Case Article:

DeLand Crayon Company: An Application of Traveling Salesman Problem to Production Scheduling with Sequence Dependent Setup Times

by

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Abstract

NP-complete problems such as the Traveling Salesman Problem (TSP) play a prominent role in most advanced undergrad/graduate courses in discrete optimization modeling. Teaching such an important topic from a purely mathematical perspective, without discussing specific applications, may result in reduced student interest and motivation. DeLand Crayon Company case introduces students to an application of the TSP to a practical production-planning scenario with realistic data. In crayon production, proper sequencing of colors of production can result in significant saving in total production time. The primary objective of the case is to provide students with hands-on experience in applying the TSP model. The case requires the students to consider variations of the basic model with the goal of encouraging them to think logically through the process of mathematical modeling and to consider the decision-making implications of their models. This case study can be used in undergraduate and graduate level courses in linear/integer programming or in production planning. It was implemented as part of a term project in a graduate operations research course with overwhelmingly positive feedback.

Key Words: Traveling Salesman Problem, Production Scheduling, Case Study, Algebraic Modeling Systems

1 Introduction

The Traveling Salesman Problem (TSP) is a very important example in integer programming and combinatorial optimization because of its intuitive and easy description, mathematical challenges associated with its solution, and its many potential applications. TSP is also an excellent means of introducing integer programming [27].

Models of TSP's many variations (e.g., multiple salesmen, multiple depots, time windows and tour length constraints), algorithms and heuristics to solve them, and their real life applications are reported extensively in operations research/management science literature. A quick (non-exhaustive) survey reveals the rich variety of applications of TSP, including drilling of printed circuit boards [25], collection of spent batteries [29], kidney transplantation [2], telescope scheduling [7], delivering meals on wheels [3], NBA team travel scheduling [4], milk collection [6], warehouse order picking [10], logistics load planning [22], optimizing delivery routes [12, 17], production scheduling [26], printing multi-edition periodicals [16], construction tower crane scheduling [32], waste management [28], scheduling games in Canadian football league [23], and scheduling umpires in major league baseball [31]. In a unique and interesting application, TSP has also been applied to minimize waste in wallpapering [15]! TSP problems with uncertainties in travel costs [19, 21] and topology [20] have also been studied.

The wide variety of actual and potential applications suggests the use of case studies in introducing and teaching the TSP. In spite of this potential, there are very few published case studies for the non-theoretically oriented students to appreciate the beauty and the practical value of TSP. A survey of all articles that appeared in the *INFORMS Transactions on Education* since its inception in 2000 revealed that there are many articles on the broader topic of Integer Programming, its formulation, solution, and applications, while there were only four articles that listed TSP in their key words. One of the four dealt with the use of AMPL (A Modeling Language for Mathematical Programming) [24] and the other three [11, 7, 8] present puzzles and teaching aids.

We believe that case studies that connect NP-complete problems such as TSP to real life applications would enhance graduate curricula by stimulating the interest of engineering and business students in the practical value of optimization techniques. We are confident that this case article makes a meaningful contribution in filling this gap in instructional support material.

2 Overview of the Case

This paper deals with a comprehensive application of the Traveling Salesman Problem (TSP) to the production planning problem faced by a fictitious crayon manufacturing company. The case is based on a project undertaken by a graduate student [18] in a crayon production facility under the supervision of one of the authors.

DeLand Crayon Company (DCC) manufactures molded wax crayons. DCC maintains a broad product line of crayons in three sizes and nearly fifty colors. Sixteen colors of the standard sized crayons account for more than 90% of total demand. This case deals with these high demand crayons. These are produced on automatic rotary molding machines, where pre-blended molten wax is poured onto cavities, cooled and hardened before labeling and packing. The variety of crayon colors, labels and package size results in a complex production-planning problem. Two key factors in planning crayon production are:

1. Sequencing of the production of crayon colors (correct sequence of colors can lead to significant saving in changeover times and the resultant loss of production capacity), and

2. Determination of batch size (number of crayons of each color to be produced during each setup).

Currently, color sequencing is done using several effective sequences developed and improved over the years. Batch sizes for the individual colors are determined based on current and past demand, inventory, and rules of thumb for the minimum batch size. Use of operations research tools offers great potential for improving operational efficiency.

