Dynamic Models Incorporating Individual Heterogeneity: Utility Evolution in Conjoint Analysis

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TECHNICAL APPENDIX

Algorithms for Simulating the Synthetic Conjoint Data

For each of the synthetic data sets, we constructed a vector of part-worths that varied by individual and by time. Let β_{it} represent the vector of part-worths used to generate an observed rating y_{it} for the ith individual at time t; then y_{it} was generated by perturbing the linear combination of the part-worth and the product description:

$$y_{it} = \beta_{it}^T X_{it} + \varepsilon_{it},$$

where ε_{it} is Normally distributed with mean zero and variance σ^2. (It is important to note, that β_{jit} is the jth element of the part-worth vector β_{it}.) Obviously, the set of covariates (product descriptions) was presented in the same order for the non-counterbalanced data and they were rotated as described in the original manuscript for the counterbalanced data. The main distinction for the three different scenarios (fatigue, learning, and rule simplification) was with respect to the method for generating the individual time-varying part-worth's. After giving a brief intuitive description of the generation approach, we will give a summary of the algorithm used to generate each of these different sets of synthetic data.

Fatigue Synthetic Data

For the fatigue scenario, we assume that the individual level part-worth used to generate ratings were centered around an individual's true part worth, but had a variance which was initially small and then increased towards the end of the sequence of profiles. This dynamic mirrors the type of behavior that one would expect from a burn-in scenario where the variance is large for the initial profiles and then decreases over time.

Fatigue Algorithm:

1. *For each individual, draw a random reference vector,* $\overline{\beta}_{i0} \sim Normal\left(\overline{\beta}_0, \sigma_0^2 I\right)$.

2. *For each individual, randomly select a time,* $\tau_i \sim Uniform\left(t_{low}, T\right)$*, where* t_{low} *is the 'earliest time' that an individual can be fatigued (we chose* $t_{low} = T/2$*) and where* T *is the total number of rating tasks (*$T = 27$*).*

3. *For* $t \le \tau_i$, $\beta_{jit} \sim Normal\left(\overline{\beta}_{ji0}, \sigma_{low}^2\right)$ *and for* $t > \tau_i$, $\beta_{jit} \sim Normal\left(\overline{\beta}_{ji0}, \sigma_{high}^2\right)$*; with,* $\sigma_{low}^2 < \sigma_{high}^2$.

Learning Synthetic Data

For the learning scenario, the individual level part-worth used to generate the ratings converges at roughly an exponentially rate to the true part-worth, and then is centered around the individual's true part-worth with a relatively small variance.

Learning Algorithm:

1. *For each individual, draw a random reference vector,* $\overline{\beta}_{i0} \sim Normal\left(\overline{\beta}_0, \sigma_0^2 I\right)$.

2. *Each subsequent element of the part-worth vector, is a perturbation around a process which follows a modified auto-regressive order 1 process, or more formally let*

$$\tilde{\beta}_{jit} = \left(\alpha_j \delta^t + \left(1 - \delta^t\right)\right)\tilde{\beta}_{ji(t-1)}$$

with $\tilde{\beta}_{i0} = \overline{\beta}_{i0}$, $\alpha_j > 0$*, and* $0 < \delta < 1$*; then* $\beta_{jit} \sim Normal\left(\tilde{\beta}_{jit}, \sigma_{low}^2\right)$.

Rule Simplification Synthetic Data

The rule simplification scenario reflects a simplification strategy where respondents suddenly determine that an attribute is no longer relevant to their utility function; the part-worth used to generate the ratings is initially centered around the individual's true part-worth with a small variance, and then at a random time, the part-worth is centered around zero and has a larger variance which quickly decreases. We used a random scheme to determine whether a part-worth was no longer relevant and hence set to zero at some point during the conjoint study, where the probability of being set to zero depends on how close the part-worth was to zero. This type of sudden change dynamic could also be related to a structural change in the part-worth where an individual suddenly realizes that the range for some of the attribute levels is different than they initially anticipated, resulting in a sudden change in some of the part-worth values.

Simplification Algorithm:

1. *For each individual, draw a random reference vector,* $\bar{\beta}_{i0} \sim Normal\left(\bar{\beta}_0, \sigma_0^2 I\right)$ *and for each element of the part-worth vector we set* $\tau_{ji} = T + 1$*, where* T *is the total number of rating tasks (*$T = 27$*).*

2. *For each individual and for each time up to a threshold (we used* $t_{up} = 20$*), randomly determine if a rule simplification occurs, or stated more formally, if* $t \le t_{up}$*,* $u_{it} \sim Uniform(0,1)$ *and* $u_{it} \le prob$ *(a pre-assigned value), then 'simplify' the reference, part-worth vector for this individual.*

3. *If the part-worth vector is to be simplified, with probability* $p_{jit} \propto 1 - \frac{\left|\bar{\beta}_{ji(t-1)}\right|}{\sum_{\ell}\left|\bar{\beta}_{\ell i(t-1)}\right|}$*, let* $\bar{\beta}_{jit} = 0$ *and we set* $\tau_{ji} = t$*.*

4. *If* $t < \tau_{ji}$, $\beta_{jit} \sim Normal\left(\overline{\beta}_{jit}, \sigma^2_{low}\right)$ *and* $t \geq \tau_{ji}$, $\beta_{jit} \sim Normal\left(\overline{\beta}_{jit}, \sigma^2_{low} + \delta^{t-\tau_{ji}} \sigma^2_{decay}\right)$, *where* $0 < \delta < 1$ *and* $0 < \sigma^2_{decay}$.