates exactly at N_F^* (which is strictly above N_R^*), it follows by continuity that N_M^* remains strictly greater than N_R^* in the immediate right-neighborhood of α_F. Consequently, when $\beta_F < \beta_R$, we obtain the full ranking $N_F^* > N_M^* > N_R^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$, corresponding to Proposition EC.1(a). ■

Proof of Proposition 3

Before proving Proposition 3(a), we first establish the relationship between the total compensation T and α under the two pure contracts. From the proof of Lemma 1, we have $T_F^* = b\beta_F d{N_F}^2 = bd\beta_F(\frac{1}{3} + \frac{1-A}{3\alpha b\beta_F})^2$. Given $0 < b < 1$ and $0 < \varphi < 1$, $\frac{\partial T_F^*}{\partial \alpha} = -\frac{(\alpha b\beta_F+1-\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1})d(\alpha b\beta_F-2+2\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1})}{9\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1}\alpha^3 b\beta_F}$. As $\alpha b\beta_F + 1 > \sqrt{(\alpha b\beta_F)^2 - \alpha b\beta_F + 1}$

and $\left|2\sqrt{(\alpha b\beta_F)^2-\alpha b\beta_F+1}\right|>|\alpha b\beta_F-2|$, it is straightforward to show that $\frac{\partial T_F^*}{\partial \alpha}<0$.

We can explain these results with the help of Proposition 1. The total compensation T_F^* under the F-Contract depends on two factors, the per-client fixed compensation amount ($b\beta_F N_F d$) and the client base (N_F). Therefore, the total expected compensation under F-Contract ($T_F=b\beta_F d{N_F}^2$) increases quadratically with N_F. Because $\frac{\partial N_F^*}{\partial \alpha}<0$ (as established in Proposition 1(a)), it follows that the total compensation under F-Contract always decreases with the security loss risk ($\frac{\partial T_F^*}{\partial \alpha}<0$).

From the proof of Lemma 2, we have $T_R^*=\frac{\alpha b\varphi(2-N_R){N_R}^2}{2}=\frac{\varphi C^2(6\alpha b\beta_R(2-\varphi)+C)}{54(\alpha b\beta_R)^2(2-\varphi)^3}$ and $\frac{\partial T_R^*}{\partial \alpha}=\frac{(4\varphi B-8\alpha b\beta_R\varphi-1)\begin{bmatrix}(16-\alpha^2b^2{\beta_R}^2(12\varphi-16)+15\alpha b\beta_R\varphi-22\alpha b\beta_R)B-32\\+\alpha^3b^3{\beta_R}^3(24\varphi-32)+6\alpha^2b^2{\beta_R}^2(\varphi-2)-6\alpha b\beta_R(9\varphi-10)\end{bmatrix}}{27B\alpha^3b^2{\beta_R}^2(\varphi-2)^3}$. Solving $\frac{\partial T_R^*}{\partial \alpha}=0$, we can obtain a unique critical point $\alpha_T=\frac{1}{b}(\frac{X2}{36\varphi-48}+\frac{75\varphi+26}{12X2}+\frac{5}{6})$, where $X2=((-375\varphi-202+27\sqrt{-\frac{2(375\varphi^2+436\varphi+124)}{3\varphi-4}})(3\varphi-4)^2)^{\frac{1}{3}}$. As $\alpha\to 0^+$, $\lim_{\alpha\to 0^+}\frac{\partial T_R^*}{\partial \alpha}=\frac{3b\varphi\beta_R}{16}>0$. Conversely, as $\alpha\to+\infty$, the expansion yields $\frac{\partial T_R^*}{\partial \alpha}=-\frac{\varphi}{4b\beta_R\alpha^2}+o\left(\frac{1}{\alpha^3}\right)<0$.

Moreover, $\frac{\partial^2 T_R^*}{\partial \alpha^2}=-\frac{\varphi}{9B\alpha^4b^2{\beta_R}^2(\varphi-2)^3\left(1+\alpha^2b^2{\beta_R}^2+\alpha b\beta_R\left(\frac{3\varphi}{2}-1\right)\right)}\Big[\big(32+\alpha b\beta_R(28\varphi-24)\big)(1+\alpha^2b^2{\beta_R}^2+\alpha b{\beta_R}^2\left(\frac{3\varphi}{2}-1\right)B-64+(9\varphi^3-54\varphi^2+52\varphi-24)\alpha^4b^4{\beta_R}^4-\frac{1}{4}(135\varphi^3-306\varphi^2-852\varphi-536)\alpha^3b^3{\beta_R}^3-(180\varphi^2-264\varphi+192)\alpha^2b^2{\beta_R}^2+\alpha b\beta_R(-200\varphi+144)\Big]<0$ ensures that the second derivative is negative. Consequently, the first derivative is monotonically decreasing and admits a unique root α_T. Given $\lim_{\alpha\to 0^+}\frac{\partial T_R^*}{\partial \alpha}>0$ and $\lim_{\alpha\to+\infty}\frac{\partial T_R^*}{\partial \alpha}<0$, we have $\frac{\partial T_R^*}{\partial \alpha}>0$ for $\alpha<\alpha_T$, whereas $\frac{\partial T_R^*}{\partial \alpha}<0$ for $\alpha>\alpha_T$, as shown in Figure A5 ($d=0.1,\varphi=0.2,\beta_F=1.23,\beta_R=1.01,b=0.5$).

To analyze how α affects total compensation $T_R^*=\frac{\alpha b\varphi(2-N_R){N_R}^2}{2}$ in detail, we apply the chain rule: $\frac{\partial T_R^*}{\partial \alpha}=\frac{\partial T_R}{\partial \alpha}+\frac{\partial T_R}{\partial N_R}\frac{\partial N_R^*}{\partial \alpha}=\frac{b\beta_R\varphi(2-N_R){N_R}^2}{2}+\frac{b\beta_R\varphi(4-3N_R)N_R}{2}\frac{\partial N_R^*}{\partial \alpha}$. The first term on the RHS of this equation captures the direct effect of rising α, which increases the per-client compensation. The second term captures the indirect effect via changes in N_R, where elevated security loss risk reduces the client base ($\frac{\partial N_R^*}{\partial \alpha} < 0$), thereby dampening the total compensation. Therefore, when the security loss ratio is relatively low ($\alpha \leq \alpha_T$), the client base remains sizable and the direct effect dominates, leading to an increase in total compensation. In contrast, when α becomes high ($\alpha > \alpha_T$), the client base contracts sharply, and the indirect effect dominates, resulting in a decrease in total compensation.

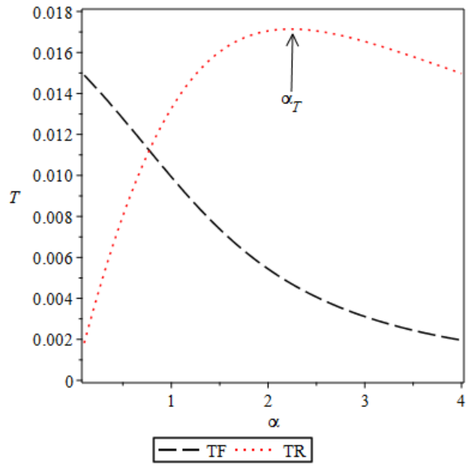


Figure A5. Variation of T_F^* and T_R^* with α.

We now prove Proposition 3(a) by assessing the impact of security loss risk α on the MSSP's profits π_F^* and π_R^*. Under the F-Contract, $\frac{\partial \pi_F^*}{\partial \alpha} = \frac{(-2\alpha^3b^3{\beta_F}^3-3\alpha b\beta_F+4)A+2\alpha^4b^4{\beta_F}^4-\alpha^3b^3{\beta_F}^3-3\alpha^2b^2{\beta_F}^2+5\alpha b\beta_F-4}{27\alpha^3b^2{\beta_F}^2}$. Let $M = ((-2\alpha^3b^3{\beta_F}^3 - 3\alpha b\beta_F + 4)A)^2 - (2\alpha^4b^4{\beta_F}^4 - \alpha^3b^3{\beta_F}^3 - 3\alpha^2b^2{\beta_F}^2 + 5\alpha b\beta_F - 4)^2 = 27\alpha^3b^3{\beta_F}^3(\alpha b\beta_F - 1)A^2$, where $A = \sqrt{(\alpha b\beta_F)^2 - \alpha b\beta_F + 1}$. On the one hand, if $\alpha b\beta_F - 1 < 0$, we have $M < 0$. Thus, $|2\alpha^4b^4{\beta_F}^4 - \alpha^3b^3{\beta_F}^3 - 3\alpha^2b^2{\beta_F}^2 + 5\alpha b\beta_F - 4| > |(-2\alpha^3b^3{\beta_F}^3 - 3\alpha b\beta_F + 4)A|$. We find that $2\alpha^4b^4{\beta_F}^4 - \alpha^3b^3{\beta_F}^3 - 3\alpha^2b^2{\beta_F}^2 + 5\alpha b\beta_F - 4$ remains consistently negative over the range of $\alpha b\beta_F \in (0, 1)$, thus $\frac{\partial \pi_F^*}{\partial \alpha} < 0$. On the other hand, if $\alpha b\beta_F - 1 > 0$, we have $M > 0$. Thus, $|(-2\alpha^3b^3{\beta_F}^3 - 3\alpha b\beta_F + 4)A| > |2\alpha^4b^4{\beta_F}^4 - \alpha^3b^3{\beta_F}^3 - 3\alpha^2b^2{\beta_F}^2 + 5\alpha b\beta_F - 4|$. We find that $(-2\alpha^3b^3{\beta_F}^3 - 3\alpha b\beta_F + 4)A$ remains consistently negative over the range of $\alpha b\beta_F \in (1, +\infty)$, thus $\frac{\partial \pi_F^*}{\partial \alpha} < 0$. Therefore, $\frac{\partial \pi_F^*}{\partial \alpha} < 0$ always holds. That is, the MSSP's profit under the F-Contract decreases as the security loss risk increases.

Under the R-Contract, $\frac{\partial \pi_R^*}{\partial \alpha} = \frac{(16-8\alpha^3 b^3 \beta_R{}^3+\alpha b\beta_R(18\varphi-12))B+(4+4\alpha^2 b^2 \beta_R{}^2+\alpha b\beta_R(6\varphi-4))(-8+4\alpha^2 b^2 \beta_R{}^2-\alpha b\beta_R(3\varphi-2))}{27\alpha^3 b^2 \beta_R{}^2(\varphi-2)^2 B}$. Furthermore, $\frac{\partial^2 \pi_R^*}{\partial \alpha^2} = \frac{(-4\alpha b\beta_R(3\varphi-2)-16)B+16\alpha b\beta_R(3\varphi-2)+32+\alpha^2 b^2 \beta_R{}^2(9\varphi^2-12\varphi+20)}{9\alpha^4 b^2 \beta_R{}^2(\varphi-2)^2 B} > 0$, because $((-4\alpha b\beta_R(3\varphi-2)-16)B)^2-\left(16\alpha b\beta_R(3\varphi-2)+32+\alpha^2 b^2 \beta_R{}^2(9\varphi^2-12\varphi+20)\right)^2 = -\alpha^4 b^4 \beta_R{}^4(\varphi-2)^2(\varphi+\frac{2}{3})^2 < 0$. This indicates that the first-order derivative is monotonically increasing with α. Additionally, we find that the limit of the first-order derivative at both extremes of α is always less than 0 ($\lim_{\alpha\to 0^+} \frac{\partial \pi_R^*}{\partial \alpha} = -\frac{b\beta_R(\varphi+2)}{16} < 0$, Conversely, as $\alpha \to +\infty$, the expansion yields $\frac{\partial \pi_R^*}{\partial \alpha} = -\frac{1}{4\alpha^2 b\beta_R} + o\left(\frac{1}{\alpha^3}\right) < 0$), leading to the conclusion that the first-order derivative remains consistently negative, $\frac{\partial \pi_R^*}{\partial \alpha} < 0$. Thus, the MSSP's profit under the R-Contract monotonically decreases with increasing security loss risk.

Next, we discuss Proposition 3(b) and 3(c) by comparing the MSSP's profits under F-Contract (π_F^*) and R-Contract (π_R^*). Similar with the analysis of N in Proportion EC.1, we divide it into four scenarios. Namely, (a) when $\beta_F < \beta_R$, $\pi_R^* < \pi_F^*$ holds (Scenario FR-1); (b) when $\beta_R < \beta_F < \left(\frac{\varphi}{2}+1\right)\beta_R$,$\begin{cases}\alpha > \alpha_V, \pi_R^* > \pi_F^* \ \text{(Scenario FR} - 3) \\ \alpha \le \alpha_V, \pi_R^* \le \pi_F^* \ \text{(Scenario FR} - 2)\end{cases}$ holds; and (c) when $\beta_F > (\frac{\varphi}{2}+1)\beta_R$, $\pi_R^* > \pi_F^*$ holds (Scenario FR-4).

By setting $\Delta\pi = \pi_F^* - \pi_R^* = 0$, we establish the critical threshold α_V, satisfying $\frac{(1-A+\alpha_V b\beta_F)(2\alpha_V b\beta_F+A-1)(2+A-\alpha_V b\beta_F)}{27(\alpha_V b\beta_F)^2} = \frac{C(-C(\alpha_V b\beta_R+1)-6\alpha_V b\beta_R(2-\varphi))}{27(\alpha_V b\beta_R)^2(2-\varphi)^2}$, where $A = \sqrt{(\alpha_V b\beta_F)^2-\alpha_V b\beta_F+1}$ and $C = -2\alpha_V b\beta_R + \sqrt{(2\alpha_V b\beta_R+2)^2+6\alpha_V b\beta_R(\varphi-2)} - 2$. Similar to Proposition EC.1, there exists a unique positive root $\alpha_V > 0$ if and only if the baseline negligence parameters satisfy $\beta_R < \beta_F < \left(\frac{\varphi}{2}+1\right)\beta_R$.

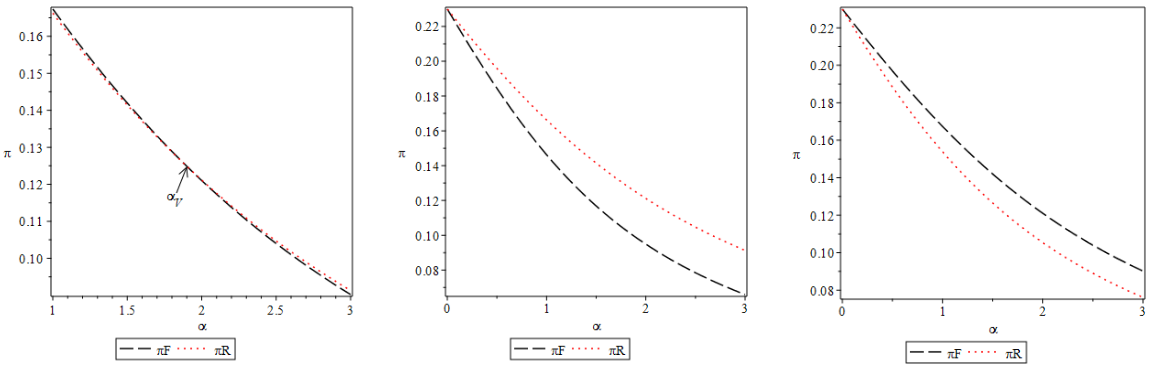
Therefore, we first examine the case where $\beta_R < \beta_F < (\frac{\varphi}{2}+1)\beta_R$ (Scenarios FR-2 and FR-3), as shown in Figure A6(a). By differentiating $\Delta\pi = \pi_F^* - \pi_R^*$ with respect to α, we obtain $\frac{\partial \Delta\pi}{\partial \alpha} =$

$\frac{1}{27\alpha^3b^2\beta_F{}^2\beta_R{}^2(\varphi-2)^2AB}\big[\big(8\alpha^3b^3\beta_F{}^3\beta_R{}^3-2\alpha^3b^3\beta_F{}^3(\varphi-2)^2\beta_R{}^2+\big(-16+\alpha b\beta_R(-18\varphi+12)\big)\beta_F{}^2-3\alpha b\beta_R{}^2(\varphi-2)^2\beta_F+4\beta_R{}^2(\varphi-2)^2-\beta_F{}^2(-16+8\alpha^2b^2\beta_R{}^2-2\alpha b\beta_R(3\varphi-2))(2+2\alpha^2b^2\beta_R{}^2+\alpha b\beta_R(3\varphi-2))\big)A+(\alpha^2b^2\beta_F{}^2-\alpha b\beta_F+1)B(2\alpha^2b^2\beta_F{}^2+\alpha b\beta_F-4)\beta_R{}^2(\varphi-2)^2\big]$.

Given the condition $\beta_R<\beta_F<(\frac{\varphi}{2}+1)\beta_R$, we have $\lim_{\alpha\to0^+}\frac{\partial\Delta\pi}{\partial\alpha}=-\frac{b(2\beta_F-\varphi\beta_R-2\beta_R)}{16}>0$ and $\lim_{\alpha\to0^+}\Delta\pi=0$. Moreover, as $\alpha\to+\infty$, $\frac{\partial\Delta\pi}{\partial\alpha}=\frac{\partial\pi_F^*}{\partial\alpha}-\frac{\partial\pi_R^*}{\partial\alpha}=\frac{\beta_F-\beta_R}{4\alpha^2b\beta_F\beta_R}+o(\frac{1}{\alpha^3})>0$ and $\lim_{\alpha\to+\infty}\Delta\pi=\pi_F^*-\pi_R^*=0^-$. Therefore, when α approaches infinity, even though $\Delta\pi$ continues to increase, it always asymptotically approaches 0 from the negative side, thus remaining less than 0. Since $\Delta\pi$ starts positive and eventually approaches zero from below, it must cross the horizontal axis. As $\alpha_V>0$ is the unique intersection point, $\Delta\pi$ transitions exactly once from positive to negative at α_V. Thus, as α increases, π_F^* is initially greater than π_R^*, and $\pi_F^*=\pi_R^*$ at α_V, then when α is sufficiently large, $\pi_R^*>\pi_F^*$, as shown in Figure A6(a). In conclusion, if $\beta_R<\beta_F<(\frac{\varphi}{2}+1)\beta_R$, we have $\pi_F^*>\pi_R^*$ when $\alpha<\alpha_V$; and $\pi_F^*<\pi_R^*$ when $\alpha>\alpha_V$.

