***ERDBEERENBURG*: MATHEMATICAL PROGRAM FORMULATION**

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——————— *Input Parameters* ———————

n = number of customers

T = maximum time (integer) in time horizon $[0, T]$

m = number of "machines" to make jams from strawberries

B_i = demand amount, in bottles of jams, of customer i

G_i = demand amount, in grams of strawberries, of customer i

g_{ik} = 1, if customer i requests for Grade k strawberries; 0, otherwise

DD_i = desired due date for customer i

d_{ij} = distance between locations i and j; location indices are 1: depot, 2: customer 1, 3: customer 2 etc.

INVENTORY

OC_k = cost per order for Grade k

UC_{jk} = unit cost of the j^{th} "cost step" for Grade k strawberries

C_{jk} = cut-off amount of Grade k strawberries for the j^{th} "cost step"

L_k = order limit for Grade k strawberries

L_T = total order limit

l_k = order lead time for Grade k strawberries

PRODUCTION SCHEDULING

SS = strawberry storage cost (per gram per day)

p_i = processing time for strawberries of customer i

DELIVERY

P = cost per pickup

α = tardiness cost

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β = earliness cost

JS = jam storage cost (per bottle per day)

DC = distance cost

$SPEED$ = truck travel speed

——————— *Decision Variables* ———————

<u>INVENTORY</u>

x_{ij} = 1, if strawberries of customer i are ordered on day j; 0, otherwise

y_{jk} = 1, if Grade k strawberries are ordered on day j; 0, otherwise

f^+_{jk} = amount of Grade k strawberries ordered (in grams) above C_{2k} for day j

s^+_{jk} = amount of Grade k strawberries ordered (in grams) between C_{1k} and C_{2k} for day j

o_i = order date for customer i

a_i = strawberry arrival date for customer i

<u>PRODUCTION SCHEDULING</u>

t_i = production start time of customer i

θ_{jk} = start time of the j^{th} operation on machine k

u_{ijk} = 1, if customer i is the j^{th} operation on machine k; 0, otherwise

c_i = completion time of customer i's jams

<u>DELIVERY</u>

p_{ij} = 1, if customer i's jams are collected on the j^{th} pickup; 0, otherwise

z_{jk} = 1, if the j^{th} pickup is on day k; 0, otherwise

PT_j = time of the j^{th} pickup (integer)

EPT_j = earliest time of the j^{th} pickup

$Time_j$ = total time taken for the j^{th} pickup = arrival time at depot - start time at depot

r_i = collection time of customer i's strawberries

w^x_{ijk} = 1, if the k^{th} arc of the tour for the x^{th} pickup goes from location i to location j; 0, otherwise

TD^x_k = total distance travelled (w.r.t depot) at the end of the k^{th} arc of the x^{th} pickup

$TotalDist_x$ = total distance travelled for the x^{th} pickup

$Dist_j$ = total distance travelled (w.r.t depot) when customer j receives his/her jams

R_i = time at which customer i receives the jam

T_i = customer i's tardiness

E_i = customer i's earliness

——————— *Problem Formulation* ———————

Minimise

(1)

$$\sum_{k=1}^{2}\sum_{j=1}^{T} OC_k y_{jk} + \sum_{k=1}^{2}\sum_{j=1}^{T}\left\{UC_{1k}\left[\left(\sum_{i=1}^{n} G_i g_{ik} x_{ij}\right) - f_{jk}^{+} - s_{jk}^{+}\right] + UC_{2k}s_{jk}^{+} + UC_{3k}f_{jk}^{+}\right\}$$
$$+SS\sum_{i=1}^{n} G_i\left(t_i - a_i\right) + P\sum_{j=1}^{n}\sum_{k=1}^{T} z_{jk} + \alpha\sum_{i=1}^{n} T_i + \beta\sum_{i=1}^{n} E_i + JS\sum_{i=1}^{n} B_i\left(r_i - c_i\right)$$
$$+DC\sum_{i=1}^{n} TotalDist_i$$

Subject to

<u>INVENTORY CONSTRAINTS</u>

(2) $$\sum_{j=0}^{T} x_{ij} = 1,\ \forall i$$

(3) $$\sum_{i=1}^{n} G_i g_{ik} x_{ij} \leq L_k,\ \forall j,k$$

(4) $$\sum_{i=1}^{n} G_i x_{ij} \leq L_T,\ \forall j$$

(5) $$\sum_{i=1}^{n} g_{ik} x_{ij} \leq n y_{jk},\ \forall j,k$$

(6) $$\left(\sum_{i=1}^{n} G_i g_{ik} x_{ij}\right) - C_{2k} = f_{jk}^{+} - f_{jk}^{-},\ \forall j,k$$