The case provides the students with sufficient details of the crayon manufacturing process to appreciate the complexities of the production planning problem. They will be guided through the demand data and the production capabilities to help them identify the appropriate mathematical tool (TSP in this case) to determine the optimal production plan. After solving the basic model, students will be directed to consider variations of the model with the goal of enhancing their ability to model progressively complex decision making problems. Through this case students will:

- **Goal 1:** Apply the TSP solution to an interesting, real life, production planning problem with sequence dependent setup times.

- **Goal 2:** Relate some extensions of the basic TSP formulations such as asymmetric TSP, and multiple TSP to different variations of the original production planning problem.

- **Goal 3:** Synthesize optimization problems and interpret their solutions into management decisions.

Question 1 of the case addresses Goals 1&3, Question 2 addresses Goal 3, and Question 3 addresses Goal 2. Question 4 is mind expanding exercise to enable students to think beyond the basic TSP considered in questions 1, 2 and 3. Please refer to the case description for complete details of the questions.

3 Pedagogical benefits of the case

A major challenge in teaching OR/MS courses, especially to students for whom OR/MS is not the primary field of study (e.g., MBA students), is to motivate them to recognize the need for OR/MS tools in decision making [5]. Examples based on specialized applications generally require the instructor and students to spend a considerable amount of time in understanding the background before the student can effectively model the decision problem under consideration. In such cases, *students lose track of the OR/MS aspects when drowning in the details* [30]. This case presents a reasonable compromise by presenting a realistic scenario in an efficient and compact manner.

Instructional exercises used in LP/IP courses fall on a spectrum. At one end of the spectrum are the end of chapter exercises. By virtue of their place in the textbook, these exercises provide a clear direction on the problem to be solved and the method to be used. In other words, they are structured in a manner that fully defines a scenario in which the objective function and constraints are clearly described. Such exercises while giving students the opportunity to practice applying the methods being taught at that moment, do little to help students recognize an opportunity to optimize a decision problem. At the other end of the spectrum, the case method of teaching goes beyond doing an exercise and helps students solve a problem. It brings the complexity of the real world and all its nuances (to the extent possible) to the classroom and places the student in the role of a decision maker. A key benefit of case analysis is for the student to wade through a lot of material and decide what is relevant and what is not. The goal of case writing is to include an occasional distraction that makes students experience the chaos in the real life and create structure out of the chaos.

In this case study, we hope to offer a comprehensive and integrative example, for the students to connect the dots and go beyond the basic model to envision extensions of the basic model and examine the scenarios for which the model may or may not be appropriate. This approach of progressively adding complexities to a basic decision problem is consistent with the commonly held wisdom that teaching new concepts and material is more effective when it is explicitly linked to material that students are already familiar with.

4 Possible Uses of the Case

Strong methodological and application components in the case makes it suitable for either a course in linear/integer programming or an advanced course in production planning in undergraduate or graduate programs in operations research, industrial engineering or business administration.

In an undergraduate course Linear/Integer Programming, the discussion can be limited to the use of TSP with a single rotary machine (Questions 1& 2). A graduate course, can include the two machine variation (Questions 1-3) . The teaching note provides answers to both these models. In an advanced production planning course, this case can be used to discuss production scheduling of several products with sequence dependent setup costs and setup times (Questions 1-4).

Since the asymmetric TSP formulation and TSP with multiple salesmen extension are usually not part of traditional textbooks, the instructor can use this case in order to motivate literature search and allow students to identify relevant literature for these extensions (detailed instructions are included with the teaching note).

In all cases, depending on the audience, one or more of the following extensions of the case can be discussed to reinforce students' modeling skills and emphasize the decision making implications addressing the third educational goal of this case.

- Can the manufacturing facility manage a 10% increase in demand?
- Is the current practice of producing all 16 colors every week the best? Can the batch size be reduced? what are the implications?
- How can the company manage if the weekly demand marginally exceeds the production capacity? (produce every two weeks? What are the cost implications? Carry additional inventory?)
- If there is excess production capacity, can that be used to produce some or all colors more than once a week to reduce the inventory?
- For the two machine case, can we optimize $Min(T_1, T_2)$ where, T_1 and T_2 are the cycle times for the two machines defined as the total of production and changeover times? What are the implications of significantly different values for T_1 and T_2? A simple approach would be to use the excess capacity on one of the machines to produce non-standard products. How else can the production facility be utilized more efficiently? By appropriately restricting the scope of discussion of the possible variations, this course can be used in undergraduate, graduate and even in Ph. D. level courses.