Next, we consider the Scenario FR-4, $\beta_F>(\frac{\varphi}{2}+1)\beta_R$, as shown in Figure A6(b). Under this condition, $2\beta_F-\varphi\beta_R-2\beta_R<0$, yielding an initial slope $\lim_{\alpha\to0^+}\frac{\partial\Delta\pi}{\partial\alpha}<0$. Since $\Delta\pi$ starts by decreasing into the negative domain and no positive intersection point α_V exists, the differential remains negative globally. Thus, $\pi_F^*<\pi_R^*$ universally holds across the feasible parameter space.

Finally, we turn to the scenario where $\beta_F<\beta_R$ (Scenario FR-1), as shown in Figure A6(c). When α tends to 0, we have $\lim_{\alpha\to0^+}\frac{\partial\Delta\pi}{\partial\alpha}=\lim_{\alpha\to0^+}(\frac{\partial\pi_F^*}{\partial\alpha}-\frac{\partial\pi_R^*}{\partial\alpha})=-\frac{b(2\beta_F-\varphi\beta_R-2\beta_R)}{16}>0$ and $\lim_{\alpha\to0^+}\Delta\pi=\pi_F^*-\pi_R^*=0$. When $\alpha\to+\infty$, $\Delta\pi=\frac{\beta_R-\beta_F}{4\alpha b\beta_F\beta_R}+o(\frac{1}{\alpha^2})>0$, meaning $\Delta\pi$ asymptotically approaches zero from above. Moreover, from Proposition 3(a), we have proven that both π_F^* and π_R^* are monotonically decreasing in α. Because $\Delta\pi$ originates with a positive trajectory, approaches zero from above at infinity, and possesses no real roots in $(0,+\infty)$, it is positive across the parameter space. Integrating these findings, we conclude that $\pi_F^*>\pi_R^*$ holds when $\beta_F<\beta_R$. ■



(a) $\beta_R < \beta_F < \left(\frac{\varphi}{2}+1\right)\beta_R$, ($\beta_R = 1.05, \beta_F = 1.1$) (b) $\beta_F > \left(\frac{\varphi}{2}+1\right)\beta_R$, ($\beta_R = 1.05, \beta_F = 1.55$) (c) $\beta_F < \beta_R$, ($\beta_R = 1.3, \beta_F = 1.1$)

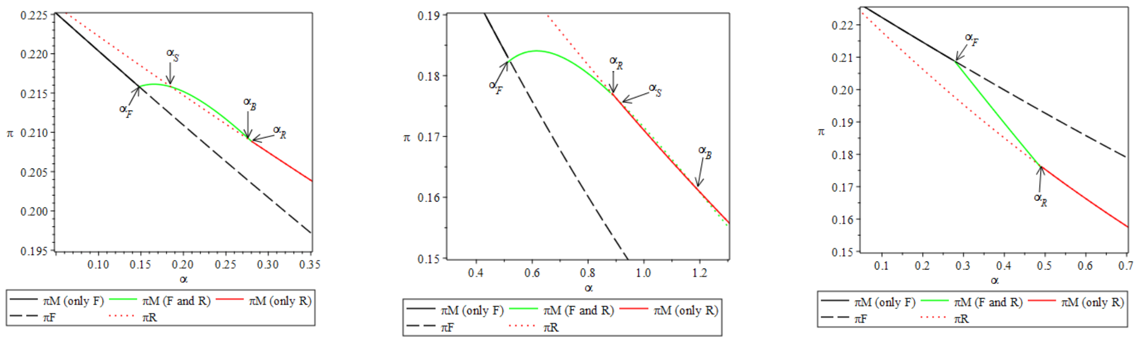
$d = 0.1, \varphi = 0.2, b = 0.5$

Figure A6. Comparison between π_F^* and π_R^*.

Proof of Proposition 4

Now we compare the profitability of the three distinct regimes: F-Contract, R-Contract, and M-Contract. Before formally discussing Proposition 4, which addresses the condition $\beta_F < (\frac{\varphi}{2}+1)\beta_R$, we first proceed to interpret the phenomena under high client negligence for the F-Contract ($\beta_F > (\frac{\varphi}{2}+1)\beta_R$). Under this condition, the inequality $\pi_R^* > \pi_F^*$ holds consistently (Proposition 3(c)), ensuring the F-Contract is never optimal here. Therefore, our analysis focuses exclusively on π_M^* and π_R^*. As shown in Figure A7(a), the profit functions π_R^* and π_M^* exhibit two intersection points, denoted as α_S and α_B. Let α_S and α_B denote the left and right intersection points, respectively, such that $\alpha_S < \alpha_B$. Since $\pi_R^* > \pi_F^*$ holds under $\beta_F > (\frac{\varphi}{2}+1)\beta_R$, and $\pi_F^* = \pi_M^*$ precisely at the boundary $\alpha = \alpha_F$ (Lemma 3(a)), it follows that $\pi_R^* > \pi_M^*$ at α_F. Consequently, the first intersection point α_S where the M-Contract overtakes the R-Contract must lie strictly to the right of α_F; that is $\alpha_F < \alpha_S$.

More specifically, we compare α_S, α_B, and α_R to obtain more detailed scenarios: (1) if $\alpha_S < \alpha_B < \alpha_R$, the M-Contract is the most profitable within the interval (α_S, α_B), as shown in Figure A7(a); (2) if $\alpha_S < \alpha_R < \alpha_B$, the M-Contract is the most profitable within the interval (α_S, α_R); (3) if $\alpha_S > \alpha_R$, then within the entire feasible range (α_F, α_R), we have $\pi_M^* < \pi_R^*$. In this scenario, the profit under M-Contract is consistently smaller than that under R-Contract, as shown in Figure A7(b). Synthesizing scenarios (1) and (2), provided the threshold $\alpha_S \in (\alpha_F, \alpha_R)$, the M-Contract generates a higher profit than the pure contracts when $\alpha \in (\alpha_S, min\{\alpha_B, \alpha_R\})$.



(a) when $\alpha_F < \alpha_S < \alpha_B < \alpha_R$ (b) when $\alpha_S > \alpha_R$ (c) when $\beta_F < \beta_R - \varphi(\beta_R - 1) < \beta_R$

Figure A7. Comparison between π_M^*, π_R^*, and π_F^*.

Next, we discuss Proposition 4, encompassing the condition $\beta_F < (\frac{\varphi}{2} + 1)\beta_R$. We begin by analyzing the scenario where $\beta_R < \beta_F < (\frac{\varphi}{2} + 1)\beta_R$ (Proposition 4 (a)). We first prove that $\pi_M^* > \pi_F^*$ holds for a local interval $\alpha \in (\alpha_F, \alpha_F^+)$. According to Lemma 3(a), at the boundary $\alpha = \alpha_F$, $\pi_M^* = \pi_F^*$ holds. Evaluating the marginal profit differential at this boundary yields $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha} = \frac{(\beta_F - \varphi + \beta_R(\varphi-1))(bd\beta_F{}^2 + \beta_F + \varphi\beta_R - \beta_R - \sqrt{Z})}{3d\beta_F{}^2}$, where $Z = b^2\beta_F{}^4 d^2 - bd\beta_F{}^3 + \left(1 - b\beta_R d(\varphi - 1)\right)\beta_F{}^2 + 2\beta_F\beta_R(\varphi - 1) + \beta_R{}^2(\varphi - 1)^2$. Since $\beta_F > \beta_R$ ensures that $(bd\beta_F{}^2 + \beta_F + \varphi\beta_R - \beta_R)^2 - Z = 3bd\beta_F{}^2(\beta_F + \varphi\beta_R - \beta_R) > 0$, the derivative difference is positive, i.e., $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} > \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha}$. This initial faster growth of π_M^* guarantees $\pi_M^* > \pi_F^*$ on the interval $\alpha \in (\alpha_F, \alpha_F^+)$.

We next prove that for $\beta_R < \beta_F < min\{\beta_3, (\frac{\varphi}{2} + 1)\beta_R\}$, the profit ranking $\pi_M^* > \pi_F^* > \pi_R^*$ holds for $\alpha \in (\alpha_F, \alpha_F^+)$. We introduce a threshold value β_3, below which $\alpha_F^+ < \alpha_V$ holds, where β_3 satisfies $\frac{(\sqrt{2S} + (b\beta_3 d + \varphi - 1)\beta_R + \beta_3)(2((b\beta_3 d + \varphi - 1)\beta_R + \beta_3) - \sqrt{2S})^2}{27b^2d^2\beta_R{}^2\beta_3{}^2(\beta_3 + \varphi\beta_R - \beta_R)(\varphi-2)^2} = \frac{(\beta_3 + \varphi\beta_R - \beta_R + b\beta_3{}^2 d - \sqrt{W})(\beta_R - \beta_3 - \varphi\beta_R + 2b\beta_3{}^2 d + \sqrt{W})(2(\beta_3 + \varphi\beta_R - \beta_R) - b\beta_3{}^2 d + \sqrt{W})}{27b^2d^2\beta_3{}^4(\beta_3 + \varphi\beta_R - \beta_R)}$, $S = \left(2b^2d^2\beta_R{}^2 + 3bd\varphi\beta_R - 2bd\beta_R + 2\right)\beta_3{}^2 + \left(3bd\varphi\beta_R{}^2 - 2bd\beta_R{}^2 + 4\beta_R\right)(\varphi - 1)\beta_3 + 2(\varphi - 1)^2\beta_R{}^2$ and $W = b^2\beta_3{}^4 d^2 - bd\beta_3{}^3 + \left(1 - bd\beta_R(\varphi - 1)\right)\beta_3{}^2 + 2\beta_3\beta_R(\varphi - 1) + \beta_R{}^2(\varphi - 1)^2$. According to Proposition 3(b), the F-Contract achieves a larger profitability than the R-Contract ($\pi_F^* > \pi_R^*$) if and only if $\beta_R < \beta_F \le (\frac{\varphi}{2} + 1)\beta_R$ and $\alpha \in (0, \alpha_V)$. Therefore, ensuring the full ranking $\pi_M^* > \pi_F^* > \pi_R^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$ necessitates two simultaneous conditions: (i) $\beta_R < \beta_F \le (\frac{\varphi}{2} + 1)\beta_R$; and (ii) $\alpha_F^+ <$

α_V, which is guaranteed by $\beta_F < \beta_3$. That is, $\beta_R < \beta_F < min\{\beta_3, \left(\frac{\varphi}{2}+1\right)\beta_R\}$. Thus, under this regime, the M-Contract will achieve a larger profitability than either pure contract ($\pi_M^* > \pi_F^* > \pi_R^*$) for $\alpha \in (\alpha_F, \alpha_F^+)$.

To construct the optimal strategy profile under $\beta_R < \beta_F < min\{\beta_3, \left(\frac{\varphi}{2}+1\right)\beta_R\}$, we synthesize the boundary analyses:

(1) Optimality of the F-Contract: For $\alpha \le \alpha_F$, the M-Contract reduces to the F-Contract ($\pi_M^* = \pi_F^*$). Since $\alpha_F^+ < \alpha_V$, the entire interval $(0, \alpha_F)$ is enveloped within $(0, \alpha_V)$. Proposition 3(b) thus guarantees $\pi_F^* > \pi_R^*$. Hence, the F-Contract uniquely maximizes profit for $\alpha \in (0, \alpha_F)$.

(2) Optimality of the R-Contract: For $\alpha \ge max\{\alpha_V, \alpha_R\}$, the dynamic shifts. If $\alpha \ge \alpha_R$, the M-Contract reduces to the R-Contract ($\pi_M^* = \pi_R^*$). We then only need to compare π_F^* and π_R^*. Because $\alpha \ge \alpha_V$, Proposition 3(b) dictates that $\pi_R^* > \pi_F^*$. Thus, taking the maximum of the two thresholds ensures that $\pi_R^* = \pi_M^* > \pi_F^*$ universally holds for $\alpha \ge max\{\alpha_V, \alpha_R\}$.

Thus, when client negligence under the F-Contract is moderate $(\beta_R < \beta_F < min\{\beta_3, (\frac{\varphi}{2}+1)\beta_R\})$, the optimal contract dynamically shifts as risk increases: the F-Contract is optimal for $\alpha \in (0, \alpha_F)$; the M-Contract is optimal for $\alpha \in (\alpha_F, \alpha_F^+)$; and the R-Contract is optimal for $\alpha \ge max\{\alpha_V, \alpha_R\}$, as listed in Proposition 4(a).

Finally, we focus on the case of $\beta_F < \beta_R$ (Proposition 4(b)), where the client negligence is lower under F-Contract. According to Proposition 3(b), the inequality $\pi_F^* > \pi_R^*$ holds for $\beta_F < \beta_R$. Hence, the R-Contract can be excluded from consideration, allowing us to focusing exclusively on comparing π_F^* and π_M^*. Under this condition, at the boundary $\alpha = \alpha_F$, we have $\pi_M^* = \pi_F^*$ and $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha} = \frac{(\beta_F-\varphi+\beta_R(\varphi-1))\left(bd{\beta_F}^2+\beta_F+\varphi\beta_R-\beta_R-\sqrt{Z}\right)}{3d{\beta_F}^2}$. Since $(bd{\beta_F}^2 + \beta_F + \varphi\beta_R - \beta_R)^2 - Z > 0$, $\left(bd{\beta_F}^2 + \beta_F + \varphi\beta_R - \beta_R - \sqrt{Z}\right) > 0$. Consequently, the sign of $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha}$ depends on the expression $(\beta_F - \varphi + \beta_R(\varphi-1))$. If $(\beta_F - \varphi + \beta_R(\varphi-1)) > 0$, i.e., $\beta_R - \varphi(\beta_R - 1) < \beta_F < \beta_R$, then $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha} > 0$ and the profits satisfy $\pi_M^* > \pi_F^* > \pi_R^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$. If $(\beta_F - \varphi + \beta_R(\varphi-1)) < 0$, i.e., $\beta_F < \beta_R - \varphi(\beta_R - 1) < \beta_R$, then $\frac{\partial(\pi_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(\pi_F^*|\alpha=\alpha_F)}{\partial\alpha} <$

0 and the profits satisfy $\pi_F^* > \pi_M^* > \pi_R^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$, as shown in Figure A7(c). In addition, within the range $\alpha \in (0, \alpha_F)$, M-Contract reduces to F-Contract, making the F-Contract optimal, $\pi_F^* = \pi_M^* > \pi_R^*$ for $\alpha \in (0, \alpha_F)$.

Thus, when $\beta_R - \varphi(\beta_R - 1) < \beta_F < \beta_R$, the F-Contract is optimal for $\alpha \in (0, \alpha_F)$, while the M-Contract becomes optimal for $\alpha \in (\alpha_F, \alpha_F^+)$. The R-Contract is never optimal. Conversely, if β_F is sufficiently small ($\beta_F < \beta_R - \varphi(\beta_R - 1) < \beta_R$), the F-Contract is always optimal. ■

Proof of Lemma 4: Social Welfare and Consumer Surplus Analysis

In the preceding sections, we demonstrated the potential optimality of the M-Contract from the perspectives of the MSSP. We now extend our analysis to evaluate the contract performance through the lens of both social welfare and consumer surplus, exploring whether the adoption of the M-Contract is also socially optimal. Since clients are heterogeneous in their valuations, the aggregate consumer surplus (CS) is computed as the integral of individual expected utilities across the participating client base. Consequently, total social welfare (SW) is defined as the sum of the MSSP's expected profit and the aggregate consumer surplus.

Therefore, under the F-Contract, the consumer surplus and social welfare are formally given by:

$$CS_F = \int_{\theta_F}^{1} [\theta - p_F - b\beta_F N_F(\alpha\theta - d)]\, d\theta \tag{A10}$$

$$SW_F = CS_F + \pi_F \tag{A11}$$

Similarly, under the R-Contract, they become

$$CS_R = \int_{\theta_R}^{1} [\theta - p_R - b\beta_R N_R(1-\varphi)\alpha\theta]\, d\theta \tag{A12}$$

$$SW_R = CS_R + \pi_R \tag{A13}$$

Under the M-Contract, the client base is segmented. Thus, the total consumer surplus is divided into two distinct components, representing the middle-valuation and high-valuation sub-markets, respectively:

$$CS_{MF} = \int_{\theta_{MF}}^{\theta_{MR}} [\theta - p_M - b\beta_F N_M(\alpha\theta - d)]\, d\theta \tag{A14}$$

$$CS_{MR} = \int_{\theta_{MR}}^{1} [\theta - p_M - b\beta_R N_M(1-\varphi)\alpha\theta]\, d\theta \tag{A15}$$

$$SW_M = CS_{MF} + CS_{MR} + \pi_M \tag{A16}$$

Based on the client base equilibrium solutions N_F^*, N_R^*, and N_M^* derived from the preceding three lemmas, we can calculate consumer surplus. Combining these with the MSSP's profit, we obtain the equilibrium social welfare under each contract, as summarized in Lemma 4.

Lemma 4. (a) *When an MSSP offers only the F-Contract, at equilibrium, the consumer surplus is* $CS_F^* = \frac{(1-A+\alpha b\beta_F)^2(2+A-\alpha b\beta_F)}{54(\alpha b\beta_F)^2}$ *and the social welfare is* $SW_F^* = \frac{(1-A+\alpha b\beta_F)(5\alpha b\beta_F+A-1)(2+A-\alpha b\beta_F)}{54(\alpha b\beta_F)^2} - \Gamma(s)$.

(b) *When the MSSP offers only the R-Contract, at equilibrium, the consumer surplus is* $CS_R^* = \frac{(-2+B-2\alpha b\beta_R)^2((1-\varphi)(B-2\alpha b\beta_R)+4-\varphi)}{54(\alpha b\beta_R)^2(\varphi-2)^3}$ *and the social welfare is* $SW_R^* = \frac{(-2+B-2\alpha b\beta_R)(-6\alpha b\beta_R(\varphi-\frac{5}{3})+B-2)(4+B-2\alpha b\beta_R-3\varphi)}{54(\alpha b\beta_R)^2(\varphi-2)^3} - \Gamma(s)$.

