TECHNICAL APPENDIX

(Not for publication)

For the paper "Delegating Pricing Decisions in Competitive Markets with Symmetric and Asymmetric Information" by Birendra K. Mishra and Ashutosh Prasad

In this appendix we provide a comparison of the results for competitive firms (in the paper) against the results for fully differentiated firms. By fully differentiated we mean that the firms in effect act like monopolies in their part of the market. We will show that the result we obtain for competitive markets (provided in Proposition 3, and proved in the manuscript) are different from the results we shall obtain in this technical appendix. This clearly shows that competition has an effect and that it is not the case that there are two competitors each behaving like a monopolist.

To further show the effects of competition, we examine the results in a collusive setting where contracts and product prices are coordinated by the firms (or alternatively, a single firm owns both products.

A. Fully differentiated Markets

We use the contract $(\alpha + \beta x, p)$ since linear contracts are optimal in the Holmstrom-Milgrom (1987) setting and since we have shown that there are equivalent delegated contracts. As before, we stay with the demand function $x_i = h - p_i + \theta_p p_j + a_i - \theta_s a_j + \delta$ and other assumptions of Bhardwaj (2001). Total demand in the marketplace can be obtained by adding the demand of the two firms:

$$X = 2h - (1-\theta_p)(p_1 + p_2) + (1-\theta_s)(a_1 + a_2) + 2\delta \, .$$

There is no guidance in the literature for how to create two monopoly demand functions from this demand function. Assuming that the size and response parameters of the total market do not change when we break it into two symmetric monopolies, we get the sales function

$$x_i = h - (1-\theta_p)p_i + (1-\theta_s)a_i + \delta$$

for each firm, where $0 < \theta_p, \theta_s < 1$ are model parameters and δ is a random noise distributed $N[0,\sigma^2]$.[1] As before, the salespersons' utilities are specified as $U(S) = 1 - e^{-r(S-C)}$ where r is the coefficient of risk aversion, S is the compensation and $C(a) = a^2$ is the cost of effort.

For the salesperson, the effort is obtained by maximizing the certainty equivalent given by $CE_i \equiv \alpha_i + \beta_i[h-(1-\theta_p)p_i + (1-\theta_s)a_i] - a_i^2 - r\beta_i^2\sigma^2/2$. The result is

$$a_i = \beta_i(1-\theta_s)/2 . \qquad \text{(A1)}$$

(Comparing this to $a_i = \beta_i / 2$ in duopoly, we see that it is different.)

Thus, the problem for firm i is:

$$\operatorname*{Max}_{\alpha_i,\beta_i,p_i} \; E[(p_i - c)x_i - \alpha_i - \beta_i x_i], \text{ s.t.,} \qquad \text{(A2)}$$

$$a_i = \beta_i(1-\theta_s)/2 , \qquad \text{(A3)}$$

$$EU(\alpha_i + \beta_i x_i - a_i^2) \ge 0 . \qquad \text{(A4)}$$

The salary α_i is chosen to make the participation constraint bind. Substituting all constraints, the objective function

$$(p_i - c)[h-(1-\theta_p)p_i + (1-\theta_s)^2\beta_i/2] - (1-\theta_s)^2\beta_i^2/4 - r\beta_i^2\sigma^2/2$$

is to be maximized with respect to β_i and p_i. We obtain the first order conditions

$$\left(\beta_i = \frac{p_i - c}{1 + 2r\sigma^2/(1-\theta_s)^2}, \quad p_i - c = \frac{2[h-(1-\theta_p)c]}{4(1-\theta_p) - (1-\theta_s)^2/\{1 + 2r\sigma^2/(1-\theta_s)^2\}}\right). \qquad \text{(A5)}$$

These can be compared to the results from duopoly. Those solutions are

[1] However, any other way of breaking it will also result in differences with the duopoly demand function since the competitor's price and effort by definition cannot affect the monopolist's demand. Hence, the results will not be the same in duopoly versus where the two firms are monopolies. The insights will depend on the demand function that is appropriate to the market in question.

$$\left(\beta_i = \frac{p_i - c}{1 + 2r\sigma^2}, \quad p_i - c = \frac{2[h - (1-\theta_p)c]}{2(2-\theta_p) - (1-\theta_s)/(1+2r\sigma^2)} \right). \tag{A6}$$

When θ_p is zero, the duopoly solution can generate a higher price than the monopoly solution. On the other hand, as θ_p increases, the monopoly prices are higher. McGuire and Staelin (1983) have some discussion regarding the interpretation of the demand functions.