Additional details and further teaching suggestions are provided in the accompanying case Teaching Notes.

5 Classroom Experience

The DeLand Crayon Company case was used as a class project in a graduate level, first course in Operations Research in the Industrial Engineering and Management Systems Department at University of Central Florida, Orlando, Florida in Fall 2017. The audience consisted of 56 masters and first year doctoral students, from the disciplines of

Economics (6), Mathematics (2) and Engineering (48, mostly from Industrial and Systems Engineering).

For most students, this course was the first formal exposure to linear programming and its extensions. A secondary goal for the case was to help students consolidate their skills in using GAMS (General Algebraic Modeling System) [14] to solve linear and integer programming problems.

There are three modules in this course devoted to linear programming models and algorithms; network flows, integer programming, and optimization solvers/software; and game theory, applied stochastic processes and queuing models. Prerequisites include basic calculus; linear algebra and matrix analysis; and probability and statistics. Classrooms are equipped with projectors that can display computer screens as well as handwritten notes, enabling the instructor to develop solutions manually and to demonstrate coding and running optimization software. All class sessions are recorded and available to the students during the semester. We spend approximately 5, 6, and 4 weeks on modules 1, 2 and 3 respectively, with three class hours each week. The DCC case was assigned in the 8th week in module 2 after two 50-minute class sessions on integer programming formulations (including TSP), and two 50-minutes sessions on the use of solvers and modeling languages, with emphasis on GAMS. Students had four weeks to complete the case analysis and typically worked in groups of two. Most of the work on the case is done outside the classroom, while the relevant theory and coding techniques are discussed in class. During each of these four weeks, about 15 minutes was devoted each week to answering student questions on the case and the associated GAMS coding. Students were encouraged to seek additional clarifications and guidance by consulting the instructor and/or the teaching assistant individually. We estimate that each group spent about eight hours on the project outside the classroom for the case analysis, coding, and report writing. All groups correctly used TSP to solve the scheduling problems in the case.

6 Software Usage

Although TSP is an NP-complete problem, solver breakthroughs enable one to solve large size TSPs on a personal computer. The instance required for the DCC case can be also be solved in Excel (even the non symmetric instance that requires double the number of nodes) using either the standard Solver in Excel or commercial Excel packages such as the Analytic Solver Platform of Frontline solvers [13]. More details are provided in the Teaching Notes.

The DCC case can also be solved using one of the algebraic modeling languages for mathematical optimization such as GAMS or AMPL, which allow users to build optimization models efficiently and solve them using various solvers such as CPLEX, BARON, etc. We used GAMS (General Algebraic Modeling System) in the course. Size limited version of GAMS (up to 300 variables and constraints) is available for free download from [14]. For larger problems, students have free access to the NEOS server, (hosted by the Wisconsin

Institute for Discovery at the University of Wisconsin in Madison) which supports GAMS and several other modeling languages.

We encourage instructors to start solution process using simple intuitive heuristics. For example, one can easily construct a feasible solution manually (or with minimal coding) using a greedy heuristic, where starting with any node, the salesman always moves to the nearest un-visited node until returning to the starting node. The improvement from the heuristic solution to the optimum solution obtained using exact algorithms can offer a sense of discovery for the students. The value of heuristics in dealing with intractable large-scale problems can also be discussed.

7 Student Responses

At the end of the semester, an online survey was administered through *Qualtrics* to the students consisting of six directed questions and four open ended questions. Responses were collected anonymously and students were informed that their response in the survey wouldn't affect their grade for the course. For the directed questions (listed below), students were asked to respond on a 5-point Likert scale, with 1=Totally agree, 2=Agree, 3= Neither agree nor disagree, 4=Disagree, 5= Totally disagree. There were a total of 52 responses (enrollment was 53).

1. The DeLand Crayon Company case gave me a good sense of a real life application of the traveling salesman model.
2. The DeLand Crayon Company case has given me insight into the use of optimization modeling for solving real-life production scheduling problems.
3. The DeLand Crayon Company case has improved my mathematical modeling skills.
4. The details provided in the DeLand Crayon Company case helped me appreciate the usefulness of optimization modeling much more than a typical textbook exercise.
5. In the DeLand Crayon Company case, progressively adding complexities to the basic decision problem improved my understanding of the modeling process.
6. The DeLand Crayon Problem helped me master the GAMS programming environment.