(c) *When the MSSP offers the M-Contract, at equilibrium, the consumer surplus is* $CS_M^* = \frac{N_M^*(\alpha b\left({N_{MF}^*}^2\beta_F+2\beta_F N_{MR}^* N_{MF}^*+{N_{MR}^*}^2(2\beta_F+\beta_R(\varphi-1))-2(\beta_F+\varphi\beta_R-\beta_R)N_{MR}^*\right)-N_M^*+2bd\beta_F N_{MR}^*)}{2}$ *and the social welfare is* $SW_M^* = \frac{N_M^*(\alpha b\left({N_{MF}^*}^2\beta_F+2\beta_F(N_{MR}^*-1)N_{MF}^*+\beta_R(N_{MR}^*-2)N_{MR}^*\right)-N_M^*+2)}{2} - \Gamma(s)$, *where* $N_{MR}^* = \frac{\alpha(\beta_F+\varphi\beta_R-\beta_R)-\beta_F d}{\alpha(\beta_F+\varphi\beta_R-\beta_R)}$ *and* $N_{MF}^* = \frac{6b{\beta_F}^2 d-(4\alpha b\beta_F-2)(\beta_F+\varphi\beta_R-\beta_R)-\sqrt{Y1}}{6\alpha b\beta_F(\beta_F+\varphi\beta_R-\beta_R)}$.

Proof of Proposition EC.2: Consumer Surplus Optimization Strategy

We now compare the consumer surplus under the three contract types, and establish the following proposition.

Proposition EC.2. *From a consumer surplus perspective, when client negligence under the F-Contract is moderate ($\beta_R < \beta_F < min\{\beta_4, (\frac{\varphi}{4}+1)\beta_R\}$), the F-Contract yields the highest consumer surplus when $\alpha \le \alpha_F$; the M-Contract maximizes consumer surplus when $\alpha \in (\alpha_F, \alpha_F^+)$; and the R-Contract maximizes consumer surplus when $\alpha \ge max\{\alpha_{CS}, \alpha_R\}$.*

By taking the first partial derivatives of CS_F^* and CS_R^* with respect to α, we can obtain $\frac{\partial CS_F^*}{\partial\alpha} < 0$ and $\frac{\partial CS_R^*}{\partial\alpha} < 0$. Therefore, as the security loss risk α increases, the consumer surplus under both pure contracts decreases monotonically.

We first analyze the scenario where $\beta_F > \beta_R$. Let $\Delta CS = CS_F^* - CS_R^* =$

$\frac{(1-A+\alpha b\beta_F)^2(2+A-\alpha b\beta_F)}{54(\alpha b\beta_F)^2} - \frac{(-2+B-2\alpha b\beta_R)^2((1-\varphi)(B-2\alpha b\beta_R)+4-\varphi)}{54(\alpha b\beta_R)^2(\varphi-2)^3}$. We find that when α tends to 0, $\lim_{\alpha\to 0^+}\Delta CS = 0$ and $\lim_{\alpha\to 0^+}\frac{\partial \Delta CS}{\partial \alpha} = \frac{b(\varphi\beta_R-4\beta_F+4\beta_R)}{32}$. As α tends to infinity, $\Delta CS = \frac{{\beta_R}^2-{\beta_F}^2-\varphi{\beta_F}^2}{16(\alpha b\beta_F\beta_R)^2} + o(\frac{1}{\alpha^3}) < 0$ always holds. On the one hand, if $\varphi\beta_R - 4b\beta_F + 4b\beta_R > 0$, that is $\beta_R < \beta_F < (\frac{\varphi}{4}+1)\beta_R$, we have $\lim_{\alpha\to 0^+}\frac{\partial \Delta CS}{\partial \alpha} = \frac{b\varphi\beta_R-4b\beta_F+4b\beta_R}{32} > 0$. As α increases, CS_F^* is initially greater than CS_R^*, and $CS_F^* = CS_R^*$ at α_{CS}, then when α is sufficiently large, $CS_R^* > CS_F^*$, where α_{CS} satisfies $\frac{(1-A+\alpha_{CS} b\beta_F)^2(2+A-\alpha_{CS} b\beta_F)}{54(\alpha_{CS} b\beta_F)^2} = \frac{(-2+B-2\alpha_{CS} b\beta_R)^2((1-\varphi)(B-2\alpha_{CS} b\beta_R)+4-\varphi)}{54(\alpha_{CS} b\beta_R)^2(\varphi-2)^3}$, with $A = \sqrt{(\alpha_{CS} b\beta_F)^2 - \alpha_{CS} b\beta_F + 1}$ and $B = \sqrt{(2\alpha_{CS} b\beta_R + 2)^2 + 6\alpha_{CS} b\beta_R(\varphi-2)}$. In conclusion, if $\beta_R < \beta_F < (\frac{\varphi}{4}+1)\beta_R$, we have $CS_F^* > CS_R^*$ when $\alpha < \alpha_{CS}$; and $CS_F^* < CS_R^*$ when $\alpha > \alpha_{CS}$. On the other hand, if $b\varphi\beta_R - 4b\beta_F + 4b\beta_R < 0$, that is $\beta_F > (\frac{\varphi}{4}+1)\beta_R$, the first-order derivative satisfies $\lim_{\alpha\to 0^+}\frac{\partial \Delta CS}{\partial \alpha} < 0$. Given $\lim_{\alpha\to+\infty}\Delta CS < 0$, ΔCS remains negative globally, and no such α_{CS} exists for $\alpha > 0$. Therefore, $CS_F^* < CS_R^*$ always holds.

We next consider the M-Contract. At $\alpha = \alpha_F$, we find that $CS_F^* = CS_M^*$ and $\frac{\partial(CS_M^*|\alpha=\alpha_F)}{\partial \alpha} - \frac{\partial(CS_F^*|\alpha=\alpha_F)}{\partial \alpha} = \frac{b(\beta_R-\beta_F)(-Z_{CS}+bd{\beta_F}^2-\frac{\beta_F}{2}+\frac{(1-\varphi)\beta_R}{2})}{6Z_{CS}}$, where $Z_{CS} = \sqrt{{\beta_R}^2(\varphi-1)^2 - \beta_F\beta_R(\varphi-1)(b\beta_F d-2) + {\beta_F}^2((bd\beta_F)^2 - bd\beta_F + 1)}$. The sign of $\frac{\partial(CS_M^*|\alpha=\alpha_F)}{\partial \alpha} - \frac{\partial(CS_F^*|\alpha=\alpha_F)}{\partial \alpha}$ is determined by the signs of $(\beta_R - \beta_F)$ and $(-Z_{CS} + bd{\beta_F}^2 - \frac{\beta_F}{2} + \frac{(1-\varphi)\beta_R}{2})$. Given that $(-Z_{CS})^2 - (bd{\beta_F}^2 - \frac{\beta_F}{2} + \frac{(1-\varphi)\beta_R}{2})^2 = \frac{3(\beta_F+\varphi\beta_R-\beta_R)^2}{4} > 0$ and $\beta_F > \beta_R$, we have $\frac{\partial(CS_M^*|\alpha=\alpha_F)}{\partial \alpha} > \frac{\partial(CS_F^*|\alpha=\alpha_F)}{\partial \alpha}$ holds. This initial faster growth of CS_M^* guarantees $CS_M^* > CS_F^*$ on the interval $\alpha \in (\alpha_F, \alpha_F^+)$.

We now prove that, for $\beta_R < \beta_F < min\{\beta_4, (\frac{\varphi}{4}+1)\beta_R\}$, the ranking $CS_M^* > CS_F^* > CS_R^*$ holds for $\alpha \in (\alpha_F, \alpha_F^+)$, as shown in Proposition EC.2. Here we introduce a threshold value β_4, below which $\alpha_F^+ < \alpha_{CS}$ holds, where β_4 satisfy $\frac{((\varphi-1)\sqrt{2S}-(2b\beta_4 d-\varphi+4)(\varphi-1)\beta_R+\beta_4(\varphi-4))(2((b\beta_4 d+\varphi-1)\beta_R+2\beta_4)-\sqrt{2S})^2}{54b^2d^2{\beta_R}^2{\beta_4}^2(\beta_4+\varphi\beta_R-\beta_R)(\varphi-2)^3} =$

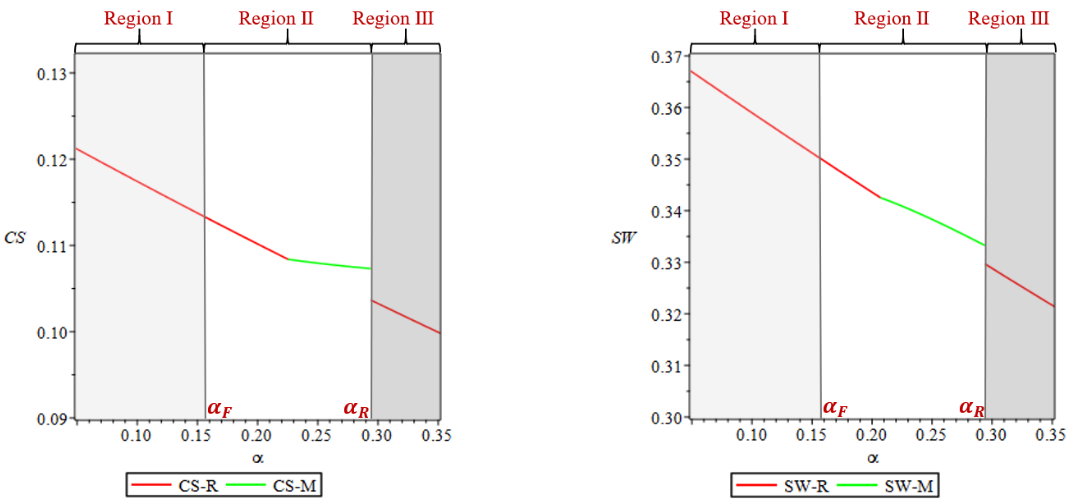
$\frac{(\beta_4+\varphi\beta_R-\beta_R+b\beta_4{}^2d+\sqrt{W})^2(2(\beta_4+\varphi\beta_R-\beta_R)-b\beta_4{}^2d-\sqrt{W})}{54b^2d^2\beta_4{}^4(\beta_4+\varphi\beta_R-\beta_R)}$, $S=\left(2b^2d^2\beta_R{}^2+3bd\varphi\beta_R-2bd\beta_R+2\right)\beta_4{}^2+\left(3bd\varphi\beta_R{}^2-2bd\beta_R{}^2+4\beta_R\right)(\varphi-1)\beta_4+2(\varphi-1)^2\beta_R{}^2$ and $W=b^2\beta_4{}^4d^2-bd\beta_4{}^3+\left(1-bd\beta_R(\varphi-1)\right)\beta_4{}^2+2\beta_4\beta_R(\varphi-1)+\beta_R{}^2(\varphi-1)^2$. As demonstrated previously, the consumer surplus under the F-Contract is higher than under the R-Contract if and only if $\beta_R<\beta_F\le(\frac{\varphi}{4}+1)\beta_R$ and $\alpha\in(0,\alpha_{CS})$. Therefore, to ensure that $CS_M^*>CS_F^*>CS_R^*$ holds for $\alpha\in(\alpha_F,\alpha_F^+)$, two conditions must be satisfied simultaneously: (i) $\beta_R<\beta_F\le(\frac{\varphi}{4}+1)\beta_R$; and (ii) $\alpha_F^+<\alpha_{CS}$, which is mathematically guaranteed by $\beta_F<\beta_4$. That is, $\beta_R<\beta_F<min\{\beta_4,(\frac{\varphi}{4}+1)\beta_R\}$. Thus, under this moderate-negligence interval, the M-Contract will yield a higher consumer surplus than either pure contract ($CS_M^*>CS_F^*>CS_R^*$) for $\alpha\in(\alpha_F,\alpha_F^+)$.

Second, we explain why the F-Contract yields the highest consumer surplus when $\alpha\le\alpha_F$, as shown in Proposition EC.2. According to Lemma 3(a), if $\alpha\le\alpha_F$, M-Contract reduces to F-Contract. As demonstrated previously, $CS_F^*>CS_R^*$ if and only if $\beta_R<\beta_F\le(\frac{\varphi}{4}+1)\beta_R$ and $\alpha\in(0,\alpha_{CS})$. Moreover, $\alpha_F^+<\alpha_{CS}$ when $\beta_F<\beta_4$. Therefore, under $\beta_R<\beta_F<min\{\beta_4,(\frac{\varphi}{4}+1)\beta_R\}$, the F-Contract yields the highest consumer surplus ($CS_F^*=CS_M^*>CS_R^*$) for $\alpha\in(0,\alpha_F)$.

Third, we analyze the optimality of the R-Contract for $\alpha\ge max\{\alpha_{CS},\alpha_R\}$. According to Lemma 3(c), the M-Contract reduces to the R-Contract if $\alpha\ge\alpha_R$. Therefore, we only need compare CS_F^* and CS_R^*. Moreover, under $\beta_R<\beta_F\le(\frac{\varphi}{4}+1)\beta_R$, the R-Contract yields the higher consumer surplus than the F-Contract ($CS_R^*>CS_F^*$) only if $\alpha>\alpha_{CS}$. Central to the analysis is a comparison between the values of α_R and α_{CS}. On the one hand, if $\alpha_R>\alpha_{CS}$, we have $CS_R^*=CS_M^*>CS_F^*$ only when $\alpha\ge\alpha_R$. On the other hand, if $\alpha_R<\alpha_{CS}$, we have $CS_R^*=CS_M^*>CS_F^*$ only when $\alpha\ge\alpha_{CS}$. Therefore, even under a tighter low-negligence regime $\beta_R<\beta_F<min\{\beta_4,\left(\frac{\varphi}{4}+1\right)\beta_R\}$, the R-Contract yields the highest consumer surplus ($CS_R^*=CS_M^*>CS_F^*$) for $\alpha\ge max\{\alpha_{CS},\alpha_R\}$, as shown in Proposition EC.2.

Moreover, when the client negligence under F-Contract is high ($\beta_F>(\frac{\varphi}{4}+1)\beta_R$), simulation analysis reveals that under moderate security loss risk conditions, the M-Contract can achieve optimal consumer surplus, as illustrated in Figure A8(a).

Finally, we focus on the case of $\beta_F < \beta_R$. When considering the M-Contract, at $\alpha = \alpha_F$, we find that $CS_F^* = CS_M^*$ and $\frac{\partial(CS_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(CS_F^*|\alpha=\alpha_F)}{\partial\alpha} = \frac{b(\beta_R-\beta_F)(-Z_{CS}+bd\beta_F{}^2-\frac{\beta_F}{2}+\frac{(1-\varphi)\beta_R}{2})}{6Z_{CS}}$. Since $(-Z_{CS})^2 - (bd\beta_F{}^2 - \frac{\beta_F}{2} + \frac{(1-\varphi)\beta_R}{2})^2 = \frac{3(\beta_F+\varphi\beta_R-\beta_R)^2}{4} > 0$, along with the condition $\beta_R > \beta_F$, it rigorously follows that $\frac{\partial(CS_M^*|\alpha=\alpha_F)}{\partial\alpha} - \frac{\partial(CS_F^*|\alpha=\alpha_F)}{\partial\alpha} < 0$. This implies that for a local interval $\alpha \in (\alpha_F, \alpha_F^+)$, $CS_F^* > CS_M^*$ holds. Therefore, under the condition of $\beta_F < \beta_R$, the M-Contract will not be optimal within $\alpha \in (\alpha_F, \alpha_F^+)$.



(a) Optimal contract for consumer surplus when $\beta_F > \left(\frac{\varphi}{4}+1\right)\beta_R$

(b) Optimal contract for social welfare when $\beta_F > \left(\frac{3\varphi}{8}+1\right)\beta_R$

$d = 0.1, \varphi = 0.5, b = 0.5, \beta_F = 1.55, \beta_R = 1.1$

Figure A8. Optimal contract strategy under high client negligence

Proof of Proposition 5

We now evaluate the social welfare under the three contracts, as established in Proposition 5. Following a methodology consistent with the analysis of consumer surplus, we first analyze the scenario where $\beta_F > \beta_R$. By taking the first partial derivatives of SW_F^* and SW_R^* with respect to α, we obtain $\frac{\partial SW_F^*}{\partial\alpha} < 0$ and $\frac{\partial SW_R^*}{\partial\alpha} < 0$. Therefore, as the security loss risk α increases, the social welfare under both contracts decreases monotonically.

Let $\Delta SW = SW_F^* - SW_R^* = \frac{(1-A+\alpha b\beta_F)(5\alpha b\beta_F+A-1)(2+A-\alpha b\beta_F)}{54(\alpha b\beta_F)^2} - \frac{(-2+B-2\alpha b\beta_R)(-6\alpha b\beta_R(\varphi-\frac{5}{3})+B-2)(4+B-2\alpha b\beta_R-3\varphi)}{54(\alpha b\beta_R)^2(\varphi-2)^3}$. We find that when α tends to 0, $\lim_{\alpha\to0^+} \Delta SW = 0$

and $\lim_{\alpha\to 0^+}\frac{\partial \Delta SW}{\partial \alpha}=\frac{b(3\varphi\beta_R-8\beta_F+8\beta_R)}{32}$. When α tends to infinity, $\Delta SW=\frac{\beta_R-\beta_F}{4b\beta_F\beta_R\alpha}+o(\frac{1}{\alpha^2})<0$ always holds. On the one hand, if $3\varphi\beta_R-8\beta_F+8\beta_R>0$, that is $\beta_F<(\frac{3\varphi}{8}+1)\beta_R$, the initial slope satisfies $\lim_{\alpha\to 0^+}\frac{\partial \Delta SW}{\partial \alpha}>0$. Therefore, as α increases, SW_F^* is initially greater than SW_R^*, and $SW_F^*=SW_R^*$ at α_{SW}, then when α is sufficiently large, $SW_R^*>SW_F^*$, where α_{SW} satisfies $\frac{(1-A+\alpha_{SW}b\beta_F)(5\alpha_{SW}b\beta_F+A-1)(2+A-\alpha_{SW}b\beta_F)}{54(\alpha_{SW}b\beta_F)^2}=\frac{(-2+B-2\alpha_{SW}b\beta_R)(-6\alpha_{SW}b\beta_R(\varphi-\frac{5}{3})+B-2)(4+B-2\alpha_{SW}b\beta_R-3\varphi)}{54(\alpha_{SW}b\beta_R)^2(\varphi-2)^3}$, $A=\sqrt{(\alpha_{SW}b\beta_F)^2-\alpha_{SW}b\beta_F+1}$ and $B=\sqrt{(2\alpha_{SW}b\beta_R+2)^2+6\alpha_{SW}b\beta_R(\varphi-2)}$. In conclusion, if $\beta_F<(\frac{3\varphi}{8}+1)\beta_R$, we have $SW_F^*>SW_R^*$ when $\alpha<\alpha_{SW}$; and $SW_F^*<SW_R^*$ when $\alpha>\alpha_{SW}$. On the other hand, if $3\varphi\beta_R-8\beta_F+8\beta_R<0$, that is, $\beta_F>(\frac{3\varphi}{8}+1)\beta_R$, the first-order derivative satisfies $\lim_{\alpha\to 0^+}\frac{\partial \Delta SW}{\partial \alpha}<0$. Moreover, $\lim_{\alpha\to+\infty}\Delta SW<0$, and no such α_{SW} exists for $\alpha>0$. Therefore, $SW_F^*<SW_R^*$ always holds.

