Channel Coordination in the Presence of a Dominant Retailer

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Technical Note: Alternative Model Specification

Consider a dominant-retailer channel characterized by the following demand functions:

$$Q_d = \gamma(\alpha - \beta p) + s, \tag{1}$$

$$Q_c = (1-\gamma)(\alpha - \beta p), \tag{2}$$

$$Q_m = \alpha - \beta p + s. \tag{3}$$

Relative to the model specified in our paper, this specification essentially increases the incentive for the dominant retailer to provide s at any given cost f, since the dominant retailer's demand can be written as $Q_d = \gamma(\alpha - \beta p + s) + (1-\gamma)s$, and eliminates any free-riding on s by fringe retailers.

1. Integrated Channel

As the channel demand remains the same under this alternative model specification, the price and profit for the integrated channel are the same as in our paper . We have for the integrated channel:

$$\Pi^*(s,f) = \frac{(\alpha - \beta c + s)^2}{4\beta} - f, \tag{4}$$

$$p^* = \frac{\alpha + \beta c + s}{2\beta}, \quad Q_m^* = \frac{\alpha - \beta c + s}{2}, \tag{5}$$

where c is the marginal cost of production.

2. Channel Conflict

By following the same procedures as in our paper, we can derive the equilibrium for this alternative model when the channel is not coordinated. Toward that end, we assume $\gamma > \frac{1}{2}$, $c > (\frac{1}{\gamma} - 1)s$, $\alpha > \beta c + 2(2n+1)s$, $n \geq 2$, and $f < f_c$. We maintain these assumptions throughout this technical note. Futher to simplify our notation, define

$$f_a = \frac{s[\gamma(\alpha - \beta c) + 2(1-\gamma)s]}{4\beta\gamma}, \tag{6}$$

$$f_b = \frac{s[a\gamma - \beta c\gamma + 2s - 2\gamma s + \sqrt{(2\gamma - 1)s(2\alpha\gamma - 2\beta c\gamma - s + 2\gamma s)}]}{4\beta\gamma}, \tag{7}$$

Table 1: Equilibrium for Decentralized Channel

	Merchandising $(0 \leq f \leq f_a)$	**Merchandising** $(f_a < f \leq f_b)$
$\tilde{p}$	$\frac{\gamma(3\alpha+\beta c+2s)+s}{4\beta\gamma}$	$\frac{4\alpha\gamma s+3s^2-4\beta\gamma f}{4\beta\gamma s}$
$\tilde{w}$	$\frac{\gamma(\alpha+\beta c+2s)-s}{2\beta\gamma}$	$\frac{s(2\alpha\gamma+s)-4\beta\gamma f}{2\beta\gamma s}$
$\tilde{\pi}_m$	$\frac{(\alpha\gamma-\beta c\gamma+2\gamma s-s)^2}{8\beta\gamma^2}$	$\frac{[4\beta f\gamma+(4\gamma-3)s^2][s(2\alpha\gamma+s)-2\beta\gamma(2f+cs)]}{8\beta(\gamma s)^2}$
$\tilde{\pi}_d$	$\frac{[\gamma(\alpha-\beta c)+(3-2\gamma)s]^2}{16\beta\gamma} - f$	$\frac{(s^2-4\beta\gamma f)^2}{16\beta\gamma s^2}$
$\tilde{\pi}_c$	$\frac{(1-\gamma)[\gamma(\alpha-\beta c-2s)-s][r(a-\beta c)+(3-2\gamma)s]}{16\beta\gamma^2}$	$\frac{(1-\gamma)(4\beta\gamma f-3s^2)(4\beta f\gamma+s^2)}{16\beta(\gamma s)^2}$

$$f_c = \frac{s(2\alpha\gamma - 2\beta c\gamma + s)}{4\beta\gamma}, \quad (8)$$

$$f_d = \frac{s[\gamma(\alpha - \beta c) + (1-\gamma)s]}{4\beta\gamma}. \quad (9)$$

Our analysis shows that when $f_b < f < f_c$, the equilibrium is such that s is not provided. In this case, we have (using the same notation as in our paper)

$$\tilde{p} = \frac{3\alpha + \beta c}{4\beta}, \quad \tilde{w} = \frac{\alpha + \beta c}{2\beta}, \quad \tilde{\pi}_m = \frac{(\alpha - \beta c)^2}{8\beta},$$

$$\tilde{\pi}_d = \frac{\gamma(\alpha - \beta c)^2}{16\beta}, \quad \tilde{\pi}_c = \frac{(1-\gamma)[\gamma^2(\alpha - \beta c)^2 - 4s^2]}{16\beta r^2}.$$

The equilibrium where s is provided is summarized in Table 1. As is the case in our paper, the decentralized channel does not achieve the maximum profit that the channel can potentially generate due to the fact that the retail price is always higher than the channel profit optimizing price p^*.