Student responses to these six questions are summarized in Table 1.

Student responses to the following four open-ended questions are summarized below. Verbatim listing of the comments is provided in the Appendix.

1. In your opinion, what are the differences between using a typical textbook exercise and a case such as the DeLand Crayon Company?

Response	1	2	3	4	5	Total
Q1	59.62	36.54	3.85	0.00	0.00	1.44
Q2	61.54	34.62	3.85	0.00	0.00	1.42
Q3	53.85	36.54	7.69	1.92	0.00	1.58
Q4	58.82	29.41	7.84	1.96	1.96	1.59
Q5	46.15	36.54	15.38	1.92	0.00	1.73
Q6	15.38	17.31	46.15	9.62	11.54	2.85

Table 1: Summary of Student Responses in percents, $n = 52$

2. Do you have suggestions on how to use cases such as this to enhance a mathematical modeling course?

3. Are there any negative aspects about using cases in an optimization course?

4. Provide any other comments on the case.

Students overwhelmingly commented on how textbook problems are directed, suggesting variables and constraints and generally led the students to the methodology to use. In contrast, DCC (and similar cases) to be more challenging, demanding and time consuming than a typical textbook exercise bcause it provides a detailed description of the circumstances without a clear structure. They felt that the additional effort in separating the relevant from the irrelevant in developing a model is well worth it and resulted in deeper understanding of the methodology. A few students expressed a desire for more direction in analyzing the case with better defined expectations. Students expressed a clear preference to using more cases to provide them more practice with using real world cases (in place of homework and tests with textbook exercises). Some students suggested that the case be assigned in parts, with each new part introduced based on a *right approach* for all previous parts. There was also a suggestion to add an additional module to the case dealing with sensitivity analysis.

The success of this approach is also evident from the request from students to add similar case studies to other sectors such as healthcare, defense and space to meet the needs of students from engineering, computer science, economics, and business. Students found the case relevant to their discipline and of potential use in their careers. There was a total of 14 responses referencing the usefulness of the case in their future career either directly, by using the term career, or indirectly by referencing terms that imply career such as real life. One student also expressed an interest in classroom discussion of papers about application of mathematical modeling in real life.

These comments stress the importance of a step-by-step approach, indicating that certain extensions are far from straightforward and, consequently, sufficient attention should be paid to explaining the corresponding constraint formulations.

Question 6 in the survey, dealing with how well the DCC case helped students in mastering the GAMS modeling language, received less than enthusiastic response (without being negative). One possible reason for this is that many students lacked adequate computer programming experience, making mastery of GAMS more challenging. It is also possible that students needed more exercises in using GAMS before attempting to model a TSP. A third reason might be the diversity in student backgrounds and educational level (the course was open to masters and doctoral students). Instructors of advanced masters or doctoral courses should consider requiring adequate prior coding experience if they plan to use GAMS or other similar modeling language. For undergraduate, MBA, or early master-level courses, use of Excel with or without add-ins such as Analytic Solver Platform may be adequate.

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Appendix

Selected Verbatim Comments from Students

(Student response was overwhelmingly positive. The following comments are selected based on the clarity of expression. A complete list of comments is available upon request.)

- My modeling skills definitely improved, and I was able to gain a functional understanding on using techniques from this course to solve a realistic problem.
- A case such as the DCC project gave me a more clear perspective of real life issues in optimization and how to apply the concepts discussed in class.
- Trying to figure out how to apply in real life seems to help you understand the subject.
- Real world problems are much more interesting and complex than textbook problems and make you have to think logically and appreciate the application of the optimization and OR apply all that we have learned from this course into a real world problems.
- It allowed me to use all sources and apply it to a real life application. This problem was more in depth than a typical textbook problem so I had to keep digging and trying new things and new ways to resolve the issue.
- It is awesome to practice such through DeLand Crayon Company. It makes it easier to relate the numbers to real world issues.
- Whereas a textbook problem typically avoids an interpretation of the problem, a case forces you to model the problem and think outside the instruction.
- It was a more realistic scheduling problem that consisted of deciding which color will be on which run and the time of each run.
- They stated that they learn best by applying the algorithms learned so I benefited a lot by using cases.
- Problems such as the DeLand Crayon Company give us a good sense of a real application of operation research.
- The DeLand Crayon Company is an EXCELLENT problem to find optimization. This is the kind of problem that I would always ask about ”real life” problems.