We next consider the menu-based contract. At $\alpha=\alpha_F$, we find that $SW_F^*=SW_M^*$ and $\frac{\partial(SW_M^*|\alpha=\alpha_F)}{\partial\alpha}-\frac{\partial(SW_F^*|\alpha=\alpha_F)}{\partial\alpha}=\frac{b(\beta_R-\beta_F)}{12{\beta_F}^2dZ_{SW}}\Big(\big(-b{\beta_F}^2d-4\beta_F-4\beta_R(\varphi-1)\big)Z_{SW}+b^2d^2{\beta_F}^4-5b{\beta_F}^3d+\big(4-5bd\beta_R(\varphi-1)\big){\beta_F}^2+8\beta_F\beta_R(\varphi-1)+4{\beta_R}^2(\varphi-1)^2\Big)$, where $Z_{SW}=\sqrt{{\beta_R}^2(\varphi-1)^2-\beta_F\beta_R(\varphi-1)(b\beta_Fd-2)+{\beta_F}^2((bd\beta_F)^2-bd\beta_F+1)}$. Since $\beta_F>\beta_R$ and $(\big(-b{\beta_F}^2d-4\beta_F-4\beta_R(\varphi-1)\big)Z_{SW})^2-(b^2d^2{\beta_F}^4-5b{\beta_F}^3d+\big(4-5bd\beta_R(\varphi-1)\big){\beta_F}^2+8\beta_F\beta_R(\varphi-1)+4{\beta_R}^2(\varphi-1)^2)^2>0$, $\frac{\partial(SW_M^*|\alpha=\alpha_F)}{\partial\alpha}-\frac{\partial(SW_F^*|\alpha=\alpha_F)}{\partial\alpha}>0$ holds. This initially faster growth of SW_M^* guarantees $SW_M^*>SW_F^*$ on the local interval $\alpha\in(\alpha_F,\alpha_F^+)$.

We next prove that, for $\beta_R<\beta_F<min\{\beta_5,(\frac{3\varphi}{8}+1)\beta_R\}$, the ranking $SW_M^*>SW_F^*>SW_R^*$ holds for $\alpha\in(\alpha_F,\alpha_F^+)$. Here we introduce a threshold value β_5, below which $\alpha_F^+<\alpha_{SW}$ holds, where β_5 satisfy $\frac{(\sqrt{2S}-2b\beta_5\beta_Rd-2((\varphi-1)\beta_R+\beta_5))((\beta_5+\varphi\beta_R-\beta_R)(4-3\varphi)-2bd\beta_5\beta_R+\sqrt{2S})(\sqrt{2S}-2(\beta_5+\varphi\beta_R-\beta_R)+2bd\beta_5\beta_R(5-3\varphi))}{54b^2d^2{\beta_R}^2{\beta_5}^2(\beta_5+\varphi\beta_R-\beta_R)(\varphi-2)^3}=\frac{(\beta_5+\varphi\beta_R-\beta_R+b{\beta_5}^2d-\sqrt{W})(\beta_R-\beta_5-\varphi\beta_R+5b{\beta_5}^2d+\sqrt{W})(2(\beta_5+\varphi\beta_R-\beta_R)-b{\beta_5}^2d+\sqrt{W})}{54b^2d^2{\beta_5}^4(\beta_5+\varphi\beta_R-\beta_R)}$, $S=\big(2b^2d^2{\beta_R}^2+$

$3bd\varphi\beta_R - 2bd\beta_R + 2)\beta_5{}^2 + (3bd\varphi\beta_R{}^2 - 2bd\beta_R{}^2 + 4\beta_R)(\varphi - 1)\beta_5 + 2(\varphi - 1)^2\beta_R{}^2$ and $W = b^2\beta_5{}^4d^2 - bd\beta_5{}^3 + (1 - bd\beta_R(\varphi - 1))\beta_5{}^2 + 2\beta_5\beta_R(\varphi - 1) + \beta_R{}^2(\varphi - 1)^2$. As demonstrated previously, the social welfare under F-Contract is higher than under R-Contract if and only if $\beta_R < \beta_F \le (\frac{3\varphi}{8} + 1)\beta_R$ and $\alpha \in (0, \alpha_{SW})$. Therefore, to ensure that $SW_M^* > SW_F^* > SW_R^*$ holds for $\alpha \in (\alpha_F, \alpha_F^{+})$, two conditions must be satisfied simultaneously: (i) $\beta_R < \beta_F \le (\frac{3\varphi}{8} + 1)\beta_R$; and (ii) $\alpha_F^{+} < \alpha_{SW}$, which is guaranteed by $\beta_F < \beta_5$. That is, $\beta_R < \beta_F < min\{\beta_5, (\frac{3\varphi}{8} + 1)\beta_R\}$. Thus, under this moderate-negligence interval, the M-Contract will yield a higher social welfare than either pure contract ($SW_M^* > SW_F^* > SW_R^*$) for $\alpha \in (\alpha_F, \alpha_F^{+})$.

Second, we explain why the F-Contract yields the highest social welfare when $\alpha \le \alpha_F$. According to Lemma 3(a), if $\alpha \le \alpha_F$, the M-Contract reduces to the F-Contract. As demonstrated previously, $SW_F^* > SW_R^*$ if and only if $\beta_R < \beta_F \le (\frac{3\varphi}{8} + 1)\beta_R$ and $\alpha \in (0, \alpha_{SW})$. Moreover, $\alpha_F^{+} < \alpha_{SW}$ when $\beta_F < \beta_5$. Therefore, under $\beta_R < \beta_F < min\{\beta_5, (\frac{3\varphi}{8} + 1)\beta_R\}$, the F-Contract yields the highest social welfare ($SW_F^* = SW_M^* > SW_R^*$) for $\alpha \in (0, \alpha_F)$.

Third, we analyze the optimality of the R-Contract for $\alpha \ge max\{\alpha_{SW}, \alpha_R\}$. According to Lemma 3(c), the M-Contract reduces to the R-Contract if $\alpha \ge \alpha_R$. Therefore, we only need compare SW_F^* and SW_R^*. Moreover, under $\beta_R < \beta_F \le (\frac{3\varphi}{8} + 1)\beta_R$, the R-Contract yields a higher social welfare than the fixed contract ($SW_R^* > SW_F^*$) only if $\alpha > \alpha_{SW}$. Central to the analysis is a comparison between the values of α_R and α_{SW}. On the one hand, if $\alpha_R > \alpha_{SW}$, we have $SW_R^* = SW_M^* > SW_F^*$ only when $\alpha \ge \alpha_R$. On the other hand, if $\alpha_R < \alpha_{SW}$, we have $SW_R^* = SW_M^* > SW_F^*$ only when $\alpha \ge \alpha_{SW}$. Therefore, even under a tighter low-negligence regime $\beta_R < \beta_F < min\{\beta_5, \left(\frac{3\varphi}{8} + 1\right)\beta_R\}$, the R-Contract yields the highest social welfare ($SW_R^* = SW_M^* > SW_F^*$) for $\alpha \ge max\{\alpha_{SW}, \alpha_R\}$.

Moreover, when the client negligence under F-Contract is high ($\beta_F > \left(\frac{3\varphi}{8} + 1\right)\beta_R$), simulation analysis reveals that under moderate security loss risk conditions, the M-Contract can achieve an optimal social welfare, as illustrated in Figure A8(b).

We now focus on the case of $\beta_F < \beta_R$. We find that when α tends to 0, $\lim_{\alpha \to 0^+} \Delta SW = 0$

and $\lim_{\alpha \to 0^+} \frac{\partial \Delta SW}{\partial \alpha} = \frac{3b\varphi\beta_R - 8b\beta_F + 8b\beta_R}{32} > 0$. When α tends to infinity, $\Delta SW = \frac{\beta_R - \beta_F}{4b\beta_F\beta_R\alpha} + o(\frac{1}{\alpha^2}) > 0$. Because ΔSW starts with a positive trajectory, approaches zero from above at infinity, and possesses no real roots in $(0, +\infty)$, it remains positive globally. Moreover, both SW_F^* and SW_R^* are monotonically decreasing in α. Therefore, we conclude that $SW_F^* > SW_R^*$ holds when $\beta_F < \beta_R$. When considering the M-Contract, at $\alpha = \alpha_F$, we find that $SW_F^* = SW_M^*$ and $\frac{\partial (SW_M^* | \alpha = \alpha_F)}{\partial \alpha} - \frac{\partial (SW_F^* | \alpha = \alpha_F)}{\partial \alpha} = \frac{b(\beta_R - \beta_F)}{12{\beta_F}^2 d Z_{SW}} \Big(\big(-b{\beta_F}^2 d - 4\beta_F - 4\beta_R(\varphi - 1) \big) Z_{SW} + b^2 d^2 {\beta_F}^4 - 5b{\beta_F}^3 d + \big(4 - 5bd\beta_R(\varphi - 1)\big){\beta_F}^2 + 8\beta_F\beta_R(\varphi - 1) + 4{\beta_R}^2(\varphi - 1)^2 \Big)$. Under the condition $\beta_F + \varphi\beta_R - \beta_R > 0$ derived in Lemma 3, the term $\Big(\big(-b{\beta_F}^2 d - 4\beta_F - 4\beta_R(\varphi - 1) \big) Z_{SW} + b^2 d^2 {\beta_F}^4 - 5b{\beta_F}^3 d + \big(4 - 5bd\beta_R(\varphi - 1)\big){\beta_F}^2 + 8\beta_F\beta_R(\varphi - 1) + 4{\beta_R}^2(\varphi - 1)^2 \Big)$ is always negative. Along with the condition $\beta_F < \beta_R$, it rigorously follows that $\frac{\partial (SW_M^* | \alpha = \alpha_F)}{\partial \alpha} - \frac{\partial (SW_F^* | \alpha = \alpha_F)}{\partial \alpha} < 0$. This implies that for a local interval $\alpha \in (\alpha_F, \alpha_F^+)$, $SW_F^* > SW_M^*$ holds. Consequently, when $\beta_F < \beta_R$, we have the full ranking $SW_F^* > SW_M^* > SW_R^*$ for $\alpha \in (\alpha_F, \alpha_F^+)$. ■

Appendix B: Model Extensions

To check the robustness of our results in a broader setting, we examine six model extensions: (i) exploring a fixed-plus-proportional compensation contract, where clients who suffer a breach receive a combination of both fixed and proportional compensation; (ii) incorporating positive security externalities, whereby the experience the MSSP gains from serving one client benefits others, thereby enhancing its defense capabilities across different scenarios; (iii) allowing client negligence to vary heterogeneously with their valuations, even under the same contract type; (iv) introducing a valuation-dependent compensation ratio, reflecting the reality that high-valuation clients demand—and are offered—stronger proportional protection due to their heightened sensitivity to security loss risks; (v) endogenizing the MSSP's multidimensional decisions, capturing its strategic ability to jointly optimize security effort and liability terms in response to market conditions; and (vi) employing Monte Carlo simulations to inject stochastic variation into key parameters, effectively capturing the inherent uncertainties in real-world security practices.

Crucially, we find that under all these diverse extensions, our central insight remains robust: the menu-based contract consistently achieves a globally optimal outcome for the MSSP, the client firms, and overall social welfare under specific conditions. The detailed analyses are presented below.

B.1. Fixed-Plus-Proportional Compensation Contract

In this section, to accommodate more intricate compensation arrangements, we analyze a standalone Fixed-Plus-Proportional Compensation Contract (FR-Contract) that combines a fixed payment with a proportional compensation component, i.e., $d + \alpha\theta\varphi$. For notational simplicity, throughout the paper we use the subscript I to denote this FR-Contract (where "I" stands for "Integrated"). Thus, under the FR-Contract, the total compensation for a client who is attacked and suffers a loss is $d_I + \alpha\theta\varphi_I$. Notably, within this integrated structure, both the fixed compensation d_I and the proportional coefficient φ_I are set lower than their counterparts in the pure contracts. That is, $d_I < d$, $\varphi_I < \varphi$. Since client behavior differs upon the selection of different contracts, we denote the client negligence under the FR-Contract as β_I. Similarly, we assume that the probability of a client being successfully breached is given by $q = bN\beta_I$ where $b = k(1-s)^2$. In the pure FR-Contract, the client's utility becomes $E[u_I] = \theta - p_I - b\beta_I N_I \alpha\theta + b\beta_I N_I d_I + b\beta_I N_I \varphi_I \alpha\theta$, and the MSSP's profit becomes $\pi_I(p_I) = p_I N_I - \int_{\theta_I}^{1} (b\beta_I N_I \varphi_I \alpha\theta)\ d\theta - b\beta_I N_I d_I N_I - \Gamma(s)$. We derive the equilibrium solutions $N_I^* = \frac{-2\alpha b\beta_I + \sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)} - 2}{3\alpha b\beta_I(\varphi_I - 2)}$ and $\pi_I^* = \frac{(-2\alpha b\beta_I + E - 2)(-2(\alpha b\beta_I)^2 + (\alpha b\beta_I + 1)E + 8\alpha b\beta_I - 6\varphi_I \alpha b\beta_I - 2)}{27(\alpha b\beta_I)^2(2-\varphi_I)^2} - \Gamma(s)$.

Proposition EC.3. *Under the FR-Contract, as the security loss risk increases, both the MSSP's equilibrium client base and profits monotonically decrease. That is,* $\frac{\partial N_I^*}{\partial \alpha} < 0$ *and* $\frac{\partial \pi_I^*}{\partial \alpha} < 0$.

We confirm that the FR-Contract is functionally analogous to the R-Contract analyzed in the main text. Both the profit and the client base monotonically decrease with rising security loss risk. This is because the FR-Contract serves only a single client segment, and thus lacks the *segmentation-driven expansion effect* identified in the M-Contract, but the *externality-driven contraction effect* gets enhanced with an increasing α, leading to a decline in the MSSP's profit. Therefore, **unlike the M-Contract, no client migration occurs within the FR-Contract framework**, precluding the possibility of expanding the client base or increasing profit under escalating security loss risks. Ultimately, the FR-Contract serves as a generalized form of the R-Contract, rather than a fundamentally new contract mechanism.

Building on this insight, we now introduce the FR-Contract as a distinct contractual form to replace the R-Contract within our new menu-based regime. We examine whether the proposed menu-based framework—where the MSSP concurrently offers both the F-Contract and this new FR-Contract—preserves its optimality. In this extended setup, the client's utility function under the new menu-based regime is expressed as:

$$E[u_M] = \begin{cases} \theta - p_M - b\beta_I N_M(1-\varphi_I)\alpha\theta + b\beta_I N_M d_I & if\ \ Accept\ \ FR-Contract, \\ \theta - p_M - b\beta_F N_M(\alpha\theta - d) & if\ \ Accept\ \ F-Contract, \\ 0 & if\ \ Reject\ \ both. \end{cases} \tag{B1}$$

Let θ_{MF} denote the valuation of the marginal client who is indifferent between accepting the F-Contract and opting out, and θ_{MI} denote the valuation of the marginal client who is indifferent between adopting the F-Contract and the FR-Contract. If $0 < \theta_{MF} < \theta_{MI} < 1$ holds, the clients' self-selection strategy profile $\tau_M(\theta)$ under M-Contract is

$$\tau_M(\theta) = \begin{cases} Accept\ \ FR-Contract & if\ \ \theta_{MI} \le \theta \le 1 \\ Accept\ \ F-Contract & if\ \ \theta_{MF} < \theta < \theta_{MI} \\ Reject\ \ both & if\ \ 0 \le \theta \le \theta_{MF} \end{cases} \tag{B2}$$

The MSSP's profit is again defined as the total service fees collected from clients minus the total compensation amount to clients and input costs $\Gamma(s)$:

$$\pi_M = p_M N_M - b\beta_F N_M d N_{MF} - \int_{\theta_{MI}}^{1} (b\beta_I N_M \varphi_I \alpha\theta)\ d\theta - b\beta_I N_M d_I N_{MI} - \Gamma(s) \tag{B3}$$

Analogous to the solution approach presented in the main text, we derive the equilibrium solutions under this extended menu-based contract, leading to Lemma EC.1.