3. Quantity Discount and Channel Coordination

The channel-coordinating quantity discount schedule can be easily derrived in the same way as in our paper. It is given by

$$t^*(q) = \left(\frac{\alpha}{\beta} - \frac{q-s}{\beta\gamma}\right)\left(1 - \frac{k_1^*}{\gamma} + \frac{k_1^*(1-\gamma)s}{q\gamma}\right) + \frac{k_1^* c}{\gamma}\left(1 - \frac{(1-\gamma)s}{q}\right) - \frac{1-k_2^*}{q}f. \quad (10)$$

We determine k_1^* and k_2^* in the above schedule following the same steps detailed in our paper (Appendix A), and they are

$$k_1^* = \frac{\gamma(\alpha-\beta c-s)[(N\gamma+\gamma-1)(\alpha-\beta c-s)+2Ns]}{(\alpha-\beta c-s-2Ns)[(2Nr+r-1)(\alpha-\beta c)+(1+2N-\gamma)s]}, \quad (11)$$

$$k_2^* = \begin{cases} 1 & \text{if } 0 \le f \le k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta} \\ k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta f} & \text{if } k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta} < f \le f_c. \end{cases} \quad (12)$$

Note that the optimal quantity discount schedule in equation (10) is nonlinear in q here, more complex than the corresponding schedule in our paper. This is due to the fact that our paper specifies a simple relationship between Q_d and Q_m or $Q_m = Q_d/\gamma$. That relationship is more complex here, given by $Q_m = s + (Q_d - s)/\gamma$. However, it is straightforward to verify that the qualitative nature of this schedule has not changed. Given our assumption about α, we can show $0 < k_1^* < \gamma$ and $\frac{dt^*(q)}{dq} \mid_{f=0} < 0$.

The payoff for the manufacturer under this quantity discount schedule is given by

$$\pi_m^d = \begin{cases} \frac{(1-k_1^*)(\alpha-\beta c+s)^2}{4\beta} & \text{if } 0 \le f \le k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta} \\ \frac{(1-k_1^*)(\alpha-\beta c+s)^2}{4\beta} - (1-k_2^*)f & \text{if } k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta} < f \le f_c. \end{cases} \quad (13)$$

4. Two-Part Tariffs and Channel Coordination

The derrivation of the channel coordinating menu of two-part tariffs follows the same process as described in Section 4 of our paper (and also Appendix B). Again, the key to coordinating this dominant retailer channel is to motivate the retailer to set its price at p^* and the manufacturer can do so only if it sets $w_d^* = \frac{\beta\gamma c-(1-\gamma)s}{\beta\gamma}$. Under the new specification, the dominant retailer has more an incentive to mark-up on the wholesale price when s is provided so that the manufacturer must lower its wholesale price to motivate the provision of s. Then, to determine F_d, F_c, and w_c, the manufacturer solves the following optimization problem:

$$\max_{(F_d,F_c,w_c)} \quad \frac{\beta\gamma(1-\gamma)(\alpha-\beta c-s)(w_c-c)-(1-\gamma)s(\alpha\gamma-\beta c\gamma+(2-\gamma)s)}{2\beta\gamma}+F_d+NF_c \quad (1)$$

$$F_d \leq \frac{[\gamma(\alpha-\beta c)+(2-\gamma)s]^2}{4\beta\gamma}-f, \quad (2)$$

$$F_c \leq (1-\gamma)\frac{(\alpha-\beta c-s)[\alpha+s+\beta(c-2w_c)]}{4\beta N}, \quad (3)$$

$$F_d-F_c \geq (1-\gamma)\frac{(\alpha-\beta c-s)[(1-\gamma)s+\beta r(w_c-c)]}{2\beta N\gamma} \quad (4)$$

$$F_d-F_c \leq \frac{[\beta\gamma(w_c-c)+(1-\gamma)s][2\alpha\gamma+(3-\gamma)s-\beta\gamma(w_c+c)]}{4\beta\gamma}. \quad (5)$$

It can be shown that the optimal solution requires that constraints (3) and (5) are both binding and constraint (2) is not. In other words, the manufacturer will take all surplus away from fringe retailers but not from the dominant retailer. From the two binding constraints, we can solve for F_c and F_d, all as a function of w_c. Substituting them into the target function (1), we can see that the target function is strictly concave in w_c. Thus, we can proceed to find the optimal w_c and then F_c and F_d. The solutions are given below:

$$F_c = \begin{cases} 0 & \text{if } 0 \leq f \leq f_d \\ \frac{(1-\gamma)(\alpha-\beta c-s)[\beta\gamma(4f+cs)-s(\alpha\gamma+(1-\gamma)s)]}{4\beta N\gamma s} & \text{if } f_d < f \leq f_c, \end{cases} \quad (6)$$