(7) $$\left(\sum_{i=1}^{n} G_i g_{ik} x_{ij}\right) - f_{jk}^{+} - C_{1k} = s_{jk}^{+} - s_{jk}^{-},\ \forall j,k$$

(8) $$o_i = \sum_{j=0}^{T} j x_{ij},\ \forall i$$

(9) $$a_i = o_i + \sum_{k=1}^{2} g_{ik} l_k, \ \forall i$$

PRODUCTION SCHEDULING CONSTRAINTS

(10) $$t_i \geq a_i, \ \forall i$$

(11) $$t_i \geq \theta_{jk} - M(1 - u_{ijk}), \ \forall i, j, k$$

(12) $$t_i \leq \theta_{jk} + M(1 - u_{ijk}), \ \forall i, j, k$$

(13) $$\theta_{jk} \geq \theta_{(j-1)k} + \sum_{i=1}^{n} p_i u_{i(j-1)k}, \ \forall j, k$$

(14) $$\sum_{i=1}^{n} u_{ijk} \leq 1, \ \forall j, k$$

(15) $$\sum_{j=1}^{n} \sum_{k=1}^{m} u_{ijk} = 1, \ \forall i$$

(16) $$\sum_{i=1}^{n} u_{ijk} \leq \sum_{i=1}^{n} u_{i(j-1)k}, \ \forall j, k$$

(17) $$c_i = t_i + p_i, \ \forall i$$

DELIVERY CONSTRAINTS

(18) $$\sum_{j=0}^{T} p_{ij} = 1, \ \forall i$$

(19) $$p_{ij} \leq \sum_{k=0}^{T} z_{jk}, \ \forall i, j$$

(20) $$\sum_{j=1}^{n} z_{jk} \leq 1, \ \forall k$$

(21) $$\sum_{k=0}^{T} z_{jk} \leq 1, \ \forall j$$

(22) $$PT_j = \sum_{k=0}^{T} k z_{jk}, \ \forall j$$

(23) $$EPT_j \geq PT_{j-1} + Time_{j-1} - M\left(1 - \sum_{k=0}^{T} z_{jk}\right), \ \forall j$$

(24) $$EPT_j \leq PT_{j-1} + Time_{j-1} + M\left(1 - \sum_{k=0}^{T} z_{jk}\right), \ \forall j$$

(25) $$PT_0 = Time_0 = 0$$

$$EPT_j \leq \sum_{k=0}^{T} kz_{jk},\ \forall j \tag{26}$$

$$r_i \geq PT_j - M\left(1 - p_{ij}\right),\ \forall i, j \tag{27}$$

$$r_i \leq PT_j + M\left(1 - p_{ij}\right),\ \forall i, j \tag{28}$$

$$r_i \geq c_i,\ \forall i \tag{29}$$

$$\sum_{j=1}^{n+1}\sum_{k=1}^{n+1} w^x_{(i+1)jk} = p_{ix},\ \forall i, x \tag{30}$$

$$\sum_{i=1}^{n+1}\sum_{k=1}^{n+1} w^x_{i(j+1)k} = p_{jx},\ \forall j, x \tag{31}$$

$$\sum_{i=1}^{n+1} w^x_{ijk} = \sum_{p=1}^{n+1} w^x_{jp(k+1)},\ \forall j, k, x \tag{32}$$

$$\sum_{j=1}^{n+1} w^x_{1j1} = 1,\ \forall x \tag{33}$$

$$\sum_{i=1}^{n+1} w^x_{i1(n+1)} = 1,\ \forall x \tag{34}$$

$$TD^x_q = \sum_{k=1}^{q}\sum_{i=1}^{n+1}\sum_{j=1}^{n+1} d_{ij} w^x_{ijk},\ \forall q \tag{35}$$

$$TotalDist_x = TD^x_{n+1},\ \forall x \tag{36}$$

$$Time_x = \frac{TotalDist_x}{SPEED},\ \forall x \tag{37}$$

$$Dist_j \geq TD^x_k - M\left(1 - w^x_{ijk}\right),\ \forall i, j, k, x \tag{38}$$

$$Dist_j \leq TD^x_k + M\left(1 - w^x_{ijk}\right),\ \forall i, j, k, x \tag{39}$$

$$R_i = r_i + \frac{Dist_i}{SPEED},\ \forall i \tag{40}$$

$$R_i - DD_i = T_i - E_i,\ \forall i \tag{41}$$

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