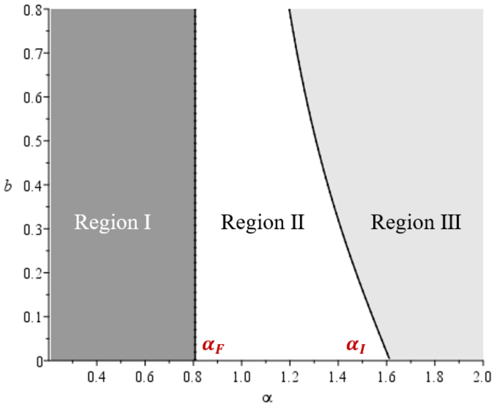
Lemma EC.1. *When an MSSP offers the* ***new*** *menu-based contract incorporating the FR-Contract:*

(a) If $\alpha \le \alpha_F$, leading to $\theta_{MI}^* \ge 1$, the equilibrium MSSP solution for the menu-based contract reduces to Case F. Formally, $p_M^* = p_F^*$, $N_M^* = N_F^*$, when $\alpha \le \alpha_F$;

(b) If $\alpha_F < \alpha < \alpha_I$, corresponding to $0 < \theta_{MF}^* < \theta_{MI}^* < 1$, the high-valuation ($N_{MI}^* =$

$\frac{\alpha(\beta_F+\varphi_I\beta_I-\beta_I)-\beta_F d+\beta_I d_I}{\alpha(\beta_F+\varphi_I\beta_I-\beta_I)}$*) clients located in* $[\theta_{MI}^*, 1]$ *will choose the FR-Contract. The MSSP will sets the service fee* p_M^* *to ensure that middle-valuation (* $N_{MF}^* = \frac{6b\beta_F(\beta_F d-\beta_I d_I)-(4\alpha b\beta_F-2)(\beta_F+\varphi_I\beta_I-\beta_I)-\sqrt{Y3}}{6\alpha b\beta_F(\beta_F+\varphi_I\beta_I-\beta_I)}$*) clients located in* $(\theta_{MF}^*, \theta_{MI}^*)$ *will choose the F-Contract, and the low-valuation clients located in* $[0, \theta_{MF}^*]$ *will not purchase the service;*

(c) *If* $\alpha \geq \alpha_I$*, resulting in* $\theta_{MF}^* \geq \theta_{MI}^*$*, the equilibrium MSSP solution for the menu-based contract reduces to Case I. That is,* $p_M^* = p_I^*$*,* $N_M^* = N_I^*$ *when* $\alpha \geq \alpha_I$.



$\beta_I = 1.05, \beta_F = 1.1, d = 0.1, \varphi = 0.2, \varphi_I = 0.15, d_I = 0.05$

Figure B1. Equilibrium client segmentation under the new M-Contract

Analogous to Figure 4 in the main text, the three regions depicted in Figure B1 correspond to distinct structural market degenerations based on the security loss risk. Specifically, Region I ($\alpha \leq \alpha_F$) represents a degeneration to Case F under a low security loss risk; Region III ($\alpha \geq \alpha_I$) illustrates a degeneration to Case I under a high security loss risk; and Region II ($\alpha_F < \alpha < \alpha_I$) features a segmented client base under a moderate security loss risk. In Region II, driven by the heterogeneity in clients' valuations of security services, the market segments into high-, middle-, and low-valuation tiers. The two critical thresholds orchestrating these structural shifts are mathematically defined as $\alpha_F = \frac{\beta_F d-\beta_I d_I}{\beta_F+\varphi_I\beta_I-\beta_I}$ and $\alpha_I = \frac{3{\beta_F}^2 bd+(\beta_F+\varphi_I\beta_I-\beta_I)-b{\beta_I}^2(1-\varphi_I)d_I-b\beta_F(d\varphi_I-d+3d_I)-\sqrt{Y4}}{(2\beta_F-\varphi_I\beta_I)(\beta_F+\varphi_I\beta_I-\beta_I)b}$.

Based on Figure B1, we confirm that our main-text findings (Figure 5) remain valid under the new menu-based regime. Although the MSSP cannot prevent high-valuation clients from self-selecting (since $N_{MI}^* = \frac{\alpha(\beta_F+\varphi_I\beta_I-\beta_I)-\beta_F d+\beta_I d_I}{\alpha(\beta_F+\varphi_I\beta_I-\beta_I)}$ is independent of p_M), it utilizes the service fee to control total market demand by regulating the adoption of middle- and low-valuation clients.

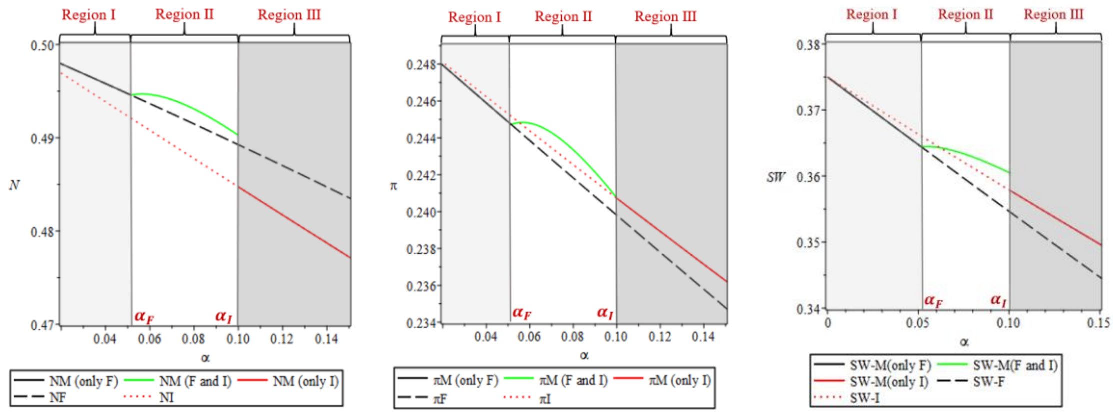
Next, we investigate whether this new menu-based contract can achieve optimality for all parties, which leads to Proposition EC.4.

Proposition EC.4. *Under* $\beta_I < \beta_F < min\left\{\beta_i, \left(\frac{3\varphi_I}{8}+1\right)\beta_I\right\}$*, where* $i = 6,7,8$*, the new menu-based contract that making both fixed and integrated compensation contracts available will yield a higher aggregate performance (in total client base, MSSP profitability, and social welfare) than either of the pure contracts. Formally,* $N_M^* > N_F^* > N_I^*$*,* $\pi_M^* > \pi_F^* > \pi_I^*$*, and* $SW_M^* > SW_F^* > SW_I^*$ *when* $\alpha \in (\alpha_F, \alpha_F^+)$*.*

Proposition EC.4 forcefully demonstrates that even when the pure R-Contract is structurally upgraded to an FR-Contract, the menu-based regime preserves its ability to foster a win-win-win equilibrium under the moderate client negligence for F-Contract, advancing the intertwined interests of the MSSP, the clients, and the social planner.

Furthermore, our numerical simulations reveal that the new M-Contract outperforms the pure M-Contract even when β_F is relatively high, as illustrated in Figure B2. Specifically, Figures B2(a) and B2(b) show that within the menu-based framework, an increase in security loss risk can still lead to an expansion of both the total client base (N_M^*) and the MSSP's profit (π_M^*). This occurs because some clients transfer from the F-Contract to the FR-Contract, confirming that the *segmentation-driven expansion effect* discussed in the main text remains valid in this extended setting.

In conclusion, comparing this extension to the main model, Proposition EC.4 establishes that replacing the R-Contract with the FR-Contract leaves our core theoretical conclusions intact. As established in Proposition EC.3, the FR-Contract alone lacks the *segmentation-driven expansion effect* and thus does not constitute the optimal strategy for the MSSP. By revealing the FR-Contract as merely a parameterized generalization of the R-Contract, this extension indirectly validates the profound necessity and strategic superiority of the M-Contract mechanism we propose.



(a) Comparison of client base under the three contracts

(b) Comparison of MSSP profitability under the three contracts

(c) Comparison of social welfare under the three contracts

$d = 0.08, \varphi_I = 0.9, b = 0.5, \beta_F = 1.65, \beta_I = 1.05, d_I = 0.05$

Figure B2. Optimal contract strategy under the new M-Contract

Proof of Proposition EC.3

In the new FR-Contract, the user's expected utility becomes $E[u_I] = \theta - p_I - b\beta_I N_I \alpha\theta + b\beta_I N_I d_I + b\beta_I N_I \varphi_I \alpha\theta$ (throughout the paper, the subscripts I refers to FR-Contract). We then derive the service fee by setting the expected utility of the marginal client (θ_I) to zero:

$$p_I = \theta_I - b\beta_I N_I \alpha\theta_I + b\beta_I N_I d_I + b\beta_I N_I \varphi_I \alpha\theta_I \tag{C1}$$

and the MSSP's profit becomes

$$\pi_I(p_I) = p_I N_I - \int_{\theta_I}^{1} (b\beta_I N_I \varphi_I \alpha\theta)\ d\theta - b\beta_I N_I d_I N_I - \Gamma(s). \tag{C2}$$

where $b = k(1-s)^2$. Here, there is only one type of client group under the FR-Contract, with a proportion N_I, and $N_I = 1 - \theta_I$. Substituting Equation (C1) into Equation (C2), we have $\max_{N_I} \pi_I(N_I) = \frac{N_I\left(2 - \alpha b\beta_I(\varphi_I - 2)N_I{}^2 - (2\alpha b\beta_I + 2)N_I\right)}{2} - \Gamma(s)$. Then, deriving the first-order condition with respect to N_I, we have

$$\frac{\partial \pi_I}{\partial N_I} = -\frac{3\alpha b\varphi_I \beta_I N_I{}^2}{2} + 3\alpha b\beta_I N_I{}^2 - 2\alpha b\beta_I N_I - 2N_I + 1. \tag{C3}$$

Solving for the optimal client base from the first-order condition, we can obtain the equilibrium solutions as: $N_I^* = \frac{-2\alpha b\beta_I + \sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)} - 2}{3\alpha b\beta_I(\varphi_I - 2)}$, and $\theta_I^* = 1 - N_I^* = \frac{3\alpha b\beta_I\varphi_I - 4\alpha b\beta_I - \sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)} + 2}{3\alpha b\beta_I(\varphi_I - 2)}$. In addition, $\frac{\partial^2 \pi_I}{\partial N_I{}^2} = 2\alpha b\beta_I(2N_I - 1) + 2(\alpha b\beta_I N_I - 1) - 3\alpha b\beta_I\varphi_I N_I < 0$ given $N_I < \frac{1}{2}$ and $\alpha q < 1$. Thus, the satisfaction of the second-order condition guarantees that the critical point N_I^* constitutes a profit maximum for the MSSP.

Next, we show that $0 < \theta_I^* < 1$ holds so that clients will accept the FR-Contract. First, we can prove that the valuation of the marginal client under FR-Contract is increasing in α, that is $\frac{\partial \theta_I^*}{\partial \alpha} = \frac{-2\sqrt{(2\alpha b\beta_I+2)^2+6\alpha b\beta_I(\varphi_I-2)}+4+b\beta_I(3\varphi_I-2)\alpha}{3\alpha^2 b\beta_I(\varphi_I-2)\sqrt{(2\alpha b\beta_I+2)^2+6\alpha b\beta_I(\varphi_I-2)}} > 0$. When $4 + b\beta_I(3\varphi_I - 2)\alpha < 0$, we have $-2\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)} + 4 + b\beta_I(3\varphi_I - 2)\alpha < 0$ and $\frac{\partial \theta_I^*}{\partial \alpha} > 0$. When $4 + b\beta_I(3\varphi_I - 2)\alpha > 0$, we have $(4 + b\beta_I(3\varphi_I - 2)\alpha)^2 - \left(-2\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)}\right)^2 = 3(\alpha b\beta_I)^2\big((3\varphi_I - 4)\varphi_I - 4\big) < 0$. Therefore, $\left|2\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)}\right| > 4 + b\beta_I(3\varphi_I - 2)\alpha$ and $\frac{\partial \theta_I^*}{\partial \alpha} > 0$. Next, since $\theta_I^*(\alpha)$ is monotonically increasing with α, $\lim_{\alpha \to 0^+} \theta_I^*(\alpha) = \frac{1}{2}$, and $\lim_{\alpha \to +\infty} \theta_I^*(\alpha) = 1$. As a result, we can conclude that $0 < \theta_I^* < 1$ holds. Note that another solution is not within the range and has been excluded.

Then, based on the equilibrium client base N_I^*, we derive the equilibrium profit $\pi_I^* = \frac{(-2\alpha b\beta_I+E-2)(-2(\alpha b\beta_I)^2+(\alpha b\beta_I+1)E+8\alpha b\beta_I-6\varphi_I\alpha b\beta_I-2)}{27(\alpha b\beta_I)^2(2-\varphi_I)^2} - \Gamma(s)$, where $E = \sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)}$.

We first prove that $\frac{\partial N_I^*}{\partial \alpha} < 0$. Under the FR-Contract, the first-order derivative with respect to α is $\frac{\partial N_I^*}{\partial \alpha} = \frac{\alpha b\beta_I(2-3\varphi_I)-4+2\sqrt{(2\alpha b\beta_I+2)^2+6\alpha b\beta_I(\varphi_I-2)}}{3\alpha^2 b\beta_I(\varphi_I-2)\sqrt{(2\alpha b\beta_I+2)^2+6\alpha b\beta_I(\varphi_I-2)}}$. Since $(\varphi_I - 2) < 0$, we have the denominator $3\alpha^2 b\beta_I(\varphi_I - 2)\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)} < 0$. Next, $(\alpha b\beta_I(2 - 3\varphi_I) - 4)^2 - \left(2\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)}\right)^2 = 3\alpha^2 b^2 {\beta_I}^2(3{\varphi_I}^2 - 4\varphi_I - 4) < 0$ since $(3{\varphi_I}^2 - 4\varphi_I - 4) < 0$ given $0 < \varphi_I < 1$. Thus, $\left|2\sqrt{(2\alpha b\beta_I + 2)^2 + 6\alpha b\beta_I(\varphi_I - 2)}\right| > |\alpha b\beta_I(2 - 3\varphi_I) - 4|$, which implies the numerator of $\frac{\partial N_I^*}{\partial \alpha}$ is positive, and consequently $\frac{\partial N_I^*}{\partial \alpha} < 0$.

We then demonstrate that $\frac{\partial \pi_I^*}{\partial \alpha} < 0$. The first-order derivative with respect to α is $\frac{\partial \pi_I^*}{\partial \alpha} = \frac{(16-8\alpha^3 b^3 {\beta_I}^3+\alpha b\beta_I(18\varphi_I-12))E+(4+4\alpha^2 b^2 {\beta_I}^2+\alpha b\beta_I(6\varphi_I-4))(-8+4\alpha^2 b^2 {\beta_I}^2-\alpha b\beta_I(3\varphi_I-2))}{27\alpha^3 b^2 {\beta_I}^2(\varphi_I-2)^2 E}$. Furthermore, $\frac{\partial^2 \pi_I}{\partial \alpha^2} = \frac{(-4\alpha b\beta_I(3\varphi_I-2)-16)E+16\alpha b\beta_I(3\varphi_I-2)+32+\alpha^2 b^2 {\beta_I}^2(9{\varphi_I}^2-12\varphi_I+20)}{9\alpha^4 b^2 {\beta_I}^2(\varphi_I-2)^2 E} > 0$, because $((-4\alpha b\beta_I(3\varphi_I -$

$2) - 16)E)^2 - \left(16\alpha b\beta_I(3\varphi_I - 2) + 32 + \alpha^2 b^2 {\beta_I}^2(9{\varphi_I}^2 - 12\varphi_I + 20)\right)^2 = -\alpha^4 b^4 {\beta_I}^4(\varphi_I - 2)^2(\varphi_I + \frac{2}{3})^2 < 0$. This indicates that the first-order derivative is monotonically increasing with α. Additionally, we find that the limit of the first-order derivative at both extremes of α is always less than 0 ($\lim_{\alpha \to 0^+} \frac{\partial \pi_I^*}{\partial \alpha} = -\frac{b\beta_I(\varphi_I+2)}{16} < 0$, and as $\alpha \to +\infty$, the expansion yields $\frac{\partial \pi_I^*}{\partial \alpha} = -\frac{1}{4\alpha^2 b\beta_I} + o(\frac{1}{\alpha^3}) < 0$). Because $\frac{\partial \pi_I^*}{\partial \alpha}$ starts negative, is monotonically increasing, yet is strictly bounded below zero at infinity, we reach the conclusion that the first-order derivative remains consistently negative, $\frac{\partial \pi_I^*}{\partial \alpha} < 0$. Thus, the MSSP's profit under the FR-Contract monotonically decreases with increasing α. ■

Proof of Lemma EC.1

Now, we propose a new M-Contract, in which the MSSP simultaneously offers both the F-Contract and the FR-Contract for clients to self-selection. The MSSP first discloses all contract terms, including the fixed compensation amount d under the F-Contract, as well as the proportional compensation ratio φ_I and the fixed compensation component d_I under the FR-Contract. It then sets the service fee p_M to achieve the desired client base N_M. Subsequently, clients decide whether to adopt the F-Contract, the FR-Contract, or opt out entirely, based on their valuations of the services provided by the MSSP. Since the MSSP offers the same security service to all clients regardless of contract type, the resulting security externality applies uniformly across the entire participating client population, denoted by $N_M = 1 - \theta_{MF}$.