B. Collusive Market

For the salespersons, the certainty equivalent is the same as in the competitive case, i.e., $CE_i \equiv \alpha_i + \beta_i(h - p_i + \theta_p p_j + a_i - \theta_s a_j) - a_i^2 - r\beta_i^2\sigma^2/2$. Thus, the choice of effort is given, as in the competitive case, by

$$a_i = \beta_i / 2. \tag{B1}$$

The collusive profit maximization problem can be written as

$$\max_{\alpha_1,\beta_1,p_1,\alpha_2,\beta_2,p_2} E[(p_1 - c_1)x_1 - (\alpha_1 + \beta_1 x_1) + (p_2 - c_2)x_2 - (\alpha_2 + \beta_2 x_2)], \text{ s.t., } \forall i \in \{1,2\} \tag{B2}$$

$$a_i = \beta_i / 2, \tag{B3}$$

$$EU(\alpha_i + \beta_i x_i - a_i^2) \geq 0. \tag{B4}$$

The participation constraints bind. Substituting the constraints, we get

$$\max_{\beta_1,p_1,\beta_2,p_2} E[(p_1 - c_1)x_1 + (p_2 - c_2)x_2] - (\beta_1^2 + \beta_2^2)(1+2r\sigma^2)/4 \tag{B5}$$

The necessary conditions are,

$-(p_1 - c_1) + (h - p_1 + \theta_p p_2 + \beta_1/2 - \theta_s\beta_2/2) + \theta_p(p_2 - c_2) = 0$, and

$(p_1 - c_1) - \beta_1(1+2r\sigma^2) - \theta_s(p_2 - c_2) = 0$

For simplicity in exposition, we assume interior solutions and symmetric firms. In that case, in equilibrium, $p_1 = p_2$ and $\beta_1 = \beta_2$. The solution is

$$\left(\beta = \frac{(p-c)(1-\theta_s)}{1+2r\sigma^2}, \quad p - c = \frac{2(h - (1-\theta_p)c)}{4(1-\theta_p) - (1-\theta_s)^2/(1+2r\sigma^2)} \right). \tag{B6}$$

The price again can be higher or lower than in competition. However, the profit of the collusive outcome cannot be lower. The objective function at the solution point, i.e., the profit, is

$$\frac{(p-c)^2}{2}[4(1-\theta_p)-\frac{(1-\theta_s)^2}{(1+2r\sigma^2)}]. \tag{B7}$$

We can compare the profit per product to the profit for the competitive outcome given by

$$(p-c)^2(1-\frac{1}{4(1+2r\sigma^2)}),\ \text{where, } p-c=\frac{2(h-(1-\theta_p)c)}{2(2-\theta_p)-(1-\theta_s)/(1+2r\sigma^2)}. \tag{B8}$$

Thus, we want to verify that

$$\begin{aligned}
&\frac{1}{2}(\frac{2(h-(1-\theta_p)c)}{4(1-\theta_p)-(1-\theta_s)^2/(1+2r\sigma^2)})^2[4(1-\theta_p)-\frac{(1-\theta_s)^2}{(1+2r\sigma^2)}]\\
&\geq 2(\frac{2(h-(1-\theta_p)c)}{2(2-\theta_p)-(1-\theta_s)/(1+2r\sigma^2)})^2(1-\frac{1}{4(1+2r\sigma^2)})\\
&\Rightarrow \frac{1}{4(1-\theta_p)-(1-\theta_s)^2/(1+2r\sigma^2)} \geq \frac{4(1-\frac{1}{4(1+2r\sigma^2)})}{(2(2-\theta_p)-(1-\theta_s)/(1+2r\sigma^2))^2}\\
&\Rightarrow (2(1+x)-\frac{y}{k})^2-(4-\frac{1}{k})(4x-\frac{y^2}{k})\geq 0
\end{aligned} \tag{B9}$$

where, $x=1-\theta_p, y=1-\theta_s$, $k=1+2r\sigma^2$. We will minimize the LHS with respect to x and y and show that the condition holds, hence it must hold (strictly) at other values. Minimizing the LHS gives the values, $x=1, y=1$ as the solution. At this point, the LHS is zero. Hence, we are done.

References

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