$$F_d = \begin{cases} \frac{3[\alpha\gamma-\beta c\gamma+(2-\gamma)s]^2}{16\beta\gamma} & \text{if } 0 \leq f \leq f_d \\ \bar{F}_d & \text{if } f_d < f \leq f_c, \end{cases} \quad (7)$$

$$w_c = \begin{cases} p^* & \text{if } 0 \leq f \leq f_d \\ w_s & \text{if } f_d < f \leq f_c, \end{cases} \quad (8)$$

$$\pi_m = \begin{cases} \frac{(4-\gamma)\gamma(\alpha-\beta c)^2+s[2\gamma(2+\gamma)(\alpha-\beta c)-(4-8\gamma+\gamma^2)s]}{16\beta\gamma} & \text{if } 0 \leq f \leq f_d \\ \bar{\pi}_m & \text{if } f_d < f \leq f_c, \end{cases} \quad (9)$$

where

$$\bar{F}_d = \frac{1}{16\beta N\gamma s^2}\{-4\beta^2\gamma[4f^2N\gamma+4cf(1-\gamma)s-c^2(N\gamma+\gamma-1)s^2]$$

$$
\begin{aligned}
&+ \quad s^2[4\alpha^2\gamma(N\gamma+\gamma-1) - 4\alpha(1-(3+4N)\gamma+2(1+N)\gamma^2)s \\
&+ \quad (4(1-\gamma)^2 + N(15-16\gamma+4\gamma^2))s^2] - 4\beta s[2\alpha\gamma(2f(\gamma-1)+c(-1+\gamma+N\gamma)s) \\
&+ \quad s(2f(2+N-2\gamma)\gamma + c(-1+(3+4N)\gamma-2(1+N)\gamma^2)s)]\} \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\pi}_m \quad &= \quad \frac{1}{16\beta N\gamma s^2}\{-4\beta^2\gamma[4f^2N\gamma+4cf(1-\gamma)s-c^2(N+\gamma-1)s^2] \\
&+ \quad s^2[4\alpha^2\gamma(N+\gamma-1)+4\alpha(-1+(3+2N)\gamma-2\gamma^2)s \\
&+ \quad (4(1-\gamma)^2+N(4\gamma-1))s^2] - 4\beta s[2\alpha\gamma(2f(\gamma-1)+c(-1+\gamma+N)s) \\
&+ \quad s(2f(2+N-2\gamma)\gamma + c(-1+3\gamma+2N\gamma-2\gamma^2)s)]\} \qquad (11)
\end{aligned}
$$

5. Winners and Losers of Channel Coordination

Due to the complexity of the payoff expressions, it is no longer possible to determine precisely under what conditions a quantity discount schedule may dominate a menu of two-part tariffs, even with the aid of *Mathematica*. However, we note that with this alternative specification, the process of the derivations has not changed qualitatively, generating similar expressions. Furthermore, with a sufficiently small s, we know that this alternative model converges to the model in our paper so that all conclusions in our paper should carry over. Although we cannot determine precisely how small s has to be relative to other parameters, we know that s does not have to be very small. For instance, if we let $\gamma = 0.6, N = 3, \alpha = 15, \beta = 0.5, s = 0.5$, and $c = 1$ (all our assumptions are satisfied), we will have $f_c = 7.45, k_1^* \frac{s(2\alpha-2\beta c+s)}{4\beta} = 2.566$, and $f_d = 3.71$. It is then straightforward to show that for all $0 < f \leq 6.5$, the menu of two-part tariffs dominates the quantity discount schedule in coordinating the channel and for $6.5 < f \leq 7.45$, the quantity discount schedule dominates. Furthermore, channel coordination with the optimally-chosen coordination mechanism is always more profitable for the manufacturer than if the channel is not coordinated. Both conclusions are qualitatively the same as in our paper, as seen from Figure 2 in our paper where one simply fixes γ and let f vary.

6. Zero-Sum Service Provision

Alternatively, consider a dominant-retailer channel characterized by the following demand functions:

$$Q_d = \gamma(\alpha - \beta p + s), \tag{12}$$

$$Q_c = (1-\gamma)(\alpha - \beta p - s), \tag{13}$$

$$Q_m = \alpha - \beta p. \tag{14}$$

In this case, the manufacturer has no incentive to encourage the provision of service for two reasons. First, the total demand for the manufacturer is not elastic with regard to the service provision. Second, the service provision will increase the sales made by the dominant retailer, an outcome that the manufacture does not want to see happening (the manufacturer can take all profit away from the competitive fringe).

There are two reasons as to why this model is not be as interesting as it may first appear. First, the incentives introduced by this model are counter-factual. Second, if s is sufficiently small, this model will converge to the previous model. Here s must be sufficiently small relative to other parameters in our model.