To maximize its profit, MSSP will always set the service fee p_M so that the expected utility of the marginal client θ_{MF} is zero, and the marginal client θ_{MI} is indifferent between accepting the F-Contract and accepting the FR-Contract, thus we have

$$\begin{cases} \theta_{MF} - p_M - b\beta_F N_M(\alpha\theta_{MF} - d) = 0, \\ \theta_{MI} - p_M - b\beta_I N_M(1 - \varphi_I)\alpha\theta_{MI} + b\beta_I N_M d_I = \theta_{MI} - p_M - b\beta_F N_M(\alpha\theta_{MI} - d). \end{cases} \tag{C4}$$

By solving Equation (C4), we obtain N_{MI} and p_M.

$$\begin{cases} N_{MI}^* = \frac{\alpha(\beta_F + \varphi_I\beta_I - \beta_I) - \beta_F d + \beta_I d_I}{\alpha(\beta_F + \varphi_I\beta_I - \beta_I)}, \\ p_M = (1 - N_{MF} - N_{MI})(1 - \alpha b\beta_F(N_{MF} + N_{MI})) + b\beta_F d(N_{MF} + N_{MI}). \end{cases} \tag{C5}$$

The MSSP's expected profit is:

$$\pi_M = p_M N_M - b\beta_F N_M d N_{MF} - \int_{\theta_{MI}}^{1} (b\beta_I N_M \varphi_I \alpha\theta)\ d\theta - \ b\beta_I N_M d_I N_{MI} - \Gamma(s) \quad (C6)$$

Substituting Equation (C5) into Equation (C6), and taking the first-order condition with respect to N_{MF}, we obtain the equilibrium solution for the client base as: $N_{MF}^* = \frac{6b\beta_F(\beta_F d - \beta_I d_I) - (4\alpha b\beta_F - 2)(\beta_F + \varphi_I\beta_I - \beta_I) - \sqrt{Y3})}{6\alpha b\beta_F(\beta_F + \varphi_I\beta_I - \beta_I)}$. Since $N_{MI}^* = \frac{\alpha(\beta_F + \varphi_I\beta_I - \beta_I) - \beta_F d + \beta_I d_I}{\alpha(\beta_F + \varphi_I\beta_I - \beta_I))}$, we have $N_M^* = N_{MI}^* + N_{MF}^* = \frac{2(\alpha b\beta_F + 1)(\beta_F + \varphi_I\beta_I - \beta_I) - \sqrt{Y3}}{6\alpha b\beta_F(\beta_F + \varphi_I\beta_I - \beta_I)}$. Therefore, $\theta_{MF}^* = 1 - N_M^* = 1 - \frac{2(\alpha b\beta_F + 1)(\beta_F + \varphi_I\beta_I - \beta_I) - \sqrt{Y3}}{6\alpha b\beta_F(\beta_F + \varphi_I\beta_I - \beta_I)}$ and $\theta_{MI}^* = 1 - N_{MI}^* = \frac{\beta_F d - \beta_I d_I}{\alpha(\beta_F + \varphi_I\beta_I - \beta_I)}$, where $Y3 = 12b^2(d^2 - d\alpha + \frac{1}{3}\alpha^2)\beta_F^4 + 6b(b((\frac{7\varphi_I}{3} - \frac{4}{3})\alpha^2 + (-4d\varphi_I + 4d + 2d_I)\alpha + d(d\varphi_I - 2d - 4d_I))\beta_I - \frac{2\alpha}{3})\beta_F^3 + (4 - 12b^2((-\frac{4}{3}\varphi_I^2 + \frac{5}{3}\varphi_I - \frac{1}{3})\alpha^2 + (\varphi_I - 1)(d\varphi_I - d - 2d_I)\alpha + d_I(d\varphi_I - 2d - d_I))\beta_I^2 - 8b\alpha(\varphi_I - 1)\beta_I)\beta_F^2 + 6(((\varphi_I - 1)\alpha + d_I)b^2((\varphi_I^2 - \varphi_I)\alpha + d_I(\varphi_I - 2))\beta_I^2 - \frac{2b\alpha(\varphi_I - 1)^2\beta_I}{3} + \frac{4\varphi_I}{3} - \frac{4}{3})\beta_I\beta_F + 4\beta_I^2(\varphi_I - 1)^2$. Accordingly, we can derive the optimal profit $\pi_M^* = p_M^* N_M^* - \ b\beta_F N_M^* d N_{MF}^* - \frac{\alpha b\beta_I\varphi_I N_M^* N_{MI}^*(2 - N_{MI}^*)}{2} - \ b\beta_I N_M^* d_I N_{MI}^* - \Gamma(s)$.

The necessary and sufficient conditions for the equilibrium solutions above are $0 < \theta_{MF} < \theta_{MI} < 1$. However, as the values of the parameters vary, the condition $0 < \theta_{MF} < \theta_{MI} < 1$ may not always hold. As a result, three scenarios may appear:

(i) When $\theta_{MI} \geq 1$, we have $\alpha \leq \alpha_F = \frac{\beta_F d - \beta_I d_I}{\beta_F + \varphi_I\beta_I - \beta_I}$, indicating that θ_{MI} is outside of the defined range [0,1], the equilibrium client strategy profile under the M-Contract reduces to that of the F-Contract. This implies that no clients select the FR-Contract, even though the MSSP offers both contract types.

(ii) when $\theta_{MI} < 1$, we have $\alpha > \alpha_F$ and heterogeneous choices of compensation contracts exist among clients. As the MSSP's pricing does not impact clients' preference between the F-Contract and the FR-Contract, its key decision is the client base purchasing its service by adjusting the fee p_M. When $\theta_{MF} \leq \theta_{MI}$ is violated, we have $\alpha \geq \alpha_I = \frac{3{\beta_F}^2 bd + (\beta_F + \varphi_I\beta_I - \beta_I) - b{\beta_I}^2(1 - \varphi_I)d_I - b\beta_F(d\varphi_I - d + 3d_I) - \sqrt{Y4}}{(2\beta_F - \varphi_I\beta_I)(\beta_F + \varphi_I\beta_I - \beta_I)b}$ and the equilibrium client strategy profile reduces to that of Case I, indicating that no clients opt for the F-Contract, despite the simultaneous availability of both contract types, where $Y4 = b^2 {d_I}^2 \beta_I^4 - 2bd_I(b(d-d_I)\beta_F + {\varphi_I}^2 - 1)\beta_I^3 + (b^2(d^2 - 4dd_I + d_I^2)\beta_F^2 + 2b(d{\varphi_I}^2 - d - 2d_I)\beta_F + (\varphi_I - 1)^2)\beta_I^2 + 2(b^2 d(d-d_I)\beta_F^2 + 2b(d + \frac{d_I}{2})\beta_F + \varphi_I - 1)\beta_F\beta_I + \beta_F^2(b\beta_F d - 1)^2$.

(iii) when $0 < \theta_{MF} < \theta_{MI} < 1$, that is, $\alpha_F < \alpha < \alpha_I$, MSSP will set a service fee p_M so that potential clients with $\theta \le \theta_{MF}$ will opt out, those with $\theta_{MF} < \theta < \theta_{MI}$ will adopt the F-Contract, and those with $\theta_{MI} \le \theta \le 1$ will adopt the FR-Contract.

Based on the above analysis, we have the following equilibrium solutions in Case M:

(a) If $\alpha \le \alpha_F$, leading to $\theta_{MI}^* \ge 1$, the equilibrium MSSP solution for the menu-based contract reduces to that for Case F. Formally, $p_M^* = p_F^*$, $N_M^* = N_F^*$, when $\alpha \le \alpha_F$.

(b) If $\alpha_F < \alpha < \alpha_I$, corresponding to $0 < \theta_{MF}^* < \theta_{MI}^* < 1$, high-valuation ($N_{MI}^*$) clients located in $[\theta_{MI}^*, 1]$ will choose the FR-Contract. The MSSP will set the service fee at p_M^* to ensure that middle-valuation (N_{MF}^*) clients located in $(\theta_{MF}^*, \theta_{MI}^*)$ will choose the F-Contract, and the low-valuation clients located in $[0, \theta_{MF}^*]$ will not purchase the service.

(c) If $\alpha \ge \alpha_I$, resulting in $\theta_{MF}^* \ge \theta_{MI}^*$, the equilibrium MSSP solution for the menu-based contract reduces to that for Case I. That is, $p_M^* = p_I^*$, $N_M^* = N_I^*$ when $\alpha \ge \alpha_I$. ■

Proof of Proposition EC.4

Through structural comparison, we observe that the expression for the equilibrium solution in Lemma EC.1 closely resembles the menu-based contract discussed in the main text, as the R-Contract is mathematically a special case of the FR-Contract. Consequently, we apply the same proof techniques used in Propositions 2, 4 and 5 in the main text to compare the optimal outcomes under the new menu-based framework. Ultimately, we obtain similar threshold points. The proof process follows the exact same structure as in the main text and is therefore omitted here. ■

B.2. Positive Security Externality

In the main text, we follow existing studies that consider negative security externalities as an increasing function of the client base size (e.g., August et al. 2022, Zhang et al. 2020, Dey and Lahiri, 2026), indicating that a larger number of clients increases the aggregate security risks. However, when an MSSP serves multiple clients, the expertise gained from mitigating vulnerabilities for one client can benefit others, thereby enhancing the MSSP's capabilities across various security scenarios. This accumulated knowledge improves both the MSSP's performance and client outcomes. We capture this effect by incorporating a positive externality representing MSSP experience in our extended model.

Specifically, as the MSSP serves more clients, its service quality improves by leveraging vulnerabilities addressed for one client and applying the lessons learned to others. As a result, the likelihood of a client experiencing a security breach decreases. To reflect this, we assume the probability of a successful attack on a single vulnerability transfers from $(1-s)$ to $(1-sN)$, where s represents the MSSP's security effort on the standardized security services as stated in the main model and $s\boldsymbol{N}$ reflects the improved service quality due to experience accumulated from serving multiple clients. This transition algebraically captures the positive externality of accumulated expertise. Therefore, the overall probability of a successful attack on a client systematically shifts from $q = k(1-\boldsymbol{s})^2 N\beta$ to $q = k(1-\boldsymbol{sN})^2 N\beta$. To the best of our knowledge, no existing literature has endogenously modelled the positive externalities arising from a client base.

The client's expected utility under the F-Contract becomes $E[u_F] = \theta - p_F - k(1 - sN_F)^2 \beta_F N_F(\alpha\theta - d)$. Similarly, under the R-Contract, the client's expected utility becomes: $E[u_R] = \theta - p_R - k(1 - sN_R)^2 \beta_R N_R \alpha\theta(1-\varphi)$. Under the M-Contract, the expected utility for clients selecting the F-Contract is $E[u_{MF}] = \theta - p_M - k(1 - sN_M)^2 \beta_F N_M \alpha\theta + k(1 - sN_M)^2 \beta_F N_M d$, while for clients choosing the R-Contract, the expected utility is $E[u_{MR}] = \theta - p_M - k(1 - sN_M)^2 \beta_R N_M \alpha\theta(1-\varphi)$.

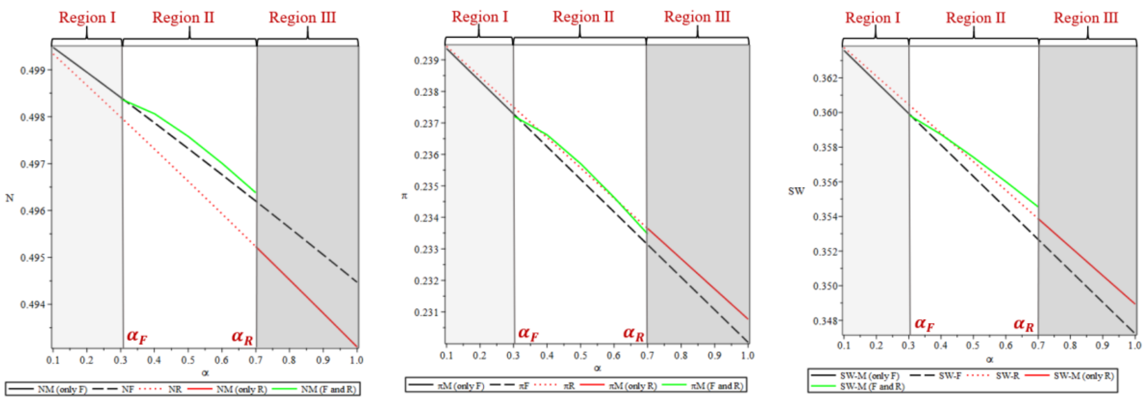
Due to the analytical complexity of solving this non-linear model, we compare the equilibrium outcomes of the three contracts through numerical analysis. By setting $d = 0.1, \varphi = 0.2, \beta_F = 1.3, \beta_R = 1.1, s = 0.4, k = 0.1, \Gamma(s) = 0.06$, we obtain the results as shown in Table B1.

Table B1. Comparison of equilibrium outcomes across the three contracts under positive externality

$\boldsymbol{\alpha}$	$\boldsymbol{N_F}^*$	$\boldsymbol{N_R}^*$	$\boldsymbol{N_M}^*$	$\boldsymbol{\pi_F}^*$	$\boldsymbol{\pi_R}^*$	$\boldsymbol{\pi_M}^*$	$\boldsymbol{SW_F}^*$	$\boldsymbol{SW_R}^*$	$\boldsymbol{SW_M}^*$
0.1	0.4994	0.4993	0.4965	0.2393	0.2394	0.2336	0.3635	0.3637	0.3593
0.15	0.4992	0.4990	0.4978	0.2388	0.2389	0.2361	0.3626	0.3629	0.3604
0.2	0.4989	0.4986	0.4982	0.2383	0.2384	0.2370	0.3617	0.3621	0.3605
0.25	0.4986	0.4983	0.4984	0.2378	0.2379	0.2372	0.3608	0.3612	0.3602
0.3	0.4984	0.4979	0.4983	0.2372	0.2374	0.2372	0.3599	0.3604	0.3598
0.4	0.4978	0.4973	0.4980	0.2362	0.2365	0.2366	0.3581	0.3588	0.3587
0.5	0.4973	0.4966	0.4975	0.2352	0.2355	0.2357	0.3563	0.3571	0.3574

α	$N_F{}^*$	$N_R{}^*$	$N_M{}^*$	$\pi_F{}^*$	$\pi_R{}^*$	$\pi_M{}^*$	$SW_F{}^*$	$SW_R{}^*$	$SW_M{}^*$
0.6	0.4967	0.4959	0.4970	0.2341	0.2346	0.2346	0.3544	0.3555	0.3560
0.7	0.4962	0.4952	0.4963	0.2331	0.2336	0.2334	0.3526	0.3538	0.3545
0.8	0.4956	0.4945	0.4957	0.2320	0.2326	0.2322	0.3508	0.3522	0.3530
0.9	0.4950	0.4938	0.4950	0.2310	0.2317	0.2309	0.3490	0.3505	0.3515
1.0	0.4944	0.4930	0.4943	0.2300	0.2307	0.2297	0.3472	0.3489	0.3500

To visually illustrate our findings, we present the results in graphical form alongside Table B1. Figure B3 shows that incorporating positive externalities into the model does not change our key results, provided that under moderate security loss risk, offering the M-Contract can still achieve a win-win-win situation that advances the interests of MSSPs, client firms, and social planners alike. Specifically, Figures B3(b) and B3(c) illustrate that, around α_R, the MSSP privately prefers the R-Contract while the social planner prefers the M-Contract. This corroborates our main-text conclusion that the MSSP's profit-maximizing choice does not necessarily maximize social welfare.



(a) Comparison of client base among the three contracts

(b) Comparison of MSSP profitability among the three contracts

(c) Comparison of social welfare among the three contracts

$d = 0.1, \varphi = 0.2, \beta_F = 1.3, \beta_R = 1.1, s = 0.4, k = 0.1, \Gamma(s) = 0.06$

Figure B3. The optimal strategy in the presence of positive externalities

Moreover, we conduct simulations on the client base N_M and find that, under the M-Contract, the client base exhibits an inverted U-shaped relationship with the security loss risk—initially increasing and then decreasing as risk rises. This result reaffirms the *segmentation-driven expansion effect* discussed in the main text, highlighting the robustness of our key findings. In addition, we find that the presence of a positive externality, which partially offsets the *externality-driven contraction effect*, leads to a more pronounced expansion of the client base than baseline model for a given level of security loss risk. As shown in Figure B4, the black curve represents the scenario **with** positive externalities while the blue curve represents the scenario **without** positive externalities, clearly illustrating the amplified positive change in N_M with the security loss risk.

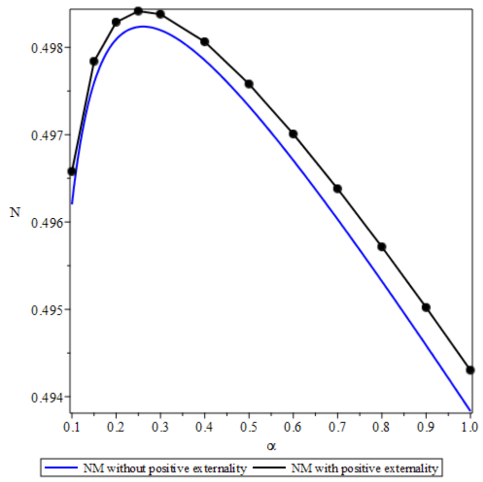


Figure B4. Client base variation with the security loss risk under the M-Contract

B.3. Heterogeneity in Client Negligence by Valuation

In the main model, we assume that client negligence varies across contract types but remains homogeneous within the same contract. That is, it is fixed at β_F in the F-Contract and β_R in the R-Contract. In this extension, we further relax the assumption of uniform negligence by allowing client negligence to vary endogenously with heterogeneous client valuations, even under the same contract type. Specifically, we posit that high-valuation clients tend to be more responsive and cautious toward security risk, and therefore exhibit lower levels of negligence.

To capture this interaction, we formally incorporate this feature into the model by specifying negligence as $\beta_F(\mathbf{1}-\boldsymbol{\theta})$ under the F-Contract and $\beta_R(\mathbf{1}-\boldsymbol{\theta})$ under the R-Contract. In doing so, we aim to examine how heterogeneous client negligence affects the equilibrium outcomes and strategic implications of the three contract types. Accordingly, the client's expected utility under the F-Contract becomes $E[u_F] = \theta - p_F - b\beta_F(1-\theta)N_F(\alpha\theta - d)$. With heterogeneous client negligence, the MSSP's profit under the F-Contract is $\pi_F(p_F) = p_F N_F - \int_{\theta_F}^{1}(b\beta_F(1-\theta)N_F d N_F)\, d\theta - \Gamma(s)$. Similarly, under the R-Contract, the client utility is $E[u_R] = \theta - p_R - b\beta_R(1-\theta)N_R\alpha\theta(1-\varphi)$, and the MSSP's expected profit is $\pi_R(p_R) = p_R N_R - \int_{\theta_R}^{1}(b\beta_R(1-\theta)N_R\varphi\alpha\theta)d\theta - \Gamma(s)$. In the M-Contract, the expected utilities for clients opting for the fixed and proportional components are respectively $E[u_{MF}] = \theta - p_M - b\beta_F(1-\theta)N_M(\alpha\theta - d)$, $E[u_{MR}] = \theta - p_M - b\beta_R(1-$

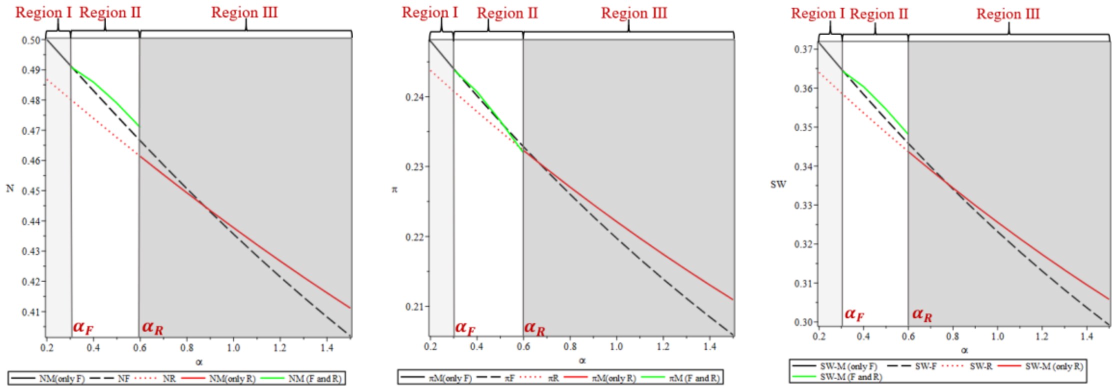
$\theta)N_M(1-\varphi)\alpha\theta$. The MSSP's expected profit under the M-Contract is then $\pi_M = p_M N_M - \int_{\theta_{MF}}^{\theta_{MR}}(b\beta_F(1-\theta)N_M dN_{MF})\ d\theta - \int_{\theta_{MR}}^{1}(b\beta_R(1-\theta)N_M\varphi\alpha\theta)\ d\theta - \Gamma(s)$.

