Appendix for "A General Theory of Pass-Through in Channels with Category Management and Retail Competition"

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Proof of Lemma 2

I will show that $[-H^{-1}]_{11} \geq [-H^{-1}]_{12}$; the others follow similarly.

By definition of an inverse,

$$[-H^{-1}]_{11} = -\begin{vmatrix} \pi^1_{1212} & \pi^1_{1221} & \pi^1_{1222} \\ \pi^2_{2112} & \pi^2_{2121} & \pi^2_{2122} \\ \pi^2_{2212} & \pi^2_{2221} & \pi^2_{2222} \end{vmatrix} \text{ and } [-H^{-1}]_{12} = \begin{vmatrix} \pi^1_{1112} & \pi^1_{1121} & \pi^1_{1122} \\ \pi^2_{2112} & \pi^2_{2121} & \pi^2_{2122} \\ \pi^2_{2212} & \pi^2_{2221} & \pi^2_{2222} \end{vmatrix}.$$

Therefore, $[-H^{-1}]_{11} - [-H^{-1}]_{12}$

$= (-\pi^1_{1212} - \pi^1_{1112})[\pi^2_{2121}\pi^2_{2222} - (\pi^2_{2122})^2] + (\pi^1_{1221} + \pi^1_{1121})[\pi^2_{2112}\pi^2_{2222} - \pi^2_{2212}\pi^2_{2122}] +$
$(-\pi^1_{1222} - \pi^1_{1122})[\pi^2_{2112}\pi^2_{2221} - \pi^2_{2212}\pi^2_{2121}],$
$= (-\pi^1_{1212} - \pi^1_{1112})[\pi^2_{2121}\pi^2_{2222} - (\pi^2_{2122})^2] - (\pi^1_{1221} + \pi^1_{1121})[-\pi^2_{2112}\pi^2_{2222} + \pi^2_{2212}\pi^2_{2122}] -$
$(\pi^1_{1222} + \pi^1_{1122})[-\pi^2_{2121}\pi^2_{2212} + \pi^2_{2112}\pi^2_{2221}],$
$> (-\pi^1_{1212} - \pi^1_{1112})[\pi^2_{2121}\pi^2_{2222} - (\pi^2_{2122})^2] - (-\pi^2_{2121} - \pi^2_{2221})[-\pi^2_{2112}\pi^2_{2222} + \pi^2_{2212}\pi^2_{2122}]-$
$(-\pi^2_{2222} - \pi^2_{2122})[-\pi^2_{2121}\pi^2_{2212} + \pi^2_{2112}\pi^2_{2221}],$ (using Assumption 2 (e))
$= [\pi^2_{2121}\pi^2_{2222} - (\pi^2_{2122})^2](-\pi^1_{1212} - \pi^1_{1112} - \pi^2_{2112} - \pi^2_{2212})$
> 0 (using Assumption 2 (e)).

Proof of Lemma 3

I will show that if $|\pi^2_{2222}| \geq |\pi^2_{2121}|$, $\pi^1_{1221} \geq \pi^1_{1122}$ and $\pi^1_{1121} \geq \pi^1_{1222}$, then $[-H^{-1}]_{13} \geq [-H^{-1}]_{14}$; the others follow similarly.

$$H \equiv \begin{bmatrix} \pi^1_{1111} & \pi^1_{1112} & \pi^1_{1121} & \pi^1_{1122} \\ \pi^1_{1211} & \pi^1_{1212} & \pi^1_{1221} & \pi^1_{1222} \\ \pi^2_{2111} & \pi^2_{2112} & \pi^2_{2121} & \pi^2_{2122} \\ \pi^2_{2211} & \pi^2_{2212} & \pi^2_{2221} & \pi^2_{2222} \end{bmatrix}$$

$$[-H^{-1}]_{14} = \begin{vmatrix} \pi^1_{1112} & \pi^1_{1121} & \pi^1_{1122} \\ \pi^1_{1212} & \pi^1_{1221} & \pi^1_{1222} \\ \pi^2_{2112} & \pi^2_{2121} & \pi^2_{2122} \end{vmatrix} \text{ and } [-H^{-1}]_{13} = -\begin{vmatrix} \pi^1_{1112} & \pi^1_{1121} & \pi^1_{1122} \\ \pi^1_{1212} & \pi^1_{1221} & \pi^1_{1222} \\ \pi^2_{2212} & \pi^2_{2221} & \pi^2_{2222} \end{vmatrix}.$$

Therefore, $[-H^{-1}]_{13} - [-H^{-1}]_{14}$

$$
\begin{aligned}
&= -(\pi^2_{2112} + \pi^2_{2212})(\pi^1_{1121}\pi^1_{1222} - \pi^1_{1221}\pi^1_{1122}) - [(-\pi^2_{2121} - \pi^2_{2221})(\pi^1_{1112}\pi^1_{1222} - \\
&\pi^1_{1212}\pi^1_{1122})] + [(-\pi^2_{2222} - \pi^2_{2122})(\pi^1_{1112}\pi^1_{1221} - \pi^1_{1212}\pi^1_{1121})], \\
&\geq -(\pi^2_{2112} + \pi^2_{2212})(\pi^1_{1121}\pi^1_{1121} - \pi^1_{1122}\pi^1_{1122}) - [(-\pi^2_{2121} - \pi^2_{2221})(\pi^1_{1112}\pi^1_{1121} - \\
&\pi^1_{1212}\pi^1_{1122})] + \quad [(-\pi^2_{2121} - \pi^2_{2122})(\pi^1_{1112}\pi^1_{1122} - \pi^1_{1212}\pi^1_{1121})], \\
&= -(\pi^2_{2112} + \pi^2_{2212})(\pi^1_{1121} - \pi^1_{1122})(\pi^1_{1121} + \pi^1_{1122}) + (-\pi^2_{2121} - \\
&\pi^2_{2221})[\pi^1_{1121}(-\pi^1_{1212} - \pi^1_{1112}) - \pi^1_{1122}(-\pi^1_{1212} - \pi^1_{1112})], \\
&= -(\pi^2_{2112} + \pi^2_{2212})(\pi^1_{1121} - \pi^1_{1122})(\pi^1_{1121} + \pi^1_{1122}) + (-\pi^2_{2121} - \\
&\pi^2_{2221})(-\pi^1_{1212} - \pi^1_{1112})(\pi^1_{1121} - \pi^1_{1122}), \\
&= (\pi^1_{1121} - \pi^1_{1122})[(-\pi^2_{2121} - \pi^2_{2221})(-\pi^1_{1212} - \pi^1_{1112}) - (\pi^1_{1121} + \\
&\pi^1_{1122})(\pi^2_{2112} + \pi^2_{2212})], \\
&> 0,
\end{aligned}
$$

because $\pi^1_{1121} > \pi^1_{1122}, (-\pi^2_{2121} - \pi^2_{2221}) > (\pi^1_{1121} + \pi^1_{1221}) \geq (\pi^1_{1121} + \pi^1_{1122})$,and $(-\pi^1_{1212} - \pi^1_{1112}) > (\pi^2_{2112} + \pi^2_{2212})$.