Due to the increased complexity of solving the model analytically, we compare the equilibrium outcomes of the three contracts through numerical analysis. The resulting equilibria under varying levels of security loss risk are presented in Table B2.

Table B2. Comparison of equilibrium outcomes across the three contracts under heterogeneous negligence

α	${N_F}^*$	${N_R}^*$	${N_M}^*$	${\pi_F}^*$	${\pi_R}^*$	${\pi_M}^*$	${SW_F}^*$	${SW_R}^*$	${SW_M}^*$
0.2	0.5000	0.2479	0.4868	0.2437	0.4985	0.2485	0.3716	0.3640	0.3709
0.3	0.4914	0.2439	0.4803	0.2407	0.4910	0.2438	0.3648	0.3587	0.3645
0.4	0.4830	0.2401	0.4738	0.2378	0.4858	0.2407	0.3583	0.3535	0.3603
0.5	0.4746	0.2364	0.4675	0.2350	0.4789	0.2365	0.3519	0.3485	0.3545
0.6	0.4664	0.2328	0.4613	0.2322	0.4709	0.2319	0.3458	0.3437	0.3481
0.7	0.4584	0.2293	0.4552	0.2296	0.4623	0.2272	0.3398	0.3389	0.3414
0.8	0.4506	0.2260	0.4492	0.2270	0.4536	0.2225	0.3340	0.3343	0.3349
0.9	0.4429	0.2228	0.4434	0.2245	0.4448	0.2180	0.3285	0.3299	0.3285
1.0	0.4355	0.2197	0.4376	0.2221	0.4362	0.2137	0.3231	0.3255	0.3224
1.1	0.4283	0.2167	0.4321	0.2197	0.4278	0.2095	0.3180	0.3213	0.3166
1.2	0.4214	0.2138	0.4266	0.2174	0.4196	0.2055	0.3130	0.3172	0.3110
1.3	0.4146	0.2111	0.4213	0.2152	0.4117	0.2017	0.3082	0.3133	0.3058
1.4	0.4081	0.2084	0.4161	0.2130	0.4040	0.1980	0.3036	0.3095	0.3007
1.5	0.4018	0.2058	0.4111	0.2109	0.3967	0.1945	0.2992	0.3058	0.2959

To facilitate a more intuitive comparison across the three contract types, we visualize the data in Figure B5. We find that our main findings continue to hold even when client negligence is valuation-dependent. The M-Contract, represented by the green part of Region II in Figure B5, outperforms both F- and R-Contract under certain conditions. Specifically, when the security loss risk is relatively moderate, the MSSP achieves the largest client base (Figure B5(a)) and the highest profit (Figure B5(b)) by offering the M-Contract, which also yields the greatest social welfare (Figure B5(c)). Moreover, Figures B5(b) and B5(c) illustrate that, around α_R, the MSSP privately prefers the F-Contract while the social planner prefers the M-Contract. Therefore, accounting for heterogeneous client negligence does not alter our core conclusions, further confirming the structural robustness of our findings.



(a) Comparison of client base among the three contracts

(b) Comparison of MSSP profitability among the three contracts

(c) Comparison of social welfare among the three contracts

$d = 0.1, \varphi = 0.2, \beta_F = 1.3, \beta_R = 1.1, b = 0.5$

Figure B5. The optimal strategy under heterogeneous client negligence

B.4. Heterogeneity in Proportional Compensation Ratio

In the main model, we assume that the proportional compensation ratio φ is uniform across all clients. In this section, we further relax that assumption by allowing the compensation ratio to depend on clients' heterogeneous valuations. Specifically, we posit that high-valuation clients receive a higher compensation ratio, consistent with their heightened aversion to security loss risks and preference for enhanced protection. This relationship captures a more nuanced form of contract design in which the MSSP can tailor compensation terms to align with client heterogeneity. We incorporate this client heterogeneity by adjusting the compensation parameter from φ to $\boldsymbol{\varphi\theta}$, ensuring that these clients receive a proportionally larger payout following a breach.

Since the proportional compensation ratio applies only to the pure R-Contract and the M-Contract, the client's utility and the MSSP's profit under the F-Contract are unaffected. Under the R-Contract, however, the client's expected utility becomes $E[u_R] = \theta - p_R - b\beta_R N_R \alpha\theta(1 - \varphi\theta)$. In the M-Contract, the expected utilities for clients opting for the fixed and proportional components are respectively $E[u_{MF}] = \theta - p_M - b\beta_F N_M(\alpha\theta - d)$, $E[u_{MR}] = \theta - p_M - b\beta_R N_M(1 - \varphi\theta)\alpha\theta$. The MSSP's expected profit under the M-Contract is then $\pi_M = p_M N_M - b\beta_F N_M d N_{MF} - \int_{\theta_{MR}}^{1} (b\beta_R N_M \varphi\theta\alpha\theta)\ d\theta - \Gamma(s)$.

Therefore, the equilibrium solutions under the F-Contract remains $N_F^* = \frac{1-A+\alpha b\beta_F}{3\alpha b\beta_F}$ and $\pi_F^* = \frac{(1-A+\alpha b\beta_F)(2\alpha b\beta_F+A-1)(2+A-\alpha b\beta_F)}{27(\alpha b\beta_F)^2} - \Gamma(s)$. However, because the compensation ratio is linked to client perceived value, solving analytically becomes challenging for both the R-Contract and the M-Contract. We therefore present the equilibrium outcomes through numerical simulation, as summarized in Table B3.

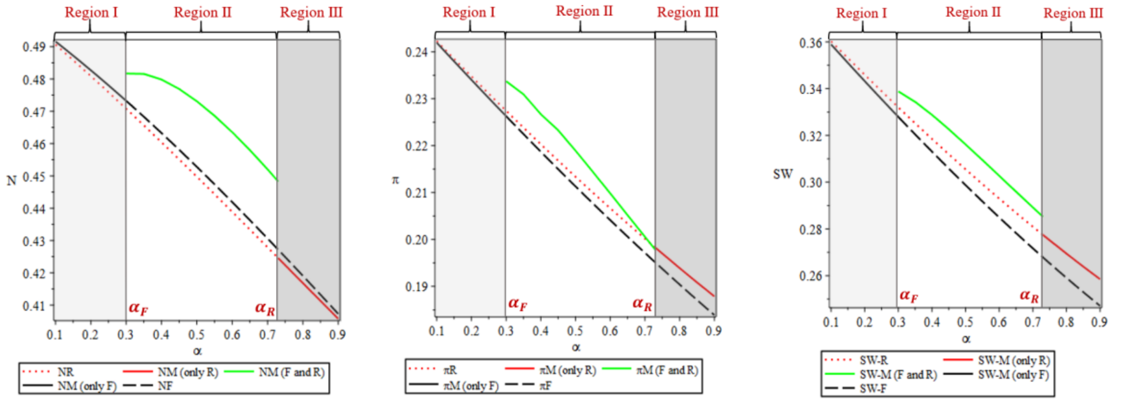
Table B3. Comparison of equilibrium outcomes across the R-Contract and M-Contract

α	${N_R}^*$	${N_M}^*$	${\pi_R}^*$	${\pi_M}^*$	${SW_R}^*$	${SW_M}^*$
0.1	0.4906	0.4234	0.2422	0.2030	0.3602	0.3005
0.15	0.4858	0.4578	0.2385	0.2260	0.3530	0.3309
0.2	0.4808	0.4726	0.2347	0.2335	0.3459	0.3401
0.25	0.4758	0.4792	0.2310	0.2350	0.3389	0.3413
0.3	0.4707	0.4816	0.2274	0.2337	0.3320	0.3388
0.35	0.4656	0.4815	0.2238	0.2309	0.3252	0.3343
0.4	0.4603	0.4798	0.2202	0.2266	0.3185	0.3288
0.45	0.4550	0.4769	0.2168	0.2232	0.3119	0.3226
0.5	0.4497	0.4731	0.2133	0.2189	0.3054	0.3161
0.55	0.4443	0.4686	0.2099	0.2144	0.2991	0.3094
0.6	0.4388	0.4635	0.2066	0.2098	0.2929	0.3027
0.65	0.4334	0.4579	0.2033	0.2051	0.2868	0.2959
0.7	0.4278	0.4520	0.2001	0.2005	0.2809	0.2892
0.73	0.4245	0.4482	0.1982	0.1978	0.2774	0.2852
0.8	0.4168	0.4391	0.1938	0.1914	0.2694	0.2760
0.85	0.4112	0.4323	0.1908	0.1869	0.2638	0.2696
0.9	0.4057	0.4252	0.1878	0.1825	0.2584	0.2633

Similarly, we present the simulation results visually to compare the differences among the three contracts when the compensation ratio varies with client valuations. As shown in Figure B6, our main findings remain robust even under the setting where the compensation ratio is tied to client valuations. Specifically, under moderate security loss risks, when the MSSP offers M-Contract, it achieves the optimal outcomes in terms of client base (Figure B6(a)), MSSP profitability (Figure B6(b)), and social welfare (Figure B6(c)), respectively. Around α_R, we also find that the MSSP's profit-maximizing contract decision can actually undermine social welfare.

Moreover, we find that under this setting, the MSSP serves an even larger client base, as shown in Figure B6(a). This is because when the proportional compensation ratio increases with client valuation, more medium- and high-valuation clients are drawn to the proportional component due to the higher compensation rate they would receive. However, this also raises the MSSP's expected compensation costs, prompting it to lower the service fee to attract additional low-valuation clients into the F-Contract. As a result, the total client base expands even further.

Taken together, these findings suggest that tailoring compensation terms to client heterogeneity not only strengthens the appeal of the M-Contract but also enables the MSSP to better segment the market and expand its reach, reinforcing the strategic value of offering a differentiated contract portfolio. This further corroborates the robustness and necessity of the proposed M-Contract framework.



(a) Comparison of client base among the three contracts

(b) Comparison of MSSP profitability among the three contracts

(c) Comparison of social welfare among the three contracts

$d = 0.1, \varphi = 0.2, \beta_F = 1.3, \beta_R = 1.1, b = 0.5$

Figure B6. The optimal strategy under heterogeneous proportional compensation ratio

B.5. A Comprehensive Decision Scheme for MSSP

Our baseline model assumes that the MSSP provides a standard security service, in which both its security effort and compensation fees are exogenously given based on industry practice. In this extension, we relax this assumption by endogenizing the MSSP's strategic decisions regarding security effort s, fixed compensation d, and proportional compensation ratio φ offered to clients. This extension captures the MSSP's ability to adjust its service effort and liability terms in a way that aligns with its contractual strategy and market conditions.

The sequence of decisions unfolds in three stages. In Stage 1, the MSSP determines the contract type and sets the associated terms—specifically, the service fee p and the compensation policy. In Stage 2, the MSSP chooses the level of security effort s. Finally, in Stage 3, clients observe the contract terms and the MSSP's effort level, and decide whether to purchase the security service. Under the F-Contract, a client's expected utility is given by $E[u_F] = \theta - p_F - k(1-s)^2\beta_F N_F \alpha\theta + k(1-s)^2\beta_F N_F d$, and the MSSP's profit becomes $\pi_F(p_F) = p_F N_F - k(1-s)^2\beta_F N_F d N_F - \varepsilon s^2$. Here, we model the effort cost as a convex quadratic function (Wu et al. 2025), with $\varepsilon \in (0,1)$ capturing the cost efficiency parameter. A higher ε implies that achieving a given level of security becomes more costly. The timing of the game is summarized in Figure B7.

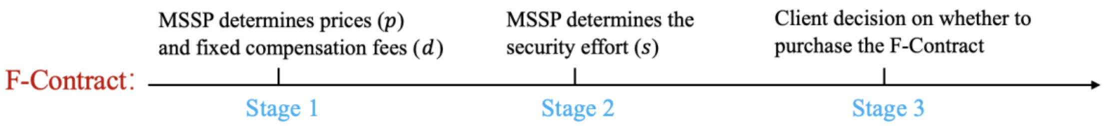


Figure B7. Sequence of events under the F-Contract

Under the R-Contract, the expected utility becomes $E[u_R] = \theta - p_R - k(1-s)^2\beta_R N_R \alpha(1-\varphi)\theta$, and the MSSP's profit becomes $\pi_R(p_R) = p_R N_R - \int_{\theta_R}^{1}(k(1-s)^2\beta_R N_R \alpha\varphi\theta)d\theta - \varepsilon s^2$. The timing of the game is summarized in Figure B8.

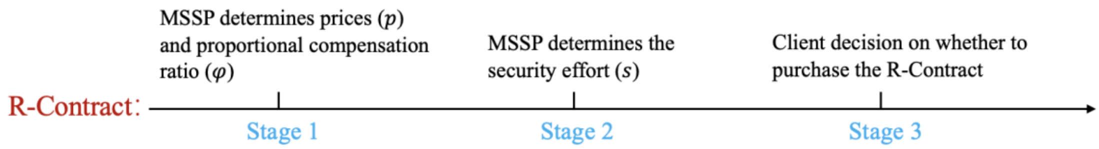


Figure B8. Sequence of events under the R-Contract

For the M-Contract, our objective is to assess whether the MSSP benefits from offering both contract types simultaneously. Specifically, we first determine the optimal outcome when the MSSP operates as a monopolist providing only a pure contract type. We then examine whether introducing the second contract type as a strategic complement can enhance the MSSP's baseline profit. The resulting equilibrium under the M-Contract is compared with the corresponding benchmark from the pure contract scenario. This analysis is conducted in two scenarios:

Scenario 1: Starting from a pure F-Contract. Suppose the MSSP initially offers only the F-Contract and achieves its equilibrium d^*. We then consider the case where the MSSP introduces the R-Contract alongside the existing F-Contract. Taking the equilibrium compensation d^* from the pure F-Contract as given, we solve for the equilibrium client base $N_M{}^*$ and the proportional compensation ratio φ^* under the M-Contract. The equilibrium outcomes are then compared with those under the pure F-Contract. The timing and sequence of this scenario are illustrated in Figure B9.

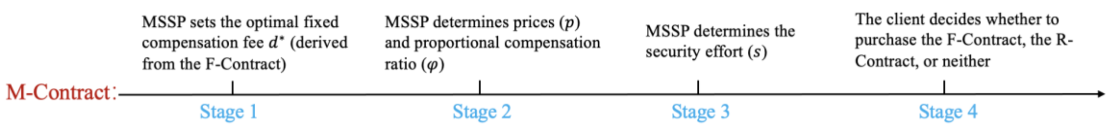


Figure B9. Sequence of events under the M-Contract

Scenario 2: Starting from a pure R-Contract. Similarly, suppose the MSSP initially offers only the R-Contract and achieves its optimal equilibrium φ^*. We then examine the case where the MSSP adds the F-Contract to its offering. Taking the equilibrium φ^* from the pure R-Contract as given, we solve for the optimal client base ${N_M}^*$ and the fixed compensation d^* under the M-Contract. The resulting equilibrium is compared with that of the pure R-Contract. The timing and sequence of this scenario are illustrated in Figure B10.

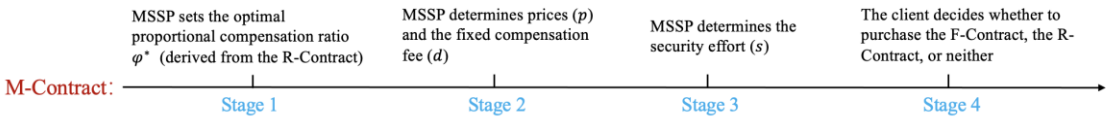


Figure B10. Sequence of events under the M-Contract

We derive the equilibrium outcomes for each contract via backward induction. Taking the F-Contract as an example, we first obtain the optimal effort level by taking the first-order condition with respect to s.

$$s^* = \frac{{N_F}^2 k\beta_F d}{{N_F}^2 k\beta_F d+\varepsilon} \tag{B4}$$

Substituting s^* into the client's utility function and the MSSP's profit function, we then solve for the price by setting $E[u_F] = 0$.

$$p_F = \frac{-2k^2{\beta_F}^2 d^2{N_F}^5+k^2{\beta_F}^2 d^2{N_F}^4-2{N_F}^3 k\beta_F d\varepsilon+{N_F}^2\alpha k\beta_F\varepsilon^2+2{N_F}^2 k\beta_F\varepsilon d-N_F k\beta_F\varepsilon^2(\alpha-d)-\varepsilon^2(N_F+1)}{({N_F}^2 k\beta_F d+\varepsilon)^3} \tag{B5}$$

Substituting Equation (B5) into the MSSP's profit, and taking the first-order partial derivatives with respect to N_F and d, we have

$$\frac{\partial\pi_F}{\partial N_F} = \frac{1}{({N_F}^2 k\beta_F d+\varepsilon)^2}\Big(k^2{\beta_F}^2 d^2{N_F}^5(N_F-2N_F-6\varepsilon)-{N_F}^4k^2{\beta_F}^2\varepsilon d(\alpha\varepsilon-3d)-$$

$$2{N_F}^3\varepsilon^2 k\beta_F d\big(3+k\beta_F(2d-\alpha)\big)+3{N_F}^2\varepsilon^2 k\beta_F(\alpha\varepsilon+d)-2N_F\varepsilon^3(\alpha k\beta_F+1)+\varepsilon^3\Big). \tag{B6}$$

$$\frac{\partial\pi_F}{\partial d} = \frac{2\varepsilon^2{N_F}^4k^2{\beta_F}^2(\alpha(N_F-1)+d)}{({N_F}^2 k\beta_F d+\varepsilon)^3}\ . \tag{B7}$$

By solving these two equations simultaneously, we can determine the equilibrium client base ${N_F}^*$ and the fixed compensation d^* for the MSSP. Given the complexity of deriving analytical solutions, we conduct numerical simulations to obtain the equilibrium outcomes. Table B5 presents the results for Scenario 1, which compares the F-Contract with the M-Contract under the same fixed compensation d^*. Table B6 reports the results for Scenario 2, where the R-Contract is compared with the M-Contract under the same proportional compensation ratio φ^*.

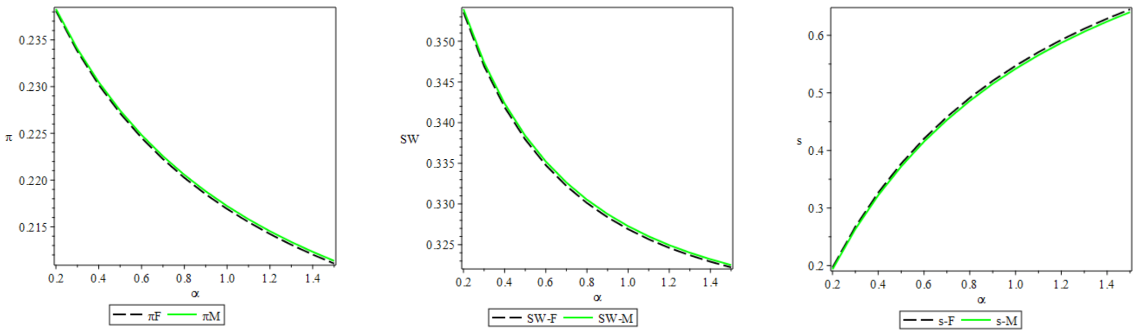
Table B5. Variation of equilibrium solutions under the F-Contract and M-Contract

α	$s_F{}^*$	$s_M{}^*$	$N_F{}^*$	d^*(F)	$N_M{}^*$	φ^*(M)	$\pi_F{}^*$	$\pi_M{}^*$	$SW_F{}^*$	$SW_M{}^*$
0.2	0.1967	0.1934	0.4899	0.1020	0.4900	0.5405	0.2380	0.2382	0.3535	0.3539
0.3	0.2675	0.2635	0.4873	0.1537	0.4874	0.5434	0.2337	0.2340	0.3469	0.3473
0.4	0.3267	0.3221	0.4856	0.2057	0.4857	0.5453	0.2301	0.2304	0.3418	0.3423
0.5	0.3770	0.3721	0.4845	0.2577	0.4846	0.5465	0.2271	0.2274	0.3379	0.3383
0.6	0.4203	0.4152	0.4839	0.3096	0.4839	0.5472	0.2245	0.2248	0.3347	0.3352
0.7	0.4581	0.4529	0.4836	0.3614	0.4836	0.5475	0.2222	0.2225	0.3322	0.3326
0.8	0.4913	0.4860	0.4835	0.4131	0.4834	0.5477	0.2202	0.2205	0.3301	0.3305
0.9	0.5207	0.5155	0.4835	0.4648	0.4834	0.5477	0.2184	0.2187	0.3283	0.3287
1.0	0.5470	0.5418	0.4836	0.5163	0.4836	0.5475	0.2169	0.2172	0.3269	0.3272
1.1	0.5706	0.5654	0.4838	0.5677	0.4837	0.5473	0.2154	0.2158	0.3256	0.3260
1.2	0.5919	0.5868	0.4841	0.6190	0.4840	0.5470	0.2142	0.2145	0.3246	0.3249
1.3	0.6113	0.6062	0.4843	0.6702	0.4842	0.5467	0.2130	0.2133	0.3236	0.3240
1.4	0.6289	0.6239	0.4846	0.7214	0.4845	0.5464	0.2120	0.2123	0.3229	0.3232
1.5	0.6450	0.6401	0.4850	0.7724	0.4848	0.5460	0.2110	0.2113	0.3222	0.3225

Table B6. Variation of equilibrium solutions under the R-Contract and M-Contract

α	$s_R{}^*$	$s_M{}^*$	$N_R{}^*$	φ^*(R)	$N_M{}^*$	d^*(M)	$\pi_R{}^*$	$\pi_M{}^*$	$SW_R{}^*$	$SW_M{}^*$
0.2	0.0688	0.1041	0.4857	0.2256	0.4904	0.0512	0.2367	0.2377	0.3504	0.3524
0.3	0.1696	0.2268	0.4811	0.4225	0.4884	0.1264	0.2315	0.2339	0.3436	0.3470
0.4	0.2497	0.3137	0.4784	0.5213	0.4860	0.1978	0.2274	0.2304	0.3386	0.3423
0.5	0.3154	0.3809	0.4769	0.5805	0.4842	0.2674	0.2241	0.2274	0.3349	0.3384
0.6	0.3707	0.4352	0.4761	0.6199	0.4828	0.3360	0.2212	0.2247	0.3319	0.3352
0.7	0.4178	0.4804	0.4759	0.6479	0.4819	0.4038	0.2188	0.2224	0.3296	0.3326
0.8	0.4585	0.5186	0.4760	0.6687	0.4814	0.4709	0.2168	0.2203	0.3278	0.3304
0.9	0.4940	0.5517	0.4763	0.6848	0.4810	0.5377	0.2150	0.2185	0.3263	0.3286
1.0	0.5253	0.5804	0.4767	0.6975	0.4809	0.6039	0.2134	0.2168	0.3250	0.3271
1.1	0.5531	0.6058	0.4772	0.7079	0.4809	0.6699	0.2120	0.2154	0.3240	0.3259
1.2	0.5779	0.6283	0.4777	0.7164	0.4811	0.7360	0.2108	0.2141	0.3231	0.3248
1.3	0.6001	0.6483	0.4783	0.7236	0.4812	0.8012	0.2097	0.2129	0.3224	0.3238
1.4	0.6202	0.6663	0.4789	0.7296	0.4815	0.8661	0.2087	0.2118	0.3217	0.3231
1.5	0.6385	0.6829	0.4794	0.7349	0.4818	0.9326	0.2078	0.2108	0.3212	0.3224

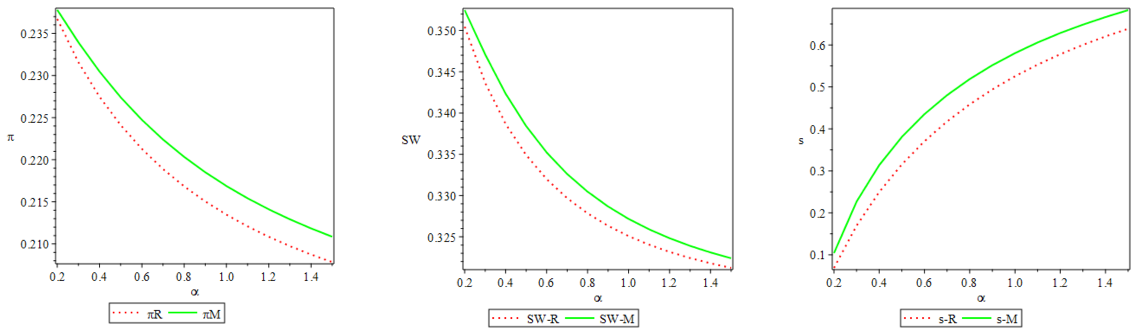
To facilitate intuition, we present simulation results in Figures B11 and B12. We find that, in both Scenarios 1 and 2, although higher security loss risk intuitively induces greater endogenous security effort by the MSSP (Figure B11(c) and B12(c)), our main results remain structurally robust. Specifically, under all three contract forms, the MSSP's profit and social welfare continue to decrease with the security loss risk, yet offering the M-Contract emerges as the optimal strategy for both the MSSP (Figure B11(a) and B12(a)) and the social planner (Figure B11(b) and B12(b)). Overall, endogenizing the MSSP's effort and compensation terms does not alter our central conclusions, underscoring the robustness and generalizability of the analysis. This result further suggests that the M-Contract serves as a flexible instrument for simultaneously managing client heterogeneity and improving the efficiency of security effort.



(a) Comparison of MSSP profitability | (b) Comparison of social welfare | (c) Comparison of MSSP effort

$\beta_F = 1.2, \beta_R = 1.1, k = 0.5, \varepsilon = 0.06$

Figure B11. Comparison between the F-Contract and M-Contract under Scenario 1.



(a) Comparison of MSSP profitability | (b) Comparison of social welfare | (c) Comparison of MSSP effort

$\beta_F = 1.2, \beta_R = 1.1, k = 0.5, \varepsilon = 0.06$

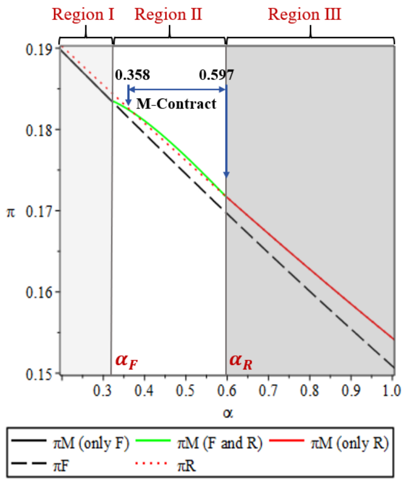
Figure B12. Comparison between the R-Contract and M-Contract under Scenario 2.

B.6. Monte Carlo Robustness Analysis

In practice, accurately measuring key parameters in security environments is often challenging. Measurement errors and unobservable environmental factors make it difficult to precisely determine values such as the hacker incentives b from an attack, the security loss risk α, or the client negligence level β. To evaluate whether our core conclusions hold under such uncertainty, this section introduces stochastic variation into these critical parameters.

As specified in our baseline model, the attack probability is given by $q = k(1-s)^2 N\beta$. For simplicity, we define $b = k(1-s)^2$ in the main text. Thus, to capture the uncertainty in the hacker's attack probability, we treat b as a random variable. In addition, we introduce stochasticity to account for uncertainty related to the client negligence (β_F and β_R) and the security loss risk α. Specifically, we assume b follows a Beta distribution, β_F and β_R follow distinct uniform distributions, and α follows a truncated normal distribution.

Before incorporating uncertainty, we first establish a deterministic benchmark using the equilibrium solutions derived in the main text. By setting the parameters to $\beta_R = 1.2, \beta_F = 1.4$, and $b = 0.3$, we identify the boundaries of the M-Contract (as defined in Lemma 3) at $\alpha_F = 0.319$ and $\alpha_R = 0.597$. We find that within the specific risk interval $\alpha \in (0.358, 0.597)$, the M-Contract yields the highest profit among the three contract types. The equilibrium outcomes under these point estimates are summarized in Figure B13, serving as the deterministic baseline for our subsequent Monte Carlo simulations.



$\beta_R = 1.2, \beta_F = 1.4, d = 0.1, \varphi = 0.2, b = 0.3, \Gamma(s) = 0.05$

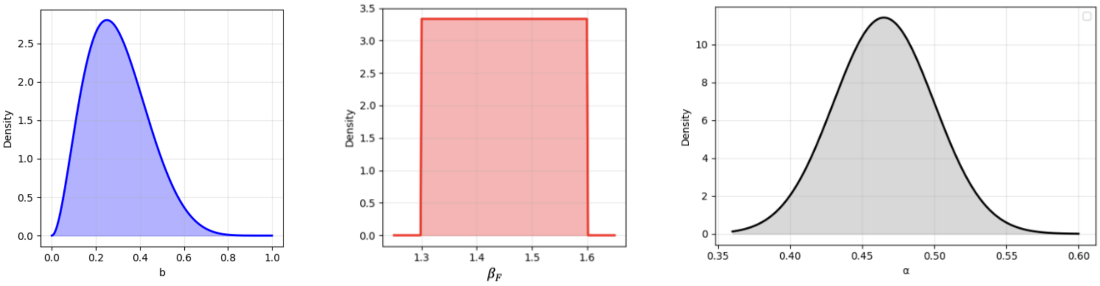
Figure B13. The optimal strategy for the MSSP

Next, we employ Monte Carlo simulation to validate the robustness of our findings. Specifically, we assume:

(i) b follows a Beta distribution with parameters (3,7), which has a mean of 0.3 and captures a moderate level of uncertainty. The majority the probability mass lies between approximately 0.07 and 0.63, reflecting the fact that the attack probability tends to be relatively low but remains subject to notable variation, as shown in Figure B14(a).

(ii) β_F is drawn from a uniform distribution over [1.3,1.6], capturing the range of possible client negligence for the F-Contract. Similarly, β_R is drawn from a uniform distribution over [1.2,1.4], representing the uncertainty in client negligence under the R-Contract, as shown in Figure B14(b). This partial overlap between the two parameter ranges reflects our underlying assumption that β_F can be either greater than or less than β_R.

(iii) α is assumed to follow a normal distribution truncated to the interval [0.358,0.597], ensuring that the draws remain within the theoretically optimal region identified in the deterministic analysis, as shown in Figure B14(c).



(a) Distribution of b (b) Distribution of β_F (c) Distribution of α

Figure B14. Probability distribution of uncertain parameter values.

All other parameters remain fixed at their baseline values ($d = 0.1, \varphi = 0.2, \Gamma(s) = 0.05$). It is worth noting that the simulation results remain robust to variations in the distribution of these parameters. We conduct a Monte Carlo simulation comprising 10,000 iterations. In each iteration, a set of random draws $(\alpha, b, \beta_F, \beta_R)$ is generated according to the distributions specified above. For each draw, we compute the corresponding MSSP profitability for the three contract types (F-Contract, R-Contract, and M-Contract). The results of the Monte Carlo simulation (hereafter Sim) are presented in Table B7.

Table B7. The results of the Monte Carlo simulation

Sim	α	b	β_F	β_R	π_F	π_R	π_M
1	0.4539	0.1407	1.5599	1.3202	0.187712	0.188602	0.189448
2	0.4842	0.1944	1.3913	1.3050	0.183915	0.183525	0.184736
3	0.4591	0.5507	1.4099	1.2912	0.157810	0.158235	0.160451
4	0.4927	0.4340	1.4823	1.2341	0.162214	0.165716	0.167585
5	0.4124	0.1605	1.4320	1.2244	0.188301	0.189036	0.189582
…	…	…	…	…	…	…	…
10000	0.4816	0.1122	1.3111	1.3518	0.191227	0.190102	0.190963

The M-Contract is considered optimal if its profit strictly exceeds those of the other two pure contracts simultaneously (such as the No.1 simulation in Table B7, where $0.189448 > max\{0.188602, 0.187712\}$). The proportion of iterations in which this condition holds is then recorded as the probability that the M-Contract remains optimal under parameter uncertainty. Overall, across all 10,000 iterations, the M-Contract is optimal in approximately **86.31%** of the cases when α is drawn from the truncated normal distribution over [0.358, 0.597]. This high probability indicates that even in the presence of measurement errors and parameter uncertainty, the M-Contract continues to dominate the other two contract types for the MSSP.

To provide a more granular comparison against the deterministic baseline in Figure B13, we partition the theoretically optimal interval $\alpha \in [0.358, 0.597]$ into 40 equally spaced sample points. For each fixed α, we independently generate 10,000 sets of random parameters (b, β_F and β_R) based on their respective distributions, and compute the profit of each contract. Based on these 400,000 profit realizations, we calculate the mean and standard deviation of the MSSP profitability for each contract. As shown in Figure B15, the solid lines represent the mean profit for each contract (green for the M-Contract, black for the F-Contract, and red for the R-Contract), and the shaded bands denote the mean ± 1 standard deviation, illustrating the dispersion of profits.

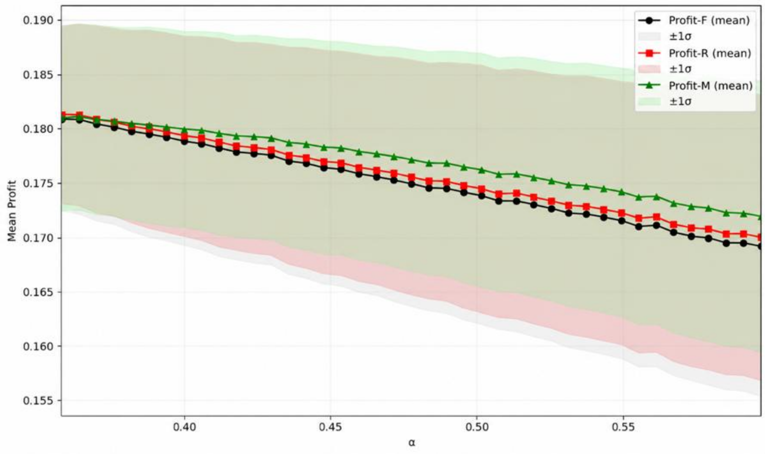


Figure B15. Expected MSSP profit under stochastic parameter variation (Monte Carlo simulation)

In the deterministic case (i.e., with fixed b, β_F and β_R, Figure B13), the M-Contract is optimal over the entire $\alpha \in (0.358, 0.597)$ interval. However, the introduction of parameter randomness reveals a nuanced boundary effect: within the low-risk interval $\alpha \in (0.358, 0.373)$, the mean profit of the M-Contract is slightly below that of the other contracts and is no longer optimal. Yet, over the broader interval $\alpha \in (0.373, 0.597)$, the M-Contract remains optimal. Nevertheless, in line with the deterministic benchmark (Figure B13), the profits of all three contracts decline as α increases, and the menu-based contract's mean profit stays the highest across most of the α range. Furthermore, considering either the upper or lower bound of the standard deviation band, the light-green shaded band of the M-Contract generally outperforms those of the other pure-strategy contracts.

Collectively, the Monte Carlo analysis demonstrates that the superiority of the M-Contract for the MSSP remains highly robust under realistic levels of parameter uncertainty. Even when key parameters such as the hacker incentives, client negligence, and security loss risk are subject to stochastic variation, the M-Contract continues to be optimal in a substantial majority of cases. These findings strengthen the main contribution of the paper by showing that the theoretical predictions are not sensitive to measurement errors or uncertain information. Consequently, the Monte Carlo analysis provides a robustness check for the conclusion that the M-Contract can serve as an effective and resilient mechanism for the MSSP in the presence of uncertainty.

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